The numerical modelling of storm surges in the Bay of Bengal

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SUMMARY

A numerical model is developed for the simulation of surges generated by a tropical cyclone in the Bay of Bengal. The analysis area extends from approximately 11°N to 22°N and, in the northeastern sector of the Bay, includes a representation of the Ganges–Brahmaputra–Meghna river system in Bangladesh. The extent of the analysis area allows three days of the surge-generating capacity of a cyclone originating in the southern Indian Ocean to be recorded before landfall at the Bangladesh coast. The incorporation of the river system permits a potentially deep inland penetration of surges originating in the Bay. The model is non-linear and this allows a determination of the interactive effect between surge and the astronomical tide. Numerical experiments are described that relate to the change in surge response resulting from a change in the cyclone track during the 24 hours preceding landfall. An account is given of the interaction between surge and tide in the Bay and the river system.

I. INTRODUCTION

A study of case histories of tropical cyclones originating in the southern Indian Ocean reveals the destructive effect that many of these have along the coastline of the Indian subcontinent. Cyclonic storms, moving with an average speed of about 20 km h⁻¹, and with associated wind speeds of over 160 km h⁻¹, exercise a continuous surge-generating influence on the underlying ocean and cause the water to be driven towards the low lying coastline of Bangladesh. A most destructive surge, resulting in great loss of life, was experienced in Bangladesh in November 1970. In the shallow water areas near Chittagong, the maximum sea surface elevation was estimated to be between 6 and 9 m above its mean level. Of this, some 1-8 m was attributed to the state of the astronomical tide and the remaining 4-2 to 7-2 m resulted from the peak surge-induced elevation. The genesis area of the tropical cyclone responsible for the 1970 surge was in the neighbourhood of the Malaysian Peninsula. After its centre crossed 10°N, the cyclone moved north, gaining in intensity, and struck the Bangladesh coastline some three days later.

The complexity of forecasting the details of a surge response is compounded by the sudden changes that frequently occur in a cyclone track. For example, a generally north-easterly track may become northerly during the 24 hours preceding landfall and the surge response may lead to severe flooding along parts of the coastline not thought to be vulnerable to the original track.

The interactive effect between surge and tide is not easily estimated in the north-
eastern sector of the Bay of Bengal. The extreme shallowness of the water, of the order of 10 m, implies that tide and surge will not be linearly additive. This is because of the non-linear character of the dynamical processes supporting the developing sea surface elevation. Additionally, the problem is further complicated by the entry into the Bay of one of the world’s major river systems. This is the Ganges–Brahmaputra–Meghna system whose headwaters rise in the Himalayas before flowing through India to meet the sea at the Ganges Delta and near Chittagong. Dynamically, this river system may be important for two reasons. First, variations in the fresh water discharge might be expected to modify the sea surface elevation and to interact with that resulting from tide and surge. Second, the presence of such an enormous waterway allows a potentially deep inland penetration of surge originating in the Bay. A consequence of this is an inland flooding hazard, and saline water intrusion, which may exist for several hundred kilometres along the river. Documented examples of inland flooding are given by the Bangladesh Meteorological Department (1977). In this, a total elevation of about 4 m was recorded at Chandpur – some 140 km inland from Chittagong – coinciding with the 1970 surge.

An ability to predict a surge, its development in time, and the spatial variation of the resulting sea surface elevation from a knowledge of the basic parameters describing the generating cyclone and its track is clearly of great practical significance. At the very least, relating the characteristics of surge response to a sequence of different cyclone intensities and tracks would yield a useful compendium of forecasting information.

In the present work we develop a numerical model of storm surge simulation in the Bay of Bengal as a step towards this forecasting objective. Earlier numerical models have been developed by Flierl and Robinson (1972) and Das et al. (1974). The objectives of these were limited by available computing capacity, and the analysis region was so restricted as to preclude consideration of the surge-generating capacity of a cyclone for longer than about twelve hours before landfall. A consequence of this is that the surge-generated sea surface elevation is assumed to be zero on the southern boundary of the model. In the case of Das et al. this is only about 140 km south of the Bangladesh coastline. Neither did the models incorporate the non-linear interactive mechanism that exists between surge and tide, or the dynamical effect of the river system.

The analysis area in our model extends as far south as about 11°N and allows a three-day history of the surge-generating capacity of a cyclone to be recorded before landfall. During the very early life of a cyclone, when it is passing over deep water, barometric forcing may make a contribution. Subsequently, however, wind stress forcing is predominant (Prandle 1975). Accordingly, in our model, we include only the latter generation mechanism. This procedure is further supported by an experiment that included the effect of barometric forcing. Near the Bangladesh coastline this indicated that the sea surface elevation would be increased by about 10% above its purely wind-stress-forced value. Moreover, this increment is of the same order as a static correction calculated on the basis of the atmospheric pressure anomaly. The additional computational effort required to include barometric forcing does not, therefore, seem justified unless the precise dynamical effects of the process are required.

The model is fully non-linear, therefore yielding an interaction between surge and tide, and there is an idealized representation of the Ganges–Brahmaputra–Meghna system joined to the northeastern sector of the analysis area. This permits an investigation of the effect of fresh-water discharge. More importantly, however, the incorporation of the river system allows a surge-generated response in the neighbourhood of Chittagong to penetrate inland. In the earlier models there was a solid latitudinal wall placed across the channels adjacent to the Bhola, Hatiya and Sandwip Islands which, in some circumstances, might result in an unrepresentatively high sea surface elevation being predicted in this region.
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Using this model, we have performed numerical experiments to investigate the effects of change of cyclone track, inland penetration of surge, and non-linear interaction between surge and tide. Fresh-water discharge has been omitted but could readily be included in future studies.

2. FORMULATION

In the formulation of the model the spherical geometry of the earth's surface is ignored. A system of rectangular Cartesian coordinates is used in which the origin, O, is in the equilibrium level of the free surface and is located at the most southerly point of the analysis area corresponding to latitude 11°N. $O_x$ points towards the east from approximately 84°E, $O_y$ towards the north and $O_z$ is measured vertically upwards. The displaced level of the free surface is given by $\zeta(x,y,t)$ and the floor of the bay corresponds to $z = -h(x,y)$. The analysis area consists of three rectangular regions bounded in the south by an open-sea boundary. The western lateral boundary consists of vertical walls approximating the east coast of India as far south as 19°6′N. South of this position, a vertical wall approximately follows the meridian 84°E to meet the southern boundary of the model about 440 km from the Indian coastline. The head of the Bay of Bengal is represented by a vertical wall along 22°N and the west coast of Burma is approximated by appropriately positioned side-wall boundaries. This configuration is shown in Fig. 1 where the dimensions of the three regions, labelled 1, 2 and 3, are respectively 540 km (east–west) × 252 km (north–south), 1080 km × 432 km and 1440 km × 648 km.

In the northeastern corner of the bay model, an idealized river system connects with the main analysis area. This is an approximation to the Ganges, Brahmaputra and Meghna river
system in Bangladesh. The configuration of the river system is shown schematically in Fig. 2. The lengths of the three rivers are respectively 342, 522 and 414 km. River 1 has a breadth of 144 km at its point of communication with the bay. Over the first landward 90 km, the breadth is adjusted to simulate crudely the effect of islands and it then reduces exponentially from 18 to 8 km at the point of junction with the other river sections. The breadth of each of these reduces exponentially from 4 to 2 km where the flow is prescribed as being due to fresh-water discharge alone.

The dynamical processes in the model are described by a system of depth-averaged equations for the velocity components. These have been used extensively in storm surge and tidal modelling; examples are to be found in Heaps (1969) and Flather and Heaps (1975). The approximations behind these equations, and their application to basic storm surge modelling in the Bay of Bengal, are discussed by Johns (1979).

In the present work we denote the depth-averaged components of velocity by $u$ and $v$ in which case the equations of motion are

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv = -g \frac{\partial \zeta}{\partial x} + (1/(\zeta + h)\rho)\{\tau_x^e - kpu(u^2 + v^2)^{\frac{3}{2}} \} \quad (1) \]

and

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu = -g \frac{\partial \zeta}{\partial y} + (1/(\zeta + h)\rho)\{\tau_y^e - kpv(u^2 + v^2)^{\frac{3}{2}} \}. \quad (2) \]

In these equations, $f$ denotes the Coriolis parameter, pressure is assumed hydrostatic, barometric forcing is omitted and the direct effect of the tide-generating forces is ignored. $(\tau_x^e, \tau_y^e)$ denotes the applied surface wind stress and the bottom friction is parametrized in terms of a conventional quadratic law. $\rho$ denotes water density and $k$ is the empirical friction coefficient, taken as $2.6 \times 10^{-3}$.

The equation of mass continuity for an incompressible homogeneous fluid is

\[ \frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}[(\zeta + h)u] + \frac{\partial}{\partial y}[(\zeta + h)v] = 0 \quad (3) \]

For subsequent numerical treatment, (1) and (2) are expressed in flux form as

\[ \frac{\partial}{\partial t}[(\zeta + h)u] + \frac{\partial}{\partial x}[(\zeta + h)u^2] + \frac{\partial}{\partial y}[(\zeta + h)uv] - f(\zeta + h)v = -g(\zeta + h)\frac{\partial \zeta}{\partial x} + \tau_x^e - ku(u^2 + v^2)^{\frac{3}{2}} \]

\[ = -ku(u^2 + v^2)^{\frac{3}{2}}, \quad (4) \]

and

\[ \frac{\partial}{\partial t}[(\zeta + h)v] + \frac{\partial}{\partial x}[(\zeta + h)uv] + \frac{\partial}{\partial y}[(\zeta + h)v^2] + f(\zeta + h)u = -g(\zeta + h)\frac{\partial \zeta}{\partial y} + \tau_y^e - kv(u^2 + v^2)^{\frac{3}{2}} \]

\[ = -kv(u^2 + v^2)^{\frac{3}{2}}. \quad (5) \]
Eqs. (4) and (5) describe the dynamical processes in the main analysis area. In each of the river systems the flow is assumed to be unidirectional with a uniform cross-sectional value. If \( b(y) \) is the breadth at position \( y \), the depth average, \( v \), satisfies
\[
b(y) \partial \zeta / \partial t + \partial \{ b(y)(\zeta + h)v \} / \partial y = 0 . \tag{6}
\]
and
\[
\partial \{ b(\zeta + h)v \} / \partial t + \partial \{ b(\zeta + h)v^2 \} / \partial y = -gb(\zeta + h) \partial \zeta / \partial y - kbv|v| . \tag{7}
\]
Dynamical conditions in each of the three river sections are determined from equations having the form (6) and (7). The solution process is one in which the solution of (6) and (7) is forced by an elevation determined from the bay model. The developing elevation at the mouth of river 1 communicates the motion into the first river from which elevations are determined at the mouths of rivers 2 and 3. These, in turn, communicate the motion into each of these sections.

The precise way in which the matching between the bay and river 1 is achieved is important. This is because of the transition from a fully two-dimensional bay model to a one-dimensional river model. Denoting the breadth of river 1 across a section PQ where it enters the sea by \( B \), the matching conditions at the mouth are
\[
\zeta_{river} = (1/B) \int_{PQ} \zeta_{sea} \, dx , \tag{8}
\]
and
\[
B \{ (\zeta + h)v \}_{river} = \int_{PQ} \{ (\zeta + h)v \}_{sea} \, dx . \tag{9}
\]
Eq. (8) implies continuity of sea surface elevation across the section, whilst (9) ensures continuity of volume flux.

On all solid side-walls the velocity component normal to the boundary is zero. Conditions on the southern open-sea boundary must be specified as a function of time. In the applications of the model, we shall consider motions originating from wind stress forcing associated with a cyclone tracking across the analysis area. In these circumstances the elevation along the open-sea boundary will be specified as zero. Other applications of the model will relate to the interaction between the astronomical tide and a wind-stress-generated response. In these, the tidal response in the model is first developed by prescribing the tidal elevation along the open-sea boundary in the general form
\[
\zeta = \sum_s a_s \sin(\sigma_s t + \gamma_s) , \tag{10}
\]
where \( a_s \), \( \sigma_s \) and \( \gamma_s \) are respectively the amplitude, radian frequency and phase of the important tidal constituents.

The equations of the model are then integrated ahead in time, from an initial state of rest, with the forcing (10) until the transient response is dissipated by the friction in the system. When this is achieved, the wind stress forcing due to the cyclone is initiated and the integration continued subject to (10) at the open-sea boundary. This procedure then enables the interaction between tide and surge to be determined.

3. Finite-difference formulation

A discrete sequence of gridpoints is defined by
\[
x = x_i = (i-1)\Delta x, \quad i = 1, 2, \ldots \tag{11}
\]
\[
y = y_j = (j-1)\Delta y, \quad j = 1, 2, \ldots
\]
where $\Delta x$ and $\Delta y$ are the grid increments. The computations are performed on a staggered grid consisting of three distinct types of gridpoint. With $i$ even and $j$ odd, the gridpoint is a $\zeta$ point at which an elevation is computed. With $i$ odd and $j$ odd, the gridpoint is a $u$ point at which $u$ is computed. With both $i$ and $j$ even, the gridpoint is a $v$ point at which $v$ is computed. This arrangement of gridpoints is shown in Fig. 3.

The lateral boundaries of the computational area are chosen so that the open-sea boundary consists of $\zeta$ points and $u$ points. Meridional side-walls consist of $u$ points and the latitudinal side-walls consist of $v$ points. Accordingly, on solid lateral boundaries, the gridpoint values of the velocity components are prescribed as zero. On the open-sea boundary, the elevation is prescribed at the $\zeta$ points as zero, or specified through use of (10).

A discrete sequence of time-instants is defined by

$$t = t_p = p \Delta t, \quad p = 0, 1, 2, \ldots \quad (12)$$

and, for any variable $\phi(x, y, t)$, we write

$$\phi(x_i, y_j, t_p) = \phi_{ij} p.$$

The discretized versions of (3), (4) and (5) are most easily described using a symbolical notation. In this, averaging operators are defined by

$$\bar{\phi}^x = \frac{1}{2}(\phi_{i+1,j}^x + \phi_{i-1,j}^x), \quad \bar{\phi}^y = \frac{1}{2}(\phi_{i,j+1}^y + \phi_{i,j-1}^y), \quad \bar{\phi}^{xy} = \bar{\phi}^{yx}$$

$$\bar{\phi}^t = \frac{1}{2}(\phi_{i,j}^{t+1} + \phi_{i,j}^{t-1}). \quad (14)$$

Difference operators are defined by

$$\delta_x \phi = (1/2\Delta x)(\phi_{i+1,j}^x - \phi_{i-1,j}^x), \quad \delta_y \phi = (1/2\Delta y)(\phi_{i,j+1}^y - \phi_{i,j-1}^y),$$

$$\delta_t \phi = (1/2\Delta t)(\phi_{i,j}^{t+1} - \phi_{i,j}^{t-1}). \quad (15)$$

Eq. (3) is then discretized in the form

$$\delta_t \zeta + \delta_x ((\zeta^x + h)u) + \delta_y ((\zeta^y + h)v) = 0. \quad (16)$$

Eq. (16) yields an updating procedure for the elevation at all the interior $\zeta$ points, the value of $\zeta$ being prescribed along the southern open-sea boundary. When applied at $\zeta$ points immediately adjacent to a solid side-wall boundary, (16) implies a reference to $\zeta$ points outside the analysis area. However, these terms make no contribution because of the
prescribed zero normal current velocities along the side-wall. Additionally, the updating scheme is formally consistent with the conservation of mass in the system.

Eqs. (4) and (5) are discretized according to

\[ \delta_t \{ (\zeta^R + h)u \} + \delta_z \{ (\zeta + h)(\bar{u}^2)^2 \} + f \{ 2(\zeta^R + h)\bar{v}_x^2 - (\zeta^R + h)\bar{v}_z \} = -g(\zeta^R + h)\delta_z \zeta + \tau_{ij}/\rho - k\{ u^2 + (\bar{v}^R)^2 \} \frac{e}{2} (2\bar{u}^2 - u) \]  

(17)

and

\[ \delta_t \{ (\zeta^R + h)v \} + \delta_z \{ (\zeta^R + h)\bar{u}^2\bar{v}_x \} + f \{ 2(\zeta^R + h)\bar{u}^2\bar{v}_z - (\zeta^R + h)\bar{v}^R \} = -g(\zeta^R + h)\delta_z \zeta + \tau_{ij}/\rho - k\{ (\bar{u}^R)^2 + v^2 \} \frac{e}{2} (2\bar{v} - v) \]  

(18)

In both (17) and (18) the Coriolis and dissipative terms are evaluated partly implicitly and contain references to the advanced time-level and the two previous time-levels. This element of implicitness guarantees stability as far as the effect of the Coriolis and dissipation terms is concerned. However, an algebraic solution of (17) and (18) is necessary for \( u_{ij}^{+1} \) and \( v_{ij}^{+1} \) before their incorporation into an updating procedure. The scheme, which avoids numerical sources or sinks of momentum within the analysis area, requires some modification at current points immediately adjacent to the open-sea boundary. Here, it is either necessary to introduce fictitious gridpoints outside the analysis area (which are referenced by the averaging operators) or, alternatively, to use an appropriate one-sided definition of \( \delta_z \). In the present work the latter course has been adopted.

The scheme given by (17) and (18) does not formally avoid numerical sources or sinks of energy, but this is not problematical in the integrations of limited length to be subsequently described.

The one-dimensional river equations (6) and (7) are discretized according to

\[ \delta_t \zeta + (1/b)\delta_z \{ b(\zeta^R + h)v \} = 0 \]  

(19)

and

\[ \delta_t \{ b(\zeta^R + h)v \} + \delta_z \{ b(\zeta + h)(\bar{v}^R)^2 \} = -gb(\zeta^R + h)\delta_z \zeta - kb|v|(2\bar{v}^R - v) \]  

(20)

Wind stress forcing is omitted in (20) since our objective is to investigate the penetration of an externally generated surge into the river system. Additionally, the meandering nature of the actual river, together with its relatively small area in comparison with that of the bay, suggest that the direct effect of wind stress forcing resulting from a weakening cyclone will be unimportant. Eqs. (19) and (20) yield procedures for updating the elevation and current in each of the three river sections. The solutions developed by this process must match the solution in the bay model.

Considering the junction between river 1 and the bay model, the first elevation point in the river (referred to as the pivot point) is chosen to coincide with a boundary \( v \) point in the bay. This configuration is shown in Fig. 4.

Supposing that there are \( M \) bay \( v \) points along the segment PQ, the \( M \) updated elevations at the \( \zeta \) points along RS are used in an extrapolation scheme to determine elevations, \( \xi \), along PQ. Eq. (8) then yields an updated average elevation at the pivot point, \( D \), given by

\[ \xi_D = (1/M)\sum_{q=1}^M \xi_q \]  

(21)

This provides a forcing for the motion in the river and its use in (19) and (20) leads to updated elevations and currents at the \( \zeta \) and \( v \) points in river 1. In particular, using the current computed at E (say \( u_{ij} \)), boundary values of \( v \) along PQ may then be determined for
the bay model. These must be consistent with (9) and, prescribing that each of the bay \( v \) values along PQ are equal (and denoted by \( v_{PQ} \)), we replace (9) by

\[
\frac{1}{2} b_E [(\zeta + h)_D + (\zeta + h)_F] v_E = v_{PQ} \int_{PQ} (\zeta + h) \, dx.
\]

(22)

the integral being evaluated by the trapezoidal rule.

Using previously updated (or initial) elevations in the bay, this matching procedure leads to updated elevations in river 1 and, additionally, yields boundary values of \( v \) across the mouth of the river to be used in the next updating of the bay variables.

Similar matching procedures are applied to the junction between rivers 2, 3 and 1 but the details are not given here.

The method of joining river 1 to the bay model in the present work is somewhat different from the techniques applied by Banks (1974) and Prandle (1974) in matching the river Thames with the North Sea. Its effectiveness has, however, been established in our numerical computations.

As previously stated, the basic bay computational area consists of three rectangular regions. In the most southerly of these, we have chosen a coarser mesh than in the two more northerly regions. Specifically, in region 3, we choose a grid increment that is four times that in regions 1 and 2. Evidently this necessitates an appropriate matching of the solutions along the common boundary between regions 2 and 3. This procedure involves the specification of an overlap area between 2 and 3 and the relating of coarse and fine grid variables by appropriate averaging formulae. The subsequent numerical technique depends on the use of the boundary coarse grid elevation values in forcing the solution in region 2. Procedures of this kind have previously been used by Jelesnianski (1965) and Das et al. (1974). Our experience has shown that this matching of coarse and fine grids introduces a localized distortion into the computed results. Accordingly, a local smoothing was introduced and this was found to have a negligible effect on the numerical solution in region 1.

4. NUMERICAL EXPERIMENTS

In all our numerical experiments the grid increment in the fine mesh regions 1 and 2 is given by \( \Delta x = \Delta y = 18 \text{ km} \), and in each of the river sections \( \Delta y = 18 \text{ km} \). This implies \( M = 9 \) in Eq. (21). In the coarse mesh region 3, we take \( \Delta x = \Delta y = 72 \text{ km} \). In the experiments which we describe here, we have used an idealized representation of the equilibrium depth which, in region 3, is uniform and equal to 500 m. In the fine mesh areas, the depth decreases towards the lateral boundaries and equals 10m along the coastline. River 1 has a uniform depth of 8 m whilst rivers 2 and 3 are of uniform depth 7 m.
The surge response is generated by an idealized cyclone tracking across the analysis area. The pressure distribution in this is represented by

$$p = p_a - \Delta p \, e^{-r/R}$$

(23)

where $p_a$ is the ambient pressure, $\Delta p$ the difference between the central pressure and the ambient pressure, $R$ the e-folding radius of the cyclone and $r$ the radial distance of any point from the centre of the cyclone.

In our experiments, the cyclone moves across the analysis area along two basic tracks, labelled in Fig. 1. 'A' corresponds to an unchanging northeasterly oriented track whilst section 'B' corresponds to a deviation from 'A' during the time preceding landfall. For reference, 'C' corresponds to the track before landfall of the November 1970 cyclone. The wind speed associated with the cyclone is calculated from a gradient balance with variable Coriolis parameter, and a corresponding surface wind stress ($\tau_x, \tau_y$) is determined from a conventional quadratic stress law with a friction coefficient of $2.8 \times 10^{-3}$. Throughout the experiments we take $\Delta p = 50 \text{ mb}$, $R = 350 \text{ km}$ (cf. Anthes 1974), and consider the initial position of the centre of the cyclone to be $432 \text{ km}$ north of the open-sea boundary. The cyclone moves with a constant speed of $14 \text{ km h}^{-1}$ and its centre reaches the Bangladesh coastline after about 3 days. The numerical solution was obtained with a timestep of $155 \text{ s}$. This was found to lead to stability of the calculations.

In the first two experiments a surge is generated by pure wind stress forcing, with no component of astronomical tide. With the track 'A', the developing sea surface response is shown in Fig. 5. The frames give the contours of equal sea surface elevation in metres in fine mesh regions 1 and 2. Figure 5(a) corresponds to $t = 43.4 \text{ h}$, $t = 0$ denoting the time of initiation of the wind stress forcing. In region 2 the general picture is of a positive surge in the east and a negative surge in the west. Sea surface elevations are generated in the northeast of region 2, corresponding to the Burmese coast in the neighbourhood of the Arakan coast. In the east of region 1, along the Bangladesh coastline, there is a negative surge with sea surface depressions amounting to about $2 \text{ m}$. In region 1 there is a steep east-west gradient in the sea surface and, on the Indian side of the bay, there is a sea surface elevation of about $2.8 \text{ m}$. Figure 5(b) corresponds to $t = 55.8 \text{ h}$. The elevation in the northeast of region 2 has now increased to as much as $6 \text{ m}$ but the water is showing a tendency to flow north into region 1 along the coastline and to generate a positive surge in this area. The surge is still negative, however, in the northeast of region 1. Figure 5(c) corresponds to $t = 68.2 \text{ h}$, by which time the centre of the cyclone is within $1.6 \text{ h}$ of landfall. The elevation in the northeast of region 2 has started to decrease and now has a maximum value of about $5.5 \text{ m}$. The northward flow of water continues and a positive surge of as much as $1.8 \text{ m}$ has developed along parts of the eastern boundary of region 1. In the extreme northeast, the surge is still negative. Figure 5(d) corresponds to about $10.9 \text{ h}$ after landfall. The elevations in the northeast of region 2 have fallen to about $4 \text{ m}$ but a positive surge has now developed in the northeast of region 1 with a sea surface elevation of about $1.8 \text{ m}$. In the west of region 1, the surge is now negative with a sea surface depression of as much as $7 \text{ m}$.

In summary, then, the development of surge along the northeastern coastline of Bangladesh, resulting from the cyclone track 'A', begins as a sea surface depression. This is because the bulge in the coastline of Burma, together with the dimensions of the forcing cyclone, cause a piling up of water in this area. As the cyclone approaches land, however, this piling up ceases and a resulting northward flow leads to the ultimate development of a positive surge along the northeastern coastline of the model. This reaches its maximum about $10 \text{ h}$ after landfall.

It is noteworthy that the process just described is also dependent on the magnitude used.
for the e-folding radius of the cyclone. The use of a reduced radius has been found to alter
the character of the response. If $R = 40$ km, the extreme piling up of water in the northeast
of region 2 does not occur. Instead, the water is driven directly towards the northeast of
region 1 and an initial positive surge develops in this area. This feature indicates the impor-
tance of an accurate incorporation of the cyclone radius into a forecasting scheme.

In the second experiment we evaluate the surge response resulting from the deviating
track 'B' in Fig. 1. The developing sea surface elevation is shown in Fig. 6. Figure 6(a)
corresponds to $t = 43.4$ h and is virtually indistinguishable from Fig. 5(a). At this time,
the response is little affected by the deviating track of the cyclone which starts at $t = 37.2$ h.
Figure 6(b) corresponds to $t = 55.8$ h and the difference between the sea surface topography
and that in Fig. 5(b) begins to show. The northward flow of water round the bulge in the
coast of Burma starts earlier and a positive surge in excess of 1 m is already developing
along the southeastern boundary of region 1. In Fig. 6(c), when $t = 68.2$ h (about 0.5 h after

Figure 5. Contours of equal sea surface displacement in regions 1 and 2 resulting from pure wind stress
forcing with cyclone track 'A'. (a)-(d) correspond to $t = 43.4(12.4)80.6$ h. The 'cliff-edges' are an artefact
of the microfilm procedure and arise as a result of a fictitious elevation being set equal to zero outside the
analysis area.
landfall), the full significance in the change of track has become apparent. A positive surge now exists along the entire east coast of region 1 and this leads to sea surface elevations of as much as 3 m. Figure 6(d) corresponds to $t = 80\cdot 6 \, \text{h}$ (about 12.9 h after landfall) and the surge elevation in the northeast of region 1 has now increased to between 4 and 5 m. This is of the same order as the maximum surge estimated at Chittagong during the November 1970 cyclone.

It is clear, therefore, that a cyclone following track 'B' will lead to far greater surges along the northeastern coastline of the model than one following the undeviating track 'A'. This is evidently because a purely northerly track does not drive as much water into the northeast of region 2 as does a northeasterly track.

Using the deviating track 'B', it is informative to calculate the spatial distribution of the maximum surge height during the $80\cdot 6 \, \text{h}$ integration period. This is shown in Fig. 7, where the contours refer to the maximum surge height in metres. We see that a maximum surge of about 5 m is generated in the northeast of region 1. In the northwest of region 1, the maximum surge is about 4 m.
The third experiment relates to the interaction between the surge generated by a cyclone following track 'B' and the astronomical tide. In the Bay of Bengal, the astronomical tide results principally from the $M_2$ and $S_2$ constituents (McCann and Wunsch 1977). In our study, we consider only the dominant $M_2$ constituent and approximate this along the open-sea boundary by using Eq. (10) with $a_s$, $s$, $y_s = 0$ for $s \geq 2$. The value of $a_1$ at $\zeta$ points varies from 0.25 m in the west to 0.5 m at the mid-point of the open-sea boundary. It is then equal to 0.55 m at the remaining elevation points to the west of the eastern boundary of the model. We take $2\pi/\sigma_1 = 12.4$ h and $\gamma_1 = 90^\circ$ in terms of the Greenwich phase. Prior to the initiation of wind stress forcing, the tidal response is established during 30 cycles of integration, after which time the solution is purely oscillatory. Given the objectives of the present study, the computed $M_2$ tide is deemed satisfactory. The predicted tidal amplitude and phase near Cox's Bazar are respectively 0.83 m and 160°. These compare satisfactorily with observed values of 1.06 m and 129°. Wind stress forcing commences at $t = 0$ when the tidally forced sea surface elevation is above its equilibrium value in the north of region 1 and amounts to about 0.6 m. Fig. 8 corresponds to the development of the sea surface topo-
Figure 8. Contours of equal sea surface displacement in metres in regions 1 and 2 resulting from astronomical tide and wind stress forcing. (a) gives the pure tidal displacement at $t = 6.2$ h; (b)–(h) give the displacement due to tide and wind stress forcing at $t = 6.2(12.4)80.6$ h. See caption to Fig. 5.
ography resulting from the simultaneous presence of both tide and surge and Fig. 8(a) gives the amplitude of the pure $M_2$ tidal response at each of the subsequent frames. Figure 8(b) corresponds to the sea surface elevation resulting from the combination of surge and tide at $t = 6.2$ h. Although the picture is largely tidally dominated, it is clear that the effect of the surge is already being felt in the south of region 1 where sea surface elevations are increased by as much as $20$ cm over the pure tidal value. This early surge response supports our choice of a large analysis area in order to record the early surge-generating capacity of the cyclone.

Figure 8(c) corresponds to $t = 18.6$ h, by which time region 1 is everywhere affected by sea surface elevations except adjacent to the east coast where the surface is depressed below its tidal position by as much as $40$ cm. Figs. 8(d)–(h) correspond to the interval $31.0–80.6$ h in steps of $12.4$ h and show the developing contribution of the surge to the sea surface response.

The final frame, 8(h), gives the sea surface topography $12.9$ h after landfall and may be compared with the corresponding pure surge response in 6(d). In the northeast of region 1 this comparison shows that the sea surface elevations resulting from surge and tide are different from a simple addition of pure surge and pure tide. This interactive effect is most readily assessed by considering the temporal variation at a fixed location of the elevation due to tide alone, surge alone, and the simultaneous presence of both tide and surge. Denoting these quantities by $\zeta_T$, $\zeta_S$ and $\zeta_{T+S+IST}$ respectively, it is convenient to define the elevation due to the interaction between tide and surge by

$$\zeta_{S+IST} = \zeta_{S+IST} - \zeta_T.$$  \hspace{1cm} (24)

Figure 9. Temporal variation of sea surface displacement at a position in region 1, $18$ km south of northern boundary and $252$ km west of eastern boundary - cyclone track 'B'. Maximum and minimum values of $\zeta_T$ during a tidal cycle are denoted by circles. $\zeta_T$ - continuous; $\zeta_{S+T+IST}$ - dotted; $\zeta_{S+IST}$ - dashed.
Clearly, in a linear model, the difference \( \zeta_{S+\text{IST}} - \zeta_S \) will be zero, but in a non-linear model a comparison of \( \zeta_{S+\text{IST}} \) with \( \zeta_S \) will yield the nature of the interactive effect.

We consider first the interactive effect at a position in region 1 18 km south of the northern boundary and 252 km west of the eastern boundary. Figure 9 shows the temporal variation of \( \zeta_S \), \( \zeta_T \), \( \zeta_{S+T+\text{IST}} \) and \( \zeta_{S+\text{IST}} \). In accordance with our earlier remarks, we see that \( \zeta_S \) starts by being positive and remains so for about 40 h after the initiation of wind stress forcing. During this period, \( \zeta_{S+\text{IST}} \) is less than \( \zeta_S \) at times of high tide and greater than \( \zeta_S \) at times of low tide. The peak values of \( \zeta_{S+T+\text{IST}} \) consistently occur after the time of high tide.

We indicate in Fig. 9 the time at which the cyclone starts to follow the deviating track 'B' and, after this, note that \( \zeta_S \) soon becomes negative. Thereafter, a negative surge continues until the time of landfall with a maximum water depression of about 2.3 m. During the

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**Figure 10.** As in Fig. 9 except that position is 18 km south of northern boundary and 18 km west of eastern boundary.
period of negative surge $\zeta_{S+1ST}$ continues to be less than $\zeta_S$ at times of high tide, and greater than $\zeta_S$ at times of low tide.

In Fig. 10, we give the corresponding information for a position 18 km south of the northern boundary of region 1 and 18 km west of the eastern boundary. This location corresponds to the extreme northeastern corner of region 1 where the maximum surge response has been found to occur. We see that $\zeta_{S+1ST}$ is negative for about 50 h after the initiation of wind stress forcing with a small positive peak of about 20 cm at $t = 21.7$ h. $\zeta_{S+1ST}$ is again greater than $\zeta_S$ at times of low tide and less than $\zeta_S$ at times of high tide. After the deviation from track 'A' has occurred the magnitude of this interactive effect can amount to as much as 1 m. The most striking behaviour, however, is the increase of $\zeta_{S+1ST}$ from zero at 24 h preceding landfall, to more than 5 m at 16 h after landfall. This extremely rapid increase in the surge-induced sea surface elevation produced in the model is compatible with surge responses estimated at Chittagong.

In Fig. 11 we give the temporal variation of the surge response at the central point of the mouth of river 1, about 108 km west of the eastern boundary of region 1. In comparison with the sea surface elevation given in Fig. 10, the response at the mouth of the river has important differences. Here, the surge is initially positive and $\zeta_{S+1ST}$ attains a local maximum of about 1 m some 24 h after the initiation of wind stress forcing. $\zeta_{S+1ST}$ then reduces and becomes negative after about 50 h. Thereafter there is an extremely rapid increase in $\zeta_{S+1ST}$ leading to a maximum surge response of about 3 m some 15-5 h after landfall. This double peaking is associated with the change of track. The interactive effect continues to have the general characteristics already described and is a maximum at times of low tide.

As the surge propagates into the river system, the interaction between surge and tide begins to change in character. Figure 12 relates to a position about 162 km from the mouth of river 1. From this we again see the double peak behaviour in the surge response. Also, during the initial build up of positive surge, it is noteworthy that $\zeta_{S+1ST}$ is less than $\zeta_S$ at all states of the tide. The elevation in the river reaches a maximum of about 20 cm above the tidal value some 26-4 h after the initiation of wind stress forcing. Subsequently, the surge response reduces, then shortly before landfall it increases rapidly during a 24 h period to produce a maximum surface elevation of about 2 m above the tidally-induced elevation. During this latter period $\zeta_{S+1ST}$ has the same relation to $\zeta_S$ as in the bay.

![Figure 11](image-url)  
As in Fig. 9 except that position is at mid-point of mouth of river 1.
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Figure 12. As in Fig. 9 except that position is 162 km inland along river 1.

In Fig. 13 we give the surge response at a position about 162 km along river 2. The dissipative effect of friction on the oscillation in the river has now considerably reduced the amplitude of the response. The double peak behaviour is still present and $\zeta_{S+1ST}$ is now less than $\zeta_S$ throughout the integration period. At the second peak the surface elevation is about 70 cm above equilibrium level. This compares with a maximum tidal elevation of about 25 cm at this position in the river.

Figure 13. As in Fig. 9 except that position is 162 km inland along river 2.

5. CONCLUDING REMARKS

Our study has shown the importance of using an analysis area large enough to allow the recording of 3 days surge-generating capacity of a cyclone before landfall at the Bangla-
desh coastline. Our experiments have shown the nature of the interaction between astro-
nomical tide and surge. The significant contribution that this makes to the developing sea surface elevation in shallow water regions demonstrates the necessity of using a non-linear model. The incorporation of a river system in our model has shown that surge may penetrate deep inland, thus leading to a flooding hazard in the inland waterways of Bangladesh. It has also been shown that the surge response depends critically on the track and diameter of the forcing cyclone.

The effect of barometric forcing has not been included in our model: we have been concerned with pure wind stress forcing. In future experiments, instead of regarding the atmospheric pressure anomaly as being equivalent to a statical correction (Prandle 1975), barometric forcing might be included in order to assess the dynamical contribution of this process. Additionally, a more detailed account might be taken of coastline and bottom topography. Further refinements might relate to an improved representation of the cyclone and the way in which it intensifies on moving north into the Bay of Bengal. These procedures, however, are probably not worth while until better data are available for comparison with the model predictions.

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