A model to derive precipitation patterns and vertical velocities in large-scale rain systems

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SUMMARY

A simple model is proposed that enables a prediction to be made of precipitation patterns and vertical velocities from single-station serial ascents. Microphysical processes are parametrized by dividing the liquid water generated by the model into cloud and rain water. The ice phase is not explicitly modelled. Rainfall is predicted reasonably well when the cloud cover is extensive and convection is not dominant. Calculations show that the precipitation developed by the model is not strongly dependent on changes in the microphysical parametrization.

LIST OF SYMBOLS

\( a \) radius of the convective parcel
\( ACC \) accretion defined by Eq. (13)
\( AUT \) autoconversion defined by Eq. (12)
\( c \) specific heat of dry air at constant pressure
\( D_u \) diffusion coefficient
\( EVA \) evaporation defined by Eq. (15)
\( F \) fractional local cover by convective cloud
\( g \) acceleration due to gravity
\( H \) wet or dry adiabatic heating rate
\( k \) entrainment coefficient
\( K_a \) thermal conductivity of air
\( L \) latent heat of vaporization
\( N_c \) number of drops per cm\(^3\) in the cloud
\( p \) pressure
\( p_0 \) base of convective activity
\( p_s \) saturation vapour pressure of water
\( \dot{q} \) rate of change of \( \bar{q} \) resulting from adiabatic heating
\( \bar{q} \) mean mixing ratio
\( \bar{q}_s \) mean saturation mixing ratio
\( Q_{lw} \) critical liquid water content, \( \frac{3}{4} \pi r_m^2 \rho_a N_c / \rho_s \)
\( Q_c \) mean cloud water mixing ratio
\( Q_R \) mean rain water mixing ratio
\( r_c \) mean cloud drop radius
\( R \) radiation term in Eqs. (1) and (5)
\( R_g \) gas constant for dry air
\( R_w \) gas constant for water vapour
\( r_m \) parameter in the Marshall–Palmer distribution
\( T_0 \) 273.16 K
\( T \) mean absolute temperature
\( t \) time
\( \bar{u}, \bar{v}, \bar{w} \) mean velocity in \( x, y, z \) direction
\[ V_r \] parametrized fall speed of rain drops, Eq. (14)

\( x, y, z \) coordinates

\( \Gamma \) \[ \delta(D\theta/Dp) - \delta\theta/\partial p \]

\( \theta_v \) mean virtual potential temperature

\( \theta \) mean potential temperature

\( \theta^* \) mean equivalent potential temperature

\( \theta^* \) potential temperature of a convective parcel

\( \theta^* \) virtual potential temperature of a convective parcel

\( \mu_c \) viscosity of air

\( \rho_w \) density of water

\( \rho_a \) density of air

\( \bar{\omega} \) \[ Dp/ Dt = -\rho_s g \bar{w} \]

\( \omega \) value of \( \omega \) for a convective parcel

\( (u', A'), (v', A'), (w', A') \) are the turbulence fluctuations of \( A \) where \( A = \theta, q, Q_c, Q_R \)

1. Introduction

It is clear from radar and aircraft observations as well as from ground-based studies (see e.g. Browning et al. 1973; Wallace and Hobbs 1977) that there is considerable mesoscale variability within extra-tropical cyclonic storms. This variability implies a need for high resolution numerical modelling in order to predict the rainfall associated with these systems. However, in such models the account which can be taken of microphysical processes which may be important for rain formation is likely to be severely limited. Nevertheless there are situations, such as cloud-seeding programmes, where microphysical processes are clearly important and in these cases there is a need for a simple dynamical model in which they can be included.

In this paper precipitation rates and vertical velocities are deduced from a series of three-hourly rawinsonde ascents and the results compared with observed rainfall rates. The data come from observations made during the Laverton Serial Sounding Experiment (Bureau of Meteorology 1968). In the initial study with the model which follows the microphysics is parametrized by dividing the liquid water into cloud and rain water; the ice phase is not explicitly modelled.

2. The model

In an Eulerian framework the mean transport equations for potential temperature, moisture, cloud water and rain water may be written:

\[ \partial \theta/\partial t + \bar{u} \partial \theta/\partial x + \bar{v} \partial \theta/\partial y + \bar{w} \partial \theta/\partial p = H/t + R - (L \delta/c T) EVA - \partial(u'v')/\partial x - \partial(u'v')/\partial y - \partial(u'v')/\partial p, \quad (1) \]

\[ \partial q/\partial t + \bar{u} \partial q/\partial x + \bar{v} \partial q/\partial y + \bar{w} \partial q/\partial p = q + EVA - \partial(u'q')/\partial x - \partial(v'q')/\partial y - \partial(q'q')/\partial p, \quad (2) \]

\[ \partial Q_c/\partial t + \bar{u} \partial Q_c/\partial x + \bar{v} \partial Q_c/\partial y + \bar{w} \partial Q_c/\partial p = -q - A U T - A C C - \partial(u'Q_c)/\partial x - \partial(v'Q_c)/\partial y - \partial(q'Q_c)/\partial p, \quad (3) \]

\[ \partial Q_R/\partial t + \bar{u} \partial Q_R/\partial x + \bar{v} \partial Q_R/\partial y + \bar{w} \partial Q_R/\partial p = A U T + A C C - E V A - \partial(u'Q_R)/\partial x - \partial(v'Q_R)/\partial y - \partial(q'Q_R)/\partial p + \rho_o g \partial(V_R Q_R)/\partial p, \quad (4) \]

where \( E V A, A U T, A C C \) and \( R \) are the evaporation, autoconversion, accretion and radiation
terms. \( \dot{H} \) and \( \dot{q} \) are given by the wet adiabatic changes in \( \theta \) and \( \bar{q} \) when the air is saturated; they are zero when the air is dry. It is convenient to use pressure rather than height coordinates because the thermodynamic equation and the thermal wind equation are then exact and because the meteorological variables which are to be used as initial conditions are readily accessible at standard pressure levels.

In the most general case these equations can be solved in conjunction with the momentum equation only by a complex three-dimensional numerical model. However, if only turbulent fluctuations arising from embedded cumulus are retained, and horizontal advection of liquid water and rain water are assumed to be second-order terms, it is possible to find solutions for \( \bar{q}, \bar{Q}_c, \bar{Q}_k \) and \( \bar{Q} \) (these assumptions are discussed below). With these approximations Eqs. (1)–(4) reduce to

\[
\frac{\partial \theta}{\partial t} + \bar{u} \frac{\partial \theta}{\partial x} + \bar{v} \frac{\partial \theta}{\partial y} + \bar{\omega} \frac{\partial \theta}{\partial z} = \dot{H} + R - (L \bar{D} / c \bar{T}) \bar{E} \bar{A} - \frac{\partial (\bar{Q} \bar{w})}{\partial p}, \tag{5}
\]

\[
\frac{\partial \bar{q}}{\partial t} + \bar{u} \frac{\partial \bar{q}}{\partial x} + \bar{v} \frac{\partial \bar{q}}{\partial y} + \bar{\omega} \frac{\partial \bar{q}}{\partial z} = \dot{q} + \bar{E} \bar{A} - \frac{\partial (\bar{Q}_c \bar{w})}{\partial p}, \tag{6}
\]

\[
\frac{\partial \bar{Q}_c}{\partial t} + \bar{u} \frac{\partial \bar{Q}_c}{\partial x} + \bar{v} \frac{\partial \bar{Q}_c}{\partial y} + \bar{\omega} \frac{\partial \bar{Q}_c}{\partial z} = - \dot{q} - \bar{A} \bar{U} - \bar{A} \bar{C} \bar{C} - \frac{\partial (\bar{Q}_k \bar{w})}{\partial p}, \tag{7}
\]

\[
\frac{\partial \bar{Q}_k}{\partial t} + \bar{u} \frac{\partial \bar{Q}_k}{\partial x} + \bar{v} \frac{\partial \bar{Q}_k}{\partial y} + \bar{\omega} \frac{\partial \bar{Q}_k}{\partial z} = \bar{A} \bar{U} + \bar{A} \bar{C} \bar{E} - \bar{E} \bar{A} + \frac{\rho_0 \bar{g}}{\partial p} \frac{\partial (V_k \bar{Q}_k)}{\partial p} - \frac{\partial (\bar{Q}_k \bar{w})}{\partial p}. \tag{8}
\]

Kreitzberg (1964) and Tucker (1973) used the thermodynamic equation in \( z \) coordinates to calculate the vertical velocities associated with a variety of synoptic situations. The local change of \( \theta \) with respect to time, \( \partial \theta / \partial t \), was determined from rawinsonde data and the thermal wind equation was used to calculate horizontal advection. Kreitzberg ignored evaporation and radiation terms, while Tucker ignored the former and parametrized the latter. With the method used by Kreitzberg the solution becomes unstable as the lapse rate approaches that for wet adiabatic ascent. Tucker overcame this problem by assuming that the latent heat released by condensation was distributed between 2 and 7 km and that the amount of heating could be deduced from rainfall amounts at several adjacent observing stations.

In the present study an iterative technique is used to find the value of \( \bar{w} \) necessary to produce the change observed in rawinsonde profiles of \( \theta \) over a three-hour period. This enables the precipitation rate to be determined. An initial value of \( \bar{w} \) is assumed and Eqs. (5)–(8) integrated to find the change in \( \bar{w} \). The initial value of \( \bar{w} \) is then increased by \( \Delta \bar{w} \), where

\[
\Delta \bar{w} = \left\{ \left( \frac{\Delta \theta}{\Delta t} \right)_{\text{calc}} - \left( \frac{\Delta \theta}{\Delta t} \right)_{\text{obs}} \right\} / \left\{ \left( \frac{\Delta \theta}{\Delta p} \right) - \left( d\theta / dp \right) \right\},
\]

being the dry or wet adiabatic lapse rate. As \( \Delta \theta / \Delta p \) approaches the wet adiabatic lapse rate the predicted change in \( \bar{w} \) over the three-hour period becomes less sensitive to changes in \( \bar{w} \). This is equivalent to the problem encountered by Kreitzberg. The iteration procedure is repeated until the observed and calculated values of \( \bar{\theta} \) differ by less than 0.3 K. This is the estimated error in the rawinsonde data used to test the model.

The terms on the r.h.s. of Eqs. (5)–(8) are all directly calculable and are found as follows:

(i) The wet adiabatic terms for potential temperature and moisture have been deduced with the aid of Tetens' equation

\[
\bar{q} = q_0 \exp\left\{ a(T - T_0)/(T - b) \right\}, \tag{9}
\]

and are as follows:

\[
\dot{H}/c = (L/c) \bar{q}_0 \left\{ (1/T) - R_k(T_0 - b)a/[c(T - b)^2] \right\} \left\{ (\bar{w}/p) \right\} \times \left\{ 1 + L \bar{q}_0 a(T_0 - b)/c(T - b)^2 \right\}^{-1}, \tag{10}
\]

\[
\dot{q} = \bar{q}_0 \left\{ a(T_0 - b)/(T - b)^2 \right\} (T/\theta)(D\theta/Dt)_{\text{wet}} + \left\{ (a(T_0 - b)R_k T)/c(T - b)^2 - 1 \right\} (\bar{w}/p), \tag{11}
\]

where \( a = 17.2694 \) and \( b = 35.86 \).
(ii) The longwave radiative cooling in Eq. (5) is approximated using a procedure developed by Manton (1978) which takes account of absorption and scattering of radiation by cloud drops.

(iii) The cloud/rain water terms for autoconversion and accretion, the terminal velocity associated with the rain water flux, and the evaporation are based on parametrizations developed by Manton and Cotton (1977) and are given by:

\[
\begin{align*}
AUT &= 0.057(\rho_w g/\mu_c N_c^{1/3})(\rho_u/\rho_o)^{1/3} Q_e^{7/3} H(Q_e - Q_{cm}), \quad (12) \\
ACC &= 0.884(\rho_w/\rho_o)^{1/2}(g_r r_m)^{1/2}(1/\rho_w g)Q_e Q_R, \quad (13) \\
V_R &= 4.13(\rho_w/\rho_o)^{1/2}(g r_m)^{1/2}, \quad (14) \\
EVA &= 0.349(\rho_w/\rho_o)^{1/4}(\rho_w^2 r_m^2 g/\mu_c^2)^{1/4}(D_{ea}/r_m^2)((\bar{a}_s - \bar{q})/\rho_w Q_R \rho_s), \quad (15)
\end{align*}
\]

where \( H(Q_e - Q_{cm}) \) is the Heaviside unit step function, \( Q_{cm} \) is the critical liquid water content, given by \( Q_{cm} = \frac{4}{3} \pi \rho_w \bar{r}_c^3 N_c/\rho_s \), \( \bar{r}_c \) and \( r_m \) are 10 and 270 \( \mu \)m respectively, \( N_c \) is 100 cm\(^{-3}\) and \( D_{ea} = D_e/(1 + L^2 D_e \rho_o/K_e R_s^2 T^2) \).

In the region where the air is saturated, that is in cloud, the horizontal gradients required to calculate the advection of vapour are directly calculable in terms of the horizontal temperature gradients. Differentiating Eq. (9) on a constant pressure surface gives \( \nabla_h \bar{q} = (\bar{q} \delta \bar{q}/\delta \bar{T}) \nabla_h \bar{T} \), where \( \nabla_h \) is the horizontal gradient.

Below cloud base the conservative property of the equivalent potential temperature, \( \bar{\theta}_c = \theta \exp(\bar{q}/L/c \bar{T}) \), is used to determine the horizontal moistent gradient. Observations suggest that during the passage of a raining low pressure system \( \bar{\theta}_c \) is approximately constant in the lower layers, and it is therefore assumed that below cloud base \( \nabla_h \bar{\theta}_s \) is also constant and equal to its value at cloud base. From this conservation relation the horizontal moisture gradient below cloud base becomes

\[
\nabla_h \bar{q} = (\nabla_h \bar{\theta}_c)_{\text{cloud base}} \exp(-\bar{q}/L/c \bar{T}) - (1 - L \bar{q}/c \bar{T}) \nabla_h \bar{T}) \nabla_h \bar{\theta} (L \bar{q}/c \bar{T})^{-1}.
\]

When there is no rain below cloud base it is assumed that \( \nabla_h \bar{q} \) is constant and equal to the value at cloud base.

Above cloud top there is no simple method of parametrizing the horizontal moisture gradients. In the model it is assumed that the moisture gradient is given by \( \nabla_h \bar{q} = \nabla_h \bar{q}_a \times RH \), where \( RH \) is the relative humidity.

(iv) The transport of heat and moisture by convective activity within the cloud system has been parametrized using a parcel model with an entrainment constant, \( k \). The transport of potential temperature, \( \theta' \), by the convective parcel of radius \( a \) is given by

\[
D\theta'/Dt = \omega d\theta'/dp = \omega \Gamma + \omega k \theta':
\]

hence \( d\theta'/dp = \Gamma + k \theta' \), where \( \Gamma = \{ \delta (D\theta'/Dp) - (d\theta'/d\rho) \} \) and is the mean value over the convective region, and \( k \), the entrainment coefficient is defined by \( k = -(1/a)(da/dp) = -0.2/\rho_0 g a \). If \( \theta' = 0 \) when \( p = p_0 \), \( \theta' = -(\Gamma/k)(1 - \exp k(p - p_0)) \). The momentum transported by the parcel is

\[
d(\rho^2)'/dp = -2 \rho_0 g^2 \theta' \theta_s + 2k(\rho')^2,
\]

and approximating \( \theta' \) by \( \theta' \)

\[
d(\rho^2)'/dp = -(2 \rho_0 g^2 \theta_s /\theta_s) (\Gamma/k)(1 - \exp k(p - p_0)) + 2k(\rho')^2,
\]

giving

\[
\rho' = (g^2 \rho_0 \Gamma /\theta_s)(k^2) \{1 - \exp k(p - p_0)\}
\]

whence

\[
\theta' \rho' = -(\Gamma/k)(g^2 \rho_0 \Gamma /\theta_s)(k^2) \{1 - \exp k(p - p_0)\}^2
\]
and the flux gradient becomes
\[ \partial(\theta'\omega')/\partial p = (2\Gamma \left(-2\Gamma \right) (1/k)(1-exp(p-p_o)) exp(p-p_o). \]

If \( F \) is the fractional cover by convective elements the net flux gradient becomes
\[ \partial(\theta'\omega')/\partial p = \{F(1-F)\} (1-k) \left(-2\Gamma \right) (1/(1/k)(1-exp(p-p_o)) exp(p-p_o). \]

The parametrization has two free parameters: \( F \) the coverage and \( k \) the entrainment coefficient. For the purposes of the present model it is assumed that there is 10% convective activity.

Since the environment is saturated, vapour and liquid water fluxes are approximated by
\[ \partial(\omega'q')/\partial p = \{q_s a(T_0-b)/(T-b)^{2}\} /T \partial(\omega'\theta')/\partial p = -\partial(\omega'Q_b)/\partial p, \]
while \( \partial(\omega'Q_b)/\partial p \) is assumed to be negligible.

3. Application of the Model to the Laverton Serial Sounding Experiment

The model has been applied to a portion of the data analysed by Tucker (1973) and known as the Laverton Serial Sounding Experiment (Bureau of Meteorology 1968). Laverton is 20 km southwest of Melbourne (see Fig. 1). Tucker discussed the need to filter the data to remove ageostrophic motion. He used a normally distributed smoothing function with a filtering interval of 6σ, where σ is the standard deviation, and found that a filtering interval of 3 km retained fluctuations with vertical dimensions of about 2 km. The data in the present paper have been treated using the same technique.

Analysis of the rainfall measured at Laverton during the course of the experiment showed that there were only two occasions when non-convective rain fell for more than three hours. The situation was judged to be convective if there was less than 8/8 cloud cover or if the meteorological observer described the rain as showers. One of these occasions was associated with very light rain and there were discrepancies between the three-hourly and one-hourly rainfall reports as well as ambiguities in the cloud cover reported, leading to rejection of this situation for testing the model. The remaining disturbance occurred on 19 October 1966; this is discussed in detail in the next section.

4. Synoptic Pattern 19 October 1966

As pointed out by Tucker, a major surface depression associated with a deep upper trough crossed the Laverton area during 18–20 October. A surface trough had existed over the entire eastern half of the continent for several days previously. Figure 1 shows surface and 500 mb maps for 09 h local time on 19 and 20 October. Figure 2 shows a time section of temperature and relative humidity as derived from the vertical soundings. A well-defined cold front extending from 750 to 900 mb passed over Laverton at noon on 19 October 1966. This was preceded by a region between 750 and 600 mb that could be described as a hyperbaroclinic zone.

The rain commenced as drizzle between 05 and 06 h and developed into continuous heavy rain by 12 h. From 15 h until rain ceased the continuous rain degenerated into shower activity. Table 1 summarizes the recorded hourly rainfall rates, together with the reported cloud cover and weather at the time of observation (present weather) as well as that during the previous hour (past weather). There are no records of cloud top height. However, for comparison with the model it was assumed that cloud top coincided with the 70% RH level obtained from the rawinsonde data.
Figure 1. Surface and 500 mb charts at 09h local time. (a) and (b) 19 October 1966. (c) and (d) 20 October 1966.
<table>
<thead>
<tr>
<th>Time (EST)</th>
<th>Cloud base (low) (mb)</th>
<th>Cloud top (mb)</th>
<th>Cloud type</th>
<th>Three-hourly average (mm h⁻¹)</th>
<th>Hourly (mm h⁻¹)</th>
<th>Weather</th>
<th>Levels at which convective instability occurs (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>03-06 900 550 5/8 Sc 3/8 As</td>
<td>0-1 0, 0, 0-3</td>
<td>Intermittent drizzle</td>
<td>700, 650</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated</td>
<td>06-09 700 600</td>
<td>0-25 0, 0-3, 0-5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>09-12 950 650 8/8 Ns</td>
<td>1-9 2-0, 0-2, 2-9</td>
<td>Continuous rain</td>
<td>750, 650</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated</td>
<td>12-15 900 600</td>
<td>5-6 4-6, 5-8, 6-4</td>
<td>Moderate intermittent rain</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>15-18 850 600 No cloud</td>
<td>0-4 1-3, 0, 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated</td>
<td>18-21 900 500 2/8 St 5/8 Sc</td>
<td>3-4 3-4, 6-3, 0-5</td>
<td>Showers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>18-21 900 1/8</td>
<td>0 0, 0, 0</td>
<td>Clouds dissolving</td>
<td>850, 800, 750</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Precipitation within sight</td>
<td>700</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 2. The vertical temperature and humidity structure from 24 h on 17 October to 18 h on 20 October 1966. In (a) temperature contours are shown every 4 K and the dashed line shows the height of cloud base. An upper level cold front passed over Laverton between 12 and 15 h on 19 October. In (b) humidity is plotted as follows: >80% - close horizontal spacing; 80-70% - wide vertical; 70-50% - wide horizontal; <50% - open. The warm moist air passing Laverton between 24 h on 18 October and 15 h on 19 October coincided with the passage of the warm sector air.

5. RESULTS FROM THE MODEL

The model was initiated using as input data from the sounding at 06 h and assuming a zero rainfall rate. Using the iterative procedure described earlier the model was run for a three-hour period with a 2 s timestep until there was agreement at all gridpoints to better than 0.3 K between the values of $\theta$ predicted by the model and those observed during the 09 h sounding. The timestep is small because the model was designed to include explicit ice phase microphysics. Although the ice phase has not been used in these experiments the model was tested with the smaller timesteps to ensure that the numerics would be suitable for later inclusion of the ice phase. There is no point in forcing better agreement than 0.3 K because of the likely errors in the rawinsonde data. At the start of calculations for the next three-hour period the 09 h sounding data for temperature and moisture were assumed, the air being assumed saturated between the observed cloud base and the cloud top. On entering cloud the observed RH increased but was generally less than 100%. Subsequently the RH fell again and cloud top was specified when it reached 70%. The values for rain and cloud water were taken to be those calculated at the end of the previous three-hour interval. This
process was repeated throughout. At the end of each three-hour period the calculated cloud depth and rainfall were compared with those observed (Table 1).

The model equations were solved on a 50 mb grid having lower and upper boundaries at 1000 and 400 mb (i.e. there are 13 gridpoints in the model). It is assumed that no mass was transported through the boundaries. The microphysical parameters have been chosen so that the initial value of liquid water required for autoconversion is approximately 0.44 g kg\(^{-1}\). With the parametrization employed this implies a mean droplet concentration of 100 cm\(^{-3}\) and a mean droplet radius of 10 \(\mu\)m. Manton and Cotton (1977) have given physical arguments why the Marshall–Palmer parameter \(r_m\) should be approximately 270 \(\mu\)m and observations by Barclay (1975) in Melbourne suggest that this is a realistic value.

Initially, calculations were run assuming that the entrainment coefficient, \(k\), was zero, i.e. convective activity was neglected. In this circumstance it was found that at both 09 and 12 h it was very difficult to find a value of \(\bar{\omega}\) in the convective region which predicted the observed value of \(\bar{\theta}\) to better than 0.5 K. A range of values of \(k\), in (N m\(^{-2}\))\(^{-1}\) were tested on the model, namely \(-3 \times 10^{-5}\) (\(a = 790\) m), \(-5 \times 10^{-5}\) (\(a = 474\) m), and \(-6 \times 10^{-5}\) (\(a = 395\) m). It was found that \(k = 6 \times 10^{-5}\) produced the most rapid convergence to the observed value of \(\bar{\theta}\) at 09 and 12 h. This value of \(k\) was chosen for all remaining experiments. Table 1 shows the levels and times where convective instability could develop.

While the convective region remained unstable, increasing \(k\) reduced the required value of \(\bar{\omega}\) and increased the rainfall. However, when a region of convective instability was generated or dissipated during a three-hour integration period the correlation between \(\bar{\omega}\) and \(k\) was not necessarily maintained. This resulted from the assumption in the model that if the local value of \(\bar{\Gamma}\) became stable during the three-hour period convective transport ceased. This indeed was the reason why from 06 to 09 h it was found that the model would not converge to give the observed temperature change when \(k\) was zero or was greater than \(-5 \times 10^{-5}\).

Figure 3 shows the calculated vertical velocities in cm s\(^{-1}\) as a function of pressure. At 06 h there is upward vertical motion between 700 and 550 mb, which accounts for the rain coming from the altocumulus layers. The calculations show that during the next six hours cloud base descends and vertical velocities exceeding 40 cm s\(^{-1}\) are present in the lower troposphere. While these velocities are large they are not inconsistent with those found by Vuorella (1957) and Kreitzberg (1964). By 15 h the vertical velocities have become negative, and this is not inconsistent with the observation of showers, since convective cloud can still occur in the absence of large-scale uplift.

It is interesting that the model predicts that rain initially forms in the altostratus layers at 06–09 h; and as vertical velocities intensify the model correctly predicts the descent of

![Figure 3. Calculated vertical velocities in cm/s as a function of pressure at 06, 09, 12, 15 and 18h.](image-url)
Figure 4  (a) Time/pressure cross-section for vertical velocities in cm s$^{-1}$ as calculated by Tucker (1973).
(b) Time/pressure cross-section for vertical velocities as calculated by the present method. The horizontally hatched area shows upward motions and the vertically hatched area the downward motion.

Figure 5. A time/pressure cross-section for a saturated parcel. Richardson numbers:

- $Ri > 1$ shown by close vertical hatching.
- $0 < Ri < 1$ shown by wide vertical hatching.
- $-1 < Ri < 0$ shown by wide horizontal hatching.
- $Ri < -1$ shown by close horizontal hatching.

The heavy dashed line shows the observed cloud base pressure.
cloud base and the rainfall maximum at 12 h as well as the rise in cloud base at 15 h (see Table 1).

Figures 4(a) and (b) compare the velocities calculated by Tucker (1973) (Fig. 4(a)) and the present results. The two calculations show significant differences, in particular, Tucker's calculation predicts a maximum in vertical velocity at about 18 h, at which time the present results show weaker vertical motions. At 12 h there is a strong maximum velocity predicted by the present model but not by Tucker's calculations. The high velocities in Tucker's calculations are a consequence of the assumption that the latent heat released by the shower activity is distributed between 700 and 400 mb, with the maximum latent heat released at about 570 mb. Consequently Tucker's maximum vertical velocity occurs at about 550 mb.

Figure 2(b) shows the air to be relatively dry after the passage of the cold front. Hence if cloud is to develop under these conditions it is more likely to be convective than widespread layer cloud. The saturated parcel Richardson number as a function of pressure is shown in Fig. 5, from which it appears that a region of large negative Richardson number centred at about 800 mb develops after 15 h. This, of course, is likely to lead to convective instability.

6. DISCUSSION

The question to examine is whether a relatively simple model is capable of predicting the presence or absence of cloud, the rainfall rate and the vertical velocity during the passage of a precipitating synoptic situation. This preliminary study suggests that the model is in good agreement with observations prior to the arrival of the cold front but that it behaves rather poorly in the convective conditions behind the cold front. The reasons for this behaviour are probably associated with the assumptions inherent in the model, of which the four most important are: (i) the geostrophic assumption implied in the thermal wind relation; (ii) the neglect of horizontal advection of cloud and rain water; (iii) the assertion that there is either 8/8 cloud or no cloud at all; (iv) the parametrization of the convection.

(i) The geostrophic assumption should be reasonable for all but the two lowest gridpoints, 1000 and 950 mb. The lowest gridpoint is assumed to be the surface and there is no transport to or from the ground or advection across it. In principle it would be possible to modify the model to allow for surface friction and exchange processes. However, since most of the rain is determined by conditions at levels well above the friction layer it is felt that such additional sophistication is not justified.

(ii) The neglect of horizontal advection of cloud and rain water is probably the most questionable aspect of the model. However, aircraft observations in such systems passing over western Victoria show that the liquid water concentration is small and uniform (generally less than 0.5 g kg\(^{-1}\)), suggesting that horizontal gradients in cloud water are indeed small (W. D. King, private communication). On the other hand, the temporal variations of surface rainfall suggest that there is also significant spatial variability, and hence unless this is random and due to embedded convection the neglect of the advection of rain water is probably a poor assumption. However, the relatively good agreement between calculated and observed rainfall rates prior to 15 h shows that the advective terms averaged over three hours in the rain water equation could not have been dominant.

(iii) The model is strongly dependent on the total cloud cover or no cloud cover assumption. After the passage of the cold front the cloud was observed to become more convective and the rain more showery. The temperature structure accompanying these conditions forced the model to dissipate rapidly all its liquid water.

(iv) The parametrization to describe convective motions is valid only for a saturated environment and therefore it is not surprising that the model breaks down as the rain
becomes more showery. It is significant that the model suggests that there must be convective activity between 700 and 600 mb when there is extensive cloud cover. This is consistent with the findings of Browning et al. (1973) and Hobbs and Locatelli (1978) that there are shallow layers of potentially unstable air at about this level. Hobbs and Locatelli suggest that this instability is a source of generating cells in the cyclonic system and, further, that there is a connection between these generating cells and observed mesoscale rain bands.

The cloud cover, $F$, and the entrainment parameter, $k$, were chosen more or less arbitrarily but they are probably not unrealistic. When additional observational data become available it will be necessary to test further the sensitivity of the model to the convective parametrization.

In steady rain the model is relatively insensitive to changes in the microphysical parameters. Whilst the autoconversion parametrization is strongly dependent on the critical liquid water, $Q_{\text{crit}}$, and is proportional to $N_e$, the total number of drops per cm$^3$, changing $N_e$ from 100 to 200 cm$^{-3}$ decreases the rainfall by only about 1 mm over a three-hour period. A change in the Marshall–Palmer parameter from 270 to 140 $\mu$m, which affects both the accretion and rain water flux terms, produces an increase of about 1 mm in three hours in the rainfall. Between 09 and 12 h this represents a change of 1 mm in 7 mm, whereas between 12 and 15 h this change is 1 mm in 16 mm.

The ice phase has not been explicitly modelled and is an obvious extension to be made to the model. However, qualitatively it is clear that above the freezing level the autoconversion term would be reduced and accretion by ice particles would become important. It is not clear how these competing processes would affect rainfall at the ground.

REFERENCES


Vuorella, L. A. 1957 A study of vertical velocity distribution in some jetstream cores over Western Europe, Geophys., 6, 68–90.