Dynamical models of two-dimensional downdraughts

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(Received 3 April 1979; revised 9 November 1979)

SUMMARY

Observational and numerical model data have indicated the crucial role of the downdraft in the motion, structure and regeneration of severe storms. This paper considers downdraft dynamics in two dimensions, using both a numerical and an analytical model. In the numerical model the downdraft is produced and maintained by a constant heat sink. The adjustment of an ambient inflow, and the resultant downdraft and potential regeneration of storm cells, is simulated.

To elucidate physical principles, steady analytical models are also described in which a simple parametrization of downdraft thermodynamics is used. Two distinct types of individual draught dynamics, labelled symmetric and jump, have been identified allowing a qualitative description of six dynamically consistent updraughts and downdraughts. In particular, a cold symmetric downdraught and a cold jump-type updraught are analysed, the former having negative, the latter positive outflow shear. These analytical solutions agree well with steady states developed by the numerical model.

The numerical model is also used to describe the propagation of downstream and upstream gust fronts, and these results are in good agreement with atmospheric and tank data.

1. INTRODUCTION

One of the most important concepts in thunderstorm research was developed by Ludlam (1963), Browning and Ludlam (1962) and Newton (1966), who drew attention to the gross properties of the airflow. It became apparent that reasonable models of thunderstorms could be derived by neglecting small-scale processes, such as mixing, and emphasizing the role of the large-scale vertical wind shear and thermodynamic structure. A conceptual model emerged of a storm as two interlocking air flows — the up- and downdraughts — which transport and process environmental air in such a manner that available potential energy is converted into kinetic energy. The draughts interact predominantly by means of the rain which falls from the updraught into the drier air below, thus producing evaporative cooling and water loading, which creates and maintains the downdraught.

The steady analytical models described by Moncrieff and Green (1972) hereafter denoted by MG, Moncrieff and Miller (1976) hereafter denoted as MM, and Moncrieff (1978), further parametrized a smaller scale by allowing pseudo-saturated (wet) adiabatic thermodynamics to control heat exchange in the system. This has proved a powerful method for understanding regimes of convective overturning (Lilly 1979).

A point has been reached where nearly all the microscale processes are either neglected or parametrized, and the equations of motion are applied to the largest scale of flow in the storm. However, despite predictions of the storm speed in terms of the pre-storm environmental conditions, it appears likely that, at least in the case of three-dimensional overturning, the detailed structure of the draughts is an intractable problem using analytical models, although certain important inferences can be made by analysing inflow and outflow profiles of temperature and velocity. Such models make it possible to predict storm speed without being able to specify details of thunderstorm motion and, in particular, their regeneration and cellular structure.

The advent of three-dimensional numerical models of thunderstorms and observational programmes designed to study the thunderstorm and its environment allowed various modes of storm motion to be identified. For storms over land, the following types are amongst those described in the literature: supercell (Browning 1968); multicell stationary (Miller 1978), multicell deviatory (Thorpe and Miller 1978), multicell splitting (Thorpe and Miller
1978, and Klemp and Wilhelmson 1978), and multicell squall line, MM. The last four categories have been shown both by observational data (Marwitz 1972, Browning 1977) and the numerical model data, to be delineated by the possible interaction of the inflow air with the downdraught outflow. This process is one of gust front regeneration, and it highlights the crucial importance of downdraught structure and dynamics in both cell and storm motion. The downdraught dynamics will be studied here as a somewhat distinct fluid dynamical problem. Although this draws attention to a smaller scale than that of the storm, the downdraught outflow links the storm to the mesoscale, and it is felt that this paper will help in understanding certain mesoscale problems. More specifically, this study attempts to quantify the mechanisms of cell regeneration and relate this concept to the steadiness hypothesis.

Observations of downdraught structure have been summarized by Miller and Betts (1977) who considered the propagating squall lines observed in VIMHEX−1972. The downdraught is created and maintained by precipitation forcing (evaporation and water drag) and produces a strong low level cooling in the subcloud layer; this is associated with an acceleration of the flow away from the storm. This outflow produces strong convergence along the direction of the cell relative inflow and upward convective motion may hence occur.

The physical picture emerges of an updraught initiated by local forcing (heat source or topography); the updraught slopes down shear and produces rain and evaporative cooling. Apart from the complexity of the initiation process, the thunderstorm problem then becomes that of the formation and maintenance of the downdraught and its interaction with the updraught. As the original updraught may well have decayed due to the evaporative cooling, it is important to ask whether new updraughts can be produced − the regeneration problem. This time-dependent description has arisen from the need to interpret steady models.

As described by Betts (1976) the thermodynamic properties of the descending air are determined by competition between evaporative cooling and adiabatic warming. Indeed, observations indicate surface temperature changes varying from the large cooling for severe storms and squall lines over land, to a warming, particularly over the ocean (Zipser 1969). As an example of the latter, Mansfield (private communication) describes a splitting system in the GATE area which gave virtually no surface changes in thermal structure. Downdraught thermodynamics are discussed in Betts and Silva Dias (1979) with reference to the two processes of pseudo saturated (WA) and dry (DA) adiabatic descent. The WA process represents evaporation and cooling, whereas the DA process has no evaporation; in the latter case the air must be forced down (potential energy being gained rather than lost by such a descent) to produce warming. Thus both warm and cold downdraughts will be considered here, using forced and free motion. The necessity of discussing forced motion also indicates that another way of producing cold air near the surface is by forced ascent so that in the study of the downdraught problem, paradoxically, updraughts also need to be considered.

This paper will, therefore, present models of idealized two-dimensional up- and downdraughts. Such a consideration of individual draughts is conceptually distinct from the dynamical models of linked updraught/downdraught systems previously described by MG and MM, although these models are all encompassed in a general convection theory (Moncrieff (1979)). Various parametrizations of an entropy sink will be used to create and maintain the draughts. It will be shown that, despite the important time dependent aspects of the downdraught, the steady analytical approach can be applied to the downdraught dynamics and that comparison between numerical and analytical models proves invaluable for a complete description of the problem.
The generality of the downdraught dynamics to be discussed makes many of the concepts applicable to other related areas of fluid dynamics, such as sea-breeze circulations, density currents, and various environmental problems, such as natural gas spillage. Appropriate reference to these topics will be made later.

2. The steady dynamics of two-dimensional downdraughts

(a) Description of models and physical constraints

The downdraught dynamics will be studied using two models: a steady model giving analytic solutions and a numerical model. The two models will be compared as well as tested against laboratory and atmospheric data.

The numerical model, which is a two-dimensional version of that described in Miller (1978), will be used with zero moisture content; thus cell regeneration will not be simulated explicitly. The evaporative and drag properties of rain will be parametrized using a time-independent heat sink \( Q_c = Q_c(x,z) \) and the adjustment of a constant inflow \( u_0 \) to \( Q_c \) will be examined. A guide to the shape and amplitude of \( Q_c \) was determined by computing the evaporative cooling from the thunderstorm simulation described in Miller (1978).

The steady analytical theory of downdraughts, to be developed in this section, is similar to those described by MG and MM, the main distinction being in the application of boundary conditions. However, there are some important differences introduced to enable downdraughts to be modelled adequately. As in MG and MM the entropy sink is parametrized as:

\[ Q = w \gamma \]

where \( \gamma \) is a representative downdraught lapse rate of potential temperature (e.g. the wet adiabat).

The justification for this form and its relationship to the constant \( Q_c \) of the numerical simulations will be described in the next section and shown to be, a posteriori, a good model for heat exchange in downdraught flow. It should be noted, moreover, that if the numerical simulations reach a steady state, which seems likely due to the constancy of the forcing function, then the two parametrizations for \( Q \) can be made identical.

Provided that \( Q = w \gamma(z) \), the vorticity and energy equations can be written in conservative form. Although in this paper the lapse rates \( B \) and \( \gamma \) will be assumed constant, this is an unnecessary restriction in the analysis; however, it enables analytical solutions to be obtained, rather than numerical ones for the more general case. Realistic variations in \( B \) and \( \gamma \) will change the present results by degree rather than kind.

The constraints on two-dimensional incompressible flow can be written as the following equations for vorticity \( (\eta) \), energy, potential temperature and continuity of mass (see MG):

\begin{align}
\frac{D}{Dt} \left\{ \frac{\eta}{\rho} + \frac{g(\gamma-B)(z-z_0)}{\rho u_0} \right\} &= 0 \quad (1) \\
\frac{D}{Dt} \left\{ \frac{1}{2} \nu^2 + \frac{\delta p}{\rho} - \int_{z_0}^{z} g \delta \phi_d dz \right\} &= 0 \quad (2) \\
\frac{D}{Dt} \left\{ \delta \phi_p - (\gamma-B)(z-z_0) \right\} &= 0 \quad (3) \\
\nabla \cdot \nu &= 0 \quad (4)
\end{align}
where

\[ \delta p = \text{deviation of pressure from a hydrostatic reference state} \]

\[ \phi = \text{log-potential temperature} \]

\[ \phi_0(z) = \text{reference state} \phi \]

\[ \delta \phi_p = \phi - \phi_0 = \text{parcel deviation} \phi \]

\[ B = d\phi_0/dz = \text{static stability, taken to be constant} \]

\[ z_0 = \text{inflow height of a streamline} \]

\[ \rho = \text{constant density} \]

\[ \eta = \partial u/\partial z - \partial w/\partial x \]

and where subscript 0 indicates inflow.

To enable the steady models to be formulated it is important to establish the types of fully developed flow to be considered. Two flow configurations which are amenable to analytical solutions are particularly interesting, and it is believed that the essential dynamics of steady downdraughts will thereby be examined. These will be called the symmetric type, which has zero net horizontal mass flux, and the jump type, which has a constant ambient inflow. The two types are illustrated in Fig. 1, in which both single updraught and downdraught flows are shown. Two values of \( \gamma \) will be used to represent the two distinct physical situations of thermodynamic instability or stability of the lower atmosphere. The first having \( \gamma > B \) corresponds, for example, to the wet adiabatic process and ensures release of available potential energy. The second having \( \gamma < B \) requires a dynamical forcing, which is not explicitly described in the model, as potential energy is not available to the motion; the most physically realizable example of this is for a dry atmosphere for which \( \gamma = 0 \). This limit will be used in the later worked example, but formally it represents an unnecessary restriction.

Figure 1. Updraughts (U) and downdraughts (D) of the symmetric (S) and jump (J) types in which \( \theta' \) positive or negative gives warming (W) or cooling (C) respectively. \( \bar{H} \) is the vertical heat flux.
as long as $\gamma < B$.

The symmetric type is stationary relative to the surface, and for downdraughts there are two possibilities (notation described in Fig. 1): (i) DSW producing warm air and a negative heat flux, and (ii) DSC producing cold air and a positive flux. DSW can only be consistent with Eq. (3) for $\gamma < B$ and thus is a warm downdraught forced, for example, by liquid water loading. DSC, however, requires $\gamma > B$ and thus is dominated by evaporative cooling. For completeness the updraught equivalents are shown and USW is free convection ($\gamma > B$) and has positive heat flux. USC requires a forcing to make the air ascend ($\gamma < B$) and is rarely observed except, possibly, in convergence lines.

The jump type has an ambient inflow, $u_0$, at all levels upstream of the system and outflow, $u_1$, at all levels behind the system. Again, apparently there are the two possibilities for downdraughts: (i) DJW, a warm forced downdraught, and (ii) DJC, a cold downdraught. However, DJC can be shown to be dynamically inconsistent, and thus not realizable for the following reasons: Equation (1) indicates that for DJC, which has $(z-z_0) < 0$, $\delta \phi_p < 0$ and zero inflow shear, the outflow must have positive shear, $\eta_1 > 0$. However, the continuity equation for this downdraught indicates low level divergence and upper level convergence, which requires $u_1(z \approx 0) > u_0$ and $u_1(z \approx H) < u_0$ or negative outflow shear. Therefore, the two equations are inconsistent and DJC is impossible, given the assumptions of the analysis. Thus the only jump type of downdraught is the forced warm flow of DJW. It is interesting to note that the warm updraught jump type (UJW), using similar arguments to those above, is also dynamically impossible. Therefore, the forced and cold updraught, which can be observed as flow over hills or over gust fronts, is the only possible jump type of updraught. The two draughts DJW and UJC, both have negative heat flux.

In summary, six single draughts have been described arising from either wet or dry (and thus forced) thermodynamic processes. By way of illustration, the complete flow structure will be calculated for DSC and UJC in the following discussion of the steady theory. It may seem paradoxical that in a paper purporting to examine the downdraught problem that an updraught will be described in detail. However, it will become apparent from the discussion of the numerical model results that the initial stages of UJC are distinctly characterized by descent and, furthermore, represent a logical progression of behaviour from that of DSC. Also, the DJW structure can be seen to be trivially related to that of UJC.

These fluid dynamical flows can be related to observations of thunderstorm draughts. When the atmosphere has small pre-convection shear, the showers which form have an initial updraught in the growth phase, modelled by USW, followed by a downdraught in the decaying phase, modelled by DSC. For environmental wind profiles more typical of severe storm situations, it can be seen that low level inflow, such as characterizes the jump type, occurs for both mid-latitude steering level and propagating squall-line storms. Thus it is likely that UJC and DJW represent aspects of the draughts in severe storms.

Limitations of the idealized flows considered here make detailed comparison with atmospheric cases unprofitable, but considerable physical insight can be gained. Perhaps the most notable limitation of the analysis is that only the symmetric and jump situations have been considered, whereas atmospheric draughts are likely to be combinations of the two. Further discussion of this point is made in section 3(e), where the numerical model results are discussed.

(b) The mathematical formulation of the steady model

In this Lagrangian approach it is appropriate to replace Eq. (4) by the mass flux conservation

$$v \cdot ds = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$
where $s$ is the cross-section of a streamtube, and since the flow is two dimensional, to
define a streamfunction $u = \partial \psi / \partial z$ and $w = -\partial \psi / \partial x$ so that the $y$-vorticity is $\eta = \nabla^2 \psi$.

It can be shown, following the discussion of the previous section, that the energy and
vorticity equations for this simple case are

$$\frac{1}{2} v^2 + \delta p/\rho - \frac{1}{2} g (\gamma - B) (z - z_0)^2 = \frac{1}{2} u_0^2 \quad . \quad (6)$$
$$\nabla^2 \psi + g (\gamma - B) (z - z_0) u_0 = d u_0 / d z_0 \quad . \quad (7)$$

where $z_0(\psi)$ is the inflow height of a streamline $\psi = \text{constant}$, and $u_0(z_0)$ is the corresponding
inflow speed. If the potential energy release along each streamline is written as

$$\text{PE}(z, z_0) = \int_{z_0}^z g \delta \phi_p dz,$$

where $\delta \phi_p$ is the value of the log-potential temperature deviation
measured along $\psi = \text{constant}$, then in this case $\delta \phi_p = (\gamma - B) (z - z_0)$ and $\text{PE}(z, z_0) = \frac{1}{2} g (\gamma - B) (z - z_0)^2$ except at inflow stagnation points, Eq. (6) may be rewritten as

$$\left(v/u_0\right)^2 - 1 = -\delta p / (1/4 u_0^2) + \text{PE}(z, z_0) / (1/4 u_0^2) \quad . \quad (8)$$

In this form the energy equation shows the relationship between the flow response and
magnitude of the heat sink; the energy available from the heat sink is related to the inflow
kinetic energy through the last term in Eq. (8) (a form of 'parcel Froude number'), while the
work done by the pressure field is also normalized by the inflow kinetic energy. Thus the net
kinetic energy generation along each streamline, expressed on the LHS of Eq. (8) is a
balance between these two effects. Likewise, the rearrangement of Eq. (7) shows that the net
generation of vorticity

$$\left(\frac{\nabla^2 \psi}{d u_0 / d z_0}\right) - 1 = -\left\{ \frac{\text{PE}(z, z_0)}{d (1/4 u_0^2) / d z} \right\} / \{1/4 (z - z_0)\} \quad . \quad (9)$$

is also closely related to the bulk properties of the heat sink and the inflow.

(c) The symmetric type (DSC)

The possibility of the circulation schematically shown in Fig. 1 existing in steady-state
is now investigated, and it is useful to first consider the flow at large distances from the sink
where the flow is hydrostatic.

At sufficiently large distances from the sink, the flow is quasi-horizontal and Eq. (7)
becomes a differential equation for the displacements, since by Eq. (5) $u_0 dz_0 = -u_i dz_1$
where $v \sim u_i f$ and subscript 1 denotes outflow. Moreover, in this region the flow must be
hydrostatic, $\frac{\partial}{\partial z_1} \left( \frac{\delta p}{\rho} \right) \sim g \delta \phi_1$ so

$$\left( \frac{\delta p}{\rho} \right)_1 = p_* - \int_{z_1}^{h_0} g (\gamma - B) (z_1 - z_0) dz,$$

where $p_*$ is the value of $\delta p/\rho$ at $z = h_p$. Since, by definition, $\delta p = 0$ on inflow and pressure
must be continuous at $z = h_p$, clearly $p_* = 0$. Consequently Eqs. (5), (8) and (10) give the
displacement equation to be:

$$\left( \frac{dz_0}{dz_1} \right)^2 = 1 - \frac{g (\gamma - B)}{1/4 (u_0(z_0))^2} \left[ \int_{h_0}^{z_0} (z_1 - z_0) dz_0 \right] \quad (h_p \leq z_0 \leq H) \quad . \quad (11)$$

The boundary conditions on the system are as in Fig. 2.

The mathematical problem is, in fact, equivalent to that considered by MG, and it is
appropriate to seek solutions of Eq. (11) of the form where $dz_0 / dz_1 = -\beta$ (a constant).
Hence $z_0 = -\beta z_1 + \alpha$ where $\alpha$ is a constant. Using the boundary conditions at $z_0 = h_p$ and
$z_0 = H$ it is clear that $\alpha = H$ and $\beta = (H - h_D)/h_D$ so $z_0 - h_D = \beta(h_D - z_1)$ is a solution of Eq. (11) if and only if

$$u_0(z_0) = (z_0 - h_D)\left[\frac{g(\gamma - B)}{\beta(\beta - 1)}\right]^\frac{1}{1-\beta},$$

(12)

so $du_0/2_z = \left[\frac{g(\gamma - B)}{\beta(\beta - 1)}\right]^\frac{1}{1-\beta} = A$, a constant.

Defining $\beta(\beta - 1) = R_D = g(\gamma - B)/A^2$, precisely the same mathematical solution as MG is obtained. There is, however, an important physical distinction. In MG it was assumed that the inflow was of constant shear, so that the inflow vorticity was as prescribed by the large-scale flow. Here the problem is different in that the inflow is considered as having been developed as a response to the sink forcing, i.e. from an initial finite-amplitude sink. The vorticity generation in the inflow (and outflow) is thus affected by gravity waves during the development of the flow, of which the above is a time asymptotic solution. This is clear in the numerical solution of the time-dependent equations. The shape of the internal flow shows the distinction between this problem and that of MG (the initial value problem appropriate to the latter is that of small amplitude waves growing in a sheared unstably stratified flow). In the above, the flow is symmetric around the $z$ axis, whereas in MG it has a large phase tilt, as shown in Fig. 7 of Moncrieff (1978). Large-amplitude initialization must, therefore, be treated carefully, since the steady result is not necessarily the same as with small-amplitude initialization. Moreover, here there is no upper bound on the size of $\beta$ in this problem, since there is zero vorticity generation on the vertical axis.

The depth of the cold air outflow $h_D$ is one of the main internal parameters. Since $h_D = H/(1 + \beta)$

$$h_D/H = 2/[3 + (1 + 4R_D)^{1/2}],$$

(13)

If $R_D > 0$ then $\gamma > B$, $h_D/H < \frac{1}{2}$ and $\beta > 1$ so cold air of enhanced kinetic energy will be produced in the layer $0 \leq z_1 \leq h_D$: the outflow lapse rate of $\phi$ is $\gamma + \beta(\gamma - B)$. It is clear that these solutions correspond to the DSC (and DSW) type. However, because of the vertical symmetry, no net vorticity or momentum has been produced in either case. The outflow speed is given by

$$u_1(z_1) = \beta(h_D - z_1)\left[\frac{g(\gamma - B)\beta(\beta - 1)}{\beta(\beta - 1)}\right]^\frac{1}{1-\beta}, \quad 0 \leq z_1 \leq h_D$$

(14)

and the outflow shear is negative ($-\beta^2 A$) with the largest speeds at the surface.

Whilst the solutions have been discussed in terms of the parameter $R_D$ quoted in MG, as has already been stated this single-draught solution is physically quite distinct. It is, therefore, somewhat misleading to treat $R_D$ as a fundamental parameter of this problem as the sheared inflow develops as a response to the cooling and is not prescribed by the large-
scale flow. Thus the solutions are more clearly expressed in terms of the two non-dimensional parameters $B^+$ and $h_D^+$ defined by $B^+ = (\gamma - B) H$ and $h_D^+ = h_D / H$ so that $R_D = (1 - h_D^+)(1 - 2h_D^+)/3(h_D^+)^2$ Downdraught variables, such as inflow and outflow, can now be written in terms of $B^+$ and $h_D^+$ which are the determining input parameters.

Since the displacements and hence the inflow/outflow velocities are specified by the asymptotic solution, the remote streamfunction, $\psi$, can be calculated. With $\psi = 0$ along the vertical boundary at $x = 0$ and along the top and bottom, the vorticity equation, Eq. (7) was solved by the iterative over-relaxation procedure used in Moncrieff (1978). The perturbed temperature and streamfunction fields are shown in Fig. 3.

![Figure 3. Streamfunction, $\psi(H^3)$, and log potential temperature perturbation, $(\phi - \phi_0)(H(H - B))^{-1}$, for one-half of the flow in DSC, as deduced by the steady theory.](image)

(d) The jump type (UJC)

In contrast to the model in section 2(c), the problem now considered is where the flow relative to the sink is everywhere in the same sense, so the flow is never stationary with respect to the heat sink. With constant static stability and rigid boundaries at top and bottom, it is obvious that the maximum displacements of particles in the vertical must be, for a two-dimensional system, in the middle of the layer. This is in contrast to the three dimensional propagating régime considered by MM in which the maximum displacements, as in the flow of section 2(c), are experienced by particles originating at either boundary. The solutions sought here are of the jump type, where particles are all displaced in the same vertical sense.

Since UJC, as described earlier, is a forced updraught, the analysis will assume the extreme limit of $\gamma = 0$. The equations relevant to this type are the same as in the last section,
namely, Eqs. (4), (8) and (9). There is one important qualification, however, in that since the inflow and outflow are on opposite sides of the system, the value of the pressure at the reference level, taken to be at the bottom boundary, is arbitrary and has to be calculated as part of the solution. This is also the case in the analytic model in MM. It is not difficult to show that the energy equation for the system yields a displacement equation

\[
\left( \frac{dz_0}{dz_1} \right)^2 = 1 - \frac{1}{2u_0^2} \left\{ \frac{\Delta P}{\rho} - gB \int_0^{z_0} (z_1 - z_0) dz_0 \right\} . \tag{15}
\]

Likewise the vorticity equation is

\[
\nabla^2 \psi - gB(z - z_0)u_0 = du_0/dz_0 . \tag{16}
\]

Since the pressure does not appear in the vorticity equation, this equation is identical to the counterpart in section 2(c). Equations (15) and (16) clearly have an infinite number of solutions since \( \Delta P \), the pressure change across the system at \( z = 0 \), is not prescribed \textit{a priori}. The problem can be simplified considerably by looking for solutions for constant inflow, \( u_0 \). If this is considered, then Eqs. (15) and (16) are simplified and

\[
\left( \frac{d\zeta_0}{d\zeta_1} \right)^2 = (1 - E) + \frac{2}{F_A^2} \int_0^{\zeta_0} (\zeta_1 - \zeta_0) d\zeta_0 \tag{17}
\]

\[
\nabla^2 \psi - \frac{1}{F_A^2} (\zeta - \zeta_0) = 0 \tag{18}
\]

where \( F_A = (u_0/H)(gB)^{-1} \), \( E = \Delta P/\rho u_0^2 \), are a Froude number and a normalized pressure change respectively, and \( \zeta = z/H \), \( \psi \rightarrow \psi/u_0H \). In fact, Eq. (17) is effectively linear and on differentiation, since \( d\zeta_0/d\zeta_1 = u_0/u_0 \) can never be zero by definition, it is readily shown that

\[
\frac{d^2\zeta_0}{d\zeta_1^2} = \frac{(\zeta_1 - \zeta_0)}{F_A^2} . \tag{19}
\]

The boundary conditions, which distinguish this problem from that considered in MM, are

\[
\zeta_1 = 0 \quad \text{when} \quad \zeta_0 = 0 \tag{20}
\]

\[
\zeta_1 = 1 \quad \text{when} \quad \zeta_0 = 1 \tag{21}
\]

that is, the particles on the boundaries are not displaced in the vertical. Since Eq. (17) has been differentiated, the additional condition is

\[
d\zeta_0/d\zeta_1 = \pm \sqrt{(1 - E)} \quad \text{at} \quad \zeta_1 = 0 \tag{22}
\]

Since jump solutions are sought, \( d\zeta_0/d\zeta_1 > 0 \) so the positive value of the square root is taken; note that the negative root was appropriate in MM. Solutions to Eq. (19), satisfying the boundary conditions Eqs. (20) and (21) are readily shown to be

\[
\zeta_0 = \zeta_1 - \varepsilon F_A \sin(\zeta_1/F_A) , \tag{23}
\]

where \( \varepsilon = 1 - \sqrt{(1 - E)} \), \( F_A \) appears as an eigenvalue in this equation and the upper boundary condition is satisfied if, and only if,

\[
F_A = 1/n\pi \quad (n = 1, 2, \ldots) , \tag{24}
\]

(Notice that the previously discussed symmetric type can be considered to be characterized by \( F_A = 0 \), where the numerator of \( F_A \) is taken to be the vertical average of \( u \), over the interval \( 0 < z < H \), which is zero.)

An upper limit to \( \varepsilon \) must exist, as can be deduced by considering the outflow velocity,
which is given by:

\[ u_1 / u_0 = 1 - ε \cos(ζ_1/F_A) \]  

(25)

For no return outflow to exist, \( u_1/u_0 \geq 0 \), so that \( ε \cos(ζ_1/F_A) \leq 1 \) for each \( ζ_1/F_A \) in the range \([0, π]\) and, in particular, at \( ζ_1 = 0, ε \ll 1 \) and at \( ζ_1 = 1, ε \gg 1 \). Consequently the value of \( E \) must lie within the limits \(-3 \leq E \leq 1\) and, moreover, the displacements must be positive so that cold air must ascend, requiring \( ε > 0 \). It is readily shown that the x-momentum budget for the region is satisfied identically (a necessary condition in two dimensions) if \( F_A = 1/(πε) \). It is important that consideration be given to the momentum, since the horizontal momentum equation is not used explicitly in the model formulation; this is crucial in more complicated problems involving free surfaces (not considered here). The parameter \( E \) defines the amplitude of the jump solutions and is responsible for the finite amplitude nature of the system. It is analoguous to the parameter \( E \) in MM and to obtain a unique solution some form of closure hypothesis has to be invoked, either by relating to the appropriate initial-value problem for the system or using an energy closure, as in MM. (Taking, as an example, the same closure as used in MM of maximum mechanical efficiency, then \( E = 1 \) and stagnation occurs at \( ζ_1 = 0 \).) It is clear that the net outflow shear is positive, the low level momentum is decreased, while the upper level momentum is enhanced.

The deviation log-potential temperature is given by:

\[ δφ = -\{εBH/(πεπ)\} \sin(ππε/H) \]  

(26)

The maximum change to the thermodynamic structure is in midlevels where the displacements are a maximum.

In this case the internal flow shape can only be determined by specifying the mechanism which forces the air to rise and cool, in this example dry adiabatically; e.g. topography. This includes both orography (Long 1965), and the lifting of air over cold outflows, as will be described in the numerical model results. The fact that such lifting occurs by initially cooling the fluid is an important feature of the following discussion, and in the wider context of storm regeneration.

3. Numerical experiments

Although a substantial research field in convective modelling has developed, surprisingly little work has been done in applying numerical modelling techniques to the physically less complex problem under investigation in this paper. Mitchell and Hovermale (1977) carried out a numerical study concentrating on the behaviour of the density current head, including experiments in which the surface drag coefficient was varied. However, no attempt was made to introduce an ambient wind.

(a) The numerical model

The model is a two-dimensional version of that described in Miller (1978) with grid dimensions of 30 km by 720 mb, grid lengths of 500 m and 40 mb and a timestep of 15 s, but improved lateral boundary conditions are used (to be described in a later paper). Higher resolution runs have also been analysed and will be mentioned when appropriate. For the present series of numerical experiments the model was run with a dry atmosphere. For the scales of motion being investigated here, it can be argued that detailed turbulence representations and the effects of the lower boundary, particularly surface friction, might be important. However, for the experiments described, simple eddy viscosity terms were included, together with a surface drag coefficient formulation (\( τ(1000 \text{ mb}) = ρC_D|u|u, \), \( τ = 0 \) elsewhere and \( C_D = 2 \times 10^{-3} \)). The value for the viscosity coefficients, \( k \), was:
$k = 500 \text{ m}^2 \text{s}^{-1}$ in the horizontal, with suitable scaling in the vertical and when running with different grid resolutions. These viscosity terms primarily control numerical ‘roughness’ and the results are insensitive to realistic variations of $k$ (or $C_d$). As will be seen later, the numerical results agree well with observations and theory and are dominated by the macro-scale with advective flow determining the principal results.

For all experiments a typical (but smoothed) atmospheric temperature sounding was used with

$\begin{align*}
B = 0 & \quad 1000 > p > 900 \text{ mb} \\
B \approx 0.3 \times 10^{-5} \text{ m}^{-1} & \quad 900 > p > 800 \text{ mb} \\
B \approx 1.6 \times 10^{-5} \text{ m}^{-1} & \quad 800 > p > 300 \text{ mb} \\
B \approx 7.0 \times 10^{-5} \text{ m}^{-1} & \quad p < 300 \text{ mb}
\end{align*}$

A simple, time-independent heat sink was used to simulate the generation mechanism. The form for this sink was based on the shape and magnitude of the evaporative cooling calculated from a storm simulation with a substantial downdraught (Miller 1978). Thus

$\begin{align*}
Q = Q_c &= -0.008^\circ \text{C s}^{-1} \quad 1000 > p > 850 \text{ mb} \\
Q = Q_c(p-750)/100 & \quad 850 > p > 750 \text{ mb} \\
Q = 0 & \quad p < 750 \text{ mb}
\end{align*}$

These values were restricted to a region of width $L = 4 \text{ km}$, with $Q = 0$ elsewhere.

If the equations of motion are non-dimensionalized using a typical horizontal scale of $L$, vertical scale of $H$, and velocity scale of $u_0$, it is easy to show that the determining non-dimensional number for this situation is $F_N$, given by:

$$F_N = u_0(-gLHQ_c/ \theta)^{1/4}.$$  \hspace{1cm} (27)

Using typical values of $H \sim 2 \text{ km}$ and $L \sim 4 \text{ km}$, the value $F_N = 1$ represents an inflow of about $10 \text{ m s}^{-1}$. (It should be noted that the specification of $Q_c$ and $L/H$ in the numerical simulations is equivalent to prescribing $B^+$ and $h^+$ in the analytic model, although the exact correspondence is difficult to obtain and obscure.)

A series of numerical experiments was carried out simulating the adjustment of the fluid to the constant $Q$. Various values of unsheared ambient inflow were used and in each case the simulation was continued until quasi-steady flows were obtained everywhere except close to the propagating head(s), an intrinsically unsteady region unless considered as a separate problem with the frame of reference transformed to move at the speed of propagation of the head. Discussion of these results follows, commencing with a classification of the regions of interest.

(b) Steady flow behind the head

In Fig. 4 the downdraught flow is schematically represented in the limit case of zero inflow $u_0 = 0$ ($F_N = 0$). In this case the outflow is symmetric to the left and right, away from the heat sink. This figure shows the five rather distinct regions of the adjustment flow.

Region I: is where the fluid is cooled in the heat sink area and flows out near the surface in the cold outflow. Above this outflow continuity requires an inflow into the sink region. There are large horizontal gradients and after a few minutes the inflow, downdraught and outflow are quasi-steady.

Region II: is one of small horizontal gradient and is again quasi-steady. In Fig. 5(a), to be described later, a vertical profile of the flow in this region is shown.

Region III: is the area of the cold outflow front. The front (between cold and ambient air) is constantly propagating, at $c_s = c_s(Q)$ away from the heat sink. Thus the front region is non-stationary with respect to the fixed heat sink. In advance of the front the fluid
is forced to rise to give a secondary updraught.

Region IV: is the undisturbed fluid.

Region V: above the top of the sink and the outflow all the dynamical and thermodynamical variables decay with height. This decay is, however, not very rapid as there are quasi-stationary gravity waves (lee waves) in this region being forced from below by the heat sink and propagating heads. Such waves can, of course, be eliminated by making \( B = 0 \) in the region and this increases the intensity of the outflow as there are no gravity waves to radiate energy upward. Note that Regions I and II are considered in section 2.

For inflow \( u_0 \) less than \( c_s \), the adjustment outflow defines both upstream and downstream fronts. Naturally enough, the upstream outflow makes less progress away from the heat sink than the downstream front. The problem of the changes in these two parts of the outflow caused by varying the size of the inflow is one of the main themes of this section. The upstream front is particularly interesting because of the large convergence produced there.

Having discussed in general the flow configuration, the steady outflow of Region II will be considered in some detail. The vertical profiles of the outflow speed and temperature 10 km downstream of the heat sink are plotted in Fig. 5(a) and (b). (The outflow speed is the total flow minus the inflow \( u - u_0 \).) In Fig. 5(c) the profile of the deviation of the height field 10 km downstream from that 5 km upstream is shown. The profiles are for \( u_0 = 17 \text{ m s}^{-1} \) and 0 m s\(^{-1}\) and are in Region II (Fig. 4) after 42 minutes of the simulation. Consider the two wind profiles in turn:

(i) \( u_0 = 0 \text{ m s}^{-1} \). The shear (or vorticity) of the cold outflow is negative, in agreement with the steady theory of section 2(c). Above the mid-height of the sink there is an inflow, with smaller shear, which decays to small values above the top of the sink. The deviation at the height field, as defined above, is zero in this case as the overturning is symmetric.

(ii) \( u_0 = 17 \text{ m s}^{-1} \). In contrast, the shear is positive in the outflow and, in fact, the outflow near the surface is moving slower than the inflow. Thus this situation is of a completely different character from that of zero or 'small' mean inflow. Although a cold downstream outflow has been produced with regions of descending motion, an analysis of fluid trajectories through the system showed that after the early adjustment stages, which are dominated by cold descent in the region of the sink, most particle trajectories leave at higher levels than they entered, consistent with the type UJC. Trajectories are not shown as the vertical displacements are small. The deviation height field shows the pressure jump at
Figure 5.  (a) Outflow profiles 10 km downstream from the heat sink at $t = 42$ min for $u_0 = 0 \text{ m s}^{-1}$ (solid line) and $u_0 = 17 \text{ m s}^{-1}$ (pecked line). $u$ is the total flow, $u - u_0$ is the outflow modification.
(b) Potential temperature deviations in the same situation as (a).
(c) Profile of the deviation of the height field between 10 km downstream and 5 km upstream of the sink at 42 min.

the surface and pressure fall aloft, as deduced in the analytic model. The value of 9 m for $h'$ at the surface gives $E \approx 0.62$ (and $e \approx 0.38$). The changeover between the two types appears to be for $u_0 \approx c_*$. The depth of the cold air is greater for the large value of $u_0$.

(c) Propagation of the head

For zero inflow, $u_0 = 0$, the head moves at a constant speed, $c_*$, which can be related to parameters of the outflow by the equation

$$c_* \approx k(-g \Delta \bar{\theta} H_*/\bar{\theta})^\dagger$$

(28)
where the constant $k \approx 1.0$, $\Delta \theta$ is an average potential temperature deficit in the outflow, and $H_*$ is a measure of its depth. $(-\Delta \theta/\theta)$ replaces $\Delta \rho/\rho$ in the standard expression for density current propagation.) However, in the present atmospheric application a complication arises due to the strong vertical variation of $\Delta \theta$ and the consequent difficulty in defining $H_*$ and $\Delta \theta$; compare the laboratory situation in which $\rho$ is a constant in the density current. Thus the above expression requires a rather arbitrary averaging to give a reasonable fit with the model results. In the present model formulation it appears preferable to try to relate $c_*$ to the input data concerning the heat sink $Q$. A scale analysis of the equations of motion yields the following quantity as the characteristic velocity of the system, which it seems reasonable to equate to $c_*$:

$$c_* \approx k' (-g Q L H \bar{\theta})^{1/3}$$  \hspace{1cm} (29)

where $k' \approx 0.8$ and, for example, $c_* \approx 8 \text{ m s}^{-1}$ for the heat sink quoted in section 3(a).

For $u_0 > 0$ the symmetrical downdraught outflow is changed to one in which the upstream and downstream fronts move at different speeds. When the mean inflow speed becomes large, the establishment of the upstream propagating outflow becomes more difficult. A way of quantifying the effects of inflow on the upstream component is to consider the evolution of the maximum surface outflow speed for various values of inflow $u_0$. When $u_{\text{max}}$ becomes less than zero, an upstream component has developed. Figure 6 shows this evolution for eight values of the inflow speed.

For $10 \text{ m s}^{-1} < u_0 > 0 \text{ m s}^{-1}$, the time taken to establish an upstream component is a slowly increasing function of $u_0$. Also, in this range of $u_0$ there is a nearly constant final value of $u_{\text{max}}$ of about $14 \text{ m s}^{-1}$, which is attained in all cases in about 30 minutes. It is clear that $u_{\text{max}}$ is essentially independent of $u_0$ and only a function of $Q$. As $u_0$ increases, the evolution of $u_{\text{max}}$ becomes increasingly slow so that for $u_0 = 17 \text{ m s}^{-1}$ there is no upstream part even after 60 min of the simulation. This behaviour again appears indicative.
Figure 7. The surface convergence as a function of the inflow speed, in which isopleths at constant time elapsed (in min.) are shown. The dashed line is the envelope of maximum convergence during any simulation. Note that $u_0 < 0$ or $u_0 > 0$ represents the upstream or downstream front respectively.

of there being two distinct types of flow such that for $u_0 \leq 10 \text{ m s}^{-1}$ (or $F_N \leq 1$) an upstream component rapidly develops, while for $u_0 > 10 \text{ m s}^{-1}$ the existence of the upstream part is unlikely.

Most attention has been paid to the various propagation characteristics of the downdraught system. However, it is worth discussing the surface convergence produced at the fronts, which is the important variable for cumulonimbus cell regeneration. Figure 7 shows the surface convergence at the upstream front as a function of the inflow speed (where, for the purposes of Figs. 7 and 8 and Eq. (30), the upstream front is characterized by $u_0 < 0$, a device which permits a continuity in the graphs but has no physical significance). Also shown is the time taken for this value to be attained. It appears that an optimum value of $u_0$ between 9–12 m s$^{-1}$ produces a maximum convergence and for inflow speeds much greater than this the convergence falls off rapidly. This interesting feature probably defines the best conditions for regeneration to occur. The peak in convergence occurs because, as was noted for Fig. 6, the maximum surface outflow speed appears independent of $u_0$ until a critical value is reached and thus convergence must be a maximum for this critical value. After this point the maximum surface outflow rapidly decreases, as does the convergence.

The propagation speed of the upstream and downstream fronts, $c$, is plotted in Fig. 8 against the inflow speed. The graph appears to give a reasonable straight line fit between $c$ and $u_0$ over the range $-10 \text{ m s}^{-1} < u_0 < 10 \text{ m s}^{-1}$ and so the following empirical equation has been deduced:

$$c \approx c_* + 0.7u_0 \quad \quad \quad \quad (30)$$

There is a surprisingly large value of inflow speed needed to reduce the advance of the upstream front to zero, although this may be modified for a sheared inflow; this may have
important implications for thunderstorm regeneration in that the stationarity of the upstream gust front relative to the parent storm is crucial for the longevity of the storm.

Finally, in this sub-section the relationship of this work to similar studies will be discussed. Published work has concentrated more on the front properties than on the generation region, e.g. the numerical model of Mitchell and Hovermale (1977), the laboratory density current experiments of Simpson (1969) and the observational data of Charba (1974) and Miller and Betts (1977).

The most detailed data concerning the front motion are from the tank experiments described in Simpson (1969). The motion of the density current is given approximately by the formula \( c/c_* \approx 1 + 0.7u_0/c_* \), where \( u_0 \leq 0 \) again represents the upstream/downstream front (Simpson: private communication). Thus there is good agreement with the results obtained from the numerical model and this gives further credence to the hypothesis that laboratory density currents may be reasonable analogue systems to thunderstorm outflows. It would be desirable to obtain conclusive atmospheric data concerning this feature but, particularly for the upstream front, the data are difficult to obtain; however, the data that are available from Charba (1974) and Miller and Betts (1977) are in reasonable agreement.

It is apparent that in the present formulation the front region may have similar dynamics to those of the generation region previously discussed (in the case of UJC). For example, ambient air is forced to rise over the front and hence experiences adiabatic cooling, as did the air in the heat sink region for UJC. It is hoped to develop these ideas more fully.

(d) Flow structure

Figures 9 and 10 show the stream function for the two types. For the symmetric type, \( u_0 = 0 \), Fig. 9(b) and Fig. 3 can be compared and it is seen that there is good agreement between the numerical and analytical models for the steady parts of the flow, i.e. Regions I, II and IV. It should be noted that there is no counterpart to Region V in the analytical model. The gust front structure shown in Fig. 9(a) gives evidence of a vortex which appears to be rather similar to the cut-off vortex described by Simpson et al. (1977) for sea-breeze fronts.
Figure 9. (a) Streamfunction for $u_0 = 0 \text{ m s}^{-1}$ at $t = 18$ minutes. Vertical grid marking is 20 mb and horizontal grid marking is 500 m. Contours in units of $10^3 \text{ m}^2 \text{s}^{-1}$. (b) As in (a) at $t = 60$ minutes. (Position of heat sink is indicated by arrows.)

For the jump type, the flow is shown in Figs. 10(a) and (b) (the ambient inflow is from the right hand side) and it is clearly more complex than that for $u_0 = 0$. There is upward motion in the sink region and for a distance of about 10 km downstream there is positive vorticity in the outflow, as predicted by the analytical model. The vertical extent of the motion is much greater in this case than for the symmetric type. The motion which exists at distances greater than 10 km downstream is more typical of the front region with negative vorticity and this downstream front is constantly propagating away from the sink region. Thus again there is good qualitative agreement between the numerical and analytical models.

Both Figs. 9(a) and 10(b) show a lee-wave pattern emanating from the flow over the heat sink/fronts. In Fig. 9(a) the waves can be seen as a symmetric pattern whose vertical phase slopes are in the direction of the inflows, while in Fig. 10(b) the waves can be seen over
Figure 10. (a) Streamfunction for $u_0 = 17\, \text{m}\, \text{s}^{-1}$ at $t = 12\, \text{minutes}$. (b) As in (a) at $t = 60\, \text{minutes}$. (Note different location of heat sink compared with Fig. 9.)

the upstream front (coincident with the heat sink). Their wavelength is much greater for $u_0 = 17\, \text{m}\, \text{s}^{-1}$ consistent with lee-wave theory which predicts that the total wavelength of the waves is given by $L = 2\pi u (gB)^{-1/2}$, which for $u_0 = 0\, \text{m}\, \text{s}^{-1}$ gives $L \approx 3.5\, \text{km}$ (relative to the front $u \approx c_* \approx 8\, \text{m}\, \text{s}^{-1}$) and for $u_0 = 17\, \text{m}\, \text{s}^{-1}$ gives $L \approx 7.5\, \text{km}$ ($u \approx u_0 = 17\, \text{m}\, \text{s}^{-1}$ as the front is stationary with respect to the heat sink in this case). Such waves may have importance to aircraft flying over such cold outflows from thunderstorms.

In the model there is a continuous behaviour in the motion as the inflow speed is altered, and this can be described by consideration of the point A in Fig. 10(a). This point represents zero surface flow, and for $u_0 = 0$ it is positioned at the centre of the sink. As $u_0$ is increased, it remains at $x = 0$ until $F_n \approx 1$, when it moves gradually downstream with a speed proportional to $u_0$. Thus it appears that the upstream current, which for $F_n \lesssim 1$ propagates upstream, is being advected downstream for $F_n > 1$. The region to the right of
A has the characteristics of the steady outflow described in the analytical theory for the jump type, whilst to the left of A the front region is steadily propagating away and contains the more time-dependent aspects of the flow.

Figures 11(a) and (b) show the potential temperature fields for \( u_0 = 0 \text{ m s}^{-1} \) and \( 17 \text{ m s}^{-1} \). The obvious difference between the two types is the greater depth of the cold air for \( u_0 = 17 \text{ m s}^{-1} \), as pointed out earlier. The symmetric type has a well-defined drop of about \(-4 \text{ K}\) across the front.

(e) Intermediate types

Although the numerical results have been compared with the two types examined by the steady theory, it is also possible to examine the behaviour of the adjustment flow for other values of \( F_N \) or the inflow speed. The numerical simulation shows a continuous range of flow adjustment between \( F_N = 0 \) and \( F_N \gg 1 \), which has not yet been amenable to the
steady analysis, although this is theoretically possible. The behaviour can be grouped into the four classes shown in Fig. 12, and the flow structure will now be discussed in order of increasing $F_N$ (or $u_0$).

For $F_N = 0$ the symmetric downdraught is illustrated, and this type was described in the numerical model and in the steady analysis. As $F_N$ is increased there are two main effects – the first is the reduction in speed of the upstream propagating gust front and the

![Diagrams](image)

(a) SYMMETRIC TYPE, $F_N = 0$

(b) $0 < F_N < 0.4$

(c) $0.4 < F_N < 1.5$

(d) JUMP TYPE, $F_N > 1$

Figure 12. Schema of intermediate types of flow produced in the numerical simulations.

second is the change in the inflow above the downstream front. The response inflow at the rear of the system is eventually reversed to become an outflow for $F_N \geq 0.4$, but in terms of relative inflow (i.e. total flow minus $u_0$) there is little change over a wide range of values of $u_0$. This suggests the following functional form

$$u_* - u_0 = f(Q)$$

where $u_*$ is an average flow above the cold air on the downstream side of the sink. In fact the numerical results show that $f$ is a relatively weak function of $Q$ with the outflow adjusting to changes in such a way as to keep $u_* - u_0$ approximately constant, i.e. the depths of the cold outflow and response inflow vary to keep the mass flux balanced. The independence of $u_* - u_0$ on $u_0$ may be an important feature for cumulonimbus structure and motion.

For $F_N > 1$ the upstream front can make little progress and its stationarity in the sink region forces the inflow to rise there. This type of cold outflow was modelled analytically in the previous discussion. It is possible to model types (b) and (c) in Fig. 12, using the steady theory despite the complex nature of the inflows and outflows on both sides of the system; this is a more complicated problem of the free boundary type.
MODELS OF DOWNDRAUGHTS

4. Conclusions

This study of the response of a two-dimensional flow to a constant localized heat sink and an analysis of possible steady solutions has demonstrated the complex fluid dynamics of a physically simple problem. However, some simplifications are possible which allow the basic behaviour to be described.

The downdraught flow can be conveniently separated into a description of the steady outflow behind the gust front and of the head structure and propagation. The former appears to be well described by both the steady analytical theory and the numerical model, the head propagation has been treated numerically, while the head structure has been considered rather crudely. Non-dimensional numbers appropriate to the analytical and numerical models are $F_A$ and $F_N$ respectively; they are equivalent but, for example, $F_A = 0$ corresponds to $F_N = 0$ while $F_A = 1/(\pi n)$ corresponds to $F_N \gg 1$. This confusion is merely due to the different forms for the heat sink function in the two models and does not, it is felt, represent any important physical difference. In fact, the good agreement between the two models, despite distinctly different $Q$ parametrizations, is seen as evidence for the generality of the results obtained.

There are two types of steady outflow which have distinct dynamics. The first, for $F_A = F_N = 0$, results in downward motion in the generation region with sheared inflow aloft and a cold outflow with large shear. The system is symmetrical about a vertical axis through the centre of the heat sink, with an outflow depth predicted to be $2H/(3 + (1 + 4R_p)^2)$. The second, for $F_N \gg 1$ or $F_A = 1/\pi$, has, in contrast to the symmetric type, inflow at all levels downstream, and thus the outflow has an effective depth of nearly the total layer depth.

It is obviously important to compare the models of downdraughts described here with any relevant observational data. For example, Miller and Betts (1977) examined the structure of the downdraughts of travelling convective storms over Venezuela. It was found that the outflow had a linear increase with height of $\Delta \theta$ and negative vertical wind shear and, therefore, they compare favourably with the structure of the symmetric type of downdraught. However, for the VIMHEX storm there is an ambient inflow (which has variable vertical shear) and the downdraught is most likely one of the intermediate types discussed in the last section. It is clear that any more detailed comparison with data should be done using the results of more complex models, which include ambient inflow shear, variable stratification of $\theta$ to describe inversion conditions and three-dimensionality. The present results give further evidence for the existence of the mesoscale downdraught described by Zipser (1969) and Miller and Betts (1977). Figure 5(b) shows that above the heat sink, i.e. $p < 750 \text{ mb}$, there is a region of warming which must have arisen from descent driven purely dynamically, as moisture is not considered in this model (cf. the explanation of Zipser for the mesoscale downdraught, which involved the evaporation of rain water falling from the anvil).

Whilst little has been deduced about cumulonimbus maintenance and regeneration, much necessary basic understanding of downdraught dynamics has been described. The basic inflow $u_0$ represents the flow in the lower half of the troposphere approaching the region affected by precipitation evaporation and drag (parametrized as $Q$). In order for this to be realized, the precipitation zone must be moving relative to this low level air. This is indeed the case for storms that move with some middle level wind in a sheared environment, or for storms which propagate relative to the flow at all levels. As shown in section 3(c), the magnitudes of $u_0$ and $Q_c$ determine the maximum attainable boundary layer convergence; this, together with the boundary layer's thermodynamic characteristics, determine the possibility of new cell growth. Likewise, $u_0$ and $Q$ determine the time taken to attain a given value of convergence. These two predictive features will be studied further, together with the inclusion of variable wind shear, moisture and, ultimately, three-dimensionality.
In general, this will require a numerical approach. However, steady non-linear models have already proved invaluable in this present study and their extension, at least to include shear, is desirable.

ACKNOWLEDGMENTS

The authors would like to thank Dr D. K. Lilly for helpful comments and constructive criticisms on the work and Mr A. G. Seaton for providing valuable programming assistance with the numerical model and graphics.

REFERENCES


Moncrieff, M. W. and Green, J. S. A. 1980 A theory of organized steady convection and its transport properties (accepted by *Quart. J. R. Met. Soc.*).


