Tropospheric gravity waves: their detection by and influence on rawinsonde balloon data

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(Received 31 August 1979; revised 20 February 1980)

SUMMARY

In this paper we discuss the interaction of a tropospheric gravity wave, detected by a radar and a microbarograph array, with a normal rawinsonde balloon. By calculating the wave properties and comparing them to balloon data, we show that such data can provide information about tropospheric gravity waves. We also draw attention to the consequences of the wave-induced differences between the measured and the actual background state of the atmosphere. The differences may be critical, for example, in predicting clear air turbulence and in evaluating remote sensing instrumentation.

1. INTRODUCTION

In this paper we analyse the ability of a rawinsonde to detect a tropospheric gravity wave. We also examine the consequences of the presence of a gravity wave on the wind and temperature rawinsonde data which have numerous uses in meteorology.

The detection of atmospheric oscillations by balloons has been reported by a number of authors (see, for example, Murrow and Henry 1965, DeMandel and Scoggins 1967, Weinstein et al. 1966, Massman 1978). An extensive review of the literature for constant-volume balloons can be found in Tatom and King (1976), and a discussion on wind finding systems in general and dropsondes in particular is given in the winter 1978–1979 issue of Atmospheric Technology. In these case studies, however, no independent measurements of the atmospheric periodic motions were available so that the accuracy with which sondes detect wave motions could not be ascertained.

Lately, developments in remote sensing instrumentation have allowed direct, continuous measurement of velocities (Van Zandt et al. 1978) and temperatures (Decker et al. 1978) in the troposphere. The velocity measurements, obtained from radars with wavelengths of the order of meters, quite often reveal periodic fluctuations over considerable height ranges and for long periods of time. Similar fluctuations are observed (Decker and Westwater, private communication) in measurements of atmospheric brightness temperatures.

These new radar measurements allow one to examine in much finer detail how the fluctuations in the horizontal and vertical velocity and in the temperature fields associated with a gravity wave would affect the motion of the rawinsonde balloon and the data that it reports back. Such an analysis can be used in the following ways: (i) to determine whether the routine rawinsonde data can be used with the recently developed analytical techniques for the study of tropospheric mesoscale waves; (ii) to better gauge the ability of the rawinsonde data to specify the status of the atmosphere; (iii) to improve the evaluation of Doppler radar and radiometric technique performance.

To this end we have examined an observed wave event that took place during a rawinsonde ascent and for which microbarograph measurements at the ground and continuous Doppler radar measurements throughout the troposphere were available. In section 2 we

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specify the atmospheric background flow from the balloon data and, through a stability analysis, we calculate the properties of the waves that the background flow can support. In section 3 we review the capabilities of rawinsonde systems to detect such wave fluctuations. We conclude that of the various measured quantities, the rate of ascent is the one best suited for revealing the presence of a gravity wave. The balloon measurements and the gravity wave vertical velocities are compared in section 4 with very good results. In section 5 we put forth some recommendations for better utilization of rawinsonde or jimsonde data in mesoscale research and for avoidance of possible misinterpretation of their accuracy.

2. Determination of Gravity Wave Characteristics

On 15 and 16 April 1976 a wave event was detected and measured by the VHF pulsed Doppler radar (Van Zandt et al. 1978), operated by the Aeronomy Laboratory of NOAA and located near Sunset, 55 km NW of Denver, Colorado and by a microbarograph array operated near Boulder, Colorado, about 55 km NNW of Denver and 25 km ENE of Sunset. The radar can sample sixteen 1-km range gates 1-km apart and measures the velocity of the atmosphere in the direction of the radar beam. The wave event was analysed by

![Figure 1. Smoothed profiles of temperature $T$ (dashed), wind speed $V$ (solid), and Richardson number $Ri$ (dot-dashed), obtained from the rawinsonde data of 00Z 16 April 1976 from Denver, Colorado (altitude 1611 m above MSL). Only values of the Richardson number less than 0.6 have been plotted. The wind has been projected in the direction of wave propagation (170° from north). In the height range between 11 900 m and 13 000 m, indicated by M.D., the balloon data were unreliable and the values shown have been obtained by interpolation. The critical levels of the gravity wave have been indicated by $C_{L1}$ and $C_{L2}$.](image-url)
Grant (1979) and Van Zandt et al. (1979) using the temperature data from the 2302 GMT of 15 April 1976 balloon sounding from Denver and the velocity data from the radar.

We have re-examined this event using the data for the velocity (as well as the temperature) from the rawinsonde ascent, rather than from the radar, to describe the background state of the atmosphere. The velocity and temperature data were smoothed using a 7-point triangular weighting function. The velocity values between 11 900 m and 13 000 m above mean sea level (MSL), where the inferred balloon wind data were unreliable because the elevation angle was less than 6°, were obtained by interpolation using a quintic polynomial spline. Figure 1 shows the profiles of the resulting wind velocity V (projected in the reported direction of propagation of the wave, 170° from north), the temperature T and the corresponding Richardson number Ri.

The stability analysis of this background against infinitesimal perturbation is carried out by the technique of Lalas and Einaudi (1976). For the reported horizontal wavelength of 10 524 m (Grant 1979) we calculate the phase velocity $v_{ph}$ and the growth rate $\omega_f$ of the wave to be 25.9 m s$^{-1}$ and 1.035 $\times$ 10$^{-3}$ s$^{-1}$, respectively. These values are in very good agreement with 27.3 m s$^{-1}$ and 1.087 $\times$ 10$^{-3}$ s$^{-1}$ that Grant (1979) calculated at Sunset. The calculated wave-induced perturbations in the horizontal and vertical velocities and in the temperature at launch time are shown in Fig. 2. The critical levels for this wave i.e. the heights at which

![Figure 2](image-url)  

Figure 2. The instantaneous amplitudes, (2a), and relative phases $\phi_{rel}$, (2b), of the perturbations due to the gravity wave at the time of the balloon launch (2302 GMT, 15 April 1976): horizontal velocity $u_i$ (solid), vertical velocity $w_i$ (dot-dashed), and temperature $T_i$ (dashed).
\( v_{ph} = V \) are at about 5730 m and 12800 m above MSL. The second critical level falls near the region of missing velocity data, and so some ambiguity about its exact location is expected. This uncertainty also casts doubt on the accuracy of the eigenvectors around that height, particularly on the accuracy of their phase since rapid changes are known to take place near the critical layer.

3. A brief description of relevant rawinsonde balloon capabilities

The accuracy with which the rawinsonde balloon measures the various atmospheric parameters is fairly well known (Vockeroth 1975). For example, Sanders and Barr (1978) report that rawinsondes are able to provide the horizontal wind to within 0.5 to 1.5 m s\(^{-1}\) below 10000 ft, whereas above 60 000 ft, the magnitude uncertainty increases to 1.0 to 8.0 m s\(^{-1}\). Danielson and Duquet (1967) in comparing the regular GMD rawinsonde with simultaneous ascents tracked by an FPS-16 radar report similar conclusions, although they point out that if measurements are made 10 times a minute, the mean error can be reduced to \( \pm 1 \) m s\(^{-1}\). Yet local errors can become as large as 10 m s\(^{-1}\). They also point out that these errors are caused by mesoscale oscillations that may even place the apparent jet maximum as much as 1 500 m away from its true height.

By comparing the two sets of data, Danielson and Duquet (1967) conclude that the rate of ascent measurements, averaged over 20 to 40 s, are much more accurate. This relative accuracy of the vertical velocity measurement of the balloon is used by Corby (1957) to investigate lee waves in England. He calculates that the ascent rate of a balloon can be determined to within 0.6 m s\(^{-1}\) even when only the pressure and temperature values are known every minute. In general, with an accuracy of 1 mb for the pressure sensor and with an average ascent rate of about 5 m s\(^{-1}\), one should expect to measure vertical ascent rate averaged over a minute to within 0.2 m s\(^{-1}\) in lower heights and to within double that value around the tropopause. When transponder adjuncts are employed, however, the accuracy increases in the near range and reaches the previously mentioned value of 0.2 m s\(^{-1}\). Finally, the radiosonde temperature sensor measures temperature that can be read after telemetering to the nearest 0.2°C. Given these uncertainties in the measurements of horizontal velocity, vertical ascent rate, and temperature, the question arises as to how well can one discern the wave and subtract its perturbations to obtain the 'true,' i.e. the unperturbed, background state of the atmosphere.

The horizontal wave velocity, whose typical magnitude is of the order of a few metres per second has to be obtained by subtracting the mean background wind from the measured total wind. The usual total horizontal wind is estimated in the presently operational algorithm (Parry 1969) by subtracting two balloon fixes two minutes apart (below 14 km) and four minutes apart (above 14 km) so that a bias due to the wave fluctuation may be already present in the raw data. In addition, the background wind is not known a priori and has to be obtained by smoothing the total wind profile, which introduces additional errors, especially when the wind changes drastically with height as is the case here. Small ambiguities in determining the total or the background wind, which are of the same order of magnitude, result in large percent errors for the horizontal wave velocity.

The temperature fluctuation which is of the order of a degree or two has to be obtained in a like fashion and is susceptible to similarly large errors.

The wave-induced vertical velocity that corresponds to a horizontal one of 5 m s\(^{-1}\) is of the order of 1 m s\(^{-1}\), whereas the background vertical velocity is approximately zero, at least in a statically stable atmosphere, which is the only one of interest here. Of course, the balloon itself moves with a vertical velocity that is of the order of 1000 ft min\(^{-1}\) (about 5.0 m s\(^{-1}\)), but this velocity remains fairly constant, with smooth changes that are associated
with density variations such as low-level inversions, the tropopause, etc. Thus, variations of the order of 1 m s\(^{-1}\) are clearly measurable and should be easily visible since they only compete with relatively smooth changes of density, the locations of which are known from the background temperature profile. Therefore, in the following section we will concentrate mainly on the ascent rate data.

4. **Comparison of ascent rate variations with wave vertical velocities**

The analysis of the balloon data and their dependence on the wave structure would be facilitated considerably if the equations of motion of the balloon could be integrated, which would be possible if the balloon were a superpressure, constant-volume sphere. (We will give an example of such a case.) The actual rawinsonde balloons, however, are not superpressurized, and so they do not retain their spherical shape but become oblate because of stagnation point effects. This deviation from sphericity increases the drag coefficient \(C_D\) for the balloon from the value (for Reynolds number \(Re = 10^5\)) of about 0.5 appropriate to a sphere, towards the limiting value of 1.2 for a semi-sphere moving with the flat surface upstream (at the same \(Re\)). In addition, the weight of the instrument package results in some stretching of the bottom part, so that a shape with a conical afterbody develops, which is known to have reduced drag. Finally, the determination of \(C_D\) is complicated by the fact that rawinsonde balloon motion in the atmosphere lies in a flow regime with Reynolds number close to the critical value of 2.5 \(\times\) \(10^5\) near which drastic changes of the drag coefficient (for a sphere at least) take place in response to small changes of the flow properties (Fichtl et al. 1972). It is not known to what extent this abrupt behaviour (which is due to the details of the boundary layer separation) also holds for the nearly spherical balloon or whether additional variability in \(C_D\) exists because of freestream turbulence and unsteadiness (Hoerner 1965).

All these complications are compounded by the dependence of the drag on the effective radius of the balloon. The radius of the balloon could be determined approximately if the temperature inside were known. However, the relatively high vertical ascent speed and the usual lapse rate of the atmosphere combine to maintain an unknown temperature differential between the ambient atmosphere and the gas inside the balloon. For typical ascent rates, the ambient temperature will drop 10°C in three minutes. A simple lumped capacity heat transfer calculation (assuming no convective motion inside the balloon) gives an e-folding time for thermal equilibrium, from an initial temperature difference of 10°C, of 16 min, in which time the balloon would have moved to an even cooler region. The actual time, although of the same order of magnitude, will be smaller because convective motion inside will result in quicker heat transfer to the centre of the balloon.

Since the method of integrating the equations of motion of the balloon is compromised by our inability to obtain a good estimate of the vertical velocity of the balloon, a modified calculation is carried out to couple the observed balloon motion with the gravity wave.

The vertical position \(z(t)\) of the balloon is obtained by interpolation from its height, reported every minute. The horizontal position is calculated by integration in time of the kinematic equation of motion

\[
\frac{dx}{dt} = u(z,t) = V\{z(t)\} + u_1\{z(t), x(t), t; \phi_0\}
\]

where \(V\) is the background velocity, \(u_1\) is the horizontal component of the wave-induced velocity, and \(\phi_0\) is the initial phase of the wave. This equation, of course, implies that the balloon is carried horizontally with the local wind and that there is no hysteresis. To test this assumption, an integration of the dynamical equation of motion, with all terms included, has been carried out and has revealed that a balloon adjusts itself to a change of velocity of 1 m s\(^{-1}\) within 0.5 s. Since we are interested in 1-minute averages and since the uncertainty of
the actual value of $V$ at a particular height is certain to produce larger errors, we consider the assumption above acceptable. To test for the effect of the uncertainty in the background velocity profile $V(z)$, the integration of (1) has been carried out without the second term $u_1$. No significant difference has been found.

With the horizontal position as a function of height (or time) thus determined, the wave-induced vertical velocity $w_1$ is calculated at every instant of the balloon trajectory. Then its average value for each minute is calculated and is compared to the observed ascent rate of the balloon. The comparison is shown in Fig. 3.

![Diagram](image)

Figure 3 (a). The observed balloon ascent rate $w_{ob}$ and the calculated wave-induced vertical velocity $w_1$ along the balloon trajectory. Also shown on the right is the number of minutes after launch that were required for the balloon to reach the corresponding height.

(b, c) The temperature fluctuation $T_1$ and the horizontal wave-induced velocity $u_1$ along the balloon trajectory. The critical levels of the wave are also marked by $CL_1$ and $CL_2$.

Since the wave magnitudes depend on the initial phase $\phi_0$ (i.e. the phase at launch time) of the wave, which is not known, the calculations are repeated for many values of $\phi_0$. The value of $\phi_0$ is chosen to maximize the correlation with the observed ascent rate. This dependence on $\phi_0$ is shown in Fig. 4. In calculating the correlation (and in subsequent statistical calculations), the first two minutes of the ascent, when the balloon is still accelerating toward its terminal velocity and simultaneously going through the existing low-level inversion, are discarded. Similarly, data after the 40th minute, when the balloon reaches 13 km MSL and the wave amplitudes are negligible, are discarded.

The visual comparison of the ascent rate variations and the vertical wave fluctuations is good, except around the tropopause. There, the observed rate of ascent lags the wave
vertical velocity by an amount that becomes progressively larger. This can be attributed to the presence of the tropopause, which slows down the balloon by an amount that is comparable to that due to the wave, as well as to the uncertainty of the data above 11900 m MSL, which will affect the apparent local vertical wavelength and hence the apparent phase of the gravity wave. The vertical wavelength is very sensitive to the location of the critical level which in this upper part of the jet stream is uncertain because of the missing velocity data.

The two most prominent features of the ascent rate variation, namely the sharp increase of 1·4 m s⁻¹ at 6640 m and of 1·1 m s⁻¹ at 9150 m above MSL, coincide rather well with the maxima of the wave vertical velocity. The magnitude of these ascent rate changes is such that it cannot be explained as an observational error.

To test whether background temperature variations may primarily cause these ascent rate changes, the dynamical equations of motion with all the pertinent terms included have been integrated forward in time from two minutes before each peak. The drag coefficient is adjusted so that the ascent rate matches the actual one for the first minute, and then it is kept the same for the next three minutes. The code used is based on the one developed by Tatom and King (1976) for constant volume balloons but is modified to account for variations in the balloon diameter and for the presence of a propagating gravity wave. The results of these calculations show that only an increase of approximately 0·15 m s⁻¹ (that is less than 20%) can be explained by the variation of the background temperature. This increase appears to be virtually independent of the manner in which the temperature inside the balloon changes.

To further ascertain the correlation between the balloon motion and the wave, a series of statistical tests were carried out, the details of which are given in the Appendix. The outcome of the tests allows us with confidence to consider that the wave is responsible for the large observed changes of the balloon ascent rate.
Additional improvements to these statistics as well as to the visual comparison of Fig. 3 could be achieved by altering the background profile in the region where the data are missing, and by assuming different wavelengths for the wave. This is not the purpose of the present work. Rather, we seek to establish that a straightforward examination of the balloon data can produce strong evidence for the existence of gravity waves.

5. Discussion

We have been able to show that gravity wave induced vertical motions are felt and can be discerned in the variations of the ascent rates of rawinsonde balloons. The effect of these waves on the horizontal wind and temperature measurements that the rawinsonde balloons provide, although more difficult to isolate, can be substantial as shown for this well-documented case in Fig. 3.

We will discuss below some of the implications of these effects.

(a) Since the wind and temperature data from these ascents are usually considered representative of the condition of the atmosphere, errors, other than those due to observational limitations, may be introduced. When these data are used as benchmarks for comparison, as for example in evaluating the performance of radiometers (Decker et al. 1978) for temperature measurements or radars (Gage and Balsley 1978) for wind fields, caution should be exercised, especially in selecting the integration time. Discrepancies between the various sets of data can be easily explained in terms of gravity wave fluctuations rather than instrumental inadequacies.

(b) The direct effect of the waves on the inferred values of the Richardson number Ri in the atmosphere should be emphasized. Quite often wave activity has been observed (by pressure measurements on the ground, wavy behaviour of clouds, repeated turbulent patches, etc.) without the simultaneous presence of regions with Ri, obtained from rawinsonde data, less than 1/4. This discrepancy is, of course, in part due to the relatively large height between data points. But, in addition, large changes in the value of Ri derive from the wave fluctuation. In the case presented, the fluctuation of the temperature field due to the wave can be as large as 1°C over 200 m, as shown in Fig. 3c. If the fluctuation were subtracted from the measured temperature, the inferred Ri value would be substantially different, even though the amplitude of the wave of 16 April 1976 is not unusually large.

(c) One of the motivations of this work has been the desire to devise a method for determining the climatology of tropospheric, as opposed to boundary layer, gravity waves. Although some evidence of gravity waves generated in the jet stream has accumulated over the years (Bull and Neisser 1976, Mastrantonio et al. 1976, Shapiro 1978), information on the frequency of occurrence, amplitude, and vertical structure is still desperately needed to fully evaluate their role in tropospheric processes. The experience acquired in this comparative study points out that likely occurrences can be quickly identified by the presence of large changes in ascent rates, both positive and negative, that are not associated with prominent temperature changes. Additional information would be available if higher acquisition rates (for example, every 1/10 minute, a rate that the standard equipment is capable of achieving without modification) were used.

Finally, detailed information on the wave properties can be obtained from constant-volume balloons ascents (jimsonde/jimspheres). Jimsondes are known to be sensitive to gravity wave motions (Tatom and King 1976, DeMandel and Scoggins 1967, Massman 1978, Weinstein et al. 1966). In addition, since the ir radius is fixed, their motion is amenable to analytical calculations by integration of the dynamic equation of motion. For example, in Fig. 5 we present the ascent rate of a jimsphere through the atmosphere that existed over Denver at 00 GMT, 16 April 1976, with and without the presence of the wave. The difference is striking and instructive in analysing the wave.
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Figure 5. The calculated ascent rates $v_z$ of a spherical helium-filled constant volume balloon (jimsphere with rough surface) with the 60 GMT, 16 April 1976 gravity wave present (dashed) and absent (solid). These ascent rates are minute averages calculated by integration of the dynamical equation of motion. The balloon is assumed to be made of mylar with a diameter $D = 2.0$ m. The drag coefficient $C_D$ is taken to be 0.72 for Reynolds number $Re > 2.5 \times 10^4$, 0.6 for $2.5 \times 10^4 > Re > 1.2 \times 10^5$, and 0.42 for $Re < 1.2 \times 10^5$. The heights at which $C_D$ changed regimes are indicated by $Rec_1$ and $Rec_2$. The vertical wave-induced velocity that the balloon encounters during its motion is also shown, with the wave critical levels marked by $CL_1$ and $CL_2$.

ACKNOWLEDGMENTS

We would like to thank Dr. J. Grant for providing helpful details of the Sunset radar observations. The financial assistance of the Cooperative Institute for Research in Environmental Sciences, University of Colorado/NOAA to one of us (D. P. L.) is gratefully acknowledged. Part of the calculations were carried out with the aid of a computing grant from the National Center for Atmospheric Research, which is sponsored by the National Science Foundation.

APPENDIX

A set of statistical tests to establish whether the apparent agreement in ascent rates and wave vertical velocities is accidental or significant was conducted. First, a simple Spearman rank correlation test was run with the following results: for 37 degrees of freedom, the value of the z-test for measurements is 2.58, which makes the correlation significant to the 99% level. Then, a linear multiple regression analysis of the observed ascent rate $w_{ob}$ was carried out with the wave vertical velocity $w_i$ and the background density $\rho_a$ as the independent variables. With the balloon rising at some constant velocity $w_{ob}$, the equation of motion in the $z$-direction simplifies to

$$g V_b(\rho_a - \rho_b) - g M_p = C_D \frac{1}{2} \rho_a w_{ob}^2 A$$

where $g$ is the gravitational acceleration, $V_b = \frac{4}{3} \pi R^3$ is the volume of the balloon, assumed spherical, with $R$ its radius, and $A = \pi R^2$ its frontal area, $C_D$ is the drag coefficient, $\rho_b$ is the density of the gas inside the balloon, and $M_p$ is the weight of the balloon and its payload. Since $M_b$, the mass of the gas inside the balloon, is fixed, $M_b = \rho_b V_b$. If $T_b = T_a$ (i.e. if there is no temperature difference inside and outside the balloon), then $\rho_a = \rho_b \alpha$ where $\alpha$ is the ratio of the molecular weights of the air and the balloon gas. Then the equation can be rewritten as
\[ w_{\text{ob}}^2 = K \rho_a^{-4/3} / C_D \]  

(A2)

where \( K = \text{constant} = 2(g(z-1) M_b - g M_p) (3 M_b z / 4\pi)^{-4/3} / \pi. \) Since we do not know the variation of \( C_D \) with \( \rho_a \) or with the radius, which itself is a function of \( \rho_a \), we assumed \( w_{\text{ob}} \) to be a function of \( (\rho_a)^{-n} \). For values of \( n \) larger than 0-5 the results changed very little, so the value \( n = 2 \) was adopted since it produced the highest correlation of \(-0.490\) with the data. The multiple regression of \( w_{\text{ob}} \) with \( \rho_a^{-2} \) and \( w_1 \) explained 36% of the variance with an overall \( F \)-value of 11.18 and partial \( F \)-values of 10.19 for \( \rho_a^{-2} \) and 6.25 for \( w_1 \), all of which correspond to a probability of significance which is higher than 99%.

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