Mesoscale energy generated in the boundary layer

By J. S. A. GREEN and G. A. DALU

Atmospheric Physics Group,
Imperial College, London

Istituto di Fisica dell'Atmosfera,
CNR, Rome

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SUMMARY

We examine the kinetic energy that may be generated from surface heating. Cumulonimbus, cumulus, mesoscale and large-scale conversion of potential into kinetic energy can be identified in an idealized way, as distinct processes. Each has a characteristic thermodynamic efficiency.

1. INTRODUCTION

We examine the flow of potential and kinetic energy between different scales of motion. We suppose that the potential energy is produced by heating the atmosphere at the earth’s surface. Part is converted into kinetic energy on the scale of shallow and of deep convection, while some is available for driving a mesoscale circulation. This latter process is examined in some detail using a mesoscale model in Dalu and Green (1980).

The atmosphere is treated as dry and the Boussinesq approximation is made. The formulation of available potential energy in terms of ρgz (Green 1970) is used because it is valid whether the atmosphere is statically stable or unstable.

2. ENERGETICS OF ORDINARY CONVECTION

Suppose a quantity of heat δq is injected at the surface into unit area of an isentropic layer of potential temperature θ₀ and depth h. If it heats a parcel of mass m at constant pressure, its temperature increases by δθ where mc_pδθ = δq. The system has acquired potential energy which is available for conversion into kinetic energy by suitable rearrangement of the fluid. Thus if the parcel were to be exchanged with a parcel of the same potential volume at height h, then the change in the potential energy of the system would be

$$\text{CAPE} = \frac{mgh\delta \theta}{\theta_0} = g\delta q/c_p\theta_0$$

which defines the Convective Available Potential Energy CAPE. There are a number of reasons why this potential would not be realized. Hydrodynamical constraints such as the difficulty in removing fluid from near an impervious boundary, balancing of internal dynamical constraints, and various aspects of the energy cascade may prevent the attainment of the optimum displacements. Energy is exported by advection and by the propagation of energy by internal and external gravity waves. Energy may be dissipated. Ideally, however, the efficiency of conversion of thermal into mechanical energy is

$$\text{CAPE}/\delta q = gh/c_p\theta_0 = h/H_0$$

where $H_0 = c_p\theta_0/g$ is the depth of an isentropic atmosphere. Remarkably, neither the energy nor the efficiency depends explicitly on m.

(a) Cumulonimbus mode

The maximum possible efficiency of unity is attained by ascent of the parcel to the upper limit of the isentropic atmosphere $h = H_0$. In this pathological case the pressure
vanishes at the finite height $H_0$. Thus the absolute temperature of the parcel vanishes at the top of the atmosphere so no thermal energy remains to be converted into mechanical.

More realistically, the height $h$ is determined by other considerations. In cumulonimbus convection the ascent of warm air can often be described rather well in terms of a constant temperature excess over the environmental air, but only up to a level near the tropopause. Thus the isentropic layer can be used as a model for cumulonimbus, in spite of the fact that the ascending air in cumulonimbus calls upon the latent heat to achieve buoyancy in an atmosphere whose potential temperature increases upwards. The appropriate value of $h$ is determined by the height of the tropopause, and the theoretical efficiency is, at $h/H_0 \sim (10\,\text{km})/(30\,\text{km})$, quite large. Comparatively little of the CAPE converted into kinetic energy is dissipated on the scale of the cumulonimbus, as shown in the numerical studies of Moncrieff and Miller (1976) and the observational study of Green, Ludlam and McIlvenn (1966). The former, however, did not calculate the efficiency and the latter assumed it to be perfect and showed that this assumption was consistent with their observations.

(b) Cumulus mode

If the ascent is unmixed, but through an environment whose potential temperature increases steadily with height, then the efficiency of conversion is roughly halved (since the buoyancy decreases to zero at the upper level). Moreover, the height of the ascent becomes dependent on the initial temperature excess, and through it, on the ratio $q/m$.

More realistically, when there is convection of small scale we find an isentropic layer capped by an inversion whose height is influenced by larger-scale processes, such as subsidence. Convection maintains the isentropic nature of the layer by delivering the energy to different heights. Thus the surface heat input is shared among many parcels, each of which ascends to a different height up to $z = h$, to make the divergence of the flux of thermal energy independent of height in the convective layer. For this 'cumulus' mode the available potential energy is therefore

$$\text{CAPE} = \int_0^h \frac{g(h-z)}{c_p \theta_0} \frac{\delta q}{h} \frac{dz}{h} = \frac{gh}{2c_p \theta_0} \delta q \quad . \quad . \quad . \quad (2)$$

We suppose that this CAPE is converted into kinetic energy, then a fraction $r$ of that kinetic energy is subsequently converted back into potential energy by entraining potentially warm air from above the inversion as visualised, for example, by Ball (1960).

(c) Development of a well-mixed convective boundary layer

Referring to Fig. 1 for illustration of the symbols, for an isentropic layer of depth $h$ potential temperature $\theta$, capped by an inversion where the potential temperature increases by $\Delta$, we have:

\[\frac{1}{2}h_1 \theta - \frac{1}{2}h_2 \Delta = q/\rho c_p \quad (\text{total thermal energy gain})\]
\[\theta = \gamma h_1, \quad \Delta = \gamma h_2 \quad (\text{continuity of potential temperature})\]
\[\text{and} \quad \delta q r gh/2c_p \theta_0 = \delta h \rho g h \Delta/2 \theta_0 \quad (\text{to relate the extension of the layer} \delta h \text{to the kinetic energy reconverted into potential})\].

Eliminating $\theta$ and $\Delta$ gives:

\[\frac{1}{2}\gamma (h_1^2 - h_2^2) = q/\rho c_p \quad \text{and} \quad \gamma h_2 \delta h = r \delta q \rho / c_p\]

Putting $h_1 = h - h_2$ and eliminating $h_2$ gives a differential equation for $h$ as a function of $q$:

\[r \delta q/\delta h + q/h = \frac{1}{2} \rho c_p \gamma h\quad . \quad . \quad . \quad (3)\]
which has the solution $h^2 = 2(2r + 1)q/\rho c_p^2$.

We note for reference the expressions:

$$h_2/h = r/(1+2r) \quad \text{and} \quad h_3/h = (1+r)/(1+2r).$$

There has been some discussion of the appropriate parameter with which to represent the entrainment. Manins and Turner (1978) use a quantity $R^* = (1+2r)/3$ which suggests that at least one-third of cumulus kinetic energy is converted into potential energy. As we understand their argument, it seems as if mixing at the inversion happens first, then convection fills in below that level afterwards. Since the thermodynamics involved is not reversible, the order in which processes take place is likely to be important, and perhaps this accounts for their different definition of efficiency. Fortunately, all the observable quantities are the same and only the interpretation of the hypothetical efficiency is different.

Within these restricted definitions Equations (2) and (3) together give the efficiency of conversion of thermal energy into cumulus kinetic energy, as

$$\text{CAPE} \frac{(1-r)}{\delta q} = \frac{1}{4}(1-r)g h/c_p \theta_0 . \quad (4)$$

The quantity $r$ is the entrained-flux ratio which, according to Cattle and Weston (1975), has a value close to 0.2. With this value and $h \sim 1 \text{ km}$, we see that the efficiency of conversion of thermal into mechanical energy on the cumulus scale is 3 to 4% and varies mainly with the depth of the convection. Note the slight ambiguity between the running efficiency as defined by (4) and the total efficiency defined as: the total cumulus kinetic energy generated and dissipated (i.e. not reconverted) up to the present, compared with the total heat input up to now. Using the relation between $h$ and $q$ from Eq. (3) to integrate Eq. (4) from $h = 0$ the total efficiency is two-thirds of the current efficiency defined by Eq. (4).
In practice the potential energy remaining in the cumulus layer could be called on, at a later stage, by cumulonimbus convection and much of the remaining potential energy converted into kinetic energy on the scale of cumulonimbus. Alternatively, inhomogeneity of energy input may cause horizontal variations of density capable of driving mesoscale motion. It is this latter aspect we explore here.

3. ENERGETICS OF THE MESOSCALE

Consider an idealized ‘sea-breeze’ situation within a two-dimensional closed region of width $L$. Over part of it, of width $D$, surface heating and convection generate an isentropic layer to height $h$, as described by Eq. (3). Adiabatic rearrangement, as illustrated in Fig. 2, leads to potential energy being made available to motion on the scale of the box. The change in potential energy brought about by rearrangement to a state of minimum potential energy defines the mesoscale available potential energy MAPE:

$$\text{MAPE} = \frac{g \gamma \rho D(L-D)}{6 \theta_0} \left( h_2^3 + h_1^3 \right) = \frac{g \rho}{6} \left( \frac{\theta_0}{\gamma} \right)^{\frac{1}{3}} \frac{D(L-D)}{L} \left( \frac{q}{\rho_c \theta_0} \right)^{\frac{1}{2}} \left( \frac{(1+r)^3 + r^3}{(r+\frac{1}{2})^3} \right)$$

(6)

where $h_1$ and $h_2$ are properties of the air over the sea defined in Fig. 1. The result follows essentially from the consideration that each slab of air has a linear variation of potential temperature with height before and after the rearrangement and that a column with constant stratification behaves like a triangular mass whose centre of gravity is one-third of its height above the base. Equation (6) accounts separately for the effects of the lapse rate $\gamma$, the geometry of the region, the heat input, and the efficiency of boundary layer entrainment.
As it appears in Eq. (6), the term in $\gamma$ suggests unbounded potential energy for a nearly isentropic initial state. We see that this cannot happen, for $h$ in Eq. (5) must be less than the isentropic limit as $\gamma \rightarrow 0$, so MAPE, as given by Eq. (5), tends to zero linearly with $\gamma$. The point is of little importance in the earth's atmosphere where the height of the convective boundary layer is nowhere comparable with that of the isentropic atmosphere, $H_o$.

From Eqs. (3) and (4) we see that CAPE is also proportional to $(q/c_p \rho \theta_0)^2$ so that for a given value of entrainment coefficient, both mesoscale and cumulus-scale energy increase together. This seems surprising at first sight because both are competing for the same potential energy - only its availability is different on the two scales.

An interesting number is the thermodynamic efficiency of conversion of heat into kinetic energy for the whole of the mesoscale region:

$$\frac{\text{MAPE}}{qD} = \left(\frac{(1+r)^3 + r^3}{12(r + \frac{1}{2})^2}\right) \frac{L - D}{L} \frac{h}{H}$$

making use of Eq. (3). The term in brackets varies from 0.67 at $r = 0$, to 0.58 at $r = 0.4$, and to 0.67 at $r = 1$, so is effectively a constant. It seems reasonable to suppose that $L \sim 2D$ for a sea breeze. It follows that the efficiency defined by (7) is, at $0.3 \, h/H_0$ comparable with that for the cumulus mode.

Another interesting number concerns the magnitude of the velocities one would expect to find. Seeing the rearrangements of Fig. 2 we can suppose the kinetic energy to be distributed across the region up to height $h$. Using $\text{MAPE} = \frac{1}{2} \rho V_*^2 L h$ to define a characteristic speed $V_*$, we get:

$$V_*^2 = \frac{1}{24} \frac{D(L-D)\gamma h}{\theta_0} g h \left(\frac{(1+r)^3 + r^3}{(r + \frac{1}{2})^3}\right)$$

$$\approx 0.11 \, (gh)(\gamma h/\theta_0).$$

Hence

$$V_* \approx 6 \, \text{ms}^{-1} \text{ for } h \approx 2 \, \text{km}$$

and

$$\approx 3 \, \text{ms}^{-1} \text{ for } h \approx 1 \, \text{km} \quad (\gamma = 2.5 \, \text{K km}^{-1})$$

The relation between these values and those obtained in a numerical model are explored in Dalu and Green (1980). We note in passing, however, that the propagation of the sea breeze has the same form as that of a gravity wave in which the density differences are determined by the thermodynamics of Eq. (8).

4. Definition of the Mesoscale

In a realistic problem the values of $D$ and $L$ are related. In our original case study of a sea breeze, $D$ represented the distance of the Appenines from the sea, and $L-D$ was chosen to represent the distance over the sea that the motion penetrated. For an extensive 'land' region it would be reasonable to take a similar value for $D$ for a period of integration of 12 hours. As the integration period is extended, however, the values of $L-D$ and $D$ should be increased to represent the greater area of energy input that can be used by the motion. Calculations such as those above suggest roughly similar velocities for the gravity wave propagating over the ocean and the gravity current propagating over land, so $L$ and $D$ should increase proportionally.

Motion of large (synoptic) scale also convert potential into kinetic energy, using horizontal temperature gradients in stably stratified fluid. We believe that one can make a fundamental distinction between the mesoscale and large scale: mesoscale motion begins to show the effect of the Coriolis acceleration, whereas the large scale motion is closely constrained by it. Equation (8), especially in its inexact form, gives an expression for the time
scale of the motion $D/V_w$ which can be compared with the inertial time scale $1/f$ to give a
Rossby number:

$$V_w/fD = (h/D)(0·11gγf^2θ_0)^{1/2}$$

(9)

Coriolis accelerations begin to become important for $V_w/fD \sim 1$ which, with the value
$γ = 2·5$ K km$^{-1}$ and $f = 10^{-4}$ s$^{-1}$ gives $D \approx 30$ h. Thus for $h = 2$ km this mechanistic
interpretation of mesoscale gives $D \approx 60$ km, $L \approx 100$ km. Thus we suppose that the sea-
breeze process starts off on the small scale of the convective boundary layer, becomes
‘mesoscale’ (defined by Coriolis forces being relevant for $L \approx 100$ km) and becomes ‘large
scale’ with along-shore geostrophically balanced winds for some larger scale.

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