A theory of organized steady convection 
and its transport properties

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**Summary**

A dynamical classification of organized convection is presented, and previously published and additional models are collated in a more general theory. The transport properties of each type are represented by flux laws, the motivation being to understand convective transports in the context of parametrization schemes for organized convection.

Five distinct models, derived as analytic solutions to a general displacement equation obtained from conservation properties of the Boussinesq equations, are necessary to describe the dynamics. These models, designated propagating, steering-level, jump, cellular and classical, have very distinctive dynamical structures, and can be combined to represent more complex types of organized convection.

The propagating and steering-level types export a significant amount of kinetic energy, momentum and entropy, while the jump and cellular types store a considerable amount of energy as work done on the environment by the pressure field. The classical type does not transport momentum, and is a representation of the classical thunderstorm in weakly sheared flow.

Apart from the cellular model, although the entropy transports are broadly similar, the momentum and kinetic energy transports are very distinctive, with counter-gradient momentum transport the rule rather than the exception. Thus by effecting organized updraught/downdraught circulations, a completely different transport problem from small-scale cumulus and mixed-layer convection is posed, with fundamental inferences regarding parametrization. The dynamical necessity of a mesoscale response in the cellular and propagating types suggests that this scale may, in certain cases, need to be represented explicitly in convective parametrization schemes for large-scale models.

These prototypes have been deliberately simplified for the sake of elucidating fundamental principles, and to give a dynamical basis for experimentation and generalization, through exploiting both cloud-scale and larger-scale numerical simulation models, as well as providing guidance in observational analyses, budget studies in particular.

1. **Introduction**

In recent years, mainly as a requirement for the subgrid-scale representation of physical processes in large-scale models of the atmosphere, there has been considerable emphasis on understanding convective transports. Substantial resources are being deployed in observational and numerical experiments, seeking to resolve this fundamental problem. However, the basic dynamics of convective transports, and particularly their interaction with the large-scale flow, are not well understood.

It is usually assumed, at least implicitly, that subgrid-scale convective fluxes of heat are of a diffusive form representing a mixing process, and the convective transports of horizontal momentum are normally neglected. A diffusive transport is probably justified for dry boundary layer and shallow cumulus convection on the basis that these appear to be of a stochastic character. In the presence of vertical shear, however, even shallow convection loses this stochastic nature and a considerable degree of cloud-scale organization exists; well-known examples are Ekman instability of stably stratified boundary layers, or cloud-street organization in the convectively unstable case. Despite these qualifications, however, it is likely that diffusive boundary layer flux representations are either not grossly enough in error to significantly affect large-scale simulation models, or these models are not sufficiently sensitive to detect a misrepresentation if it exists.

While diffusive flux laws may be sufficient for many types of small-scale convection or mixed-layer models, this paper demonstrates that more complex transports are charac-
teristic of most types of deep convection. The basic reason for this distinction is the relationship between the distribution of latent heating and evaporative cooling in the presence of vertical wind shear, which effects a dynamical organization of the convection, and hence fundamentally influences the convective transports. It has been known for some time that vertical shear has an organizing influence, and the paper by Browning and Ludlam (1962) represents a classical example.

This paper seeks to quantify these transports and effectively examines the response of a flow to sources and sinks of heat, recognizing that a number of different organizations are possible. The systems considered here are archetypes and hybrids of these types represent more complicated organized convection. Due to the very large fluxes of both heat and momentum involved in organized flows, it is likely that these are important in large-scale and mesoscale numerical models. It is, however, beyond the scope of this paper to examine the important problem of a feedback mechanism, where a time-dependent mean flow changes the convective transports either by influencing the type of convection, or otherwise.

2. DEEP CONVECTION MODELS

The definition of a parametrization scheme for convection can be considered in three distinct parts. First, and central to the problem, is the type of convection model used; second, the closure hypothesis, that is, how the cloud transports are related to the large-scale flow in magnitude and areal distribution; third, the criteria for implementing or initiating the parametrization scheme. The main concern here is with the first of these, as it is considered that this is the main shortcoming of present schemes, at least when convection is organized. The closure and initiation aspects require a direct examination from model/large-scale interaction studies.

The cloud models used in current parametrization schemes are based on one-dimensional considerations, and most of their complexity arises in the relationship between the cloud model and the large-scale fields – the closure hypothesis. From a dynamical viewpoint, particularly when used to represent deep (cumulonimbus) convection, one-dimensional models are, in many ways, inadequate. For instance, the influence of the vertical shear requires multi-dimensional analysis, particularly through effecting flow organization, cloud-scale momentum transports, downdraught transports and cloud-scale pressure fields.

It is useful to outline the basic distinctions between deep and shallow convection dynamics. There are two main aspects to consider, of a thermodynamic and dynamic nature respectively. The basic thermodynamic feature of deep convection is the prevalence of downdraughts and cloud-scale organization. Precipitation-cooled downdraughts have two main effects. First, a low-level heat sink is produced, and second, a mass balance is effected predominantly on the scale of the cloud. These two features cause an entirely different heat transport from shallow convection, where the compensating descent (subsidence) produces dry-adiabatic warming on a horizontal scale greater than that of the cloud. Most current cloud models produce this effect exclusively. The sophisticated Arakawa–Schubert (1974) scheme, with a closure which requires a balance between the large scale and convective scale, is probably the most physically reasonable scheme for small-scale convection, but the cloud-model used is not intended to reproduce the downdraught cooling typical of deep convection. It is considered that no existing cloud model is capable of adequately representing cloud-scale downdraughts in a parametrization scheme. The majority of deep convective systems produce a substantial cooling of the lower troposphere, even in the tropics where the environment lapse rate is nearly wet adiabatic. The production
of this low-level heat sink evidently has substantial feed-back effects on the boundary layer transports and the synoptic-scale flow.

The dynamical distinction between deep and shallow convection is also important. In order to effect an organization on an updraught/downdraught system, powerful dynamical constraints operate to impose a shape and character to the cloud-scale circulations. Hence the transport of momentum, basically a feature of the flow orientation, is therefore characteristic of deep convection. In any transport problem, heat and momentum transports should not be considered in isolation when a consistent description of the interaction with the large-scale flow is required. This is formally clear in the energy transports, since the rate of conversion of potential to cloud-scale kinetic energy is effected by the correlation of entropy and velocity perturbations, while the conversion of cloud kinetic energy into the kinetic energy of the large-scale flow is through the correlation of the velocity perturbations. In this paper, therefore, momentum and heat transports will be considered in conjunction to give consistent energy budgets. The dynamical control on the cloud transports is effected by the vertical shear of the large-scale flow.

The cloud models incorporated in most convective parametrization schemes are derivatives of parcel models, where the energy release is equal to the convective available potential energy derived from the 'parcel buoyancy'. Indeed, this simple model is the only practicable one in one-dimensional representations, since the horizontal pressure gradients and the vertical shear are not formally represented. As will be seen later, this has basic implications, particularly regarding the convective transport of energy. A parcel type of cloud model is used in the Arakawa–Schubert scheme. This scheme is unique since it represents an ensemble of clouds of non-uniform depth, the cloud spectrum responding to the overall constraint that the ensemble is in a statistical equilibrium with the large-scale fields.

The CISK theory proposed by Charney and Eliassen (1964) is an alternative interpretation which relates the convective and large-scale fields by linking the boundary layer convergence to the cloud transports, and hence has an explicit physical closure. The transport models considered later are fundamentally distinct from both CISK and parcel models in that the boundary layer convergence is on the cloud scale and is mainly a result of cloud propagation rather than a large-scale feedback. Moreover, the downdraughts implicit in CISK are warm and effectively large scale, as opposed to cool and cloud scale in the majority of the following models.

The ensemble formulation (Arakawa–Schubert) and CISK are both useful for representing convection when evaporatively driven downdraughts are unimportant and a mutual adjustment to a characteristic thermodynamic profile is observed to take place (Ludlam 1966). The representation of isolated, vigorous, open and propagating cumulonimbus cells and ensemble of cumulonimbus with predominant cloud-scale downdraughts and large localized heat and momentum transports, is an example of a dynamically distinct problem which requires a different approach, both in terms of cloud modelling and large-scale feedback. For this purpose a general theory is described.

\( a \) Dynamical models of steady convection

Some of the cloud models used in the transport formulation have been published in previous papers: Moncrieff and Green (1972), Moncrieff and Miller (1976), Thorpe, Miller and Moncrieff (1980) (hereafter referred to as MG, MM and TMM, respectively) and in Moncrieff (1978). It is useful to summarize the basic aspects of these models in the present context, particularly since they are special cases of a more general formulation.
The main common assumption is that the flow is steady in a reference frame moving at the translation speed of the system. In addition, subcloud-scale turbulence is neglected, so that the motion is effectively inviscid. If absolute precision is required, it is clear that the steady-state hypothesis and the neglect of small-scale dissipative processes is unrealistic. However, due to the inherently complex nature of meteorological processes, particularly those arising from the interaction of multiple time and space scales, such accuracy is seldom of primary importance and it is more useful to illuminate dominant processes in a given situation; the cumulonimbus problem is one example of a physical system amenable to this philosophy.

The steady-state and inviscid assumptions are particularly useful because under these constraints the Boussinesq equations are integrable in a Lagrangian reference frame, and quite general properties which are conserved by the flow are defined. These conservation principles are particularly useful for calculating the transport properties and can also be used to define the energy exchanges such as the proportion of the convective available potential energy being realized as mean flow kinetic energy and the work done by the pressure field on the environment. In addition to steadiness and inviscid dynamics, another supposition necessary to obtain the conservative variables is that the entropy source $Q(x,y,z)$ is of the form $w \Gamma(z)$, where $w$ is the vertical velocity and $\Gamma$ is a 'lapse rate'. Consequently, special cases can be shown to be dry-adiabatic and pseudo-saturated-adiabatic thermodynamic systems, although it is important to realize that the theory is not restricted to these cases. With these assumptions steady-state (Lagrangian) conservation properties in relative coordinates are:

$$\frac{1}{2}v^2 + \frac{\delta P}{\rho} - \int_{z_0}^{z} g \delta \phi_p \, dz = C_1(\psi)$$

$$\frac{\delta \phi_p}{\rho} - \int_{z_0}^{z} (\Gamma - B) \, dz = C_2(\psi)$$

$$\rho \nu \cdot dS = C_3(\psi)$$

$$\frac{1}{\rho} \frac{\partial \omega}{\partial z} \cdot \nabla C_2 = C_4(\psi)$$

$$\frac{\eta}{\rho} - g \int_{z_0}^{z} \left( \frac{\partial \phi_p}{\partial \psi} \right)_z \, dz = C_5(\psi)$$

$$u - \int_{z_0}^{z} \left( \frac{\partial}{\partial \psi} \right)_z \frac{\delta P}{\rho} \, dz = C_6(\psi)$$

where $B = d\phi_0/dz_0$ is the static stability of the unmodified environment, $\phi_p$ the log-potential temperature following a fluid particle, $\phi_0$ the base state log-potential temperature, $\delta \phi_p$ is $\phi_p - \phi_0$ the deviation log-potential temperature, $\omega = (\zeta, \eta, \zeta)$ the vorticity, $S$ a streamtube cross-section, $\delta P$ the pressure deviation from the hydrostatic basic state, $C_1, C_2, C_3, C_4, C_5, C_6$ streamline variables, and $z_0(\psi)$ a reference (inflow) level of an arbitrary streamline $\psi$. These properties respectively represent the conservation equations for energy, entropy, mass, potential vorticity, $y$-vorticity and momentum production. In particular, $C_5$ and $C_6$ are conserved only in two-dimensions. Due to its more complex nature, the potential vorticity conservation is not as useful as the form defined in a flow dominated by vortex stretching, for instance in large-scale dynamics. In particular, on the convective scale, twisting and stretching terms are locally large and the flow more complex, and so the potential vorticity equation is generally intractable. The derivation of the conservation principles can be found in MG and MM.
It should be noted that the horizontal scale in the above equations is not specified, and in principle they can be applied to any system in which the production of horizontal vorticity is dominated by horizontal gradients of potential temperature. Since the way in which the systems finally process air is the main requirement, only the structure of the remote outflow, where the flow is hydrostatic, need be determined. Define \( z = z_1 \) as the outflow level of the streamline passing through the inflow level \( z = z_0 \). Once the displacements, \( z_1(z_0) - z_0(\psi) \) are calculated, since the static stability and the parcel lapse rates are specified, the outflow profiles of \( \phi \) can be determined. Moreover, the momentum change can also be found from mass continuity. It follows that all the fluxes can be calculated from these kinetic and thermodynamic changes; the crux of the problem, therefore, lies in obtaining the displacement solutions.

(b) The displacement equation

In this section a general equation relating the inflow and outflow heights of streamlines is derived of which the solutions in MG, MM and TMM are special cases; the main distinction between these lies in the boundary conditions applied to the equation for the particle displacements.

By definition, on inflow \( \delta p = 0 \) and \( \delta \phi_p = 0 \), so it follows that \( C_1(\psi) = \frac{1}{2} u_0^2 \) and \( C_2 = 0 \). Thus, energy and entropy conservation applied to outflow can be combined to give

\[
\frac{1}{2} u_1^2 = \frac{1}{2} u_0^2 - \left( \frac{\delta p}{\rho} \right)_1 + g \int_{z_0}^{z_1} \int_{z_0}^{z'} (\Gamma - B) \, dz \, dz' \tag{2}
\]

where the subscripts 0 and 1 refer to inflow and outflow respectively. Now the mass continuity equation applied on outflow defines \( u_1 \):

\[
u_1 = -\frac{\rho_0 \, dS_0}{\rho_1 \, dS_1} u_0 \quad . \tag{3}
\]

where in both equations \( u_0, \, u_1 \) are the inflow and outflow speeds normal to the streamtube cross-sections \( dS_0, \, dS_1 \) with inflow and outflow having negative and positive signs respectively.

Substitution into Eq. (2) gives

\[
\frac{1}{2} \left( \frac{\rho_0 \, dS_0}{\rho_1 \, dS_1} u_0 \right)^2 = \frac{1}{2} u_0^2 - \left( \frac{\delta p}{\rho} \right)_1 + g \int_{z_0}^{z_1} \int_{z_0}^{z'} (\Gamma - B) \, dz \, dz'.
\]

Consequently, except at inflow stagnation points

\[
\left( \frac{\rho_0 \, dS_0}{\rho_1 \, dS_1} \right)^2 = 1 + \frac{1}{2} u_0^2 \left( g \int_{z_0}^{z_1} \int_{z_0}^{z'} (\Gamma - B) \, dz \, dz' - \left( \frac{\delta p}{\rho} \right)_1 \right)
\]

If the outflow is hydrostatic,

\[
\left( \frac{\delta p}{\rho} \right)_1 = \Delta P + \int_{z_0}^{z_1} g \, \delta \phi_1 \, dz_1 = \Delta P + g \int_{z_0}^{z_1} \int_{z_0}^{z'} (\Gamma - B) \, dz \, dz',
\]

where \( \Delta P \) is the Boussinesq 'pressure' perturbation at the outflow reference level \( z = z^* \). Therefore, it follows that

\[
\left( \frac{dS_0}{dS_1} \right)^2 = \left[ 1 - \frac{\Delta P}{\frac{1}{2} u_0^2} \left( g \int_{z_0}^{z_1} \int_{z_0}^{z'} (\Gamma - B) \, dz \, dz' - \int_{z^*}^{z_1} \int_{z_0}^{z'} (\Gamma - B) \, dz \, dz' \right) \right] e^{2(z_0 - z_1)\rho_0}
\]

where density has been expressed in the scaled form \( \rho = \rho_0 e^{-(z_0 - z_1)/H_0} \).

An important property of Eq. (4) is that it contains no constraints regarding the scale of
motion to which it applies; the constraints imposed are that the motion should be steady (in an appropriate reference frame), inviscid, the rate of change of \( \phi \) along streamlines should be of the form \( w \Gamma \) and the remote flow should be hydrostatic. Since it arose from an energy equation, the effects of the earth’s rotation does not appear, since the Coriolis force can do no work, independent of the space scale. This property suggests that the theory used for the cumulonimbus scale could be modified for use in certain larger-scale organized systems.

It is stressed that Eq. (4) is only applicable to relative outflow and the inflow shear is specified. If the internal flow is required, then a different calculation is necessary. For instance, in the non-hydrostatic interior, a balance equation could be used to define the pressure perturbation contained in the energy equation. Alternatively, in two dimensions the equation for the \( y \)-vorticity can be used to give a complete solution for the internal streamlines, using the asymptotic solution as boundary conditions, a procedure used in Moncrieff (1978).

The generality of Eq. (4) obscures the basic dynamical principles and makes the mathematical problem difficult, consequently the models described in this paper have been formulated for the special cases of constant density \( (\rho = \rho_s) \), constant static stability \( (B) \) and constant parcel lapse rate \( (\Gamma = \gamma) \). Letting \( dS = dy \, dz \), then for these special cases Eq. (4) becomes

\[
\left( \frac{dy_0 \, dz_0}{dy_1 \, dz_1} \right)^2 = 1 - \frac{\Delta P}{\frac{1}{2} u_0^2} + \frac{g(\gamma - B)}{\frac{1}{4} u_0^2} \left[ \frac{(z_1 - z_0)^2}{2} - \int_{z_0}^{z_1} (z_1 - z_0) \, dz_1 \right].
\]

(5)

Since Eq. (5) contains a double differential, a second equation is necessary to determine \( y_1(y_0, z_0, z_1) \), which the potential vorticity constraint could yield in principle. However, in order to define a tractable problem, it is useful to write \( dy_1/dy_0 \equiv Y_1(z_1) \), a specified function of outflow height.

With this simplification, Eq. (5) gives a relationship between the outflow and inflow streamline heights, and so the vertical displacements can be determined from solution of the following displacement equation,

\[
\frac{dz_0}{dz_1} = \left[ Y_1(z_1) \right]^{-2} \left[ 1 - \frac{\Delta P}{\frac{1}{2} u_0^2} + \frac{g(\gamma - B)}{\frac{1}{4} u_0^2} \left[ \frac{(z_1 - z_0)^2}{2} - \int_{z_0}^{z_1} (z_1 - z_0) \, dz_1 \right] \right]^{-1/2}
\]

(6)

subject to appropriate boundary conditions; the transport properties can hence be deduced as described in section 4.

3. Solutions of the Displacement Equation: Dynamical Regimes

The displacement equation is extremely useful for the formulation of dynamical models of organized convection. The basic mathematical distinction between these models lies in the nature of the boundary conditions and this has related physical and dynamical implications. In this way distinct types of organized convection can be identified and the transport properties represented in terms of variables defined on the mean flow scale, useful for defining cloud models in parametrization schemes. Useful results can be obtained for the simplest case of \( Y_1 \equiv 1 \), so that the inflow and outflow widths of each streamtube are equal. This is not the same as imposing two-dimensionality, because as in the following model, the internal flow must be of a three-dimensional character.
(a) The propagating model

This type, described in MM, represents a convective system travelling relative to the undisturbed flow at all levels, so that inflow is confined to the front of the system, outflow to the rear and no reversal of the relative flow exists. With inflow and outflow on opposite sides, there can (indeed has to) be a net cross-system pressure gradient, so in particular \( \Delta P \) is non-zero in Eq. (6). This has implications regarding the momentum transport and budget, as will be discussed later. Unless this pressure change is specified by some suitable closure, the solution is not unique.

It was shown in MM that shear has little effect on this type of convection, provided that it is not large enough to violate the approximate condition that \( \text{CAPE}/\frac{1}{2}(\text{\Delta}u)^2 \geq 2.7 \), where \( \text{CAPE} \) is the convective available potential energy of the system, and where \( \Delta u \) represents the undisturbed shear between cloud top and bottom. The presence of a reverse shear in the wind profile (jet) is, however, important in the initial growing stages because this can effect a backward-orientated anvil outflow and a propagating system structure; this aspect is explained in MM. For this reason, this type of convection is more typical of tropical regions, where such a profile frequently exists. The schematic flow structure is shown in Fig. 1. The unsheared case is, however, adequate for describing the nature of the dynamics.

![Figure 1](image)

Figure 1. (a) Schema of the relative flow in the propagating model. \( \Delta P \) is the normalized pressure change at the mid-level, \( z = 0 \). (b) The relationship between the propagation speed \( c \) and the undisturbed flow relative to the ground \( U_\alpha \).

The boundary conditions on the mathematical problem represent the simplest unsheared model, so that the flow is symmetric around the mid-level, the centre streamline is undisplaced and the outflow at the top boundary originates from the bottom boundary, and vice versa; the internal flow, significantly, has to have a three-dimensional form.

On imposition of these conditions the solution to the displacement equations is as follows:

\[
\delta(z_0) = z_1 - z_0 = \epsilon_1 FH \sinh(z_1/FH)
\]

provided that the value of \( F \) satisfies the condition

\[
\epsilon_1 F \sinh(1/2F) = 1
\]

where \( F = (c - U_\alpha)/(\text{CAPE})^3 \) is a normalized propagation speed relative to the mean (unsheared) flow \( U_\alpha \), \( \epsilon_1 = 1 + \sqrt{(1 - E)} \), where \( E = 2 \Delta P/(c - U_\alpha)^2 \) is a normalized 'pressure' change across the system at the mid-level, \( \text{CAPE} = \int_{-H/2}^{H/2} g \delta \phi_p \, dz \), \( H \) being the convection
depth. The solution given by Eq. (7) is not unique unless a closure assumption is made, such as a maximization of the amount of CAPE realized as outflow kinetic energy (mechanical efficiency) given by $E = 1$, the assumption used in MM.

(b) The steering-level model

The asymptotic solution of this two-dimensional model has been described in MG and the full solution in Moncrieff (1978). It was shown in these papers that if the vertical shear is constant and the undisturbed speed change between top and bottom ($\Delta u$) is sufficiently large that $R = \text{CAPE}/(1/4(\Delta u)^2)$ lies within the range $-1/4 \leq R \leq 1$, then a type of steady convection can exist which moves at the speed of the undisturbed wind at a 'steering-level' $z = z_s$. Moreover, since in this case the inflow and outflow is on the same side of the system and pressure must be continuous in a fluid, $\Delta P = 0$. As in the propagating model, the maximum vertical displacements are experienced by particles originating from the top and bottom boundaries, with a zero displacement at the updraught and downdraught steering levels. The travel speed relative to the surface wind ($U_s$) is given by

$$c - U_s = \Delta u \{1 + \sqrt{1 + 4R}\}/\{3 + \sqrt{1 + 4R}\}$$

It was shown by Moncrieff (1978) that in constant vertical shear the orientation of the internal flow is distinctive in that the warm updraught underlies the cold downdraught air. The vertical tilt of the system is downshear, consequently horizontal momentum is transported upwards against the mean velocity gradient, upper-level mean flow momentum is increased and low-level momentum decreased. This counter-gradient transport is effected by constraints on the generation of transverse vorticity for steady, two-dimensional flow in a steering-level model; vorticity with the same sign as that of the basic flow is produced. The schematic flow is shown in Fig. 2.

![Figure 2](image)

Figure 2. (a) Schema of the relative flow in the steering-level model. The level $z = z_s$ is the height at which the travel speed equals the environment windspeed. (b) The relationship between the travel speed $c$ and the undisturbed flow relative to the ground.

The displacement equation solution in this case is given in terms of an external parameter $R$

$$\delta(z_0) = z_1 - z_0 = (z_s - z_0)(1 + \beta)/\beta$$

where $\beta = \{1 + \sqrt{1 + 4R}\}/2$. Solutions exist only if $-1/4 \leq R \leq 1$, the negative values
representing forced convection with the energy supply originating from the mean flow, thereby creating potential energy.

The above flow structure does not give a satisfactory thermodynamically consistent explanation for deep convection in a strictly two-dimensional flow, because the downdraught cannot be maintained; three-dimensional effects, particularly the directional shear, are evidently important. Many of the severe storm observational models, such as Browning (1964), indicate a circulation which contains indications of both propagating and steering-level convection, and the numerical simulation in Miller (1978) quantified this aspect. It can be demonstrated from remote flow considerations, at least where the vertical component of vorticity is sufficiently small, that a three-dimensional circulation in a vector wind shear can be represented by two orthogonal quasi two-dimensional projections, the downshear orientated steering-level model applied in the direction of maximum environmental wind shear, and the propagating model in the transverse direction. This gives a method of calculating the system motion vector (Tam 1979). The downshear slope is not an embarrassment in a three-dimensional flow, since it can pass underneath the updraught and hence be maintained by the evaporation of precipitation. The motion of a cumulonimbus in a vector shear can thus be represented as a hybrid of the above two models. It would, however, be impossible to solve the internal flow analytically in three space dimensions; a numerical simulation approach is required for this degree of detail.

(c) The jump model

In the previous two cases the convective layer was overturned and particles at the upper and lower boundaries experienced the maximum vertical displacement, while particles at mid-levels had a minimum displacement. In the propagating model, the non-zero pressure change across the system is vital. Solutions to the equation exist which exhibit such a pressure change but have the opposite displacement structure – maximum displacements in mid-levels and zero displacements at both upper and lower boundaries. A schematic representation of this model is shown in Fig. 3.

![Diagram of jump and drop models](image)

**Figure 3.** (a) Schema of the relative flow in the jump (drop) models. $\Delta P$ is the normalized pressure change at the surface $z = 0$, positive for the drop solution and negative for the jump case. (b) The relationship between the propagation speed ($c$) and the undisturbed flow relative to the ground ($U_b$).

An analytic model was presented in TMM and a necessary condition for the mathematical solution was that the vertical displacements should have the same sign throughout the entire system; for this reason it is called the 'jump' model. There are certain similarities between hydraulic jumps in stratified flow over obstacles and these jump solutions although
an important distinction is that in hydraulic jumps kinetic energy is dissipated by turbulence, while in the jump models this energy is used as work against a pressure field. This jump solution refers to unsheared inflow of constant static stability. It can be shown that no convective circulation can, in this case, exist, that is, potential energy is always created, the source being the work done by the pressure field; hence cold air rises and warm air sinks, a feature of the vorticity production, as discussed in TMM.

The displacement solution is given by

\[ \delta(z_0) = z_1 - z_0 = \varepsilon_2 F \sin(z_1/FH) \]  

where \( \varepsilon_2 = 1 - \sqrt{(1 - E)} \), so that the opposite sign for the square root from that of the propagating model is appropriate. The boundary conditions of zero displacement at the top and bottom boundaries require that

\[ F = 1/n\pi, \quad n = 1, 2, 3 \ldots \]

where \( F = (c - U_0)/\sqrt{(-2 \text{ CAPE})} \), the normalized propagation speed relative to the (unsheared) flow; \( n = 1 \) is used in this paper as an example.

Clearly, solutions which satisfy the displacement equations and the boundary conditions can only exist if \( \delta \) is real, so \( E \leq 1 \). Moreover, since the flow is always of one sense, the outflow always satisfies \( u_i \geq 0 \), so this gives a lower limit to \( E \). Consequently \( -3 \leq E \leq 1 \), and it follows from Eq. (9) that positive values of \( \varepsilon_2 \) (\( 0 < \varepsilon_2 \leq 1 \)) represent cold updraughts and negative values (\( -1 \leq \varepsilon_2 < 0 \)) represent warm downdraughts.

\[ (d) \quad \text{The cellular model} \]

A physically different regime in which the descending air is on a scale substantially larger than the cloud can be formulated using the displacement equation. This type of convection has features in common with the classical Rayleigh cellular convection, except in this case the macroscale circulation is being described and dissipation of energy is neglected. A two-dimensional, finite-amplitude circulation with a cell spacing of \( 2L \) and depth \( H \) is assumed, and conditions under which this motion is steady are sought. The mean flow is assumed to be unsheared, but the system may move with the speed of a constant ambient flow. The form of this circulation is shown schematically in Fig. 4. The release of potential energy in the updraught (CAPE) effects an inflow in the region \( 0 \leq z_0 < h \) and an outflow in \( h < z_1 \leq H \), with a stagnation point at \( z = h \).

It can be shown that the solution in the updraught interior is equivalent to that in Moncrieff (1978) with a vertical axis, although the internal flow will not be calculated here. The asymptotic solution in the region where the flow is horizontal is analogous to that of

![Figure 4. Schema of the relative flow in the cellular model. Region 1 is convectively driven, while Region 2 is forced by the kinetic energy generated in Region 1. Radiation, evaporation and surface fluxes are parametrized in terms of parcel lapse rates.](image-url)
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MG and this flow has constant inflow shear \( du_0 / dz_0 = A \) and constant outflow shear \( du_1 / dz_1 = \beta^2 A \) as well. The height of the stagnation point is given by

\[
h = H\beta/(1 + \beta)
\]

where

\[
\beta = \tfrac{1}{2}(1 + \sqrt{(1 + 4R_1)})
\]

(10)

The value of \( R_1 = g(y_1 - B)/A^2 \) therefore completely determines the flow. It is equivalent to the parameter \( R \) of MG, with the qualification that it is now an internal parameter (indeed, only a convenient expression) since the inflow is considered as having been established as a response to the potential energy release instead of having been specified from the large-scale flow.

Since the outflow from the updraught region also has constant shear \((\beta^2 A)\), considering this as inflow into the descent region, it can be shown that the solution which gives an outflow from the descent region equal to \( A \), and required to maintain a steady overturning specifies a value of \( R_2 = -g(\gamma_2 - B)/\beta^4 A^2 \) and also

\[
2/\beta = 1 + \sqrt{(1 + 4R_2)}
\]

(11)

Now \( R_2 \) is negative because the descent is forced and it is readily seen from Eq. (11) that a limiting condition must exist. Since \( \beta \) is a real number, \( R_2 \geq -\frac{1}{4} \) and because Eq. (11) relates \( R_2 \) to \( R_1 \) through the value of \( \beta \), clearly a powerful constraint on the convective overturning must exist. It follows that this condition is

\[
R_2 = (1 - \beta)/\beta^2
\]

Since Eq. (10) is a root of \( R_1 = \beta(\beta - 1) \), the equation for \( R_2 \) can be re-expressed as

\[
R_1 = -\beta^3 R_2
\]

In order to be consistent with Eq. (11), clearly \( \beta \geq 2 \), and hence \( R_1 \geq 2 \). Steady overturning can therefore exist only if \( R_1 \geq 2, R_2 \geq -\frac{1}{4} \) and \( R_1/R_2 = -\beta^3 \). These conditions have important implications regarding the physical nature of the cellular convection, including the convective heat flux, the convection aspect ratio, the physical nature of the descent cooling (radiation) and the maintenance of the convection (boundary-layer moisture and heat transports). A full discussion is, however, beyond the scope of this paper.

It is important to realize that the condition \(-\frac{1}{4} \leq R \leq 1 \) in the regime represented in MG is irrelevant for cellular type of convection; the upper limit in that case was determined by the concentration of negative vorticity generated in the updraught/downdraught boundary layer. There is no such generation in the cellular model, and hence the upper limit of \( R_1 = 1 \) does not apply; the lower limit of \( R_1 = 2 \) is due to the forced recirculatory form of the convection and the subsequent need to have a continuous vorticity distribution in each half of the convective system.

(e) The classical model

In unsheared ambient flow, the cellular model represents one extreme where the downdraught is forced, resulting in a warming of the convection layer, due to both heat release and subsidence warming. Another extreme is represented by the case where a strong precipitation-driven downdraught develops, and where the convection has a bimodal, but transient form. While this motion is basically unsteady, it is likely that each mode exists sufficiently long enough for particles of air to traverse the system, and so each can be described by a steady hypothesis. The motivation is to produce a cloud model for representing this type of cumulonimbus convection in parametrization schemes, although it is
realized that idealization is being pushed to its limit in this case.

This form is represented in Fig. 5, and during the first stage a vertically orientated updraught circulation exists and discharges precipitation, which by evaporative and drag effects leads to the development of a strong downdraught; this leads to a parametric representation of the classical Byers-Braham (1949) observational model. The transport properties are quite distinct from the cellular case, since there is mass compensation on a scale comparable with that of the updraught and low-level cooling.

Consider the first stage where only an updraught exists; this is analogous to the updraught branch of the cellular model, and a solution to the displacement equation is

\[ h_u = H\beta_u/(1 + \beta_u) \]

\[ z_1 - z_0 = (h_u - z_0)(1 + \beta_u)/\beta_u \]  \hspace{1cm} (12)

with \( \beta_u = [1 + \sqrt{(1 + 4R_u)}]/2 \) and \( R_u = g(\gamma_u - B)/A_D^2 \) where \( A_u \) is the inflow shear affected by the potential energy release and \( \beta_u^2 A \) the corresponding outflow shear.

Likewise, in the second (downdraught) stage, a solution is defined. For the sake of simplicity, consider the case where the upper level of the downdraught inflow is given by \( z = h_u \), a physically reasonable value. It can be shown that the solution is completely defined by the value of \( R_D = g(\gamma_D - B)/A_D^2 \) where \( \gamma_D \) is the downdraught lapse rate and \( A_D \) is the (constant) downdraught inflow shear generated by the downdraught cooling. Since downdraught thermodynamics are, in practice, distinct from that of the updraught, being a combination of evaporation-driven and dynamically forced motion (Miller and Betts 1977), there is much to be understood by further modelling of this process. In particular for this problem, the ratio of the updraught and downdraught lapse rates and the inflow shear ratio to define \( R_u \) and \( R_D \) are necessary.

Taking only as an example the case of \( R_u = R_D = R \), it can be shown by analogy with the updraught solution that

\[ h_u/h_D = (1 + \beta_u)/\beta_u \]

and

\[ z_1 - z_0 = \begin{cases} (h_D - z_0)(1 + \beta_u)/\beta_u & \text{(updraught)} \\ (h_u - h_D - z_0)(1 + \beta_D)/\beta_D & \text{(downdraught)} \end{cases} \]  \hspace{1cm} (13)

The model displacements are in terms of \( R \) where the inflow shear \( (A) \) is established as
a response to the horizontal gradients of potential temperature effected by the convection. Consequently, \( R \) is an internal parameter, contrasting with the steering-level case where it is specified from the large-scale flow and the potential temperature lapse rates. More practical internal parameters to use are the stagnation levels \( h_p \) and \( h_n \), since these are more easily measured from data or numerical models.

Note that small-scale cumulus convection is excluded from the model classification because mixing effects are dominant and so the theory developed in this paper is inapplicable. However, a macroscale circulation reminiscent of the Ludlam (1966) representation of cumulus/environment interaction, is effectively a form of the cellular model.

### 4. FORMULATION OF THE CONVECTIVE FLUXES

Since the momentum and thermodynamic changes can be calculated from the displacement solutions, convective flux divergences can be inserted in the appropriate mean-flow (grid-scale) equations. These fluxes are defined in terms of mean flow variables such as CAPE, \( H \) and \( \Delta u \), vertical profile functions and dynamical parameters defined from the displacement solutions.

The standard method of representing convective fluxes in mean-flow equations is to write a scalar property (\( q \) say) as the sum of a horizontal mean (\( \bar{q} \)) and a perturbation from that mean (\( \delta q \)). Since the fluxes originate from the non-linear terms, it is sufficient to consider the advection equation in the form \( \partial(q\rho q)/\partial t + \text{div}(q\rho \nabla) = 0 \) in which case it is readily shown that with the horizontal average \( \bar{q} = \left( \int q \, dx \, dy \right)/A \) where \( A \) is a grid area,

\[
\frac{\partial(\bar{q} \rho)}{\partial t} + \text{div}(\bar{q} \rho \nabla) = -\text{div}(\bar{q} \nabla \nabla) \quad .
\]

(The time-dependent perturbations have been neglected since the cloud models are steady.) The term on the right-hand side of Eq. (14), the convective flux divergence, contains vertical and horizontal components. Current cloud models used in parametrization schemes neglect horizontal transports; the vertical flux divergence is, for example, represented as the vertical gradient of the product of the convective mass flux and the convective perturbations from a cloud model, as in Betts (1975).

Cloud models in current use are one-dimensional and laterally detrain cloud properties, implying a level-by-level horizontal mixing with the environment. Such a formulation is reasonable if, say, a bubble or entraining jet is used to represent shallow convection in which mixing dominates. Deep convection, however, is envisaged as a macroscale process dominated by dynamical effects involving the vertical exchange of entire layers of air by updraughts and downdraughts. Air particles can therefore experience large vertical displacements. This interchange is the most fundamental distinction between the dynamical models considered here and conventional lateral detrainment concepts. The corresponding fluxes are likely to result in a quite different convective/mean flow interaction.

The mean flow representation of the fluxes is straightforward because the outflow modifications are calculated. Since the transverse (\( y \)-direction) fluxes are zero from the two-dimensional assumption and the inflow fluxes are zero by definition, it follows that the convective flux divergence can be represented by

\[
\text{div}(\bar{q} \rho \nabla) = \rho_x \delta q(\delta u_x)/L_x \,
\]

at each outflow level \( z = z_1 \). \( L_x \) is the horizontal scale length of the convection and in practical application \( L_x \approx H \). As usual, the subscript \( \delta \) denotes outflow, and the velocity perturbation is defined as the relative outflow speed \( \delta u = u - u_i \). It follows that Eq. (14) reduces to
Figure 6. The modification effected by the propagating model: (a) thermodynamics (log-potential temperature); (b) momentum.

\[
\frac{\partial(\bar{\rho}q)}{\partial t} + \text{div}(\bar{\rho}q\mathbf{v}) = -F_q(z_1)/L_x
\]  

where the convective flux \( F_q \) is defined as

\[
F_q(z_1) = \rho_j \delta q_1(z_1) \delta u_1(z_1)
\]

(15)

(16)

It is interesting to note that formulated in this way, the right-hand side of Eq. (15) depends on horizontal fluxes, contrasting with detrainment models which seek to represent vertical fluxes. In Eq. (15) it is assumed that the convection covers the entire (grid) area, clearly an extreme case except in limited areas. In application, therefore, the flux divergences would be multiplied by the fractional area occupied by the convection (of the order 1% globally).

The formulation of a moisture budget is an important aspect not considered in the present prototype theory. Although heating by cloud-scale condensation and cooling by rain evaporation releases convective available potential energy which drives the circulation and effects the fluxes, the model geometry is too idealistic to provide realistic rainfall; this can be rectified by allowing shallower downdraughts.

\( (a) \) Propagating model fluxes

From the displacement solutions given by Eq. (7), the convective perturbations are

\[
\delta u_1(z_1) = F \sqrt{(CAPE)} \left( 1 - \epsilon_1 \cosh(z_1/F \gamma) \right); \quad \delta \phi_1(z_1) = \epsilon_1 F \gamma \sinh(z_1/F \gamma)
\]

where \( \epsilon_1 = 1 + \sqrt{(1-E)} \) and \( F \) is, in general, a function of \( E \). In particular, when \( E = 1 \), then \( F \approx 0.32 \) and these modifications are shown in Fig. 6. Substituting these values into the flux divergence given by Eq. (16), the entropy and momentum transports can be shown
to be represented respectively by

$$F_H(z_1) = \frac{2\beta c_p}{gH} \epsilon_1 F^2(CAPE)^2 \left\{ 1 - \epsilon_1 \cosh(z_1/FH) \right\} \sinh(z_1/FH)$$

and

$$F_M(z_1) = 2\rho_c \epsilon_1 F^2(CAPE) \left\{ \epsilon_1 \cosh(z_1/FH) - 1 \right\}^2 \cosh(z_1/FH)$$

(b) Steering-level model fluxes

It can easily be shown from the displacement solutions of Eq. (8) that the outflow velocity and the flow changes are given by

$$\delta u_1(z_1) = \Delta u \beta^2(z_1 - z_*)/H; \quad \delta \phi_1(z_1) = (\gamma - B)(1 + \beta)(z_1 - z_*)$$

and are shown in Fig. 7. Substitution into Eq. (16) shows that the heat flux is given by

$$F_H(z_1) = \rho c_p \frac{2\Delta u(CAPE)}{gH^3} \beta^2(\beta + 1)(z_1 - z_*)^2 \quad z_* \leq z_1 \leq H$$

Due to the symmetry of this model, clearly

$$F_H(z_1) = 0 \quad H - z_* \leq z_1 \leq z_*$$

$$F_H(z_1) = \rho c_p \frac{2\Delta u(CAPE)}{gH^3} \beta^2(\beta + 1)(z_1 - H + z_*)^2 \quad 0 \leq z_1 \leq H - z_*$$

Similarly, the momentum divergence

$$F_M(z_1) = \begin{cases} 
\rho \frac{(\Delta u)^2}{H^2} \beta^4(z_1 - z_*)^2 & z_* \leq z_1 \leq H \\
0 & H - z_* \leq z_1 \leq z_* \\
\rho \frac{(\Delta u)^2}{H^2} \beta^4(z_1 - H + z_*)^2 & 0 \leq z_1 \leq H - z_* 
\end{cases}$$
Figure 8. The modification effected by the jump model for \( \gamma = 0 \) (dry-adiabatic flow). (a) The broken line is the unmodified (inflow) profile of log (potential temperature) and \( \phi^* = (\phi_0 - \phi_1)/BH \); the full line is the modified (outflow) profile of \( \phi^* = (\phi_0 - \phi_1)/BH \); the dash-dotted line shows the modification. (b) The momentum change. The drop solution is of similar structure except that the outflow profile is a mirror image with the inflow profile as an axis and the modification is a mirror image around the z-axis.

(c) **Jump model fluxes**

The displacement solutions for this case, shown in Eq. (9), define the flow modifications as follows:

\[
\delta u_1(z_1) = F(-2 \text{CAPE})^{1/2} \left(1 - \varepsilon_2 \cos(\pi z_1/H)\right) ; \quad \delta \phi_1(z_1) = (\gamma - B) \frac{\varepsilon_2 H}{\pi} \sin(\pi z_1/H)
\]

The modifications are shown in Fig. 8 where \( \gamma \leq B \) and \(-1 \leq \varepsilon_2 \leq 1 \). Cold updraughts correspond to \( \varepsilon_2 > 0 \), and warm downdraughts to \( \varepsilon_2 < 0 \). The former tend to destabilize the lower part of the convective layer and stabilize the upper part; the latter have exactly the opposite effect.

Substitution of these modifications into the equations for \( F_H(z_1) \) and \( F_M(z_1) \) above shows that the heat and momentum transports are respectively represented by

\[
F_H(z_1) = \frac{\varepsilon_2 \rho c_p (-2 \text{CAPE})^{1/2}}{gH^{1/2}} \left(\varepsilon_2 \cos \frac{\pi z_1}{H} - 1\right) \sin \frac{\pi z_1}{H}
\]

\[
F_M(z_1) = \frac{\varepsilon_2 \rho c_p^2 \text{CAPE}}{\pi^2} \left(\varepsilon_2 \cos \frac{\pi z_1}{H} - 1\right)^2 \cos \frac{\pi z_1}{H}
\]

(d) **Cellular model fluxes**

The transport properties of this closed system are conceptually quite distinct from the open cases considered above. The descending air is on a scale much greater than the cloud scale – either on a mesoscale in the case of periodic cellular convection typical of cold air masses flowing over relatively warm oceans, or on a larger scale as in the case of isolated ‘Hadley-type’ circulations. It is an example of a system where the cloud-scale fluxes do not interact directly with the synoptic scale, but are involved in a mesoscale response.
For the system to be steady, both updraught and downdraught must have the same lapse rate, but not necessarily on the same scale. The physical details of how this is achieved involves small-scale boundary layer transports and also radiative effects; these are effectively parametrized in the lapse-rate specification in the macroscale dynamical model described in section 3(d). In particular, in region 2 there must be a heat export, presumably through long-wave radiative cooling, in order that the necessary parcel lapse rate is achieved. Moreover, in this region there must be a moisture transport through the lower boundary, in order that the necessary value of \textit{CAPE} is generated to drive the updraught circulation in region 1. The details of this model have not yet been fully explored.

(e) Classical model fluxes

This is an open system, so there is an export of heat, momentum and energy to the large-scale flow, contrasting with the above closed system which exchanges energy between the convective and meso-scales. This energy is therefore directly available for the large-scale flow.

In the updraught mode the displacement equation solutions given by Eq. (12), define for $h_u \leq z_1 \leq H$

$$\delta u_1(z_1) = A_u \beta_u^2(z_1 - h_u); \quad \delta \phi(z_1) = (\gamma_u - B)(1 + \beta_u)(z_1 - h_u)$$

while in the downdraught mode the solutions of Eq. (13) give in the region $0 \leq z_1 \leq h_D$

$$\delta u_1(z_1) = A_D \beta_D^2(h_D - z_1); \quad \delta \phi(z_1) = (B - \gamma_D)(1 + \beta_D)(h_D - z_1)$$

In the region $h_D \leq z_1 \leq h_u$ clearly $\delta \phi_1 = \delta u_1 = 0$.

Consequently the flux formulae are as follows:

$$F_H(z_1) = \begin{cases} 4\rho c_p A_u(CAPE)_u \beta_u^2(1 + \beta_u)(z_1 - h_u)^2/gH^2 & h_u \leq z_1 \leq H \\ 0 & h_D \leq z_1 \leq h_u \end{cases}$$

$$F_D(z_1) = \begin{cases} 4\rho c_p A_D(CAPE)_D \beta_D^2(1 + \beta_D)(h_D - z_1)^2/gH^2 & 0 \leq z_1 \leq h_D \end{cases}$$

Since the system is vertically orientated, clearly the momentum flux is zero. Note that the case where no downdraught exists is obtained by setting $F_H = F_M = 0$ in the region $0 \leq z_1 \leq h_D$.

5. Energy budgets

The calculation of the energy budgets are useful for determining the influence of the convective fluxes on the large-scale flow. The energy equation, written in flux form in streamline variables, is

$$(\rho v E) \cdot dS = 0$$

with

$$E = \frac{1}{2}v^2 + \frac{\delta P}{\rho} - \int_{z_0}^z g \delta \phi_p dz$$

Integrating Eq. (17) over the domain of the appropriate model, a relationship between outflow and inflow is obtained:

$$\Delta(puK) + \Delta(uP) = \Delta(puA)$$

where $\Delta$ is the difference between inflow and outflow integrated over the whole system, that is
\[ \Delta(\rho uK) = \int \int_{\text{outflow}} \rho_1 u_1 \frac{1}{2} u_1^2 \, dy_1 \, dz_1 - \int \int_{\text{inflow}} \rho_0 u_0 \frac{1}{2} u_0^2 \, dy_0 \, dz_0 \]

\[ \Delta(uP) = \int \int_{\text{outflow}} u_1 \delta p_1 \, dy_1 \, dz_1 \]

\[ \Delta(\rho uA) = \int \int_{\text{outflow}} \rho_1 u_1 \left\{ \int_{z_0}^{z_1} g \delta \phi_r \, dz \right\} dy_1 \, dz_1 \]

The expressions in Eq. (18) are, respectively, the kinetic energy flux, the rate of working of the pressure field, and the available potential energy flux.

Since the left-hand side of Eq. (18) can be evaluated from the displacement solutions, the problem can be expressed in analytic form, at least for constant lapse-rates and density.

(a) Propagating model

The energetics of the propagating model for the case of \( E = 1 \) are given by

\[ \Delta(\rho uK) \approx 0.034 \rho_1 (\text{CAPE}) \]

\[ \Delta(uP) \approx 0.029 \rho_1 (\text{CAPE}) \]

\[ \Delta(\rho uA) \approx 0.063 \rho_1 (\text{CAPE}) \]

So since \( \Delta(\rho uA) \) is the total available potential energy flux the expressions

\[ \varepsilon_K = \frac{\Delta(\rho uK)}{\Delta(\rho uA)} \approx 0.54 ; \quad \varepsilon_p = \frac{\Delta(uP)}{\Delta(\rho uA)} = 0.46 \]

respectively, give the fraction of the total available potential energy released as convective scale kinetic energy and the fraction stored as work done by the pressure field. The outflow pressure and temperature are in hydrostatic balance, so it is possible to calculate the work done by the non-hydrostatic pressure field as a residual. It can be shown that this is an order of magnitude smaller than the hydrostatic term and so the convective available potential energy is used mainly to provide outflow kinetic energy and to change the outflow entropy, while a small proportion is unavailable as nonhydrostatic pressure work. In the steering-level, classical and cellular models, there is no net work done by the non-hydrostatic pressure, since \( \Delta p \) is zero.

(b) Steering-level model

In this case the formulae can be shown to be

\[ \Delta(\rho uK) = \rho_0 \beta^4 (\beta - 1) (\Delta u)^3 / 4(1 + \beta)^3 \]

\[ \Delta(uP) = \rho_0 \beta^3 (\beta - 1) (\Delta u)^3 / 4(1 + \beta)^3 \]

\[ \Delta(\rho uA) = \rho_0 \beta^2 (\beta^2 - 1) (\Delta u)^3 / 4(1 + \beta)^3 \]

Therefore it is easily shown that \( \varepsilon_K = \beta / (1 + \beta) \) and \( \varepsilon_p = 1 / (1 + \beta) \). For example, when \( R = 1, \beta = 1.62 \) so \( \varepsilon_K = 0.62 \) and \( \varepsilon_p = 0.38 \).
(c) Jump model

For this case of forced convection, potential energy is created mainly from the work done by the pressure field:

\[
\Delta(puK) = \frac{4}{3} \rho \varepsilon_2^3 (-2 \, CAPE)^3 \pi^{-3}
\]
\[
\Delta(uP) = -\rho \varepsilon_2^3 (-2 \, CAPE)^3 \pi^{-3}
\]
\[
\Delta(puA) = -\frac{4}{3} \rho \varepsilon_2^3 (-2 \, CAPE)^3 \pi^{-3}
\]

so that \( \varepsilon_K = -3.0 \) and \( \varepsilon_p = 4.0 \). This shows that the system is a forced regime driven by the pressure field, to the extent that it generates both kinetic and potential energy.

(d) Cellular model

Since this is a closed system, no net energy is available to change the kinetic energy of the large-scale flow, and the net work done by the pressure field is zero. The potential energy released in the updraught generates outflow kinetic energy, but this is used to drive the descending branch of the circulation, which is effectively on a mesoscale. The thermodynamics of the descending branch and physical processes such as radiative cooling and boundary layer moisture fluxes, are quite complex. In the model these effects are parameterized by specifying the lapse rate. It is clear that a more complicated representation is necessary, but this is beyond the scope of this paper.

(e) Classical model

In this case the energetics of the model is closely dependent on the coupling between the updraught and the downdraught and, in particular, on the respective depths of these two branches of the circulation as follows:

\[
\Delta(puK) = \frac{\rho \varepsilon H^3}{2} \left( \frac{\beta_u^4(\beta_u^4 - 1)}{(1 + \beta_u)^3} A_u^3 + \frac{\beta_D^4(\beta_D^4 - 1)}{(1 + \beta_D)^3} A_D^3 \right)
\]
\[
\Delta(uP) = \frac{\rho \varepsilon H^3}{2} \left( \frac{\beta_u^2(\beta_u^2 - 1)}{(1 + \beta_u)^3} A_u^3 + \frac{\beta_D^2(\beta_D^2 - 1)}{(1 + \beta_D)^3} A_D^3 \right)
\]
\[
\Delta(puA) = \frac{\rho \varepsilon H^3}{2} \left( \frac{\beta_u^3(\beta_u^3 - 1)}{(1 + \beta_u)^3} A_u^3 + \frac{\beta_D^3(\beta_D^3 - 1)}{(1 + \beta_D)^3} A_D^3 \right)
\]

and therefore

\[
\varepsilon_K = \frac{(1 + \beta_D)^3 \beta_u^4(\beta_u^4 - 1) A_u^3 + \beta_D^4(\beta_D^4 - 1)(1 + \beta_u)^3 \left( \frac{A_D h_D}{H} \right)^3}{(1 + \beta_D)^3 \beta_u^2(\beta_u^2 - 1) A_u^3 + \beta_D^2(\beta_D^2 - 1)(1 + \beta_u)^3 \left( \frac{A_D h_D}{H} \right)^3}
\]
\[
\varepsilon_p = \frac{(1 + \beta_D)^3 (\beta_u^2 - 1) \beta_u^3 A_u^3 + (1 + \beta_u)^3 (\beta_D^2 - 1) \beta_D^3 \left( \frac{A_D h_D}{H} \right)^3}{(1 + \beta_D)^3 (\beta_u^2 - 1) \beta_u^3 A_u^3 + (1 + \beta_u)^3 (\beta_D^2 - 1) \beta_D^3 \left( \frac{A_D h_D}{H} \right)^3}
\]
6. DISCUSSION AND CONCLUSIONS

The dynamics of organized convection have been quantified and the results qualitatively agree with many present numerical and observational models of deep convection; detailed comparisons require many further refinements. It is considered that these five archetypes describe the basic dynamics of relevant atmospheric systems, which are complex in detail.

A general conclusion is that the dynamics are crucial to the macroscale organization, assuming that subcloud-scale transports are of secondary importance. While this assumption provides a most useful simplification, it would be interesting to obtain a quantitative estimate of the role of cloud-scale dissipation in deep convection energetics, for instance, by using a simulation model with realistic turbulence closures. However, by modeling the macroscale, important dynamical features have been identified. With the exception of the cellular model, the cold, cloud-scale downdraughts play a crucial role in the dynamical structure of the convective systems, quantifying the conclusions of observational experiments and numerical simulations. As a result, the convective transports of heat, momentum and energy are fundamentally different from those considered typical of small-scale convection. Much of the horizontal momentum transport is counter-gradient, the form depending on the particular model. Kinetic energy is thereby generated on the scale of the mean flow and vertical shears are increased in these models.

Regarding the parametrization of organized convection, it is clear that this must be considered with care, since no single model is likely to be universally applicable. It is common in present parametrization schemes to use a single, often inadequate, one-dimensional model to represent the entire spectrum of convection; this philosophy should be changed for deep convection. Since it is clear that atmospheric convection is to a major degree organized, this is likely to be important in large-scale simulation. It would be instructive to establish the sensitivity of these models to a broad spectrum of parametrization schemes, such as ones based on the widely different cloud models discussed in this paper.

The interaction and feedback of these models to the large-scale flow is clearly an important problem and as such is not treated in any detail here. However, some general comments can be made. Clearly the low-level cooling produced by the cloud-scale downdraught will have a feedback effect on the boundary layer transports. Present parametrization schemes do not adequately represent downdraughts, and it would be interesting to examine this aspect more closely in large-scale models. Evidently the generation of kinetic energy on the cloud scale, together with the associated (counter-gradient) momentum transports, means that there is a direct communication between convective and large-scale energetics. At least 50% of the total convective available potential energy can be directly exported as cloud-scale kinetic energy. In disorganized convection where the dynamics are relatively unimportant, a mean flow interaction is usually effected by geostrophic adjustment through the temperature and pressure fields, and the above direct interaction is absent. It is not clear what effect this direct interaction might have on the structure and energetics of the large-scale flow, and this aspect has not been fully studied. However, preliminary work by Vincent and Schlatter (1980) indicates that the role of the kinetic energy generated by counter-gradient momentum transport in deep convection is important in observational budgets, and that the magnitude of the energy transfer compares favourably with those calculated in section 5 of this paper.

Another aspect which is implicit in the results of this paper is the dynamical necessity for a mesoscale forced downdraught, which uses energy otherwise directly available to the large-scale flow. For example, in order to satisfy a momentum budget, the propagating
model must be involved in a mesoscale interaction; this aspect is considered in Moncrieff (1980). Although the horizontal dimensions are not defined, it is clear from observations that this scale is a few hundred kilometres. Likewise, the cellular model should be considered in terms of a convective/mesoscale interaction. Consequently, due to dynamical processes, the parametrization problem for organised convective systems may be more complicated than a direct cloud/large-scale interaction.

The next step in quantifying this interaction is to use the cloud models for a full parametrization scheme of deep convection in a large-scale model. Such practical use of the theory will benefit from exploiting numerical simulation models of deep convection, since these are much more realistic (albeit with more obscure dynamics). For instance, the budgeting of detailed simulations of distinct types of convection and the stratification of those data with regard to the dynamical models represented here, is one obvious combination of theoretical and numerical approaches.

Large-scale interaction studies should be interesting and instructive. Present cloud models are exclusively one-dimensional and so do not represent the dynamical effects of pressure work and vertical shear. The dynamical models represented here are quite distinct in that the relevant dynamical terms are included because the transport laws are formulated from complete (steady state) analytic solutions of the full non-linear equations of motion and hence the macroscale structure of the convection is well represented. In principle, the transports are easily incorporated into the large-scale equations as forcing terms on the 'right-hand side', the normal way of representing the physics. Since the flux laws are formulated in terms of large-scale effects, such as convective available potential energy and shear, the magnitude of these fluxes can change throughout a large-scale simulation. Moreover, since the basic classification is in terms of the size of the shear compared to the available potential energy, it is envisaged that more than one type of model will be used throughout a simulation. Apart from the latter qualification, a standard implementation of the cloud models is sufficient.

The important aspects of closure and initiation are envisaged to be similar to those used in other deep convection schemes, depending on such properties as boundary-layer moisture convergence (Kuo 1974) and conditional instability, with the added constraint of the magnitude of the vertical shear.

The cloud/large-scale interactions are too complex to predict without experimentation using a large-scale model, but the effect of direct changes to the windfield by momentum transports, in addition to the usual indirect changes caused by gravity wave and geostrophic adjustment on the structure of large-scale systems such as baroclinic waves, long waves and the Hadley circulation, will be an important aspect to test.

It is likely that it will be necessary to improve the flux laws by introducing more realistic lapse rates, inflow velocity fields and downdraught heights into the analytic models. In this sense the models and the transports reported in this paper should be regarded as prototypes.

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