The climate at maximum entropy production by meridional atmospheric and oceanic heat fluxes

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(Received 28 December 1978; revised 26 May 1980, Communicated by Professor J. T. Houghton)

Summary

In a zonally-averaged energy balance climate model (principal features as with Paltridge 1975) with the four unknowns — surface temperature $T$, cloud amount $N$, total meridional heat fluxes within the ocean and the atmosphere $O_m + A_m$ and total vertical heat flux $LE + H$, the maximum entropy production by meridional heat fluxes is used as a constraint to solve the system for the four unknowns in all 10 boxes of the model. Since the solution at maximum entropy production by meridional heat fluxes agrees quite well with present mean conditions, this maximum principle is used as a working hypothesis for climate sensitivity studies avoiding the use of fixed cloud amount and meridional heat fluxes. The resulting sensitivities partly agree and disagree with those of similar energy balance climate models. Disagreement is particularly high if an ice-albedo feed-back is included. The feed-back is strongly reduced because of opposing effects of cloud amount in high latitudes.

1. Introduction

Systems with many degrees of freedom may be governed by an extremum principle. Unfortunately no a priori estimate of the parameter operating at extremum conditions can be given. For all systems within the non-equilibrium domain of thermodynamics (the atmosphere—ocean—land—cryosphere system is an example) entropy production within the system may be such an extremum parameter. At first glance the principle of minimum entropy production applicable to stationary systems within the linear range of irreversible thermodynamics (Prigogine 1947) could be used as a working hypothesis. However, all systems with known structures are far from the linear range. Therefore, the above-mentioned principle cannot be used.

Stimulated by Paltridge (1975) and a comment to this publication by Rodgers (1976) the author of this paper tried to calculate entropy flux and entropy production components in a zonally averaged, 10-box, simple energy balance climate model in order to find one term in the entropy balance equation showing an extremum value near to present climatic conditions. This extremum in turn could be used as a working hypothesis for the operation of a climate model avoiding the use of fixed cloud amount and eddy diffusivities for meridional heat fluxes.

2. The entropy balance equation

The first law of thermodynamics leads to energy conservation which has been extensively used in many simple climate models (Adem 1965, Budyko 1969, Sellers 1969, 1973 and others). The second law of thermodynamics postulates a positive entropy production $\sigma$ within real systems. Both laws and the continuity or mass conservation equation as well as the equation of motion are necessary for the derivation of the entropy balance equation.

Since the total entropy $S$ of a system with volume $V$ is not conserved, the time derivative $dS/dt$ can be separated into a divergence term $dS_e/dt$ and an internal production term $dS_i/dt$:

$$\frac{dS}{dt} = \frac{dS_e}{dt} + \frac{dS_i}{dt} = -\nabla \cdot J_s + \sigma \quad . \quad . \quad . \quad (1)$$

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with $\frac{dS_e}{dt} = -\nabla \cdot \mathbf{J}_s$ = rate of entropy change due to the interaction with the environment; $\mathbf{J}_s$ = entropy flux

$\frac{dS_i}{dt} = \sigma$ = rate of entropy production by internal processes.

The first term in Eq. (1) may contain entropy changes due to fluxes of heat and radiation as well as diffusion. Since the energy exchange of the planet earth with space is only via electromagnetic radiation (if we neglect a small mass exchange)

\begin{equation}
\frac{dS_e}{dt} = \int_V -\nabla \cdot \left( \frac{\mathbf{R}^*}{T} \right) dV = \int_F -\frac{R_a}{T} dF
\end{equation}

with $F$ = closed boundary of the volume $V$

$\mathbf{R} = $ radiative flux; $\mathbf{R}^* = \frac{3}{4} \mathbf{R}$ because the entropy of the radiation is $\frac{3}{4}$ times the total radiative energy in a volume divided by the temperature of the emitter of the photons (Levich 1971).

The temperature $T$ by which the outward normal of the radiative flux $R_a$ has to be divided is the temperature at the boundary of the system or the temperature at which the emission of radiation occurred. We will show that by minimizing Eq. (2) using some temperature $T_a$ of the ocean–atmosphere system one approximately maximizes the entropy production by meridional heat fluxes. Introducing into Eq. (2) the relation $R_a dF = -\nabla \cdot \mathbf{J}_m dV$ which only expresses the energy balance in a sub-system and using the relation $(\nabla \cdot \mathbf{J})/x = \nabla (J/x) + (1/x^2) \mathbf{J} \cdot \nabla x$ from vector calculus we get

\begin{equation}
\int_F -\frac{R_a}{T_a} dF = \int_V \nabla \cdot \frac{\mathbf{J}_m}{T_a} dV = \int_V \nabla \left( \frac{\mathbf{J}_m}{T_a} \right) + \frac{1}{T_a^2} \mathbf{J}_m \cdot \nabla T_a \right) dV
\end{equation}

Since the term $\int \nabla \cdot (\mathbf{J}_m/T_a) dV$ which is equal to $\int_F (J_{mn}/T_a) dF$, vanishes because of $J_{mn} = 0$ at the boundary we find

\begin{equation}
\int_F -\frac{R_a}{T_a} dF = \int_V \frac{1}{T_a^2} \mathbf{J}_m \cdot \nabla T_a dV = -\sigma_m
\end{equation}

The term $\int \{(\mathbf{J}_m \cdot \nabla T_a)/T_a\} dV$ is the entropy production by a heat flux $\mathbf{J}_m$ in a medium with a temperature gradient $\nabla T_a$.

A paper by Paltridge (1978), which came to the author’s knowledge after completion of the first version of this paper, expresses a similar view. However, Paltridge writes

\begin{equation}
\frac{dS_e}{dt} = -\frac{dS_i}{dt} = \int \{(\nabla \cdot \mathbf{J}_m)/T_a\} dV,
\end{equation}

which will only be correct if temperatures associated with solar radiation are those of the atmosphere. Minimizing $\int (-R_a/T_a) dF$ (Eq. (3)) therefore corresponds to maximizing the entropy production rate by meridional heat fluxes $\sigma_m$. The minimization procedure follows Nelder and Mead (1965). If the temperature associated with the incoming solar radiation were the temperature of the photosphere of the sun rather than of the earth’s atmosphere, we could no longer apply only one Gibbs equation to the earth atmosphere system. We then have to split $R_a/T$ in Eq. (2) into

\begin{equation}
\frac{dS_e}{dt} = \int -\left( \frac{Q_a}{T_0(1-a)} + \frac{L_n}{T_a} \right) dF
\end{equation}

with $Q_a = $ outward normal solar flux

$\frac{L_n}{T_a}$ = outward normal long-wave flux

$T_0$ = temperature of the sun’s photosphere
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\[ T_a = \text{temperature at which the outgoing long-wave flux is emitted} \]
\[ a = \text{planetary albedo} \]

We did not, however, pursue this line of reasoning in the present calculations.

3. **A SIMPLE ZONALLY AVERAGED CLIMATE MODEL**

The basis of the model is a publication by Paltridge (1974) and an up-dated version by the same author (Paltridge 1975). The latter gave the stimulation for this paper. After a short review of Paltridge’s climate model, variations introduced by the author of the present paper will be discussed.

Separate energy balance equations for 10 ocean and 10 atmosphere boxes covering equal areas of the globe with boundaries at 53.1°N, 36.8°N, 23.5°N, 11.5°N, 0, 11.5°S, 23.5°S, 36.8°S, 53.1°S are established. A solution for the three unknowns namely, surface temperature \( T \), cloud amount \( N \), and the sum of sensible and latent vertical heat fluxes \( H/LE \), needs three independent equations: two energy balance equations for the ocean and the atmosphere and a third relating the vertical heat fluxes at the ocean–air interface, \( H \) and \( LE \), to the long-wave emission in cloud-free areas close the system. However, the meridional transports within the ocean \( O_m \) and the atmosphere \( A_m \) have to be given a priori. If we could apply an extremum principle to any function of the variables of the system, the meridional transport could be an additional unknown. Exactly this procedure has been pursued by Paltridge (1975). The model then would be free of assumptions about the relation between dynamics and meridional transport.

The energy balance of the ocean part of one latitude zone, i.e. for one of the 20 boxes, reads with losses on the left and gains on the right:

\[ LE + H + \xi \sigma T^4 = \left\{ Q - gQ(1 - N) - QNd - mQ \right\} / \xi + f \xi \sigma T^4 N + O_m \]  

with \( Q = \) incoming solar flux
\( \xi = \pi / \cos \phi \) with \( \phi = \) geographical latitude
\( d = \) cloud albedo plus additional absorption of solar radiation within clouds due to liquid water
\( g = \) surface albedo
\( m = \) absorption of solar radiation by atmospheric gases and aerosol particles
\( \sigma = \) Stefan-Boltzmann constant
\( \xi = \) net long-wave flux at the surface related to the blackbody emission at surface temperature \( T \)
\( f = \) fraction by which blackbody radiation from clouds is reduced due to their lower temperature

The second term on the left side describes the long-wave net flux at the surface under clear skies, the first term on the right side the absorbed solar radiation, the second the reduction of the net long-wave loss by radiation from clouds, and the third term the divergence of the meridional heat transport in the ocean.

The energy balance of the atmosphere within one latitude zone (again energy losses on the left and gains on the right) can be expressed by

\[ \varepsilon f \sigma T^4 N + G \sigma T^4 (1 - N) = LE + H + Q(m + aN) / \xi + (1 - f) \xi \sigma T^4 N + A_m \]  

with \( \varepsilon' = \) empirical measure for the long-wave loss from a cloudy atmosphere
\( G = \) empirical measure for the long-wave loss of the clear atmosphere
\( a = \) extra short-wave absorption by the liquid water in clouds.
The third equation necessary to solve for the three unknowns $T$, $N$, $LE + H$ has also been adopted from Paltridge (1975):

$$LE + H = k_1 G \sigma T^4 (1 - N)$$  (8)

with $k_1$ = constant empirically determined from global mean values of $LE + H$, $T$, and $N$. The argument for the derivation of Eq. (8) is: the lower layers of the atmosphere and the ocean surface lose energy to space only in cloud-free parts of the globe, the sensible heat flux $H$ heats only the lowest part of the atmosphere, while the latent heat flux heats at the net condensation level of the atmosphere, and the meridional transports are mainly restricted to the higher layers.

The values of the parametrization coefficients ($\varepsilon^*, G, k, f, \ldots$) and the differences from those of Paltridge (1975) will be discussed in the appendix.

4. RESULTS FOR MAXIMUM ENTROPY PRODUCTION BY MERIDIONAL TRANSPORT

For a zonally averaged 10-box model Eq. (4), which expresses the functional used for the extremum calculations, reduces to

$$\sigma_m = \sum_{i=1}^{10} J_{mi}^* / T_{ai}$$  (9)

with $J_{mi}^*$ = net meridional heat flux into box $i$

$T_{ai}$ = emission temperature of the earth $\approx (L_m/\sigma)^{1/4}$

If the latitudinal distribution of the unknowns $T, N, LE + H$, and $O_m + A_m$ belonging to a maximum of $\sigma_m$ approaches present climate conditions, the maximum of $\sigma_m$ can be used as a working hypothesis for sensitivity studies of global climate. Searching for $\sigma_{m, \text{max}}$ when changing external variables like extraterrestrial solar flux, CO$_2$-content or aerosol burden of the atmosphere we can determine corresponding changes in cloud amount, surface temperature, vertical heat fluxes and meridional heat fluxes. Table 1 shows a comparison with Paltridge’s results while Figs. 1–3 demonstrate the agreement between calculated latitudinal distribution of the unknowns and related variables and empirical values or measurements. This confirms the working hypothesis mentioned above. All results presented

<table>
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<th>Mean latitude</th>
<th>Paltridge $T$ (1)</th>
<th>Paltridge $N$ (1)</th>
<th>Paltridge $A_m$ (2)</th>
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<td>301·11</td>
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<td>259·52</td>
<td>260·37</td>
<td>53°S</td>
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Table 1. SURFACE TEMPERATURE $T$, CLOUD AMOUNT $N$, AND MERIDIONAL NORTHWARD ATMOSPHERIC HEAT FLUX $A_m$. COMPARISON BETWEEN PALTRIDGE’S AND PRESENT RESULTS AND TWO DIFFERENT PARAMETRIZATIONS FOR $\varepsilon^*$; (1) $\varepsilon^* = (170·9 - 0·195 \sigma T^4^*) / \sigma T^4^*$; (2) $\varepsilon^* = (165·9 - 0·195 \sigma T^4^*) / \sigma T^4^*$
Figure 1. Observed (— — — continents; · · · oceans; — — — mean) and calculated (⋆) zonally averaged surface temperature depending on geographical latitude \( \phi \). Observations are taken from Lee and Snell (1977).

Figure 2. Observed (..... Landsberg 1945, quoted by Winston 1969) and calculated (— — —) cloud amount \( N \) as a function of geographical latitude \( \phi \). Additionally a smoothed calculated temperature distribution is shown.

were calculated using Eq. (9); however, results from the term \( \int \left( (J_m \cdot \nabla T_a) / T^2 \right) dV \) could also have been used though the accuracy would be reduced because of the uncertainty in giving a correct temperature gradient in polar areas from a box model.

Figure 4 displays the \( \sigma_m \) values as a function of \( J_m \) related to Seller's values which were derived from measurements and reported in Palmer and Newton (1969). The dashed curve represents the \( \sigma_m \) values if we split Eq. (8) into an atmospheric and oceanic contribution in
order to account for the higher temperatures at oceanic meridional transport.

\[
s^{*}_{m} = \sigma_{m,0} + \xi = \sum_{i=1}^{10} \left( \frac{A'}{T_{al} + O_i^{*}} \right)
\]

with \( T_{i} \) = mean ocean surface temperature in latitude zone \( i \). The corresponding \( \sigma_{m,\text{max}} \) is higher than the former one, the associated surface temperatures and all other unknowns remain almost unchanged. All non-equilibrium systems try to reach either thermodynamic
equilibrium or a stable steady state in accordance with the boundary conditions which force the system to a non-equilibrium steady state (see Glansdorff and Prigogine 1972). The state of maximum entropy $S_{\text{max}}$ which would be an isothermal, static state is not far from the present entropy value of the atmosphere as demonstrated by Dutton (1973). A condition for the existence of a distinct maximum of $\sigma_m$, as shown in Fig. 4, is according to Eq. (4) a strong non-linear relationship between the fluxes $J_m$ and the temperature gradient $\nabla T_y$. This again means inapplicability of the linear irreversible thermodynamic theory with a linear relationship between 'fluxes' and 'forces'.

Other components of the entropy production rate (for instance, entropy production by vertical turbulent heat fluxes $LE$ and $H$) did not show clear extrema under physically reasonable meridional fluxes and their derivation is omitted here. In this context reasonable means meridional fluxes from 50 to 200% of the estimated present values.

5. Climate sensitivity studies

If a climate model can approach present-day conditions as demonstrated in Figs. 1–3, this is by no means a justification to proceed at once to forecast the climate. A first small step in this direction is a sensitivity test showing the reaction of the model to variations of the external parameters as for instance incoming solar radiation, composition of the atmosphere, distribution of land and oceans with a possible comparison to known past climatic conditions. Since the parametrization coefficients apply to present conditions only small excursions about the present status should be tested. We therefore stick to small variations in external parameters not at all aiming at a climate stability test.

The following tables and Fig. 5, if compared to many existing sensitivity tests, partly show agreement and partly show complete disagreement.

Figure 5. Latitude dependent changes in surface temperature $\Delta T$ in K and cloud amount $\Delta N$ for a doubling of CO$_2$-content with and without (---) ice-albedo feed-back and for neglecting the latitude dependence of the factors $\varepsilon^*$ and $\varepsilon'$ (-----).
<table>
<thead>
<tr>
<th>Mean latitude</th>
<th>1% increase of $Q$</th>
<th>1% decrease of $Q$</th>
<th>Latitude boundary</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Without ice-albedo feed-back</td>
<td>With ice-albedo feed-back</td>
<td>Without ice-albedo feed-back</td>
</tr>
<tr>
<td>64°N</td>
<td>Δ$T$</td>
<td>Δ$N$</td>
<td>Δ$A_m^*$</td>
</tr>
<tr>
<td>44</td>
<td>0.66</td>
<td>0.000</td>
<td>0.60</td>
</tr>
<tr>
<td>30</td>
<td>0.73</td>
<td>0.001</td>
<td>0.81</td>
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<tr>
<td>17</td>
<td>0.74</td>
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<td>0.67</td>
<td>0.000</td>
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<tr>
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<td>0.34</td>
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<tr>
<td>44</td>
<td>0.72</td>
<td>0.001</td>
<td>0.53</td>
</tr>
<tr>
<td>64°S</td>
<td>0.53</td>
<td>-0.001</td>
<td>0.66</td>
</tr>
<tr>
<td>Mean</td>
<td>0.71</td>
<td>0.0004</td>
<td>0.75</td>
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</table>

* It is sufficient to show $A_m$ only, since the oceanic meridional heat flux has been fixed to the atmospheric flux according to Seller's empirical values quoted by Palen and Newton (1969).
Table 2a presents sensitivities of the present, simulated climate system for a $\pm 1\%$ change in solar flux at the upper boundary. The usual decrease or increase in surface temperature with decreasing or increasing solar flux is accompanied by a negligible positive or negative change in cloud amount respectively. Simultaneously, the meridional heat flux change in the atmosphere $\Delta A_m$ is positive for increasing and mainly negative for decreasing solar flux. The global mean decrease of $-0.71$ K and increase of $+0.70$ K is lower than nearly all existing results from pure energy balance models (see Oerlemans and Van Den Dool 1978). It agrees, as it should, with Paltridge (1975). If we allow for an ice-albedo feed-back (IAF), i.e. we introduce a temperature dependence of albedo for all latitude zones with $\phi \approx 37^\circ$ as derived empirically by Lian and Cess (1977) for the northern hemisphere and a reduced (factor 2) sensitivity for the southern hemisphere, we have to iterate using the temperature values at the end of one minimization for the next step. The result is an enhanced heating or cooling in higher latitudes and a further increase or reduction in the meridional heat fluxes if compared to the non-IAF case. The enhancement is far smaller than in all hitherto known sensitivity experiments, since the cloud amount, still only slightly changing, reacts in an opposite way. If we calculate the feed-back coefficient $\gamma = \beta \beta_0 - 1$ with $\beta = (Q\,dT/dQ)_{IAF}$ and $\beta_0 = Q\,dT/dQ$, we get $\gamma = +0.07$. This value is even lower than the value of the model by Gal-Chen and Schneider (1976) with $\gamma = +0.22$ as given by Lian and Cess (1977).

Table 2b shows changes in surface temperature $\Delta T$, cloud amount $\Delta N$ and meridional atmospheric heat fluxes $\Delta A_m$ for a doubling of CO$_2$-content of the atmosphere from 300-600 ppmv. This doubling may well be realistic if the present energy consumption of mankind remains unchanged. The parametrization coefficients altered by an increase in CO$_2$-content are: $\varepsilon^*$, $\varepsilon'$ and $G$. From the author's calculations in a wavelength-integrating radiative transfer model $\varepsilon^*$, the loss of the surface to space, has been found to be decreased by 0.014 in a mid-latitude atmosphere for a doubling of CO$_2$-content. This value is near to 0.016 derived from a simple formula given by Bryson and Dittberner (1976). The coefficient $\varepsilon'$, a measure of the long-wave loss from a cloudy atmosphere, has been found to be decreased to 60% of the $\Delta \varepsilon^*$ value, if mean cloud top is at 4 km height. The coefficient $G$ (a measure of the long-wave loss of the cloudless atmosphere only) remains nearly unchanged ($\Delta G$ is $-0.001$), since the main loss of a distinct atmospheric layer is at slightly shifted wave-

\begin{table}[h]
\centering
\caption{Results of sensitivity studies; doubling of CO$_2$-content}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
\textbf{Mean latitude} & \textbf{Without ice-albedo feed-back} & \textbf{With ice-albedo feed-back} & \textbf{Latitude boundary} \\
\hline
\textbf{64°N} & 0.62 & 0.002 & -0.58 & 0.78 & 0.002 & 2.99 & -0.45 & 53°N \\
\textbf{44°} & 0.94 & 0.012 & -0.77 & 1.01 & 0.012 & 6.47 & -0.69 & 37° \\
\textbf{30°} & 1.20 & 0.019 & -0.39 & 1.23 & 0.019 & 8.43 & -0.28 & 23° \\
\textbf{17°} & 1.20 & 0.018 & -0.15 & 1.22 & 0.018 & 8.64 & -0.06 & 11° \\
\textbf{6°} & 1.12 & 0.015 & 0.01 & 1.16 & 0.014 & 8.14 & -0.00 & 0° \\
\textbf{6°} & 1.19 & 0.014 & -0.00 & 1.15 & 0.015 & 8.10 & -0.12 & 11° \\
\textbf{17°} & 1.24 & 0.019 & -0.31 & 1.24 & 0.018 & 8.78 & -0.48 & 23° \\
\textbf{30°} & 1.22 & 0.019 & -0.75 & 1.20 & 0.019 & 8.28 & -0.91 & 37° \\
\textbf{44°} & 0.96 & 0.013 & -0.65 & 0.96 & 0.012 & 6.42 & -0.64 & 53°S \\
\textbf{64°S} & 0.62 & 0.003 & 0.71 & 0.002 & 3.05 & & & \\
\hline
\textbf{Global mean} & \textbf{ΔT} & \textbf{ΔN} & \textbf{ΔT} & \textbf{ΔN} & \textbf{Δ(LE+H)} & \textbf{ΔA_m} & & \\
\hline
\textbf{1.03} & \textbf{0.0135} & \textbf{1.06} & \textbf{0.0133} & \textbf{6.93} & & & \\
\hline
\end{tabular}
\end{table}
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<tr>
<th>Mean latitude</th>
<th>Temp. $\Delta T$</th>
<th>Cloud cover $\Delta N$</th>
<th>Meridio. heat flux $\Delta A_m$</th>
<th>$\Delta m = 0.04$ With ice-albedo feedback</th>
<th>$\Delta g = 0.0$ With ice-albedo feedback</th>
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<td>+0.044</td>
<td>-0.88</td>
<td>+0.022</td>
<td>-0.89</td>
</tr>
</tbody>
</table>

$\Delta T$: Temperature change; $\Delta N$: Cloud cover change; $\Delta A_m$: Meridional heat flux change.
lengths and thus the influence on net flux as well as on heating or cooling rates nearly vanishes. Therefore $G$ has been held constant.

Due to a latitude dependent overlap between water vapour and CO$_2$ emission the $\Delta e^*$ and $\Delta e'$ values become latitude dependent too. The latitude dependence has been introduced by reducing the $\Delta e^* = 0.016$ for polar areas stepwise to 0.0125 in the boxes near to the equator. Figure 5 also shows results for the $\Delta e^* = 0.016$ everywhere.

The main feature in Fig. 5 and Table 2(b) is a marked reduction of the heating in all latitudes ($\Delta T = 1.06$ K) if compared to other energy balance model results. Especially low is the heating in polar areas although an ice-albedo feed-back with coefficients given by Lian and Cess (1977) has been introduced. The differences can be explained by an increase in cloudiness ($\Delta N = 0.0133$) and a reduction in meridional heat flux. Increased cloudiness (within this model) reduces surface temperature and reduced meridional heat flux strengthens the equator-to-pole temperature gradient. Accounting for ice-albedo feed-back additionally reduces cloud amount in polar areas thus increasing the long-wave loss in this deficit area – as far as net radiative flux is concerned – simultaneously damping the positive albedo feed-back.

A reduction of the accuracy criterion from $\Delta n = 10^{-9}$ to $2 \times 10^{-9}$ where $\Delta n =$ mean difference of all sets of the simplex calculation between steps $n-1$ and $n$, did not show temperature variations above 0.03 K, even using a different starting condition. However, lowering the accuracy limit $\Delta n$ to $10^{-8}$ already leads to temperature variations of one tenth of a degree.

Table 2(c) treats the possible influence of changing aerosol particle concentrations and characteristics. Additional aerosol particles may change the coefficients $m$ and $g$ indicating an influence on the absorption of solar radiation and the reflection of solar radiation respectively. The values chosen $\Delta m = 0.04$, $\Delta g = 0.0$, and $\Delta m = 0.02$, $\Delta g = 0.01$ are in accordance with rough estimates relying on measured characteristics of man-made aerosol particles. Especially the $\Delta g$ value determines whether aerosol particles increase (negative values) or decrease the available energy for the earth. $\Delta g$ is strongly dependent on the surface albedo (see Yamamoto and Tanaka 1972, Eschelbach 1973, Grassl 1978) and negative values are more likely with a high surface albedo. Since we have used $\Delta g = 0.01$ a reduction in surface temperature is predetermined. Astonishingly ice-albedo feed-back does

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**TABLE 2(d). Results of sensitivity studies, combination of CO$_2$ and aerosol variations: doubling of CO$_2$, $\Delta g = 0.01$, $\Delta m = 0.02$**

<table>
<thead>
<tr>
<th>Mean latitude</th>
<th>Without ice-albedo feed-back</th>
<th>With ice-albedo feed-back</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta T$</td>
<td>$\Delta N$</td>
</tr>
<tr>
<td>64°N</td>
<td>-0.17</td>
<td>+0.016</td>
</tr>
<tr>
<td>44</td>
<td>+0.20</td>
<td>+0.026</td>
</tr>
<tr>
<td>30</td>
<td>+0.59</td>
<td>+0.034</td>
</tr>
<tr>
<td>17</td>
<td>+0.52</td>
<td>+0.034</td>
</tr>
<tr>
<td>6</td>
<td>+0.58</td>
<td>+0.032</td>
</tr>
<tr>
<td>6</td>
<td>+0.98</td>
<td>+0.034</td>
</tr>
<tr>
<td>17</td>
<td>+0.98</td>
<td>+0.035</td>
</tr>
<tr>
<td>30</td>
<td>+0.78</td>
<td>+0.034</td>
</tr>
<tr>
<td>44</td>
<td>+0.33</td>
<td>+0.027</td>
</tr>
<tr>
<td>64°S</td>
<td>-0.08</td>
<td>+0.016</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Delta T$</th>
<th>$\Delta N$</th>
<th>$\Delta T$</th>
<th>$\Delta N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0.47</td>
<td>+0.0288</td>
<td>+0.77</td>
<td>+0.0282</td>
</tr>
</tbody>
</table>
not increase this temperature reduction and has nearly no influence on the drastic increase in cloud amount which occurred in its absence.

Since both a CO$_2$ and aerosol particle increase are probable in the future, changes discussed separately in Tables 2(b) and 2(c) are combined in Table 2(d). Simply adding the separate results for $\Delta T$ and $\Delta N$ would give $\Delta T = \Delta T_{CO_2} + \Delta T_{aerosol} = 1.06 - 0.64 = +0.42$ K and $\Delta N = +0.0401$ as compared to $+0.40$ K and $+0.0414$ for the calculation changing all necessary coefficients at once.

6. CONCLUSIONS

An extremum principle related to maximum entropy production by meridional heat fluxes in oceans and atmosphere gives satisfactory agreement with observations. Partly similar and partly different reactions of the zonally averaged climate model to variations in external parameters occur if compared with earlier studies in pure energy balance climate models. This clearly shows that all forecasts derived from sensitivity tests of different simple climate models should be considered as pure tests on our way to an understanding of climate. Looking for instance at the CO$_2$-problem we find: many models with fixed cloud amount and parametrized meridional heat fluxes, including however ice-albedo feed-back (see for instance Manabe 1975), show a dramatic heating in polar areas for a doubling of CO$_2$-content, whereas the model presented here with no fixed cloud amount and meridional fluxes according to a maximum entropy production by these fluxes gives a small nearly negligible reaction in polar areas (a slight additional warming). The positive ice-albedo feed-back is strongly reduced in all cases calculated and is non-existent for changed aerosol characteristics.

APPENDIX

PARAMETRIZATION COEFFICIENTS

Table A1 contains all values of those coefficients given as constants either with or without latitudinal dependence. Some coefficients (independent of latitude in Paltridge 1975) have been parametrized latitude dependent, relying on empirical relationships.

One of these coefficients is $\varepsilon$, the net long-wave flux at the surface in units of the black-body radiation at surface temperature. $\varepsilon$ follows an empirical relationship given by Swinbank (1963).

$$\varepsilon = \frac{(\sigma T^4 - F)}{\sigma T^4} = \frac{(\sigma T^4 + 170.9 - 1.195 \sigma T^4)}{\sigma T^4} \quad (A1)$$

$\sigma T^4$ and the downward long-wave emission $F$ have to be given in W m$^{-2}$. $\varepsilon$, in accord with A1, has been directly introduced into Eqs. (6) and (7), leading to no change for the quadratic expression in $\sigma T^4$, if solved for $\sigma T^4$. One of the two solutions has been discounted because it is unphysical with, for example, negative cloud amount. An $\varepsilon = \varepsilon(\phi)$ causes a $G = G(\phi)$, since satellite measurements in clear areas can give the sum $G + \varepsilon$ showing a latitudinal dependence for $G$. The very simple empirical relation used is:

$$G + \varepsilon \approx 0.7 + (296 - T)(0.0043) \quad (A2)$$

The value $\varepsilon' = 0.75$ as used by Paltridge has been lowered and made latitude dependent. From satellite measurements published by Von der Haar and Suomi (1971) the total relative emission $G + \varepsilon(1 - N) + \varepsilon'N$ is between 0.54 and 0.65 except within the central polar areas, where values higher than 0.65 may occur. In low latitudes the values near 0.54
even with \( N \leq 0.5 \) indicate high-reaching clouds at very low temperatures. We therefore write for \( \varepsilon' \) in box i with mean latitude \( \bar{\varphi}_i \),

\[
\varepsilon' (\bar{\varphi}_i) = 0.70 + \left( |i - 5.5|/100 \right) \left( \frac{\sum_{i=1}^{4} A_m}{140} \right).
\]

(A3)

showing a first term smaller than Paltridge’s and a second term accounting for the dependence of the width of the infrared window (\( \varepsilon' \) is a measure of the radiation to space only from the cloud tops and the upper atmosphere) on temperature which in turn depends on the mean meridional atmospheric flux \( \left( \sim \sum_{i=1}^{4} A_m \right) \) of one hemisphere. The division of this sum by 140 is somewhat arbitrary only relying on current values of \( A_m \).

The short-wave albedo of the planet earth – determined from satellite measurements for instance by Raschke et al. (1973) – together with ground-based measurements of the short-wave net flux give an estimate of the absorption of solar radiation \( m + aN \) by gases, aerosol particles and liquid water. Values published by Major (1976a, 1976b) for some locations within the northern hemisphere are significantly higher than those by Fritz (1954) used by Paltridge. Therefore, we have also changed the \( m \) values for some calculations simply by adding a constant.

The values \( \zeta \) given in Table A1 have been changed too, accounting for the changing distance between sun and earth within one year leading to a higher insolation for the southern hemisphere.

### Table A1. Zonally Averaged Input Parameters

<table>
<thead>
<tr>
<th>Mean latitude</th>
<th>64°N</th>
<th>44</th>
<th>30</th>
<th>17</th>
<th>6</th>
<th>6</th>
<th>17</th>
<th>30</th>
<th>44</th>
<th>64°S</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
<td>0.55</td>
<td>0.50</td>
<td>0.45</td>
<td>0.43</td>
<td>0.41</td>
<td>0.41</td>
<td>0.43</td>
<td>0.45</td>
<td>0.50</td>
<td>0.55</td>
</tr>
<tr>
<td>( m )</td>
<td>0.20</td>
<td>0.18</td>
<td>0.17</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>0.17</td>
<td>0.18</td>
<td>0.20</td>
</tr>
<tr>
<td>( g )</td>
<td>0.22</td>
<td>0.18</td>
<td>0.16</td>
<td>0.15</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.16</td>
</tr>
<tr>
<td>land fraction</td>
<td>0.70</td>
<td>0.50</td>
<td>0.40</td>
<td>0.25</td>
<td>0.20</td>
<td>0.22</td>
<td>0.20</td>
<td>0.17</td>
<td>0.03</td>
<td>0.05</td>
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<tr>
<td>( g_{\text{land}} )</td>
<td>0.24</td>
<td>0.21</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td>( g_{\text{sea}} )</td>
<td>0.16</td>
<td>0.14</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.14</td>
<td>0.16</td>
</tr>
<tr>
<td>atmospheric flux</td>
<td>6.6</td>
<td>3.1</td>
<td>1.2</td>
<td>0.57</td>
<td>0.29</td>
<td>0.62</td>
<td>1.6</td>
<td>3.5</td>
<td>7.3</td>
<td></td>
</tr>
</tbody>
</table>

\( Q = 1353 \, \text{W m}^{-2} \); \( f = 0.80 \); \( a = 0.04 \)

### Acknowledgment

The author is very grateful to both Prof. Hasselmann and Prof. Hinze Peter in Hamburg for their criticism and encouragement helping to clarify some aspects, however simultaneously opening new questions, which could not be answered.

### References


Manabe, S. 1975 *The use of comprehensive general circulation modelling for studies of the climate and climate variation*, GARP Publication Series No. 16, WMO, Geneva, Switzerland.


