Observations of condensate profiles over
Arctic leads with a hot-film anemometer

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SUMMARY

During the AIDJEX Lead Experiment on the Arctic Ocean large-amplitude, high-frequency spikes were observed on the voltage signals of hot-film sensors being used for turbulent velocity measurements near the water surface. Because over leads in winter temperature gradients are very steep and the super-saturation extreme, condensate droplets can form and grow to measurable size even in the few seconds the air is over the lead. The voltage spikes resulted when these condensate droplets struck the hot-films. We could, therefore, use the hot-film information to determine not only turbulence quantities but also condensate concentration profiles and the flux of condensate droplets at the surface of leads. Condensate droplets were roughly 10 μm in diameter, concentrations within 20 cm of the surface were usually 20–80 mg kg⁻¹ (50–220 droplets cm⁻³), and the condensate flux was of order 3 mg m⁻² s⁻¹.

Non-dimensional condensate profiles are well modelled by

\[ \left( \frac{-C(z)}{c_0} \right) = -0.34 \ln(z/h) \]

where \( c_0 \) is a concentration scale and \( h \), an estimator of the internal boundary layer height, is fetch dependent. The condensate flux is related to average quantities through the Nusselt number, \( \text{Nu}_c \), and the fetch Reynolds number, \( R_e \):

\[ \text{Nu}_c = 0.14 R_e^{0.72} \]

Because \( c_0 \) also depends on the condensate flux, the simplicity of the profile relation and this Nusselt number equation suggest two alternative ways to estimate condensate flux at the surface of leads from the measurement of bulk environmental parameters.

1. INTRODUCTION

A lead is a linear area of open water within the pack ice of the Arctic Ocean. In March and April of 1974 the AIDJEX Lead Experiment (ALEX) focused on the turbulent transfer in the internal boundary layer (IBL) over leads (Andreas et al. 1979, Andreas and Paulson 1979). While measuring turbulent velocities with hot-film sensors during the course of ALEX, we noticed conspicuous voltage spikes in oscilloscope traces of the signals. These spikes were intermittent and had large amplitude and high frequency. They were thus easy to distinguish from the turbulent fluctuations. Because temperatures were typically -25 °C and the sensors were very near the water surface, we hypothesized sea-salt particles, super-cooled spray or condensate droplets, or ice crystals were striking the small hot-film sensors. The spikes were proof that these impacting particles had disturbed the thermal balance of the sensor.

Leads are often suggested as a source of aerosol particles in the Arctic atmosphere: the evidence supporting this conclusion, however, has always been indirect. In studying the precipitation of ice crystals at Barrow, Alaska, Ohtake and Holmgren (1974) and Ohtake et al. (1978) inferred leads were supplying the moisture for the growth of these crystals. Radke et al. (1976) correlated sudden increases in the concentration of sodium-containing particles in the aerosol at Barrow with the opening of leads in the vicinity. Holmgren et al. (1974) and Shaw (1975) attributed high atmospheric turbidity evident from optical transmission measurements near Barrow to lead-related aerosol particles. Despite the importance of the aerosol in the Arctic radiation balance (Holmgren et al. 1974; Shaw 1975, 1976) and these implications that leads are an important source of the aerosol particles, no one has
looked closely at the rate at which leads can supply particles. We decided, therefore, to investigate the hot-film voltage spikes further in hopes of determining the mass flux from leads.

2. Experimental procedure

Despite the 50-year history of the hot-film anemometer in turbulence research (Frey-muth 1978), hot-wires and hot-films have only recently been recognized as valuable airborne-particle samplers. Goldschmidt (1965; see also Goldschmidt and Householder 1968, 1969) has been primarily responsible for developments in the use of hot-wires for aerosol sampling, though Vonnegut and Neubauer (1952) suggested this application a decade earlier. Hollingshead et al. (1973) and Lai and Shemdin (1974) investigated the use of hot-film sensors for sampling spray above a water surface. Although Goldschmidt (1965) saw the unique potential of these small, heated cylinders for measuring turbulent velocity fluctuations and aerosol concentration simultaneously, we are evidently the first to use hot-films for both measurements.

(a) Sampling instrumentation

The sensing elements on all the hot-film probes used during ALEX were Thermo-
Systems Inc.-20 (TSI dash 20) sensors. Each consists of a cylindrical quartz rod of diameter 2 mil (50-8 μm) coated with a thin layer of platinum (~0.1 μm) and a protective layer of high-purity alumina (~1 μm). The rods have a diameter of 53 μm and are about 1.67 mm long with gold plating at each end to define a 1 mm platinum sensing region.

During the experiment voltage spikes caused by the impacting particles were evident on signals from three different hot-film sensors. Two of these sensors, the sensing elements on a TSI 1241–20 crossed-sensor probe used for measuring longitudinal and vertical velocity fluctuations, were positioned at the downwind edge of leads about 20 cm above the water surface. The third hot-film, the sensor on a TSI 1210–20 single-sensor probe, was mounted at the end of a tower arm and sampled the mean downwind velocity profile at five logarithmically spaced heights between 10 cm and 2 m as the arm cycled up and down through the internal boundary layer. The crossed sensors were operated with TSI anemometers (power supply, 1051–2; anemometer-linearizer, 1054A; bridge, 1056); the single sensor, with a DISA anemometer (main unit, 55M01; standard bridge, 55M10). All signals were recorded in FM mode on a Hewlett-Packard 3960 analog tape recorder, although the profiling hot-film signal was first electronically differentiated (Williams 1974). This differentiation – done in anticipation of a dissipation calculation related to the main purpose of ALEX – changed the character of the voltage spikes but did not make them any less distinct. Because the profile measurements are the more interesting, we used the crossed-sensor particle data only for checking their consistency. Andreas (1977) gives a more complete description of ALEX transducers, sampling methods, and data handling.

Figure 1 is a composite picture of several oscilloscope traces showing particle events. The lower frequency turbulent fluctuations have been filtered out of the (a) and (c) traces; the unfiltered (b) trace, nevertheless, shows the spikes as clearly because the oscilloscope sweep frequency, 100 Hz, was at or beyond the high frequency end of the turbulence spectrum. The (d) trace is the differentiated profile velocity signal. Differentiation removed the mean, making filtering unnecessary; amplified the noise, as evidenced by the longer-period oscillations between the three events; and inverted the signal.

The oscilloscope traces of particle events reported by Goldschmidt and Householder (1968, 1969) did not have the oscillations that characterize ours. Goldschmidt and House-
holder used a hot-wire anemometer specifically to sample aerosol particles, while we optimized our anemometers for turbulence measurements. Because the frequencies involved in the two uses are very different, optimizing the stability of the anemometer for one measurement does not necessarily optimize it for the other. We believe the positive-going first peak in each of our aerosol events corresponds to the spikes Goldschmidt and Householder saw; subsequent oscillations are ringing due to underdamping in our anemometer.

(b) Particle counting

The next section establishes that the voltage spikes were caused by supercooled water droplets. With this information we could develop equations relating the duration and amplitude of a voltage spike on the anemometer bridge to the size of the droplet that produced it (Appendix, section (a)). Retrieving the bridge voltage and, in turn, size information for individual droplets was prohibitively involved, however, because we linearized the bridge signal before recording it to optimize for the ALEX turbulence measurements. Fortunately, the droplets had a limited size range (section 3(b)); we could determine concen-
traction by assuming all droplets were the same size and then simply counting voltage spikes. This counting involved converting the voltage spikes recorded on magnetic tape into countable pulses. Because the spikes were of much higher frequency than the turbulent fluctuations they were mixed with, we isolated them using a second-order, high-pass filter with a $-3\text{ dB}$ point at 250 Hz. The output of this filter went into a pulse-generating network in which spikes were converted to pulses that could increment a digital counter.

From the voltage spike rate $S$ (counts per unit time) we computed the droplet concentration (number per unit volume),

$$ N = \frac{S}{1UDH}. \quad (2.1) $$

Here $U$ is the average wind speed at the sensor; $D$ the sensor diameter; $H$ the sensor length; and $l$ the impaction coefficient, the specification of which we discuss in section (b) of the Appendix. We also defined the volume concentration (mass per unit volume),

$$ V = (4\pi/3)(d/2)^3 \rho_w N, \quad (2.2) $$

where $d$ is the droplet diameter assumed to be the same for all droplets, and $\rho_w$ is the density of water. Lastly, the mass concentration (mass of droplets per unit mass of moist air), which has the same units as the specific humidity, is

$$ C = \frac{V}{\rho}, \quad (2.3) $$

where $\rho$ is the density of the moist air. In most of what follows we refer to the mass concentration $C$. Since all the water droplets are assumed to have the same diameter, however, $V$ and $N$ differ from $C$ only by multiplicative constants; the $V$ and $N$ profiles, consequently, have the same shape as the $C$ profile we will describe later.

3. The nature of the particles

(a) What are the particles?

Anyone who has seen fog above a lead in winter has probably remarked, 'The lead is steaming.' Thus, initially, we might presume that impacting liquid or frozen condensate droplets caused the hot-film voltage spikes. Liquid condensate droplets, in fact, were responsible for the overwhelming majority of the spikes, but this conclusion is not as obvious as the casual observer might think. The aerosol over leads could consist of any or all of the following: the ambient aerosol, sea-salt particles, and liquid or frozen spray or condensate droplets. It will be instructive to discuss how we settled on liquid condensate droplets alone as the cause of the spikes.

Because signals from hot-film sensors upwind of leads showed no particle spikes and because spike counts increased monotonically as the downwind sensors approached the surface, the background aerosol could not have been the cause of the spikes. Although the background aerosol near Barrow in March and April can have a concentration of several hundred particles per cubic centimetre (Turner and Cotton 1975; Radke et al. 1976), evidently the particles are too small to be caught by the hot-film sensor or do not disturb it appreciably if they do impact on it. A lead-derived aerosol must have caused the spikes.

Equation (2.1) shows that even without complete information about the impacting particles we can make an estimate of their concentration from spike counts by assuming $l = 1$. That estimate, $100\text{ cm}^{-3}$, $12\text{ cm}$ above the surface at the downwind side of leads, implies sea-salt particles and spray droplets could not have been important spike producers: neither has ever been observed in concentrations this high near a sea water surface, as we will see.
Spray and sea-salt particles are produced by the same surface processes. Waves break and trap air bubbles; when these submerged bubbles rise to the surface they burst, ejecting spray droplets that evaporate and leave salt particles (Kientzler et al. 1954; Blanchard and Woodcock 1957). The efficiency of this generation process clearly increases with wind speed and wave height. With the low wind speeds we encountered during ALEX and the limited fetch in the leads – both precluding a well-developed wave field – we initially suspect that spray generation could not have been important.

Measurements in wind tunnels are the best available source of data for estimating spray concentration over leads because the flow in them, as over leads, is fetch-limited. Wu (1973), Lai and Shemin (1974), and Wang and Street (1978) have all consistently measured spray droplet concentrations in wind–water tunnels of less than 0.1 cm$^{-3}$ regardless of wind speed. Even with a 24 m fetch over salt water, a wind speed of 18 m s$^{-1}$, and a collection height of 15 cm, Lai and Shemin (1974) found a spray droplet concentration of only 0.02 cm$^{-3}$. Because these concentrations are three to four orders of magnitude less than the particle concentrations we observed, spray droplets could not have caused the spikes.

Although a spray droplet may yield several sea-salt particles, the spray concentration would have been so low that – even with 10-fold multiplication – sea-salt particles could not have produced the requisite number of spikes. Many studies of sea-salt particles over the ocean confirm this. For example, none of the samples taken by Monahan (1968) at 50 and 100 cm above the ocean, by Frank et al. (1972) at heights between 2 and 14 km, or by Lovett (1978) at heights between 5 and 15 m ever indicated sea-salt particle concentrations higher than 0.5 cm$^{-3}$, although some of these measurements were made in winds of 20 m s$^{-1}$. Toba's (1965) numerical model of the sea-salt distribution in the marine surface layer similarly predicted concentrations at the sea surface of 0.5 cm$^{-3}$ or less in all but Beaufort 12 winds (30 m s$^{-1}$).

Particle concentrations in Arctic water and ice fogs, on the other hand, are of the same order as our estimate of particle concentrations over leads, 100 cm$^{-3}$. Gathman and Larson (1974) recorded concentrations of about 10–20 cm$^{-3}$ in marine fogs between Greenland and Norway. Kumai (1973) reported an average summer fog concentration on the Arctic coast near Pt Barrow, Alaska, of 14.5 cm$^{-3}$. The data collected in ice fogs in Fairbanks, Alaska, however, are best for comparison with our lead data because of the similarities in horizontal scales and in air temperatures. In an ice fog at −39°C Kumai (1964) found a particle concentration of 155 crystals cm$^{-3}$ in downtown Fairbanks near a section of the Chena River kept open by discharge from a power plant. Sampling nearer this open water site and also near hot springs in air temperatures below −40°C, Ohtake (1970) measured ice crystal concentrations of 300 to 2000 cm$^{-3}$. Since all other causes have been eliminated and because fogs formed at low temperatures over limited open water areas can have particles concentrations comparable to those observed, we conclude that the spikes must be condensate-derived.

The remaining question is, 'Are the particles liquid or solid?' Water droplets can freeze by either heterogeneous or homogeneous nucleation (Hobbs 1974). Heterogeneous nucleation is freezing activated by a foreign particle, an ice nucleus; homogeneous nucleation requires no foreign nucleus. Small water droplets (less than 50 μm) condensed from clean air can be supercooled to temperatures between −37°C and −41°C (Cwilong 1947; Schaefer 1948; Fournier D’Albe 1949; Mason 1952; Kuhns and Mason 1968) before they freeze by homogeneous nucleation. Droplets condensed in 'outdoor air' freeze at about −32°C (Cwilong 1947; Fournier D’Albe 1949; Mason 1950); this must be heterogeneous nucleation. In a natural environment as the temperature decreases from the heterogeneous nucleation threshold, water droplets and ice crystals coexist; but only below the homogeneous nucleation threshold do ice crystals proliferate (Fournier D’Albe 1949; Mossup
The best example of this spectacular increase in the number of ice particles near −40°C is Alaskan ice fog. Because of the high concentration of hydrocarbon pollutants in Fairbanks, slight ice fog is usually present at −30°C; when the temperature descends to −40°C, however, ice fog is always severe (Kumai and O'Brien 1965; Ohtake 1970). In all but one of our 21 experimental runs, the downwind temperature 50 cm above the surface of the lead, $T_{50}$, was above −30°C. Thus, even if the heterogeneous freezing threshold were as high as −30°C, water droplets would still have been about one thousand times more numerous over the leads than ice crystals (Fournier D'Albe 1949; Mossup 1956).

(b) Droplet size

Because of the difficulty in obtaining size information for individual droplets from the linearized anemometer signal, we assumed that the droplets were all the same size in our computations of concentration and mass flux. This assumption is fairly good since the droplets had such a short time to grow. In this subsection we will discuss the evidence on which our choice of droplet size is based.

One size estimate comes from studies of Fairbanks ice fog. Because ice-fog particles and the water droplets we are studying form by like processes – water vapour condensing into droplets at very low temperature and in conditions of high supersaturation – the particles should be of similar size. Crystals collected during ice-fog events in Fairbanks generally range from 2 to 15 μm in diameter (Kumai 1964; Ohtake 1967, 1970). Those collected near open water sources, however, have a narrower distribution centred at a diameter of about 10 μm (Ohtake 1970; Huffman and Ohtake 1971). Our first estimate of droplet diameter, therefore, is 8 to 12 μm.

The next size estimates are based on a model for a water droplet impacting on a hot-film that we discuss in the Appendix. There we derive three equations, (A5), (A8) and (A15), that relate $r_0$, the initial radius of a (hemispherical) droplet sitting on the sensor to, respectively: $t_b$, the time needed for an impacting droplet initially at ambient temperature to reach its boiling point; $t_e$, the time required for the impacting droplet to evaporate; and $e_{max}$, the maximum deviation of the voltage spike from the mean voltage level of the anemometer bridge.

Oscilloscope traces of droplet events indicate the ranges of $t_e$ and $e_{max}$ (Fig. 1). We assume that the time elapsed during the first half cycle of the spike corresponds to $t_e$. These times are generally 0-4 to 1 ms. Estimating $e_{max}$ necessitated converting the linearized voltage displayed on the oscilloscope to the corresponding bridge voltage. In the velocity network we sampled, excursions in bridge voltage were amplified about five times by the linearizing network, so values of $e_{max}$ range from 0-04 to 0-2 V.

Table 1 summarizes how spike characteristics are related to the droplet diameter in free air when typical anemometer values are used in Eqs. (A5), (A8) and (A15). The $t_e$ values indicate 6 to 10 μm diameter droplets were striking the sensors. The $e_{max}$ values show that the droplets could have had diameters of 6 to 25 μm. These results agree with our inferences from ice-fog studies: droplets formed over water at low temperature have a mode diameter near 10 μm.

As a third method of estimating droplet size and the width of the size distribution spectrum we modelled the growth and evaporation of water droplets in conditions typical over leads using an equation developed by Pruppacher and Klett (1978, Eq. (13–28)) to describe diffusional growth of an aqueous solution droplet at rest. We solved this equation numerically for droplet radius as a function of time by a second-order, single-step method.
TABLE 1. THE CORRESPONDENCE BETWEEN SPIKE CHARACTERISTICS AND DROPLET SIZE ON THE BASIS OF Eqs. (A5), (A8) and (A15). $d$ is the DIAMETER OF THE DROPLET IN FREE AIR AND $r_0$ IS THE RADIUS OF THE DROPLET ON THE SENSOR, $r_0 = 2^{-2/3}d$ (SEE APPENDIX).

<table>
<thead>
<tr>
<th>$d$ ($\mu$m)</th>
<th>$r_0$ ($\mu$m)</th>
<th>$t_k$ (ms)</th>
<th>$t_o$ (ms)</th>
<th>$e_{\text{max}}$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>15.7</td>
<td>0.6</td>
<td>7.3</td>
<td>0.19</td>
</tr>
<tr>
<td>20</td>
<td>12.6</td>
<td>0.4</td>
<td>4.7</td>
<td>0.15</td>
</tr>
<tr>
<td>15</td>
<td>9.45</td>
<td>0.2</td>
<td>2.6</td>
<td>0.11</td>
</tr>
<tr>
<td>12</td>
<td>7.56</td>
<td>0.14</td>
<td>1.69</td>
<td>0.090</td>
</tr>
<tr>
<td>10</td>
<td>6.30</td>
<td>0.10</td>
<td>1.17</td>
<td>0.075</td>
</tr>
<tr>
<td>8</td>
<td>5.04</td>
<td>0.06</td>
<td>0.75</td>
<td>0.050</td>
</tr>
<tr>
<td>6</td>
<td>3.78</td>
<td>0.04</td>
<td>0.42</td>
<td>0.045</td>
</tr>
<tr>
<td>4</td>
<td>2.52</td>
<td>0.02</td>
<td>0.19</td>
<td>0.030</td>
</tr>
<tr>
<td>2</td>
<td>1.26</td>
<td>0.004</td>
<td>0.047</td>
<td>0.015</td>
</tr>
<tr>
<td>1</td>
<td>0.630</td>
<td>0.0010</td>
<td>0.0117</td>
<td>0.0075</td>
</tr>
</tbody>
</table>

Although growth will be faster for moving droplets than for ones at rest, for the very small droplets we are studying this difference is negligible (Beard and Pruppacher 1971). The droplet condensation nucleus is water soluble and assumed pure.

Most information on potential condensation nuclei in the Arctic comes from K. A. Rahn and his colleagues in the Arctic Air-sampling Network (Kerr 1979). In winter the Arctic aerosol is pollution derived, coming predominantly from Eurasia via a short path over the North Pole (Kerr 1979; Rahn and McCaffrey 1979; Ottar 1980). Sulphate constitutes most of this aerosol, having an average peak concentration of 2 $\mu$g m$^{-3}$ at Barrow in March and April (Kerr 1979; Rahn 1979). Although the corresponding anion is still in question, we hypothesize it is ammonium on the basis of studies of the composition of both marine and continental aerosols reported by Mészáros (1968), Mészáros and Vissy (1974) and Jaenicke (1978). On reviewing some of this same evidence, Pruppacher and Klett (1978) also concluded that ammonium sulphate is a primary component of the atmospheric aerosol; and Shaw (1975) based an explanation of the high optical turbidity values he measured at Barrow in April and July of 1972 on the presence of high concentrations of ammonium sulphate particles.

From the measured value, 2 $\mu$g m$^{-3}$ of sulphate at Barrow in March and April, we estimate a concentration of 2-8 $\mu$g m$^{-3}$ of ammonium sulphate in the aerosol during ALEX. If particles of average radius 0.05 \(\mu\)m constituted this ammonium sulphate, there would have been 3000 particles cm$^{-3}$; for a radius of 0.1 $\mu$m, 380 particles cm$^{-3}$; or for a radius of 0.2 $\mu$m, 47 particles cm$^{-3}$. These particle concentrations at radii from 0.1 to 0.2 $\mu$m, the probable size range of the Barrow aerosol (Rahn 1979, personal communication; Bigg 1979), are similar to those reported by Mészáros and Vissy (1974) and Jaenicke (1978) and are completely compatible with our measurements of droplet concentrations.

Figure 2 shows some results of our numerical model of the growth of condensate droplets from diluence of air, ammonium sulphate particles of radii 0.1 $\mu$m \((8 \times 10^{-15} \text{ g})\) and 0.2 \(\mu\)m \((6 \times 10^{-14} \text{ g})\). For comparison we also computed growth on a sea-salt nucleus of radius 0.1 $\mu$m \((10^{-14} \text{ g})\). It is clear that the humidity conditions in which the growth is occurring are as important as the nature of the nucleus.

Growth for a relative humidity of 200% is modelled as such conditions are encountered over leads. In section 4 we will discuss a profile relation that describes the condensate concentration over leads (Eq. (4.25)). Because Andreas (1981) showed an equation of the same form models humidity profiles in the absence of condensation downwind of a step-increase in surface moisture flux, we used this equation to estimate the downwind humidity.
TABLE 2. THE ESTIMATED RELATIVE HUMIDITY PROFILE FOR RUN 262. T(z) IS THE MEASURED TEMPERATURE PROFILE; Q_{sat}(z), THE SPECIFIC HUMIDITY OF AIR IN SATURATION WITH WATER AT TEMPERATURE T(z); AND Q(z), THE HUMIDITY ESTIMATED BY A RELATION ANALOGOUS TO EQUATION (4.25), ASSUMING NO CONDENSATION

<table>
<thead>
<tr>
<th>z (cm)</th>
<th>T(z) (°C)</th>
<th>Q_{sat}(z) (g kg^{-1})</th>
<th>Q(z) (g kg^{-1})</th>
<th>R.H. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2.3</td>
<td>3.063</td>
<td>3.063</td>
<td>100</td>
</tr>
<tr>
<td>12</td>
<td>-26.47</td>
<td>0.415</td>
<td>1.412</td>
<td>340</td>
</tr>
<tr>
<td>23</td>
<td>-27.42</td>
<td>0.380</td>
<td>0.945</td>
<td>249</td>
</tr>
<tr>
<td>53</td>
<td>-28.13</td>
<td>0.356</td>
<td>0.510</td>
<td>143</td>
</tr>
<tr>
<td>100</td>
<td>-28.62</td>
<td>0.339</td>
<td>0.302</td>
<td>89</td>
</tr>
<tr>
<td>204</td>
<td>-28.88</td>
<td>0.331</td>
<td>0.197</td>
<td>60</td>
</tr>
<tr>
<td>254</td>
<td>-28.91</td>
<td>0.330</td>
<td>0.192</td>
<td>58</td>
</tr>
</tbody>
</table>

profile for Run 262 (Table 2), first using Eq. (4.12) to compute the surface moisture flux needed in the computation. From Table 2 it is obvious that, because of the severe temperature gradient near the surface of leads, remarkable supersaturation can occur. Ohtake (1970) described similar supersaturation over open water areas during ice-fog events in Fairbanks. It can be relieved only if sufficient condensation nuclei are present: homogeneous nucleation would play an insignificant role even at these saturation levels (Pruppacher and Klett 1978). We can see from the measured condensate profiles and will show again in section 4(b) that only a small fraction of the escaping vapour recondenses, however; evidently too few suitable nuclei are present. The supersaturation must persist until mixing occurs over the ice downwind of the lead.

In the extreme supersaturation over leads droplets can grow to a diameter of 10 μm in 2 seconds on any of the three condensation nuclei used to obtain Fig. 2. The transit times of air blowing across the leads we observed range from 2 to 12 seconds with an average of about 7 seconds. Thus, for a constant relative humidity of 200%, for example, droplets at the downwind sensors could have had diameters of 10 to 25 μm. It is, of course, unlikely
that the humidity environment remained constant around the droplet; Fig. 3 shows, however, that even if 10 \( \mu m \) diameter droplets were swept upward into less humid air, they would change little in size for several seconds.

Our modelling clearly has not treated many of the complexities of the nucleation and growth of condensate droplets over leads, but such is beyond our scope. This rudimentary study of the microphysics has been enough to substantiate our earlier size estimate of 10 \( \mu m \) for the diameter of the condensate droplets. We henceforth assume that all the droplets impacting on the hot-film sensors had a diameter of 10 \( \mu m \).

4. PROFILE CONCENTRATION MEASUREMENTS

Figure 4 shows 21 condensate concentration profiles collected over leads of fetches.
from 6.8 to 85 m. Total averaging time at each of the five profile levels was usually one and a half to two minutes. These profiles, in general, are indicative of flow over a surface moisture source: concentration is high near the surface and almost zero at the upper measurement height. Several profiles at the longest fetches are anomalous, however; they have concentration maxima in the mid-region of the profile. During these runs ice was forming on the downwind side of the leads, so the surface below the sensors was no longer a vapour source.

(a) The mass flux

From the concentration and downwind velocity profiles we can calculate the condensate flux. Suppose the wind is blowing in the positive x-direction, the lead is parallel to the y-axis, and the z-axis is vertically upward. Let \( x = 0 \) be the upwind edge of the lead; the sensors are at a fetch \( x = X \). Consider a control volume over the lead with a front side in the y-z plane at \( x = 0 \) and a back side at \( x = X \). The height of this volume, \( z' \), is greater than the height of the internal boundary layer, \( \delta(x) \), for all \( x \) out to \( X \). If conditions are at steady state, the moisture content of this control volume will be constant. Because the moisture has two forms, vapour and condensate, the moisture budget of the volume is then

\[
0 = \rho \int_0^{z'} U(z) (Q(z) + C(z)) \, dz + \rho \int_0^X (\bar{wq}_0 + \bar{wc}_0) \, dx \\
- \rho \int_0^X W(x, z') (Q(z') + C(z')) \, dx - \rho \int_0^X (\bar{wq}_i + \bar{wc}_i) \, dx \\
- \rho \int_0^{z'} U(X, z) (Q(X, z) + C(X, z)) \, dz
\]

(4.1)

Here \( Q(z) \) and \( C(z) \) are the specific humidity and condensate concentration profiles upwind over the pack ice; \( U(z) \) is the upwind velocity profile; \( Q(X, z) \), \( C(X, z) \) and \( U(X, z) \) are the corresponding downwind profiles at \( x = X \); \( W(x, z') \) is the vertical velocity over the lead at height \( z' \); \( \bar{wq}_0 \) and \( \bar{wc}_0 \) are the turbulent vapour and condensate fluxes at the surface of the lead; and \( \bar{wq}_i \) and \( \bar{wc}_i \) are the turbulent fluxes upwind, constants with height if the upwind ice is horizontally homogeneous.

We can make several simplifications in Eq. (4.1). Substituting for \( W(x, z') \) using the integrated, two-dimensional continuity equation,

\[
W(x, z') = - \int_0^{z'} \partial U/\partial x \, dz,
\]

(4.2)

makes the third integral in (4.1) easy. Since there were no droplets upwind, \( C(z) \) and \( \bar{wc}_i \) are zero. \( \bar{wq}_i \) is also zero: the moisture flux over ice in winter is negligible (Badgley 1966; Maykut 1978). Finally, because there is no evidence that moisture is leaving the surface as droplets, \( \bar{wc}_0 \) is also zero. All that remains of Eq. (4.1) is

\[
\rho \int_0^X \bar{wq}_0 \, dx = \rho \int_0^{z'} U(X, z) (Q(X, z) + C(X, z)) \, dz - \\
- \rho \int_0^{z'} U(z) Q(z) \, dz - \\
- \rho \int_0^{z'} (U(X, z) - U(z)) Q(z') \, dz
\]

(4.3)

On adding and subtracting
to (4.3) we get
\[ \rho \int_0^z U(X,z) Q(z) \, dz \]

\[ \rho \int_0^X wq_q \, dx = \rho \int_0^z U(X,z)[(Q(X,z) - Q_i) + C(X,z)] \, dz \]

\[ + \rho \int_0^z (U(X,z) - U_i(z)) [Q(z) - Q_i(z')] \, dz \quad (4.4) \]

Implicit in our earlier statement that \( wq_i \) is zero is the corollary that \( Q_i(z) \) is constant with height; \( Q_i(z) - Q_i(z') \) therefore is zero. The moisture budget of the control volume is then simply
\[ \rho \int_0^X wq_q \, dx = \rho \int_0^{\delta_q(X)} U(X,z)[Q(X,z) - Q_i] \, dz + \rho \int_0^{\delta_c(X)} U(X,z) C(X,z) \, dz \quad (4.5) \]

In this, \( \delta_q \) and \( \delta_c \), the respective heights of the vapour and condensate internal boundary layers at fetch \( X \), have replaced \( z' \) since \( Q - Q_i \) and \( C \) are zero above these heights by definition.

The left-hand side of Eq. (4.5) is the total amount of moisture escaping from the surface of the lead per unit length. The first term on the right-hand side is the portion of that flux passing through the downwind plane of the lead as vapour; the second term is the portion carried by the liquid phase. On dividing Eq. (4.5) by \( X \), we can approximate the average moisture flux per unit area by
\[ F_E = \rho X^{-1} \int_0^{\delta_q} U(Q - Q_i) \, dz + \rho X^{-1} \int_0^{\delta_c} U C \, dz \quad (4.6) \]

The moisture flux is, thus, composed of two measurable components, a vapour flux and a condensate flux,
\[ F_E = F_Q + F_C, \quad (4.7) \]

where
\[ F_Q = \rho X^{-1} \int_0^{\delta_q} U(X,z)[Q(X,z) - Q_i] \, dz, \quad (4.8) \]

\[ F_C = \rho X^{-1} \int_0^{\delta_c} U(X,z) C(X,z) \, dz. \quad (4.9) \]

Although the condensate is forming somewhere above the surface, Eq. (4.9) for condensate flux has exactly the same form as the equations used by Andreas et al. (1979) to determine the true surface fluxes, sensible and latent heat. We henceforth treat the condensate as a surface flux.

Because we have measured \( U(X,z) \) and \( C(X,z) \) only at discrete points, for purposes of the integration in Eq. (4.9) we interpolated between profile levels with semi-logarithmic arcs. \( z = 0 \) therefore became \( z_0 = 0.02 \) cm (Kondo et al. 1973; Smith and Banke 1975), the surface roughness, where \( U(z_0) = 0 \). To find \( C(z_0) \) we extrapolated the concentration measurements at the two lowest levels down to \( z_0 \) using Eq. (4.22). With our lowest sampling levels so near the surface, errors in \( z_0 \) or \( C_0 \) could lead to errors in the computed fluxes of no more than 10%. Table 3 lists values of \( F_C \) calculated using Eq. (4.9).

Andreas et al. (1979) showed that sensible heat and evaporative fluxes from leads in winter can be predicted from bulk environmental parameters. If these fluxes are non-dimensionalized in the form of Nusselt numbers, the Nusselt numbers correlate with a fetch (length) Reynolds number.
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where $U_{200}$ is the wind speed 200 cm above the lead at $x = X$ (little different from the upwind speed) and $v$ is the kinematic viscosity of air evaluated at the temperature of the water surface, $T_w$. Non-dimensionalizing fluxes as Nusselt numbers is a judicious way to analyse scalar fluxes in a developing boundary layer because of the many theoretical and experimental studies that have used this approach. This body of theoretical work shows, for example, that in forced convection – the transfer regime over the leads we studied – the Nusselt number will depend on only the fetch Reynolds number and the Prandtl or Schmidt numbers and, in particular, will not depend on a stability parameter (Schlichting 1968). Because guidelines like these clearly simplify our analysis, we attempt the same correlation of Nusselt and Reynolds numbers for the condensate flux data.

The Nusselt number based on condensate flux is

$$\text{Nu}_c = F_c X / \left( \rho D_e (C_0 - C_{200}) \right) \quad . \quad . \quad . \quad (4.11)$$

Here $D_e$ is the molecular diffusivity of water vapour in air, and $C_{200}$ is the downwind droplet concentration 200 cm above the surface. Figure 5 shows a plot of $\text{Nu}_c$ versus $R_x$.

![Figure 5. The condensate flux Nusselt number correlated with fetch Reynolds number. The line is Eq. (4.12).](image)

for our highest quality profiling runs. Andreas et al. (1979) suggested that the Nusselt numbers formed from the sensible heat flux, $\text{Nu}_H$, and the evaporative flux, $\text{Nu}_E$, are equal and that

$$\text{Nu}_H = \text{Nu}_E = 0.14 \, R_x^{0.72} \quad . \quad . \quad . \quad (4.12)$$

Eq. (4.12) is the line drawn in Fig. 5; the data points, though scattered, collect around it.

The above-mentioned Nusselt number formed with the evaporative flux – the total moisture flux – is

$$\text{Nu}_E = F_e X / \left( \rho D_e (Q_w - Q_{200} - C_{200}) \right) \quad . \quad . \quad (4.13)$$
where $Q_w$ is the specific humidity of air in saturation with a sea water surface at temperature $T_w$, and $Q_{200}$ is the downwind humidity of air 200 cm above the surface—estimated by assuming saturation in air at temperature $T_{200}$ (Andreas et al. 1979). We can likewise define a vapor flux Nusselt number,

$$ \text{Nu}_Q = F_Q X / \{ \rho D_v (Q_0 - Q_{200}) \} , \quad \quad (4.14) $$

in which $Q_0$ is the specific humidity at $z = z_0$ and

$$ Q_w \approx Q_0 + C_0 . \quad \quad (4.15) $$

Using Eqs. (4.11) and (4.13–4.15) in (4.7), we find that

$$ 0 = (Q_0 - Q_{200}) (\text{Nu}_E - \text{Nu}_Q) + (C_0 - C_{200}) (\text{Nu}_E - \text{Nu}_C) . \quad \quad (4.16) $$

Figure 5 suggests that $\text{Nu}_E$ and $\text{Nu}_C$ are equal; Eq. (4.16) thus implies that $\text{Nu}_E$ and $\text{Nu}_Q$ are also equal. Apparently all Nusselt numbers non-dimensionalizing scalar fluxes from a lead are equal:

$$ \text{Nu}_H = \text{Nu}_E = \text{Nu}_C = \text{Nu}_Q . \quad \quad (4.17) $$

Recent work by Andreas (1981) substantiates this Nusselt number equality.

Equation (4.17) is a powerful result; from it we can estimate any of the turbulent fluxes by measuring only one of them and a few environmental parameters. It also lets us estimate how the fluxes are partitioned. For example, the total evaporative flux is partitioned between vapor and condensate fluxes according to

$$ F_Q / F_C = (Q_0 - Q_{200}) / (C_0 - C_{200}) \quad \quad (4.18) $$

On the basis of our measurements of the quantities on the right-hand side of (4.18), $F_Q / F_C$ ranges from 4 to 20: the vapor accounts for the great majority of the evaporative flux.

(b) The generalized profile

The velocity and temperature profiles collected over leads by Andreas et al. (1979) did not have a semi-logarithmic form. The condensate concentration profiles shown in Fig. 4 also are not linear when plotted against log $z$. Considering the pronounced instability in the airflow over leads and the fetch dependence of the turbulence, the deviation of these velocity and temperature profiles from a semi-logarithmic profile is not unexpected; what is unexpected is our discovery that when $C(z)$ is plotted against log $z$ the result is nearly linear (Fig. 6). We try with the following model to explain this simple and consistent behaviour of the condensate profiles.

The standard approach to representing profiles of meteorological variables in the atmospheric surface layer is to assume that the profile gradient scales with $z$, the measurement height, and with some scaling parameter having the same units as the variable of interest. For the condensate profiles we will denote this scaling parameter by $\hat{\theta}$; then

$$ \partial_z C(z) \propto \hat{\theta} / z , \quad \quad (4.19) $$

where $\partial_z$ is the partial derivative with respect to $z$. Because the IBL is developing downwind, $z$ is not the only length scale important over leads, however; there must also be a fetch dependence in the profile. This manifests in $\hat{\theta}$. If $\hat{\theta}$ were independent of position, it would also be constant with height, and we would derive the semi-logarithmic profile, $C(z) \propto \ln z$, from Eq. (4.19). This profile form is not accurate since $\hat{\theta}$ is a function of both height and fetch.

We can build on Eq. (4.19) by including a dependence on both length scales in $\hat{\theta}$.
$|\partial_z C(z)|$ and $|\hat{C}(z)|$ must decrease with height. And because the lead is the source of the vapour from which the condensate forms, there can be no condensate above some fetch-dependent $h$ representative of IBL height. $C(z)$ and $\partial_z C(z)$ must, thus, go to zero at $h$. A simple model for $\hat{C}(z)$ that satisfies these restrictions is

$$\hat{C}(z) \propto \ln(z/h), \quad z_0 \leq z \leq h$$

(4.20)

Inserting (4.20) into (4.19), we get

$$\int_{C(z)}^{C(h)} dC \propto \int_{z}^{h} (z')^{-1} \ln(z'/h) dz'$$

(4.21)

Since $C(h) = 0$, the integration yields

$$C(z) \propto \{\ln(z/h)\}^2$$

(4.22)

or

$$C(z)^{1/2} \propto \ln(z/h)$$

(4.23)

Figure 6. The condensate concentration profile for Run 262. Notice how $h$ is defined.

Figure 6 shows the accuracy of (4.23) in describing the condensate profiles. $h$ is defined as the height at which a straight line drawn through the $C(z)^{1/2}$, log $z$ data intersects the log $z$-axis and so estimates the height of the IBL.

For (4.23) to be a useful prognostic equation we must evaluate the proportionality constant. Following the usual practice in surface layer meteorology, we define a concentration scale $c_*$ as

$$c_* = -(F_c/\rho) k u_\star,$$

(4.24)

where $k$ is the von Kármán constant (0.4), and the $u_\star$ values are those discussed by Andreas et al. (1979). $c_*$, $u_\star$ and $h$ are tabulated in Table 3. Plots of condensate profiles non-dimensionalized with $c_*$ and $h$ are very consistent (Fig. 7). The line in Fig. 7 intersects the $(z/h)$-axis
at one and has a slope of $-0.34$. The general form of the condensate profiles over leads is consequently

$$\left\{ -C(z)/c_m \right\}^{1/2} = -0.34 \ln(z/h), \quad z_0 \leq z \leq h. \quad (4.25)$$

There is some evidence that the multiplicative constant, $-0.34$, may depend weakly on stability; Andreas (1981) reported a form like (4.25), but with a constant of $-0.37$, for humidity profiles collected in a stably stratified IBL.

Andreas et al. (1979) showed that over leads

$$u_* = 0.0643 U_{50} - 0.77, \quad \quad (4.26)$$

with $u_*$ in cm s$^{-1}$ for $U_{50}$, the velocity 50 cm above the lead, also in cm s$^{-1}$. This result combined with Eqs. (4.24) and (4.25) implies that the condensate flux, $F_c$, can be estimated from one wind speed measurement, condensate measurements at two heights within the 1BL, and temperature and pressure measurements for determining $\rho$.

Figure 8 shows that, as expected, $h$, our estimate of the height of the condensate boundary layer, increases with the fetch, $X$; the line is drawn by eye so that $h$ is proportional to $X^{0.8}$ (Sutton 1934, 1953; Calder 1949; Elliott 1958; Philip 1959). If we estimate $z_w$ from Fig. 1 in Garratt and Hicks (1973) using average values for roughness length, $z_0$ (0.025 cm),
and roughness Reynolds number, $R_{\kappa} (2.5)$, $z_w = 0.025 \text{cm}$. The line in Fig. 8 is then

$$h/z_w = 1.31 (X/z_w)^{0.8} \quad \ldots \quad (4.27)$$

We know of only one comparable experimental study of a moisture boundary layer of geophysical scale with which we can compare Eq. (4.27) -- that by Rider et al. (1963). They, unfortunately, said nothing about how the IBL develops with fetch; our reanalysis of their data set, with emphasis on this point, will be the subject of a forthcoming paper (Andreas 1981).

On discovering how well Eq. (4.25) represents the condensate profiles, we tested it on the temperature profiles measured over leads by Andreas et al. (1979). Because these profiles were incompatible with this $(\ln z)^2$ model, there must be a fundamental difference in the ways condensate and temperature are transported in the IBL. There have been other reports of such differences in heat and moisture transfer (e.g. McBean and Miyake 1972; McIntosh et al. 1975). On the basis of a theoretical study, Warhaft (1976) concluded that the transports of heat and moisture will be similar only if the turbulent fluctuations in temperature and moisture are correlated and, in particular, will be dissimilar if the vertical gradients of the scalars have opposite signs or if either temperature or moisture -- but not both -- is acting as a passive contaminant. In the IBL over leads condensate and humidity are passive contaminants while temperature is an active contaminant. The temperature profile can thus not satisfy an equation like (4.25) but must depend strongly on yet a third length scale that characterizes the stability.

5. CONCLUSIONS

The versatility of hot-film sensors is unique among turbulence transducers; during the AIDJEX Lead Experiment we have used them to measure average and turbulent velocity components and condensate concentration. Because anemometer voltage fluctuations produced by the turbulence and by the impacting condensate droplets occurred at very different frequencies, records of the two phenomena were easily separable with proper filtering.

Conditions over leads are extraordinary. The temperature can drop 20–30 °C between
the water surface and a height of two metres. This extreme temperature gradient, the large moisture flux from leads, and the paucity of condensation nuclei foster supersaturations of over 100% near the surface. In such a humidity environment condensate droplets can grow rapidly; the aerosol we observed consisted of condensate droplets of roughly 10 μm diameter.

Profiles of condensate concentration at the downwind edge of leads have the non-dimensional form

\[-\frac{C(z)}{c_s} = -0.34 \ln(z/h),\]

where \(c_s\) is a concentration scale and \(h\) is a fetch-dependent estimator of internal boundary layer height. From the measured concentration and velocity profiles we also calculated the condensate flux from leads, evidently the first measurement of this quantity. When non-dimensionalized in the form of a Nusselt number, this flux is compatible with a relation found to describe the sensible heat flux from leads,

\[\text{Nu}_H = \text{Nu}_C = 0.14 R e^{0.72}.\]

In fact, Nusselt numbers non-dimensionalizing the turbulent flux from leads of any scalar quantity seem to obey Eq. (5.2); consequently, for a given \(R\), all such Nusselt numbers are equal. Because \(c_s\) and \(Nu_C\) both contain the condensate flux, Eqs. (5.1) and (5.2) imply two different ways of estimating that flux by measuring only average quantities. And the equality of Nusselt numbers permits evaluation of the partitioning of energy or mass among the various fluxes.

Appendix

The Cylindrical Hot-Film as an Aerosol Sampler

(a) Equations relating spike characteristics to droplet size

With a simple model we can show how characteristics of a voltage spike indicate the size of the water droplet that produced the spike. Suppose a droplet at ambient temperature strikes the sensor, becomes attached to it, and evaporates since the sensor temperature is 200°C. Because the diameter of the hot-film is 53 μm, a droplet with a radius in free air, \(r_f\), of 4 to 6 μm will be virtually hemispherical sitting on the sensor. The initial radius, \(r_0\), of the attached droplet is thus \(2^\frac{3}{2}r_f\). Since the sensor resistance is held constant, the cooling effect of this droplet necessitates a voltage increase through the sensor. This is the voltage spike.

Assume evaporation occurs in two distinct steps (Goldschmidt and Householder 1968, 1969): (1) the droplet attaches and is heated to its boiling temperature, \(T_b\), without evaporation (without changing radius); (2) evaporation then occurs with the droplet at \(T_b\). Simple equations describe each of these processes. During the heating the flux rate through the base of the hemispherical droplet must equal the change in energy content of the droplet:

\[\pi r_0^3 \kappa (T_s - T_b)/(2 r_0) = \frac{8}{3} \pi r_0^3 \rho_w c_p d_i(T(t))\]

As the droplet vaporizes the latent heat loss will equal the flux rate:

\[\pi r(t)^3 \kappa (T_s - T_t)/(2 r(t)) = -2 \pi r(t)^2 \rho_v L_v d_i(r(t))\]

In (A1) and (A2) \(\kappa\), \(\rho_w\), \(c_p\), and \(L_v\) are the thermal conductivity, density, specific heat at constant pressure, and latent heat of vaporization of water; \(T_s\) is the sensor temperature; \(T(t)\), the droplet temperature during the heating; \(r(t)\), the droplet radius during evaporation; and \(d_i\) denotes differentiation with respect to time. In the left term of each equation we have
approximated the temperature gradient normal to the sensor surface by \( \Delta T/(2r) \), where \( \Delta T \) is the difference between the sensor and droplet temperatures, \( r \) is the droplet radius, and the denominator, \( 2r/3 \), is the average depth of the droplet.

Solution of the two equations is straightforward. Equation (A1) reduces to

\[
(4r_0^2/9)(\rho_w c_p/\kappa) d_T + T = T_s \quad . \quad . \quad . \quad (A3)
\]

At \( t = 0 \) the droplet is at ambient temperature, \( T_a \); hence,

\[
T(t) = T_s - (T_s - T_a) \exp(-9D_w t/(4r_0^2)), \quad . \quad . \quad . \quad (A4)
\]

where \( D_w \) is the thermal diffusivity of water. The time \( t_b \) at which the droplet reaches its boiling temperature, \( T_b \), is consequently

\[
t_b = -(4/9D_w) \ln\left\{ (T_s - T_b)/(T_s - T_a) \right\} r_0^2 \quad . \quad . \quad . \quad (A5)
\]

This is one relation between spike characteristics and droplet size.

The solution of Eq. (A2) yields a second one. Simplify (A2) to

\[
r d_T r = 3\kappa(T_s - T_b)/4\rho_w L_v \quad . \quad . \quad . \quad (A6)
\]

The radius \( r \) at time \( t \geq t_b \) therefore is

\[
r(t) = \left\{ r_0^2 - (3\kappa/2\rho_w L_v)(T_s - T_b)(t - t_b) \right\}^{1/2}, \quad . \quad . \quad . \quad (A7)
\]

and the time \( t_e \) required for the droplet to evaporate (i.e. \( r(t_e) = 0 \)) is

\[
t_e = t_b + \left\{ 2\rho_w L_v/3\kappa(T_s - T_b) \right\} r_0^2 \quad . \quad . \quad . \quad (A8)
\]

Again a spike characteristic depends on droplet size.

Following a method similar to that used by Goldschmidt (1965) for hot-wires, we can show spike amplitude is also dependent on droplet size. The heat transfer relation for the hot-film sensors used during ALEX is (Andreas 1979)

\[
\text{Nu} = \text{Nu}_0 - 0.31 + 0.55 R^{0.46} \quad . \quad . \quad . \quad (A9)
\]

where \( \text{Nu} \) is the Nusselt number,

\[
\text{Nu} = \bar{E}^2 R_s/\pi R_t^2 H \kappa_s(T_s - T_a), \quad . \quad . \quad . \quad (A10)
\]

and \( R = \bar{U}/D_v \) is the Reynolds number. Here \( \bar{E} \) is the instantaneous anemometer bridge voltage; \( R_s \), the hot-film sensor resistance; \( R_t \), the total resistance in the sensor leg of the anemometer bridge; \( H \), the sensor length; \( \kappa_a \) and \( \nu \), the thermal conductivity and kinematic viscosity of air; \( \bar{U} \), the instantaneous cooling velocity at the sensor; and \( D_v \), the sensor diameter. To compute \( \text{Nu}_0 \), \( \bar{E} \) in Eq. (A10) becomes the bridge voltage with the sensor shielded from the wind.

In the standard Reynolds decomposition, effective cooling velocity consists of an average, \( U \), and a turbulent component, \( u \); the instantaneous bridge voltage then also has an average, \( \bar{E} \), and a turbulent component, \( e \). In addition, however, there are voltage fluctuations, \( e \), occurring because the anemometer electronics respond to the heat extracted by the impacting condensate droplets. Because on impact a droplet apparently throws the bridge out of equilibrium for a moment, there is no obvious heat budget expression for the droplet-sensor system as the droplet warms. By the time the droplet reaches temperature \( T_s \), however, equilibrium has been re-established. The energy balance as the droplet evaporates is therefore

\[
(E + e_u + e)^2 R_s/\nu^2 = \pi H \kappa_s(T_s - T_a)[\text{Nu}_0 - 0.31 + 0.55[(U + u)D_v/\nu]^{0.46}] -
\]

\[
-2\pi\{r(t)^2\} \rho_w L_v d_T \{r(t)\} \quad . \quad . \quad . \quad . \quad (A11)
\]
Figure 1 shows that for short time intervals ($\sim 10$ ms) the turbulent voltage fluctuations simply add to the average voltage to make an effective DC level above which the droplet spikes prominently stand. Hence, because the amplitude of these spikes is our only interest, we can write $E + e_u \sim E$ and $U + u \sim U$. With the substitution of Eq. (A9) for the term in large brackets in Eq. (A11), we consequently get

$$ (E + e)^2 p = E^2 p - 2\pi\{r(t)\}^2 \rho_w L_e d_e \{r(t)\}, \quad \ldots \quad (A12) $$

where $p = R_e/R_f^2$.

Now using Eqs. (A6) and (A7) to evaluate $d_e \{r(t)\}$ and $r(t)$ in (A12), we obtain

$$ e(t) = E[1 + {3\pi\kappa(T_s - T_b)/2pE^2}\{r_0^2 - (3\kappa/2\rho_w L_e)(T_s - T_b)(t - t_b)\}^4] - E \quad (A13) $$

for $t \geq t_b$. Since the second term in the large brackets in (A13) is small compared to one, we make a binomial expansion, keeping only the first order term:

$$ e(t) = 3\pi\kappa(T_s - T_b)/4pE \{r_0^2 - (3\kappa/2\rho_w L_e)(T_s - T_b)(t - t_b)\}^{\frac{3}{2}}. \quad (A14) $$

The maximum amplitude of the voltage spike seems to occur when the droplet just reaches its boiling point (Goldschmidt 1965; Goldschmidt and Householder 1968): that is, at $t = t_b$. Hence,

$$ e_{\text{max}} = (3\pi\kappa/4pE)(T_s - T_b)r_0 \quad \ldots \quad (A15) $$

This is a third relation between a spike characteristic and droplet size.

\(b\) The collection efficiency of hot-film sensors

A hot-film sensor is not a perfect droplet collector. Not all droplets seemingly on a collision course with the cylinder are counted; some are diverted around the cylinder when the streamlines turn to satisfy mass continuity, others might strike the sensor but fail to cling, and still others might shatter on impact. We will treat here only the first source of inefficiency, inertial effects; investigations by Goldschmidt and Householder (1969) showed that for our experimental conditions the second and third sources, failure to cling and droplet shattering, would have been important only for droplets of diameter 100 $\mu$m or greater.

In computing the droplet concentration, $N$ (droplets per unit volume), from the number of voltage spikes, $S$ (counts per unit time), recorded on magnetic tape, we must introduce an inertial impaction coefficient, $I$, reflecting the collection efficiency of the hot-film sensors. Then

$$ N = S/UDH, \quad \ldots \quad (B1) $$

where $U$ is the average wind speed at the sensor; $D$, the sensor diameter; and $H$, the sensor length. This impaction coefficient, $I$, is defined simply as the ratio of the area at infinity containing droplets that will hit the sensor to the total area of the target.

Attempts to find $I$ either analytically or experimentally begin with the specification of two non-dimensional parameters, the Reynolds number,

$$ R = UD/v, \quad \ldots \quad (B2) $$

and a particle parameter,

$$ P = \rho_w d^2 U/18\mu D \quad \ldots \quad (B3) $$
(e.g. May and Clifford 1967). In these ν and μ are the kinematic and dynamic viscosities of air, and ρw is the density of the aerosol substance – in our case, water. For condensate droplets 10 μm in diameter, P values during our experiment were between 3 and 23 and R values were 2 to 16. The ratio d/D was 10/53 ~ 0.2.

No experimental or theoretical study of I we have encountered (Langmuir and Blodgett 1945; Wong et al. 1955; Davies and Peetz 1956; May and Clifford 1967; Householder and Goldschmidt 1969) has duplicated the parametric ranges of our experiment, however. Davies and Peetz did predict I in the P and d/D ranges of our experiment for one particular value of R in our range, R = 10, while Householder and Goldschmidt (1969) formulated an expression for I as a function of R, P, and d/D for R and P in our experimental range but for d/D values from 1 to 7. To get an analytic expression for the computer analysis of our data, we adapted the equation of Householder and Goldschmidt (1969) to a d/D ratio of 0.2 using the predictions of I at R = 10 and d/D = 0.2 by Davies and Peetz (1956). The resulting equation for I is

\[ I = \frac{1}{2} + \frac{1}{2} \tanh[-0.150 + 0.1921 \ln(1 + 2P) + 0.0066 \ln(1 + R) + 0.0287(\ln(1 + R))^2], \]  

(B4) good for d/D = 0.2, P in [3, 23], and R in [2, 16].

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