Synoptic-scale forcing of coastal lows: forced double Kelvin waves in the atmosphere*

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SUMMARY

Gill (1977) has interpreted the coastal lows of southern Africa as atmospheric coastal Kelvin waves. The lows are trapped horizontally next to the steep slopes of the Great Central Plateau by Coriolis forces and vertically by a low-level inversion. This hypothesis is tested and extended using two linear models.

A barotropic primitive-equation analysis on an equatorial beta-plane models the generation of these waves. An impermeable eastern boundary is included to reflect the blocking of the low-level flow below the coastal inversion by the orography. Results of the linear analysis suggest that the formation of the coastal lows may be explained in terms of the scattering of eastward-propagating synoptic-scale disturbances at the meridional boundary. The boundary response is structurally similar to a Kelvin wave. At the frequencies of interest, no zonally propagating Rossby waves are excited. Further, in contrast to the case of a western boundary, the equatorial response is negligible.

A two-layer, f-plane model enables the dynamics of the layer below the interior inversion to be incorporated. At the period of the synoptic forcing (six days), the boundary response is a maximum along the escarpment and decays exponentially from it. The e-folding distance is slightly less than the appropriate Rossby radius of deformation. These results, together with observations, suggest that the coastal low and its companion low over the interior may be interpreted as forced internal double Kelvin waves.

The possible application of these results to foehn (chinook) winds and lee cyclogenesis is mentioned.

1. INTRODUCTION

Gill (1977) has interpreted the coastal lows of southern Africa as atmospheric coastal Kelvin waves. He suggested that the waves are produced by the blocking action of the escarpment of southern Africa on the low-level flow associated with synoptic-scale migratory cyclones and anticyclones. A strong low-level inversion traps the response vertically and prevents significant flow over the escarpment which rises above the mean level of the inversion. Gill presented a forced non-linear Kelvin wave analysis that suggests that lows, rather than highs, are preferentially generated by the synoptic-scale forcing.

Figure 1 provides an example of a coastal low, located on the west coast at 25°S. The low has its maximum strength at the coast and weakens rapidly out to sea. Later maps (see Gill 1977) detail the southward propagation of the low, its rounding of the Cape, and its eventual northward propagation along the east coast. Typical phase speeds are 5–10 m s⁻¹ with a period of six days (Preston-Whyte and Tyson 1973). The passage of the low is associated with a drop in the height of the coastal inversion at the 1 km level. For a more complete discussion of the observational aspects of the coastal low, the reader is referred to Gill (1977) and the literature cited therein.

In Fig. 1 a middle-latitude high-pressure cell lies to the south of the continent. This cell propagates eastward during the described sequence and is eventually replaced by a migratory cyclone. The coastal low lags behind the mid-latitude anticyclone but rounds the Cape before the arrival of the cyclone.

Typically the coastal low is accompanied by a similar cell located over the interior plateau. This companion low is most intense near the coast and decays with distance.

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inland. It propagates together with the coastal low. Over the interior an inversion is present 1–2 km above the surface (which is 1–1.5 km above sea level). Characteristics of both coastal and plateau inversions are given in Preston-Whyte et al. (1977).

The present study refines and extends the work of Gill (1977) using two linear analyses. A barotropic primitive-equation model on an equatorial beta-plane is used to study the generation of the lows. An impermeable eastern boundary is included to model the blocking of the low-level flow by the orography. The forcing of the motion by middle-latitude disturbances is modelled by a prescribed eastward-propagating zonal wind field. The forcing has a six-day period. It is shown that the boundary response is trapped next to the eastern boundary and is structurally similar to a coastal Kelvin wave. In contrast, Gill (1977) assumed a priori that the boundary response is a Kelvin wave on an $f$-plane.

Some comment on the choice of an equatorial beta-plane is necessary. The $f$-plane analysis of Gill (1977) is overly restrictive in that all forcings with periods greater than the pendulum day will generate a horizontally trapped response along the meridional boundary. Freely propagating Rossby waves could compose part of the boundary response but are excluded in that analysis. A beta-plane analysis is therefore required to explore this possibility and determine under what conditions a Rossby wave could be excited. In section 3 it is shown that a zonally propagating response is more likely to occur on an equatorial beta-plane than a mid-latitude one. However, at the frequencies and parameter settings of interest here, no propagating Rossby waves are excited. In addition, with the synoptic forcing extending equatorward of 15–20°S, an equatorial model is suggested. Lastly the choice of an equatorial beta-plane allows one to examine the possibility of an interhemispheric connection. The analysis therefore complements the work of Bannon (1979) in which the scattering at a western boundary is considered.

In addition, a two-layer $f$-plane analysis is performed to include the response of the layer of fluid inland from the coast above the Great Central Plateau. This upper layer is also capped by an inversion. At the period of the synoptic forcing (six days), the boundary response is a maximum along the escarpment and decays exponentially away from it in qualitative agreement with Fig. 1. The two-layer results have some features of the double Kelvin (seascarp) waves of Longuet-Higgins (1968). These barotropic modes have periods less than the inertial period and are laterally trapped next to a discontinuity in the fluid.
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depth. Here the deeper layer is divided in two in order to reflect the stratification of the coastal region of southern Africa. The shallower layer of fluid is not similarly subdivided. In this way the present analysis differs from the oceanographical literature (e.g. Wang 1975) of baroclinic coastally trapped waves along the continental shelf.

In the next section the general features of the two models are described. Section 3 presents the barotropic beta-plane analysis for the layer of fluid lying below the coastal inversion. Section 4 includes the dynamics of the fluid below the inversion of the interior plateau. The paper concludes with a summary of the model’s results and a discussion of their limitations and further applications.

2. THE GENERAL MODEL

A rotating Cartesian coordinate system \((x, y, z)\) is assumed (see Fig. 2). The angular rotation vector is \(\frac{1}{2}fz\) and is parallel to gravity, \(-gz\). The unit vectors \(\hat{x}\) and \(\hat{y}\) denote eastward and northward directions, respectively. The west coast of Africa lies along the line \(x = 0\). To the east the Great Central Plateau rises discontinuously from the sea surface \((z = 0)\) to the height \(z_B\).

![Figure 2. Geometry of model.](image)

The stratification of the region is modelled using two homogeneous layers. The fluid below the coastal inversion has a depth \(h_0(x, y, t)\), a mean depth \(H_0 < z_B\), and a uniform potential temperature \(\theta = \theta_0\). Similarly, the plateau inversion marks the top of a layer of fluid of depth \(h_2(x, y, t)\), mean depth \(H_2\), and potential temperature \(\theta_1\). For simplicity the plateau inversion is assumed to extend continuously out to sea where it lies at the height \(z = h_0(x, y, t) + h_1(x, y, t)\). Preston-Whyte et al. (1977) note that the occurrence of multiple upper-level inversions along the west coast is not unusual. An inert semi-infinite layer of potential temperature \(\theta_T\) lies above this second inversion.

The values of these parameters are given in Table 1. The heights of the west coast and interior plateau inversions follow Preston-Whyte et al. (1977). The strength of the coastal inversion \((\theta_1 - \theta_0) = 12\ K\) agrees with Gill (1977). Here \(\theta_T\) is the average (mass-weighted) tropospheric potential temperature from 700 to 200 mb.
TABLE 1. Parameter settings

\begin{align*}
H_0 &= 1.0 \text{ km} & \theta_0 &= 290 \text{ K} \\
H_1 &= 2.5 \text{ km} & \theta_1 &= 302 \text{ K} \\
H_2 &= 2.0 \text{ km} & \theta_T &= 326 \text{ K} \\
z_B &= 1.5 \text{ km} & \\
\end{align*}

Each layer is an inviscid Boussinesq fluid in hydrostatic balance. Vertical integration of the hydrostatic relation downward from \( z = \infty \) yields the fluid pressure \( p \) at a point \( z \) in each layer:

\begin{align}
\rho^2 = P_0 + \rho_0 g_1 (h_2 + z_B) - \rho_1 g z & \quad \text{(2.1a)} \\
\rho^1 = P_0 + \rho_0 g_1 (h_0 + h_1) - \rho_1 g z & \quad \text{(2.1b)} \\
\rho_0 = P_0 + \rho_0 g_1 (h_0 + h_1) + \rho_0 g_0 h_0 - \rho_0 g z & \quad \text{(2.1c)}
\end{align}

where \( P_0 (\rho_0) \) is a constant reference pressure (density). Here \( g_0 \) and \( g_1 \) are reduced gravities for the lower and upper layers, respectively:

\[ g_0 = \frac{\theta_1 - \theta_0}{\theta_0} g, \quad g_1 = \frac{\theta_T - \theta_1}{\theta_0} g \quad \text{(2.2)} \]

With the values of Table 1, \( g_0 = 0.40 \text{ m s}^{-2} \) and \( g_1 = 0.80 \text{ m s}^{-2} \).

The linearized, depth-averaged equations of motion and of continuity for each layer are

\begin{align}
\frac{\partial u_i}{\partial t} - f v_i &= - \frac{1}{\rho_0} \frac{\partial p_i}{\partial x} \quad \text{(2.3a)} \\
\frac{\partial v_i}{\partial t} + f u_i &= - \frac{1}{\rho_0} \frac{\partial p_i}{\partial y} \quad \text{(2.3b)} \\
\frac{\partial h_i}{\partial t} + H_i \left( \frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} \right) &= 0 \quad \text{(2.3c)}
\end{align}

where \( i = 0, 1, \text{ or } 2 \) refers to the particular region of fluid in Fig. 2 and \( p_i \) is defined by (2.1). In its most general form, the set (2.3) comprises nine equations in nine unknowns.

Motion is forced by a prescribed mass flux incident on the escarpment in each layer. This forcing represents the eastward-propagating middle-latitude disturbances. The frequency and meridional wavenumber is set by the forcing. The phase, magnitude, and zonal structure of the escarpment response is determined. In the next section attention is restricted to the lowest layer of fluid. An equatorial beta-plane is assumed to analyse this barotropic case. In section 4 the two-layer problem is solved on an \( f \)-plane.

3. Barotropic model on an equatorial beta-plane

Here we ignore the upper-level stratification and formally set \( g_1 = 0 \). Geometrically an equatorial beta-plane is assumed: \( f = \beta y \) where \( \beta = 2.3 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \). The line \( y = 0 \) denotes the equator while \( y = -y_0 = -3.5 \times 10^3 \text{ km} \approx 35^\circ \text{S} \) denotes the southern extent of the African continent. (One degree of latitude or longitude is about 100 km.) Then (2.3) with \( i = 0 \) becomes

\begin{align}
\frac{\partial u}{\partial t} - f y v &= - g_0 \frac{\partial h}{\partial x} \quad \text{(3.1a)} \\
\frac{\partial v}{\partial t} + f y u &= - g_0 \frac{\partial h}{\partial y} \quad \text{(3.1b)}
\end{align}
\[ \frac{\partial h}{\partial t} + H_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \quad \text{(3.1e)} \]

where the subscripts on the dependent variables have been dropped for convenience.

The synoptic forcing consists of a zonal wind \( u_F \) incident normal to the meridional boundary of the form

\[ u_F(0, y, t) = i U_S \exp[i \omega t - a(y + b)^2], \quad \text{(3.2)} \]

where \( U_S = 1 \text{ m s}^{-1}, 2\pi/\omega = 6 \text{ d}, b = 3.0 \times 10^3 \text{ km}, \) and \( a^{-1/2} = 1.5 \times 10^3 \text{ km}. \) With this Gaussian structure in \( y, u_F \) has a maximum of \( 1 \text{ m s}^{-1} \) at about \( 30^\circ \text{S} \) and drops to \( e^{-1} \text{ m s}^{-1} \) at \( 15^\circ \text{S}. \) Equation (3.2) represents a more realistic forcing than that used previously by the author (Bannom 1979, 1980). The qualitative aspects of the results presented below are not particularly sensitive to the latitudinal shape of (3.2) provided the forcing is confined to the southern hemisphere. The sensitivity of the response to the frequency \( \omega \) is discussed later in this section.

The problem is to determine the response at the escarpment. Along the escarpment, the kinematic condition

\[ u(0, y, t) = u_F(0, y, t) + u_B(0, y, t) = 0 \quad \text{(3.3)} \]

must be satisfied. The boundary response \( u_B \) consists of an infinite set of the free modes of (3.1) which may be expressed in terms of parabolic cylinder functions. The mathematical treatment follows Bannom (1979) where a western boundary is considered and is not reproduced here. Bannom (1979) showed that condition (3.3) may be cast in the form of a matrix equation for the magnitudes of the free modes. In the results to be presented, the set of free modes is truncated after 60 terms and resolves 99% of the power in the forcing \( u_F. \)

Figure 3 depicts the zonal variation of the free surface height \( h_B \) associated with the boundary response at several latitudes. As in Gill (1977), an inversion height variation of
100 m corresponds to about 0.5 mb in the surface pressure variation. At a given latitude, \( h_b \) is a maximum at the boundary and decays exponentially from the boundary. The local Rossby radius of deformation \( R = (g_0 H_0 / \beta^2 y^2)^{1/2} \) is 580, 348, 248 km at \( y = 15, 25, 35 \times 100 \) km, respectively, while the equatorial radius \( R_E = (g_0 H_0 / \beta^2)^{1/4} \) is 932 km. Inspection of Fig. 3 indicates that the appropriate zonal scale of the disturbance is roughly the local deformation radius.

The trapped behaviour of Fig. 3 arises because all the free modes of the system are non-propagating and have complex zonal wavenumbers at the frequency of interest. The zonal wavenumber \( k \) is given by

\[
k = \left( \frac{\beta^2}{g_0 H_0} \right)^{1/2} \left[ -\frac{1}{2\hat{\omega}} - i \left( 2\alpha + 1 - \left( \hat{\omega}^2 + \frac{1}{4\hat{\omega}^2} \right) \right)^{1/2} \right] .
\]  

(3.4)

where \( \hat{\omega} = \omega / (g_0 H_0 \beta^2)^{1/4} \) is a non-dimensional frequency and \( \alpha \) is the meridional node number of the free mode. Here \( \alpha \geq 1 \) and the radicand is positive provided

\[
(1 - \frac{1}{2} \sqrt{2}) < \hat{\omega} < (1 + \frac{1}{2} \sqrt{2}) .
\]  

(3.5)

If the lower (upper) bound is violated, zonally propagating Rossby (inertia-gravity) waves are generated at the boundary. Here \( \hat{\omega} = 0.57 \). Note that this result depends only weakly (by a fourth root) on the choice of \( g_0 \) or \( H_0 \). In comparison, divergent quasi-geostrophic Rossby waves on a mid-latitude beta-plane satisfy

\[
\omega = \frac{-\beta k}{k^2 + l^2 + f_0^2 / g_0 H_0} .
\]  

(3.6)

where \( l \) is a meridional wavenumber and \( f_0 = -\beta y_0 \). For \( l \) real, (3.6) implies real zonal wavenumber if

\[
\hat{\omega} < \frac{(g_0 H_0 \beta^2)^{1/4}}{2 |f_0|} = 0.13
\]  

(3.7)

This criterion is more restrictive than the equatorial result (3.5); the response on an equatorial beta-plane is less likely to be trapped than on a mid-latitude one.

Figure 3 shows that the magnitude of the response increases monotonically with latitude. The maximum response occurs not at 30°S, where the forcing \( u_F \) is largest, but at 35°S. [The magnitude of \( h_b \) at 30°S at the coast is 184 m.] As the response becomes more closely trapped, its magnitude must increase in order to conserve the same mass flux. This mechanism and the increase in synoptic forcing \( u_F \) with distance from the equator explains the poleward growth in the magnitude of the response of the height field.

Figure 4 displays the temporal variation of the height or pressure field of the response at three latitudes along the boundary. The pressure of the synoptic forcing at \( y = -y_0 \) is also plotted. Comparison of the curves indicates that a low is present along the coast as the synoptic high pressure cell passes to the south of the continent. This phase relation is in qualitative agreement with observations (see Fig. 1 and discussion). Incorporation of friction would presumably further increase the phase lag. The synoptic zonal wind is offshore (negative) during the formation of a coastal low-pressure cell.

Figure 4 also demonstrates that the oscillatory forcing (3.2) generates a poleward propagating disturbance. The response at a given latitude, therefore, represents an integrated reply to the forcing both at and equatorward of the chosen latitude. This superposition of responses prohibits the use of Fig. 4 in determination of the meridional wavelength of the propagating disturbance. Calculations with a forcing of the form

\[
u_F(0, y, t) = iU_s \exp(\text{i} \theta t)
\]

\[
\begin{cases} 
1, y = 0 \\
0, y \neq 0
\end{cases}
\]  

(3.8)
suggests a wavelength of $10^{-0} \times 10^{3}$ km. In comparison a Kelvin wave of six-day period has a wavelength of $10^{-4} \times 10^{3}$ km. This discrepancy with the observed wavelength of about $3 \times 10^{3}$ km is discussed in section 5.

The meridional velocity field of the boundary response is likewise trapped next to the coast and attains its maximum value of $0^{-0} m s^{-1}$ at $(0, -y_0)$. In addition the $v$-field is about $180^\circ$ out of phase with the boundary $h$-field. Thus low pressure is associated with equatorward flow along the coast. This phase relation and the coastal trapping of the response are consistent with the features of a coastal Kelvin wave and with the observations. These results and the southward phase propagation agree with the analyses of Moore (1968) and Anderson and Rowlands (1976).

A final issue concerns the boundary response north of the equator. Though Fig. 3 depicts a northward decrease in the response, this trend is reversed north of the equator. However, the magnitude at $35^\circ$N is $3^{-6} m$. In addition the magnitude of the maximum cross-equatorial flow is $0^{-1} m s^{-1}$.

This absence of a significant equatorial response should be contrasted to that for a western boundary (Bannon 1979). For the present parameter settings and forcing, the results for a western boundary indicate a maximum meridional wind at the equator of $2{-5} m s^{-1}$. The western boundary response consists of coastally trapped modes in the South Hemisphere which propagate equatorward to excite zonally propagating mixed Rossby-gravity and equatorial Kelvin waves. The pressure of the coastal modes is in phase with the synoptic pressure field and with the meridional wind field. A synoptic high-pressure cell at the tip of the African continent generates onshore winds for a western boundary and a boundary ridge develops.

These differences between a western and an eastern boundary arise for two reasons. First, neither an equatorial Kelvin wave nor a mixed Rossby-gravity wave is generated at
an eastern boundary since the modes have eastward energy propagation. They are therefore not included in the set of free modes for an eastern boundary. [The spurious anti-Kelvin wave is always excluded since it is not a physically realistic solution.]

Second, a coastal Kelvin wave propagates poleward (equatorward) along an eastern (western) boundary. The choice of boundary condition at \( y = -y_0 \) reflects this physical difference. For an eastern boundary, the southern boundary should be an open one; the free modes are assumed to satisfy

\[
\frac{\partial v}{\partial y}\bigg|_{y = -y_0} = 0 \quad (3.9)
\]

as well as being bounded at \( y = +\infty \). For the western boundary study, the condition

\[
u(x, -y_0, t) = 0 \quad (3.10)
\]

is employed rather than (3.9). Use of the Dirichlet condition (3.10) for an eastern boundary leads to a matrix equation which can be shown to be singular as \( N \to \infty \) where \( N \) is the number of free modes retained in the analysis. The Neumann condition (3.9) eliminates this behaviour and produces a convergent solution as \( N \) increases. In the eastern boundary results presented here, the free modes satisfy (3.9).

4. TWO-LAYER MODEL ON AN \( f \)-PLANE

The results of the previous section indicate that the effect of \( \beta \) is small: no zonally propagating Rossby waves can be excited at the frequencies of interest. Hereafter we ignore the variation in the Coriolis parameter and set \( f = f_0 = -10^{-4} \text{ s}^{-1} \) in (2.3). We assume solutions of the form \( \exp\{i(ly - \omega t)\} \) and seek trapped modes like \( \exp(\pm x \kappa) \) for \( x \leq 0 \) where \( \text{Re}(\kappa) > 0 \). Without loss of generality \( \omega \) is taken to be positive. Using (2.3a) and (2.3b) the horizontal velocity components may be expressed in terms of the layer depths \( h_i \) as follows:

\[
\begin{pmatrix}
  u_0 \\
  u_1
\end{pmatrix} = -i \frac{(f_0 l + \omega \kappa_1)}{(f_0^2 - \omega^2)} \begin{pmatrix}
  g_1(h_0 + h_1) + g_0 h_0 \\
  g_1(h_0 + h_1)
\end{pmatrix} \quad (4.1a,b)
\]

\[
u_2 = -i \frac{(f_0 l + \omega \kappa_2)}{(f_0^2 - \omega^2)} g_1 h_2 \quad (4.1c)
\]

and

\[
\begin{pmatrix}
  v_0 \\
  v_1
\end{pmatrix} = -\frac{(\omega l - f_0 \kappa_1)}{(f_0^2 - \omega^2)} \begin{pmatrix}
  g_1(h_0 + h_1) + g_0 h_0 \\
  g_1(h_0 + h_1)
\end{pmatrix} \quad (4.2a,b)
\]

\[
u_2 = -\frac{(\omega l + f_0 \kappa_2)}{(f_0^2 - \omega^2)} g_1 h_2 \quad (4.2c)
\]

Substitution of these expressions into (2.3c) yields

\[
\omega^2 = f_0^2 + g_1 H_2(l^2 - \kappa_2^2) \quad (4.3a)
\]

\[
g_0 g_1 H_0 H_1(\kappa_2^2 - l^2)^2 - (f_0^2 - \omega^2)(g_0 + g_1)H_0 + g_1 H_1(\kappa_2^2 - l^2) + (f_0^2 - \omega^2)^2 = 0 \quad (4.3b)
\]

\[
r = \frac{h_1}{h_0} = \frac{(l^2 - \kappa_2^2)g_1 H_1}{\omega^2 - f_0^2 - g_1 H_1(l^2 - \kappa_2^2)} \quad (4.3c)
\]

Equation (4.3a) is the dispersion relation for the free modes in the region \( x > 0 \). With \( \omega \) and
l set by the forcing (see below), (4.3a) determines $\kappa_2 : \kappa_2 > 0$. The quartic in $\kappa_2$, (4.3b), represents exponentially growing and decaying, baroclinic and barotropic modes. Here only the two decaying solutions $-\kappa < 0$ are retained. The baroclinic and barotropic modes are denoted by a caret ($\hat{}$) and overbar ($\bar{}$), respectively. The relative strengths of the upper and lower components for a given mode are determined by (4.3c). Use of (4.3b) in (4.3c) indicates that this ratio depends only on the ratios of the depths $H_0/H_1$ and of the relative stratification $g_0/g_1$ and is not dependent on $f_0$ or $\omega$. For the parameter settings of Table 1, $\hat{\rho} = 2 \cdot 15$ and $\bar{\rho} = -1 \cdot 15$.

Motion is forced by synoptic-scale winds incident normal to the escarpment in both the upper and lower layers of the form $(F_1 + F_0) \exp\{i(l y - \omega t)\}$, respectively. Continuity of mass flux above and below the plateau and continuity of pressure require that, at $x = 0$,

\[
\begin{align*}
H_0(\bar{u}_0 + \bar{u}_0 + F_0) &= 0 \quad (4.4a) \\
H_1(\bar{u}_1 + \bar{u}_1 + F_1) &= H_2(\bar{u}_2 + F_2) \quad (4.4b) \\
\bar{h}_0 + \bar{h}_0 + \bar{h}_1 + \bar{h}_1 &= h_2 \quad (4.4c)
\end{align*}
\]

Conditions (4.4a) and (4.4b) may be written in terms of the $\bar{h}$'s using (4.1). Then, with the relations

\[
\bar{h}_1 = \hat{\bar{h}}_0, \quad \hat{\bar{h}}_1 = \hat{\bar{h}}_0
\]

the set (4.4) consists of five equations in five unknowns and is solved using standard techniques.

Tables 2–4 summarize the results when the meridional wavelength is $2\pi/l_0 = 3\cdot55 \times 10^7$ km and the period is six days. The disturbance phase speed along the

<table>
<thead>
<tr>
<th>Table 2. Zonal decay distances (in km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l = \pm l_0$</td>
</tr>
<tr>
<td>$l = -i l_0$</td>
</tr>
<tr>
<td>$\bar{\kappa}_1$</td>
</tr>
<tr>
<td>$392$</td>
</tr>
<tr>
<td>$2260$</td>
</tr>
<tr>
<td>$\bar{\kappa}_2$</td>
</tr>
<tr>
<td>$160$</td>
</tr>
<tr>
<td>$175$</td>
</tr>
<tr>
<td>$\kappa_2$</td>
</tr>
<tr>
<td>$328$</td>
</tr>
<tr>
<td>$582$</td>
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</table>

<table>
<thead>
<tr>
<th>Table 3. Layer depth responses to lower-level convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0 F_0 = 1$ km s$^{-1}$</td>
</tr>
<tr>
<td>$\text{Magnitude is in 100 m and phase (in parenthesis) in degrees}$</td>
</tr>
<tr>
<td>$l = +l_0$</td>
</tr>
<tr>
<td>$l = -l_0$</td>
</tr>
<tr>
<td>$l = +i l_0$</td>
</tr>
<tr>
<td>$\bar{h}_0$</td>
</tr>
<tr>
<td>$0.1 (90)$</td>
</tr>
<tr>
<td>$0.2 (-90)$</td>
</tr>
<tr>
<td>$0.1 (0)$</td>
</tr>
<tr>
<td>$\bar{h}_0$</td>
</tr>
<tr>
<td>$0.9 (90)$</td>
</tr>
<tr>
<td>$1.7 (-90)$</td>
</tr>
<tr>
<td>$0.9 (32)$</td>
</tr>
<tr>
<td>$\bar{h}_2$</td>
</tr>
<tr>
<td>$0.1 (90)$</td>
</tr>
<tr>
<td>$0.3 (-90)$</td>
</tr>
<tr>
<td>$0.3 (-17)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4. Same as Table 3 but for upper-level convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(H_1 F_1 - H_2 F_2) = 1$ km s$^{-1}$</td>
</tr>
<tr>
<td>$l = +l_0$</td>
</tr>
<tr>
<td>$l = -l_0$</td>
</tr>
<tr>
<td>$l = +i l_0$</td>
</tr>
<tr>
<td>$\bar{h}_0$</td>
</tr>
<tr>
<td>$0.2 (90)$</td>
</tr>
<tr>
<td>$0.3 (90)$</td>
</tr>
<tr>
<td>$0.4 (44)$</td>
</tr>
<tr>
<td>$\bar{h}_0$</td>
</tr>
<tr>
<td>$1.9 (-90)$</td>
</tr>
<tr>
<td>$5.5 (-90)$</td>
</tr>
<tr>
<td>$3.9 (-110)$</td>
</tr>
<tr>
<td>$\bar{h}_2$</td>
</tr>
<tr>
<td>$1.0 (90)$</td>
</tr>
<tr>
<td>$1.8 (90)$</td>
</tr>
<tr>
<td>$1.9 (53)$</td>
</tr>
</tbody>
</table>
escarpment is 6.8 m s\(^{-1}\). For either northward \((l = +l_0)\) or southward \((l = -l_0)\) forcing, the response is trapped next to the escarpment with the baroclinic (') modes having the smallest decay distance (Table 2). This trapping is greater than that for a one-layer f-plane model where \(\{(f_0^2 - \omega^2)g_1H_0 + l_0^2\}^{-1} = 190\) km.

Tables 3 and 4 also give the magnitudes of the responses to unit forcings (1 km m s\(^{-1}\)) in the bottom and top layers, respectively. In each case the response is stronger for southward-propagating forcing than northward. In addition, the baroclinic mode \((h_0)\) response is larger than the barotropic \((h_0)\). Though the forcings are relatively weak, the deviations in the height of the coastal inversion are large. For southward forcing, lower (upper) levels convergence of 6 (2) km m s\(^{-1}\) results in the surfacing of the inversion.

Tables 3 and 4 also show the phase lags of the responses relative to the time of maximum convergence of the forcing along the escarpment. For the case of southward forcing, a lowering of the inversions lags the low-level convergence by a quarter cycle. This result disagrees with the barotropic \(\beta\)-plane analysis of the previous section where lowering of the coastal inversion lags low-level divergence. This discrepancy arises because the sinusoidal response is appropriate only for an infinitely long escarpment. As in section 3, a semi-infinite boundary may be modelled with an exponentially decaying forcing of the form \(\exp(-l_0y)\). The tables display the results for this case \((l = il_0)\). The phase lag is then in qualitative agreement with section 3. In contrast, upper-level convergence produces a lowering (raising) of the coastal (interior) inversion with a phase lag of 90° for both northward and southward forcings. This phase relation is relatively insensitive to the above geometrical considerations.

Figure 5. Surface pressure (4.5) and surface velocity response to southward low-level forcing of 1 km m s\(^{-1}\). Contour interval is 10 m and maximum vector length corresponds to 3.4 m s\(^{-1}\). Negative contours are dashed.
Figure 6. Surface pressure (4.5) and surface velocity response to southward upper level forcing of 1 km/s. Contour interval is 20 m and maximum vector length corresponds to 5.7 m/s. Negative contours are dashed.

A contour plot (not shown) of the height of the interior inversion reveals maximum displacement along the escarpment and exponential decay away from it. By pressure continuity (4.4c), the field is continuous at $x = 0$ and is structurally similar to the barotropic displacements of Longuet-Higgins (1968). The upper-level velocity pattern has cyclonic and anticyclonic circulation about the low and high height fields, respectively. As in Longuet-Higgins (1968), the velocity tangential to the escarpment is discontinuous and a vortex sheet is present. Northward (southward) flow occurs aloft over the sea (plateau) when the upper-level inversion descends.

Figures 5 and 6 depict the surface pressure fields of the response

$$p^* = \begin{cases} 
(1 + \tilde{g}_o)(\tilde{h}_0 + \tilde{h}) + (\hat{h}_1 + \hat{h}) = (1 + \tilde{g}_o + \hat{r})\tilde{h}_0 + (1 + \tilde{g}_o + \hat{r})\tilde{h}_0, & x < 0 \\
\hat{h}_2, & x > 0
\end{cases} \quad (4.5)$$

where $\tilde{g}_o = g_o/g_1$ and the surface velocity fields. Each figure is for southward forcing. Here a 100 m height variation corresponds to about a 1 mb change in surface pressure. The pressure field is discontinuous at $x = 0$ as the plateau values are not reduced to sea level. This feature aids comparison with Fig. 1. The barotropic mode dominates the surface pressure response over the ocean since there is strong cancellation within the baroclinic mode ($1 + \tilde{g}_o + \hat{r} = 0.35$) and the latter decays more rapidly. Figures 5 and 6 are suggestive of the observations (see Fig. 1). Superposition of both upper- and lower-level forced solutions is left to the reader’s imagination. Detailed comparison requires both better obser-
vations, which are not presently available, and better knowledge of the forcing functions and is not attempted here.

It should be noted that the substantial responses obtained in this section and the preceding one are not the result of a resonant excitation. Observations indicate phase speeds of 5–10 m s⁻¹ for the coastal lows. For a period of six days a wavelength of (2.6–5.2) × 10³ km is implied. In contrast, a barotropic Kelvin wave has wavelength (g₀H₀)⁻¹ × 6d or 10.4 × 10³ km and southward propagation. A search for the free modes of the set (4.4) yielded only one resonant response for a six-day period. That mode has a wavelength of 10.0 × 10³ km and southward propagation. (The latter feature explains the stronger response for southward forcing.) Thus the responses of Tables 3 and 4 are far from resonance. In addition, these results are not particularly sensitive to variations in H₀, H₂, zₘ, or g₀/g₁ of a factor of 2. Interestingly, neglect of the coastal inversion (setting g₀ = 0) yields the barotropic model of Longuet-Higgins (1968) whose Fig. 3 shows a free mode of six-day period for γ = (H₀ + H₁)/H₂ ≃ 1.75 and e = f₀²/(1²g₁(H₀ + H₁)) ≃ 2, indicating a wavelength of 4.7 × 10³ km for the present parameters. However, the inclusion of the coastal inversion removes this resonance possibility.

5. DISCUSSION

This paper has presented two linear models of the coastal lows of southern Africa. The first is a barotropic primitive-equation analysis of the scattering of an eastward-propagating middle-latitude disturbance at an impermeable eastern boundary on an equatorial beta-plane. The boundary response is exponentially trapped next to the boundary, with the typical e-folding distance being the local Rossby radius of deformation. No zonally propagating Rossby waves excited at the frequencies of interest. A coastal low (high) is generated in response to synoptic divergence (convergence) from the coast and has equatorward (poleward) flow. The coastal lows are predicted to 'round the Cape' behind the passage of a migratory anticyclone but before a cyclone. In contrast to the western boundary results of Bannon (1979), the equatorial response for this eastern boundary is negligible. These results are in qualitative agreement with observations and with the hypothesis of Gill (1977) that coastal lows form in response to synoptic-scale forcing and have the structure of coastal Kelvin waves.

The second model consists of a two-layer f-plane analysis which includes the response of the layer of fluid inland from the coast lying above the Great Central Plateau and below the interior inversion. Again the boundary response to an incident mass flux is maximum along the escarpment and decays exponentially with distance from it, both inland and seaward. The e-folding distance is always slightly less than the appropriate Rossby deformation radius. A coastal low (high) has equatorward (poleward) flow while the reverse holds for disturbances over the interior plateau. Aloft there is a phase reversal that leads to the presence of a vortex sheet along the escarpment. Comparison of results for sinusoidal and exponential forcing highlights the importance of a semi-infinite escarpment in obtaining the correct phase relations. Low-level (i.e. below the plateau) synoptic divergence generates a coastal low while upper-level convergence leads to a low over the interior. Together, these results and the observations suggest that the coastal low and its companion over the interior may be interpreted as forced internal double Kelvin waves analogous to the free barotropic modes of Longuet-Higgins (1968).

The present study is not comprehensive in scope. For example, the synoptic forcing is poorly modelled. Proper determination of such forcing requires knowledge of the evolution of the synoptic disturbance as it passes over the escarpment. The strong escarpment response
of the linear models suggests the need of a non-linear calculation. Such an approach would enable the mutual interaction of the forcing and boundary response to be ascertained. In addition the proper determination of the forcing requires the projection of the synoptic winds onto the curved coastline of southern Africa. Gill (1977) has crudely incorporated this aspect. Currently this geometrical consideration is the only explanation of the preferred wavelength (~3000 km) of the coastal lows. Bottom friction and its land-sea contrast should also be important in these low-level flows. These shortcomings should be addressed in future modelling efforts.

Another question concerns the more frequent occurrence of coastal lows rather than highs. Gill (1977) suggests that, due to non-linear effects (the lows are slower and more in phase with the forcing), coastal lows are preferentially excited. The large boundary responses found in the present studies also suggest the importance of non-linear terms. Here we mention some additional considerations. Synoptic anticyclones, which excite the coastal lows along eastern boundaries, are associated with large-scale subsidence. This subsidence helps to lower and strengthen the coastal and plateau inversions. Synoptic cyclones will have the opposite influence. A weaker inversion will be less effective in vertically containing the escarpment response. Figure 7 illustrates the spilling of the coastal marine air over the escarpment onto the interior plateau. This process, called a leader or intercell front in South Africa (Taljaard, Schmitt and van Loon 1961), is interpreted here as a coastal high. As Tables 3 and 4 suggest, equatorward-moving cyclones will produce a weaker response than, say, poleward displacements of the subtropical high of the South Atlantic Ocean. Further, migratory anticyclones are more prevalent in the subtropical belt (Taljaard and

Figure 7. Surface pressure chart for 27 July 1957 (top) and cross-section AB along 30°S (bottom) (from Taljaard 1972).
van Loon 1962, 1963). These factors provide additional reasons for the more frequent occurrence of coastal lows.

The presence of coastal lows and highs is not restricted to southern Africa. Taljaard (1972, pp. 156–158) notes that the Australian cool change is similar to the leader front. In the northern hemisphere, Noonkester (1979) presents evidence (see his Figs. 14 and 15) of coastal lows and their inland companions along southern California. Thus coastal lows are world-wide phenomena.

Two final applications of the present study are foehn winds and lee cyclogenesis. Tyson (1964, 1965) has documented the association of coastal lows with the foehn (foehn) winds of southern Africa. In the present model, the foehn is interpreted as the surfacing of the coastal inversion. Table 4 indicates that the height of this inversion is very sensitive to upper-level convergence. Thus the two-layer analysis of section 4 is believed applicable to the meso-scale meteorology of foehn winds (e.g. the Santa Ana of California and the chinook of Colorado).

More speculative is the possible relation of this study to lee cyclogenesis. Buzzi and Tibaldi (1978) describe a case study of the explosive formation and development of a cyclone near the Alps. The initial phase of the cyclogenesis is a low-level (below 3–4 km) and meso-scale (around 250 km) phenomenon. The time-dependent process of the blocking of the low-level flow by the Alps could be modelled in a manner similar to that in section 3. Here, however, the synoptic wave would be scattered by a circular barrier (the Alps) on an f-plane. Trapped barotropic modes similar to those of Longuet-Higgins (1969) would be generated by this mechanism. They may provide a qualitative description of the early stage of cyclogenesis and act as a trigger for future growth of the cyclone. Further research should address these questions.

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