Inertial capture of particles by obstacles in form of disks and stellar crystals

By F. PRODI, M. CAPORALONI, G. SANTACHIARA and F. TAMPIERI

Laboratorio FISBAT-CNR, Reparto Nubi e Precipitazioni, Bologna, Italy
and Osservatorio Geofisico dell'Università, Modena, Italy

(Received 27 March 1980; revised 5 January 1981. Communicated by Dr J. S. A. Green)

SUMMARY

The process of cloud droplet capture by falling plate-like ice crystals has been reproduced in the laboratory by impacting streams of monodisperse spherical particles \(2 < R < 22 \mu m\) onto disk-shaped and star-shaped fixed obstacles in the range between viscous and potential flow. The particles collected on the upwind surface of the obstacles are counted using an optical microscope. A collection efficiency is determined for the different planar shapes under the specific experimental conditions.

For the disk shape the results have been compared with those from the potential and viscous flow theories. They show that collection efficiencies computed from the potential flow theory are not representative of the actual efficiencies. Comparison with the viscous flow theory shows that in the range \(0.8 < N_{\text{stk}} < 2\) the theory fits the experimental data well. Non-zero efficiencies have been found for \(N_{\text{stk}}\) lower than the theoretical cut-off, while for \(N_{\text{stk}} > 4\) the collection efficiencies are lower than theoretically predicted. This difference has been interpreted in terms of the hydrodynamic interactions between the particle and the obstacle.

For the planar shapes, the collection efficiencies regularly increase with the Stokes number and depend on the surface area distribution in the models. The values of the collection efficiencies are always higher than those of the disks of equal area.

1. INTRODUCTION

The capture of supercooled droplets by the different types of ice crystals is an important mechanism of growth of precipitation. However, the quantitative treatment of this stage of growth in numerical models of precipitating cloud is limited by the incomplete knowledge of the collection efficiencies of the different crystal shapes for the various sizes of water droplets.

The cut-off diameters of droplets accreted on real columnar and dendritic crystals (Ono 1969) and plate-like and dendritic (Harimaya 1975) have been observed to be between 10 and 20 \(\mu m\). For particles of this size the only important capture process is the inertial or aerodynamic capture, as phoretic forces are not effective and electrostatic effects are considered to be negligible. In the conditions in which the aerodynamic capture is effective the characteristics of the fluid motion have a wide range of variability due to the different flow velocities (in this case the terminal velocities of droplets and crystals) and sizes of droplets and ice crystals.

The flow characteristics are determined by a set of numbers which indicate the relative importance of the acting forces (Fuchs 1964):

\[
N_{\text{Re}} = 2LU\rho/\mu \quad \text{the Reynolds number},
\]

\[
N_{\text{stk}} = U\tau/L \quad \text{the Stokes number, with } \tau = 2R^2\rho_F/(g\mu) \text{ the relaxation time of the particle},
\]

\[
N_{\text{Fr}} = U^2/2Lg \quad \text{the Froude number}.
\]

\(L\) and \(U\) are a linear dimension and a velocity characteristic of the flow.

\(N_{\text{Re}}\) describes the balance between inertial and viscous forces of the fluid near the obstacle. At low \(N_{\text{Re}}(\lesssim 1)\) the viscous effect is dominant and the flow is described by the Navier Stokes equation in which the viscous dissipation terms are maintained. At \(N_{\text{Re}}\) in
the range 100–3000 the fluid motion is still laminar but the viscous terms can be neglected, and the flow field is well described by the Euler equation: as the velocity field is expressed in terms of a gradient of a scalar function the corresponding flow regime is named potential flow. The range $1 < N_{Re} < 100$ defines a region of transition from the viscous flow to potential flow and is typically the range of the inertial capture of droplets by ice crystals in clouds.

$N_{Stk}$ is the ratio of the stopping distance of a particle initially moving with a velocity $U$ to the cross linear dimension of the obstacle.

When gravity can be neglected with respect to the other forces $N_{Stk}$ is the basic parameter in determining the capture of the particle by the obstacle: high $N_{Stk}$ indicates that the particle, due to its inertia, tends to leave the streamlines in proximity of the obstacle.

$N_{Fr}$ becomes important when gravity is important, as it is the ratio of the fluid velocity to the settling velocity of the particle.

Besides these parameters, the ratio between the size of the particle and that of the obstacle, the interception parameter, has to be taken into account in the collision problem.

In the transition region of $N_{Re}$, the solutions for viscous or potential flow are derived as approximations of the actual flow (Ranz and Wong (1952) for potential flow, and Pitter and Pruppacher (1974), for viscous flow). The main problem in this region is to determine the fluid-obstacle interaction, e.g. through the drag coefficient $C_D$ which is not given by the theory. Similarity experiments allow the drag coefficient to be determined as a function of $N_{Re}$. However, in complex systems such as the particle–obstacle interaction, similarity conditions cannot be satisfied simultaneously for all the numbers characterizing the fluid motion and the particle–obstacle interaction. This is the reason for simulation experiments to be performed (Starr and Mason 1966, Staviskaya 1972, Pedori et al. 1973) in which the capture process is reproduced in the laboratory for particles and obstacles of various shapes, to derive experimental collection efficiencies to be compared with the values given by potential or viscous flow theories.

The experiments performed by the above-mentioned authors have inherent limitations in the range of $N_{Stk}$ and sizes and shapes of the collecting model tested. The purpose of the present investigation is to provide experimental values of the collection efficiency by performing a set of experiments in a wide range of $N_{Stk}$ with planar and stellar models as obstacles in the intermediate range of flow, taking advantage also of the recent improvements in the generation of monodisperse spherical particles.

2. Experimental part

The experiments have been performed in a downward continuous vertical flow with the obstacle fixed in the rear on the tip (40 μm in diameter) of a long vertical needle along the axis of the cylindrical pipe. The supporting needle itself was fixed to the wall downstream in a way not to perturb the flow in the region of the obstacle. The working section of the pipe had constant diameter for 60 cm above and below the obstacle, in order to avoid convergence or divergence of the flow. The vertical pipe was connected to a monodisperse aerosol generator (TSI 3050) for test particles less than 12 μm in size, while a device for releasing pollen grains was set up at the top of the vertical duct, for particles greater than 12 μm in size. The monodisperse particle generator operates at fixed flow, and the different flow velocities were obtained by changing the working section of the vertical pipe. Velocities in the range 32–90 cm s$^{-1}$ have been obtained, with pipes of internal diameter from 9.5 to 5 cm. As the model was on the axis of the pipe and had small sizes compared with the flow diameter, the velocity profile of the unperturbed flow was considered uniform in the region of the model. The flow velocity has been measured by the flow rate, with the correction for
**TABLE 1. MODELS OF THE PLANAR SHAPES AND EXPERIMENTAL CONDITIONS TESTED**

<table>
<thead>
<tr>
<th>Planar shape</th>
<th>External size $\mu$m</th>
<th>Internal sizes $\mu$m</th>
<th>Surface area $\text{cm}^2$</th>
<th>Flow velocity $32 \text{ cm s}^{-1}$</th>
<th>Flow velocity $64 \text{ cm s}^{-1}$</th>
<th>Flow velocity $90 \text{ cm s}^{-1}$</th>
<th>Flow velocity $176 \text{ cm s}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disk</td>
<td>620</td>
<td>—</td>
<td>$3 \times 10^{-3}$</td>
<td>—</td>
<td>$N_{Re} = 26$</td>
<td>O$_2$, O$_6$, L, P</td>
<td>—</td>
</tr>
<tr>
<td>Disk</td>
<td>800</td>
<td>—</td>
<td>$5 \times 10^{-3}$</td>
<td>—</td>
<td>$N_{Re} = 34$</td>
<td>R</td>
<td>$N_{Re} = 48$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>O$_2$, O$_3$, O$_6$, P</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>O$_2$, O$_3$, L, P</td>
</tr>
<tr>
<td>Disk</td>
<td>1200</td>
<td>—</td>
<td>$11 \cdot 3 \times 10^{-3}$</td>
<td>$N_{Re} = 25$</td>
<td>$N_{Re} = 51$</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Stellar (P1d)</td>
<td>3380</td>
<td>Size of central area:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>920 $\mu$m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch width: 350 $\mu$m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordinary dendritic</td>
<td>3200</td>
<td>Size of central area:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P1c)</td>
<td></td>
<td>1100 $\mu$m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch width: 80 $\mu$m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stellar with plates</td>
<td>3500</td>
<td>Size of central area:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>at ends (P2a)</td>
<td></td>
<td>500 $\mu$m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hexagon size: 800 $\mu$m (at ends)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Branch width: 150 $\mu$m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$O_2$, $O_3$, $O_5$, $O_6$, oil droplets of 2, 3, 5, 6 $\mu$m radius respectively. R, Ragweed pollen. L, Lycopodium spores. P, hycory Pecan pollen.
the velocity profile in a cylindrical duct \( u(y) = 2\overline{u}(A^2 - y^2)/A^2 \); along the axis, \( u = 2\overline{u} \). The Reynolds number of the flow is much below the critical value of about 2000, usually indicated for the transition to turbulent regime in cylindrical pipes. The regime of laminar flow during the experiments was also checked by visualizing the flow with smoke.

The disk shape has been taken to simulate hexagonal plates and three stellar shapes to simulate planar crystals. Disk diameters were 620, 800 and 1200 \( \mu \text{m} \), while the sizes of the planar shapes were 3380, 3200, 3500 \( \mu \text{m} \) for the stellar, the ordinary dendritic and the stellar with plates at ends respectively. The characteristics of the models are shown in Table 1 together with the experimental combinations of flow velocities and particles tested. In Fig. 1 the models are shown, with details of captured particles.

The 800 and the 1200 \( \mu \text{m} \) models have been punched in thin aluminium foil (about 30 \( \mu \text{m} \) thick) while 620 \( \mu \text{m} \) disks and the planar shapes have been prepared by a photo-chemical process similar to a standard printed-circuit board technique, which is described as the models obtained are suitable for use also in investigating the dynamics of fall of natural shapes. The models are drawn on a transparent paper magnified ten times or more, and photographed on film (Ilford contact IC 4) at high contrast, in such a way that the image on the film is the final real size of the model. From the negative a positive is derived by contact, on a film of the same type. A thin aluminium sheet (20 \( \mu \text{m} \)) has been previously painted with a photostatic varnish (Positiv 20 Kontakt Chemie) and dried at \( T = 70 \degree \text{C} \) for 20 min. The positive film is placed on the aluminium sheet under a U.V. light source, which illuminates the surface outside the models. The aluminium sheet is then dropped into a 7% solution of NaOH which dissolves away the varnish from the surface exposed to U.V. light. Finally a chemical etching is performed with a solution having the following volume concentrations: 80% of 85% \( \text{H}_3\text{PO}_4 \), 10% of distilled water, 5% of 65% \( \text{HNO}_3 \), 5% of \( \text{CH}_3\text{COOH} \), at a temperature of 40 \degree \text{C}.

The particles 12 \( \mu \text{m} \) or less in diameter were olive oil droplets, while Ragweed pollen (19-2 \( \mu \text{m} \) size), Lycopodium spores (32 \( \mu \text{m} \) size) and Hicory Pecan pollen (44 \( \mu \text{m} \) size) have been used as larger size particles. Particle physical and dynamical characteristics are reported in Table 2. The uniformity of the particle distribution in the airstream has been verified by interrupting the flow and letting the particles settle on a surface at the bottom of the test section. The particle distribution on the surface has been systematically inspected with an optical microscope and found to be uniform.

### Table 2. Characteristics of the particles used in the tests

<table>
<thead>
<tr>
<th>Particle</th>
<th>Size ( \mu \text{m} )</th>
<th>Density ( \text{g cm}^{-3} )</th>
<th>Terminal velocity ( \text{cm s}^{-1} )</th>
<th>Relaxation time s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Olive oil</td>
<td>4</td>
<td>0-90</td>
<td>0-04</td>
<td>4-41 ( \times 10^{-5} )</td>
</tr>
<tr>
<td>Olive oil</td>
<td>6</td>
<td>0-90</td>
<td>0-10</td>
<td>9-92 ( \times 10^{-5} )</td>
</tr>
<tr>
<td>Olive oil</td>
<td>10</td>
<td>0-90</td>
<td>0-27</td>
<td>2-76 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>Olive oil</td>
<td>12</td>
<td>0-90</td>
<td>0-39</td>
<td>3-97 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>Ragweed pollen</td>
<td>19-2</td>
<td>0-63</td>
<td>0-70</td>
<td>7-10 ( \times 10^{-4} )</td>
</tr>
<tr>
<td>Lycopodium spores</td>
<td>32</td>
<td>1-17</td>
<td>2-14</td>
<td>3-68 ( \times 10^{-3} )</td>
</tr>
<tr>
<td>Hicory Pecan pollen</td>
<td>44</td>
<td>0-79</td>
<td>4-6</td>
<td>4-68 ( \times 10^{-3} )</td>
</tr>
</tbody>
</table>

Since preliminary tests have shown bouncing effects on the surface of the models (a possibility also pointed out by Chamberlain (1970) studying deposition of pollen on vegetation), the models have been previously covered with a diluted solution of liquid silicone. The total retention of the surface covered with liquid silicone has been tested by

\* \( \overline{u} \) is the average velocity, \( A \) the radius of the duct and \( y \) the distance from the axis.
Figure 1. (a) Different planar shapes tested in the experiments. From right to left: disk, stellar, ordinary dendritic and stellar with plates at ends.
(b) Detail of the disk shaped obstacle (1200 µm size) with captured Lycopodium spores.
(c) Detail of a tip of the stellar shaped obstacle with captured oil droplets.
(d) Detail of the ordinary dendritic shaped obstacle, with captured Lycopodium spores.
(e) Detail of the planar shaped obstacle with hexagons at ends with captured Hicory Pecan pollens.
taking photographs of the model with flowing particles, at open shutter. The particle trajectories are revealed as streaks on the picture; no bouncing trajectory has been found in the conditions tested in our experiments.

The measurements of the concentrations in air of the monodisperse particles have been performed with a Royco Counter (Model 218) for the oil droplets and with isokinetic sampling on filters of a known volume of air for spores and pollen grains.

The captured particles have been counted using an optical microscope and the local distribution on the model surface has also been recorded.

The capture efficiency for a model of surface area $S$ for the monodisperse particles is:

$$E = \frac{N}{S u \Delta t}$$

where $N$ is the number of particles counted on the model surface, $n$ the particle concentration in air, $u$ the flow velocity and $\Delta t$ the time interval.

Figure 2. Collection efficiency of disks vs. Stokes number. Open symbols refer to the present experiments. The error bars are reported for sets of more than three experimental runs. Other experimental results and theoretical curves from potential and viscous flow are also drawn for comparison.

3. RESULTS AND DISCUSSION

The experimental collection efficiencies as a function of the Stokes number are presented in Fig. 2 and Fig. 3 for the disks and the planar shapes respectively. The theoretical curve by Ranz and Wong (1952) for potential flow and two theoretical curves by Pitter (1977) for intermediate $N_{Re}$ are also drawn. These latter curves correct previous curves derived from a numerical solution of the Navier Stokes equations, in the hypothesis of superposition of the two flows by Pitter and Pruppacher (1974). Hereinafter we shall refer to the Pitter and Pruppacher model as the one presented by Pitter and Pruppacher (1974) including drag on accelerating bodies introduced by Pitter (1977).
INERTIAL CAPTURE OF CLOUD PARTICLES

Figure 3. Collection efficiency of planar shapes vs. Stokes number. Open symbols refer to the present experiment. Results from different authors are also reported for comparison.

The results for the disks (Fig. 2) show that the data are much better fitted, in the 0.7 < N_{Stk} < 4 range, by the viscous flow curves relating to nearly the same sizes of the obstacles and to Reynolds numbers slightly lower than those of our experiments. This range of N_{Stk} covers the conditions of most interest in real clouds, both in droplet and crystal sizes, with high values of the collection efficiency. The data, therefore, support the validity of the approach of the viscous flow approximation used in the Pitter and Pruppacher model. As the two theoretical curves show, in this range the collection efficiencies are a unique function of the Stokes number, for a wide range of Reynolds numbers.

As far as the N_{Stk} < 0.7 are concerned, the theoretical curves show a cut-off for N_{Stk} larger than 0.5, while our experimental results present low but non-zero collision efficiencies down to N_{Stk} around 0.2. The data by Starr and Mason (1966), also drawn in Fig. 2, show too non-zero and scattered values of the collision efficiencies in this range. Their non-zero collection efficiencies (which relate to small particles less than 13 \mu m in size) can be largely accounted for by electrostatic and phoretic forces. In the present simulation experiment and in this size range only the electrostatic forces can be active due to the higher mobility of the particles and to the longer times spent in the proximity of the obstacle (Schlamp et al. 1976).

In the range N_{Stk} > 4 our experimental conditions depart from those described by the theoretical curves of the Pitter and Pruppacher model. In fact the theoretical curves refer to velocities of the droplets relative to the falling oblate spheroid; these relative velocities are therefore lower than the fall velocities of the spheroids. While the Stokes number accounts for the relative velocity, the Reynolds number is based on the fall velocity of the obstacle. So the curves for different Reynolds numbers depart, when drawn as a
function of the Stokes number, and the comparison of our experimental data with the theoretical curves is no longer possible. Our experimental results obtained using a fixed obstacle depart in this range of $N_{Stk}$ from the real conditions in clouds, but contribute to the knowledge of capture by pure impaction. In this range our values of the collection efficiency decrease at $N_{Stk} > 3$, while we would expect the collection efficiency to grow asymptotically to one. By visual inspection of the obstacles in these tests we noticed that the particles did not display a uniform distribution on the obstacle surface (see Fig. 1(b), referring to an experiment at $N_{Stk} = 4$), and rather they showed a preferential capture in the outer region. With an efficiency close to 1 we would expect a uniform distribution, on the reasonable assumption that the deflection from the mean flow streamline is continuous with the displacement from the axis, an assumption supported by the computations of Pitter (1977) which for large droplets do not indicate annular collection. This behaviour is a further support of the absence of experimental biases, and must be accounted for in terms of hydrodynamic interaction between the particle and the plate. By comparing our data with the theoretical curves for viscous flow it appears that this interaction should be of a kind not described by the superposition approximation used by Pitter and Pruppacher, which does not include the changing boundary conditions at the surfaces of the two bodies during interaction.

For comparison, two curves from the experimental data by Kajikawa (1974) are also reported in Fig. 2. They are not parametrized by the Stokes number alone, and the trend of the collection efficiency with crystal size is hardly explicable; the combination of natural and laboratory hydrometeors in his experiments was probably not under complete control.

The results for the planar shapes are presented in Fig. 3. The shapes we have chosen are the most frequently occurring in real cases, as it is not possible to introduce a parameter by which a general description of the aerodynamic behaviour of the different shapes can be accounted for. Among the possible characteristic lengths the radius of the disk of an area equal to that of the corresponding model has been used to calculate the Stokes number.

For each shape the trend of the collection efficiency is a regular function of the Stokes number, but they differ markedly from one shape to another. The highest efficiency is shown by the stellar shape. The dendritic one displays a relatively high cut-off Stokes number (close to the value of the cylinders, see Davies and Peetz 1956) but the efficiency increases markedly. The stellar shape with plates at ends shows a different behaviour with the lower cut-off and the lower efficiencies at $N_{Stk} > 0.4$. In this case the behaviour of the captured droplets is similar to that of the six independent hexagons; in fact if the Stokes number is calculated using the characteristic length of the individual hexagons, the experimental points move close to the experimental points for the disks shown in Fig. 2.

The shapes we have chosen differ markedly in the areal distributions of different parts of the obstacles, but all of them have a central area. The deviation of the flow by the obstacle is strongly dependent on this area displacement therefore we introduced the ratio of the total surface to the surface of the central part as a new parameter describing this characteristic of the models. Some efficiencies as a function of this parameter are reported in Fig. 4; the stellar shape with plates at ends is not reported because the peripheral area is gathered in six individual centres. The efficiency increases for all the Stokes numbers chosen as this parameter increases. The same behaviour is shown by the data of Stavitskaya (1972) referring to stellar shapes with different branch widths and fixed central area, in spite of the systematically lower collection efficiencies found by this author (Fig. 4).
Analysis of the efficiency trend

The interaction between particle and obstacle is a quite complex dynamical problem, so that the numerical integration of the equation of motion seems to be the only way to take into account all the relevant physical features of the problem. However, lacking the possibility of using the Pitter and Pruppacher (1974) and Pitter (1977) computations to explain some features of the present results, an heuristic approach has been attempted to understand the trend of the collection efficiencies of the fixed disks in our experiments. The equation of motion of a spherical particle in the flow field near a plate obstacle should be considered. By scaling the linear dimensions with the obstacle radius \( A \) and the velocities with the unperturbed flow velocity \( v_\infty \), assumed to be positive and \( y \)-directed, we obtain the following non-dimensional equation of motion (a tilde indicates non-dimensional quantities):

\[
\tilde{\tau}_p = - \frac{C_D N_{Rep}}{24} (N_{Stk})^{-1} (\tilde{p} \tilde{v}_p - \tilde{v}_p) + \frac{\tilde{g}}{2} \frac{N_{Re}}{v_\infty^2} + \frac{L}{v_\infty^2 m_p} (F_w + \tilde{F}_s) .
\]

where \( C_D \) is the drag coefficient for the sphere, \( F_w \) represents the wall interaction forces and \( \tilde{F}_s \) the drifting force due to shear flow (Magnus effect). The subscript \( p \) refers to particle and \( f \) to fluid properties and \( m_p \) is the particle mass. We should note that the drag coefficient is dependent on the Reynolds number; from Beard and Pruppacher (1969) we can obtain the following approximate relationship:

\[
C_D N_{Rep}/24 \approx 1.0 + 0.1 N_{Rep} .
\]

but the particle Reynolds number is usually small, so that its variations do not affect strongly the behaviour of the particle near the disk (see for example the results of Pitter and Pruppacher 1974, and Pitter 1977).
The force due to the particle-plane wall interaction is difficult to obtain in the present case. For an infinite plane wall Soo (1967) proposes the following approximate equations:

\[ F_{wx} = -\frac{\pi}{8} \rho_f \frac{a^6}{y^3} (\dot{u}_p - \dot{u}_f) + \text{terms of the order of } (a/y)^4 \]  

\[ F_{wy} = -\frac{\pi}{4} \rho_f \frac{a^6}{y^3} (\dot{v}_p - \dot{v}_f) + \text{terms of the order of } (a/y)^4 \]  

\( y \) being the particle-wall distance. In our case the fluid flow is steady, so that equations for \( \vec{F}_w \) further simplify. In fact this force has the effect of increasing the drag: the component of the velocity of the particle towards the obstacle is reduced with respect to the case in which \( \vec{F}_w \) is neglected, and the particle is accelerated in the transverse direction (the fluid velocity being higher near the periphery of the obstacle).

The force due to the presence of a shear in the fluid velocity field, say \( \frac{d\dot{v}_f}{dx} \), can be parametrized following the Saffman’s formulation (see Soo 1967) for low \( N_{Re}^2 \):

\[ F_s \approx 80 \eta \frac{3}{4} \rho_f a^2 \dot{v}_p (d\dot{v}_f/dx)^4 \]  

and is directed towards the region of higher velocity.

Neglecting the wall and shear forces, Eq. (1) suggests that at high \( N_{St} \) the particle velocity is weakly influenced by the drag, so that it depends only on the gravitational acceleration term. As \( N_{St} \) becomes larger than one, the efficiency due to pure impaction should rapidly increase towards one.

In order to evaluate the effects of the other terms, we analyse the two components of Eq. (1) giving the particle velocity toward the obstacle (\( \phi \)) and the transverse component (\( \eta \)) in the geometry of our experiment (\( y \) pointing downward, with the origin in the obstacle centre):

\[ \left( 1 + \frac{3}{32} \rho_f \frac{a^3}{\rho_p y^3} \right) \ddot{\gamma}_p = -\frac{C_D N_{Re}^2}{24} (N_{St})^{-1} (\ddot{u}_p - \ddot{u}_f) + \frac{120 \rho_f A}{\pi \rho_p a} \left( \frac{d\ddot{v}_f}{dx} \right)^4 \ddot{v}_f (N_{Re})^4 \]  

\[ \left( 1 + \frac{3}{16} \rho_f \frac{a^3}{\rho_p y^3} \right) \ddot{\gamma}_p = -\frac{C_D N_{Re}^2}{24} (N_{St})^{-1} (\ddot{v}_p - \ddot{v}_f) + \frac{120 \rho_f A}{\pi \rho_p a} \left( \frac{d\ddot{u}_f}{dx} \right)^4 \ddot{u}_f (N_{Re})^4 + \frac{1}{4} N_{Fr}^{-1} \]  

The boundary conditions appropriate for Eq. (6) and (7) are:

\[ \ddot{y} = -\infty, \quad \ddot{u}_p = 0, \quad \ddot{v}_p = 1 \]  

In these equations \( N_{Re} \) is the Reynolds number of the obstacle. The sign of the last but one term of Eq. (7) depends on the shear direction; by inspection of the velocity patterns it results that the shear effect on the \( \eta \) component is significant only in a relatively small region near the plate edge.

In the present formulation wall forces can be neglected because the coefficient is always less than 1; for an order of magnitude evaluation, the drag term variations can be neglected too, actual values of \( N_{Re}^2 \) being always less than 5.

The Froude number attains values larger than 10 in the actual conditions, so that gravity effects are not important.

In order to evaluate the lift force coefficient we note that a value of the transverse shear \( d\ddot{v}_f/d\xi \approx 0.2 \) can be derived from inspection of Fig. 7 of Pitter and Pruppacher (1974), as well as a maximum \( |d\ddot{u}_f/d\xi| \approx 2 \) near the plate edge. Of course the shear intensity is a function of the Reynolds number; the proposed values have to be considered as indicative, remembering that they refer to \( N_{Re} = 20 \). For our purposes the influence of the shear can
be evaluated at high Stokes numbers accounting for a lateral displacement that turns out to be function of \(N_{Re}\) of the fluid and of the reciprocal of the interception parameter \(a/A\). In particular it should be noted that the two data at \(N_{Stk} = 1.9\) and \(4.0\) are obtained with the same type of particle and obstacle, but with a \(N_{Re}\) doubled for the point at higher \(N_{Stk}\), in agreement with the previous discussion. The minimum of the collection efficiency at \(N_{Stk} \approx 10\) might be related to the combined effects, whereas the efficiency should increase (as the experimental efficiency does) for higher \(N_{Stk}\).

4. Conclusions

The present results confirm the validity of the Pitter and Pruppacher model based on the integration of the Navier Stokes equations in a wide range of conditions encountered in real clouds. However, for large Stokes numbers, our results indicate an effect of hydrodynamic interaction lowering the collection efficiency, well before the 'large radius cut-off' due to the settling velocity of the particle becoming comparable with that of the falling crystal. This lowering occurs, for example, with droplets of radii around 20 \(\mu m\) and plate ice crystals of radius around 500 \(\mu m\) and, if confirmed, should have remarkable consequences in modelling the growth of the hydrometeors. An heuristic explanation for this behaviour is proposed, in terms of hydrodynamic forces acting on the particle approaching the obstacle. The increased drag near a wall and the lift force in a shear flow appear to vary in such a way as to define the existence of a minimum of the collection efficiency. The exact position of the minimum is not a unique function of \(N_{Stk}\), but depends also on the interception parameter and on the Reynolds number of the flow.

Small but non-zero experimental efficiencies can be found for Stokes numbers well below the cut-off value indicated by the theoretical model. This may result from our specific experimental conditions, for example by possible electrostatic effects. However, if this effect is not an experimental bias, it could have important consequences in the growth of precipitating particles, due to the relatively long residence times of the crystals in the cloud.

For the planar shapes, the collection efficiency regularly increases with the Stokes number and is dependent on the surface area distribution in the models. The values of the collection efficiencies are always higher than those of the disks of equal area. The need for determining the riming efficiency for different crystal shapes, as well as the critical size and time for the onset of riming, ranks high in the priority list. In fact the riming process has direct effects on the evolution of the cloud, its water budget and precipitation efficiency, and has indirect effects on other cloud processes such as ice multiplication. The shapes that we have used as obstacles are representative of the most common shapes encountered in natural clouds, and the experiments performed can contribute in this area. The efficiencies experimentally determined can be introduced into numerical models of precipitating clouds and in the computations of microphysical and dynamical processes determined by riming.

Acknowledgments

Partial support of the Piano Finalizzato 'Promozione della qualita dell' Ambiente' (CNR) and of a research contract ENEL(Ente Nazionale per l'Energia Elettrica)-Università di Modena (Osservatorio Geofisico) is gratefully acknowledged. Mr Marcello Tercon and Mr Andrea Malossini provided technical assistance during the experiments.
REFERENCES


Stavitskaya, A. V. 1972 Capture of water-aerosol drops by flat obstacles in the form of star-shaped crystals, Izv. Atmospheric and Ocean Physics, 8, n. 7, 768–772.