Kolmogorov constants for structure functions in turbulent shear flows

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SUMMARY

Kolmogorov constants in the inertial sub-range expressions for the second-order velocity structure function and the one-dimensional spectrum of the streamwise velocity fluctuation have been estimated for both laboratory turbulent free shear flows and the atmospheric surface layer. A unique relation between the spectral constant \(a_1\) and the structure function constant is expected when the Reynolds number is infinitely large. The relation is found to be inadequate in both the laboratory and the atmosphere where values of \(a_1\), obtained via the structure function are, on average, smaller by about 15\% than spectral estimates of \(a_1\). For the present atmospheric data the average estimate, from the second-order structure function data, of \(a_1\) is about 0.54 in reasonable agreement with an estimate from the skewness of the structure function. The additive constant in the \(r^4\) law is approximately zero for the laboratory data.

1. INTRODUCTION

The first and second Kolmogorov (1941a) hypotheses have led to the well-known result that, in any turbulent flow with sufficiently large Reynolds number, the mean square of the velocity difference between two points separated by a distance \(r\) should be proportional to \(r^4\) provided \(\eta \ll r \ll L_0\). Here \(L_0\) is a length scale representative of the energy containing eddies and \(\eta\) is the Kolmogorov microscale \((\nu^3/\langle \varepsilon \rangle)^{1/4}\) where \(\langle \varepsilon \rangle\) is the mean turbulent energy dissipation and \(\nu\) the kinematic viscosity of the fluid. In the inertial subrange \((\eta \ll r \ll L_0)\), the second order structure function of the velocity fluctuation \(u\) does not depend on \(\nu\) or \(L_0\) and

\[
\langle (\Delta u)^2 \rangle = \langle (u(x+r) - u(x))^2 \rangle = f(r, \langle \varepsilon \rangle).
\]

Dimensional analysis then yields

\[
\langle (\Delta u)^2 \rangle = C \langle \varepsilon \rangle^{3/4} r^{4/3}
\]

or

\[
\frac{\langle (\Delta u)^2 \rangle}{\nu_k^2} = C (r/\eta)^{4/3}
\]  \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\)

where \(\nu_k\) is the Kolmogorov velocity \((\nu \langle \varepsilon \rangle)^{1/4}\) and \(C\) is a constant of proportionality, to be determined experimentally. The inertial expression for the normalized spectral density \(\phi(k,\eta)\) of \(u\), corresponding to (1) is given by

\[
\phi(k,\eta) = \alpha_1 (k_1 \eta)^{3/8},
\]  \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\)

where \(k_1\) is the one-dimensional wave number \(2\pi n/U\) \(n\) is the circular frequency, \(\alpha_1\) is a constant and \(\phi(k,\eta)\) is equal to \(F_1(k_1)/(\langle \varepsilon \rangle \nu^3)\) or \(F_1(k_1)/\eta \nu_k^2\) where \(F_1(k_1)\) is the one-dimensional spectrum such that the longitudinal turbulent intensity is

\[
\langle u^2 \rangle = \int_0^\infty F_1(k_1) \, dk_1.
\]  \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\) \(\ldots\)

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Monin and Yaglom (1975) noted that the equivalence of (1) and (2) is expected in the idealized situation of locally isotropic turbulence with \( L_0 \) and \( \eta \) equal to infinity and zero respectively. In this case, the Kolmogorov constants \( C \) and \( \alpha_1 \) are related by (e.g. Monin and Yaglom 1975, p. 355)

\[
\alpha_1 = 18C/(27 \Gamma(\frac{1}{3})) \\
\text{or} \\
C = 4.02 \alpha_1.
\]  

(4)

It should be emphasized that the validity of (2) and (4) implies that spectral contributions to \( \langle (\Delta u)^2 \rangle \) from outside the inertial sub-range are negligible, a condition that should be nearly satisfied when the turbulence Reynolds number \( R_\lambda \) (\( \equiv \langle u'^2 \rangle^{\frac{1}{2}} \lambda / \nu \), where \( \lambda \) is the Taylor microscale \( \langle u'^2 \rangle^{\frac{1}{2}} / \langle (\partial u/\partial x)^2 \rangle^{\frac{1}{2}} \)) is sufficiently large. Dickey and Mellor (1979) recently presented values of \( \alpha_1 \) obtained directly from the spectrum and using (4) over a relatively wide range of \( R_\lambda \). For \( R_\lambda < 100 \), there was, not surprisingly, a significant departure between the two determinations of \( \alpha_1 \), the value obtained from \( \langle (\Delta u)^2 \rangle \) using (4) being consistently lower than the spectral value obtained using (2). Values of \( \alpha_1 \), obtained using either determination, increase with increasing Reynolds number up to \( R_\lambda \approx 1000 \). For \( R_\lambda > 1000 \), the scatter in the data is large and Dickey and Mellor concluded that the asymptotically large \( R_\lambda \) value of \( \alpha_1 \) is in the range 0.5–0.7 with the structure function and spectral estimates favouring the lower and upper ends of the range respectively. Williams and Paulson (1977) also found that there was no significant variation in spectral estimates of \( \alpha_1 \) over the range 1000 < \( R_\lambda \) < 4000.

It is not intended here to review estimates of \( \alpha_1 \) available in the literature. Townsend (1976) has summarized (pre-1967) values of \( \alpha_1 \), indicating that \( \alpha_1 = 0.50 \pm 0.03 \). A more thorough discussion of values of \( \alpha_1 \), obtained from several different methods, is given in Monin and Yaglom (1975, pp. 479–485) who consider more recent data than Townsend. Monin and Yaglom conclude that the data suggest a value of \( C \) equal to approximately 2, or \( \alpha_1 \approx 0.50 \) using (4). It is worth noting however that Dickey and Mellor only include in their summary one point where independent estimates of \( C \) and \( \alpha_1 \) were made using the same data. The particular data (\( R_\lambda = 6000 \)) were obtained in the atmosphere by Van Atta and Chen (1970) who estimated \( C = 2.3 \) (or \( \alpha_1 = 0.58 \) using (4)) and \( \alpha_1 = 0.70 \) from spectra of the same data. These authors noted that the difference in these estimates, which is in the same direction as for Dickey and Mellor's summarized data at \( R_\lambda < 100 \) (where the existence of the inertial sub-range must be in question), indicated a non-negligible spectral contribution to \( \langle (\Delta u)^2 \rangle \) outside the inertial sub-range. However, by replotting Van Atta and Chen's structure function data, Dickey and Mellor found that \( C = 2.94 \), which suggests a value of \( \alpha_1 \), using (4) of 0.73. This value is not significantly different from the spectral estimate of \( \alpha_1 \) but is appreciably larger than the average value of about 0.57 from structure function data obtained by Paquin and Pond (1971) for the same experimental period and conditions (BOMEX) for which Van Atta and Chen's data were measured. In this context, it is also worth noting that all structure function estimates (Townsend 1976) of \( C \) in the atmospheric boundary layer indicate, via (4), that \( \alpha_1 \) is less than or equal to 0.50. For these estimates, however, direct measurements of the mean turbulent energy dissipation rate were not available and only indirect estimates of \( \langle \varepsilon \rangle \), inferred by closure of an incompletely measured turbulent energy budget, were made. A major source of uncertainty in estimates of \( C \) and \( \alpha_1 \) is the poor reliability of \( \langle \varepsilon \rangle \), a point which has already been discussed in detail by Monin and Yaglom (1975, p. 479). This uncertainty must be kept in mind when these estimates are used to estimate \( \langle \varepsilon \rangle \) by applying (1) or (2).

The purpose of the present paper is to present and discuss estimates of both \( C \) and \( \alpha_1 \).
obtained from laboratory and atmospheric data over a relatively wide range of $R_e$. Laboratory data were obtained in circular and plane jets. The atmospheric data were obtained in the surface layer over land at a nominal height of 4 m. For all these flows the isotropic relation $\langle \varepsilon \rangle = 15 \nu \langle (\partial u/\partial x)^2 \rangle$ was assumed to determine $\langle \varepsilon \rangle$.

It is reasonable to assume that a more reliable estimate of $\langle \varepsilon \rangle$ can perhaps be obtained from the third-order structure function $\langle (\Delta u)^3 \rangle$ since the behaviour of $\langle (\Delta u)^3 \rangle$ in the inertial sub-range can be obtained directly (e.g. Kolmogorov 1941b) from the Kármán–Howarth equation, and is written as

$$\langle (\Delta u)^3 \rangle = -(4/5) \langle \varepsilon \rangle r,$$  \hspace{1cm} (5a)

or

$$\langle (\Delta u)^3 \rangle = -(4/5)(r/\eta).$$  \hspace{1cm} (5b)

There have been relatively few attempts in the literature to verify (5a), perhaps because, as noted by Paquin and Pond (1971) and Van Atta and Chen (1970), third-order quantities converge relatively slowly and long records are required to achieve reliable averages. It should also be noted that the use of modern digital data systems instead of analogue instrumentation would have resulted in an improvement in the accuracy of third and high order moments. Van Atta and Chen observed that their data clustered around linear relation (5b), although for any particular run the data do not convincingly follow this relation, and suggested that their data provided support for Kolmogorov’s local isotropy. Park’s (1976) measurements of $\langle (\Delta u)^3 \rangle$ (see also Park and Van Atta 1980), also over the ocean, seem to provide more convincing support for (5b). Using Eqs. (1) and (5b), the skewness $S$ can be written for inertial sub-range values of $r$,

$$S = \frac{\langle (\Delta u)^3 \rangle}{\langle (\Delta u)^2 \rangle^{3/2}} = -(4/5)C^{-1/4}$$  \hspace{1cm} (6a)

or, using (4)

$$S = -0.1 \sigma_1^{1/4}.$$  \hspace{1cm} (6b)

These equations have been used to estimate $C$ (or $\sigma_1$) but the experimental scatter in $S$ (e.g. Monin and Yaglom 1975, pp. 471–472) is rather large.

Equations (1) and (2) require modifications when fluctuations in the instantaneous rate $\varepsilon$ of turbulent energy dissipation are taken into account. When the probability density function of $\varepsilon$ (subscript indicates averaging over a volume of characteristic dimension $r$) is assumed to be log-normal (Kolmogorov 1962; Obukhov 1962; Gurvich and Yaglom 1967) the exponent of $\nu$ in (1) is increased to $2/3 + \mu/9$ while the exponent of $k_1$ in (2) is correspondingly decreased to $-5/3 - \mu/9$. While the bulk of the experimental evidence suggests that $\mu$ is in the range $0.3-0.5$, a recent attempt (Van Atta and Antonia 1980) to reconcile predictions of the log-normal model with the experimentally determined dependence on $R_e$ of the skewness and flatness factors of $\partial u/\partial x$ has indicated a value of $\mu$ of about 0.2. For this value, modifications to (1) and (2) would be almost impossible to detect experimentally. It should also be noted that neither the log-normal model nor other models (e.g. Novikov and Stewart 1964, and Frisch et al. 1978), modify relation (5), which is a direct consequence of the Navier-Stokes equations. Unlike (5), relation (6) is slightly affected by the modifications since (1) is slightly affected. In section 3, experimental values of $\langle (\Delta u)^3 \rangle$ are compared with (5a) and an estimate of $\sigma_1$ is also made using (6b).
2. EXPERIMENTAL ARRANGEMENTS AND CONDITIONS

The laboratory data considered here were obtained in both plane and circular jets, described in Antonia et al. (1980). The data were measured on the axis of two circular jets of nozzle diameter \( d \) equal to 2.54 cm and 18 cm, at Reynolds numbers \( R_j \) based on \( d \) and nozzle exit speed \( U_j \) of 5.56 \( \times 10^4 \) and 4.71 \( \times 10^5 \) respectively. For the plane jet, the nozzle width \( d = 3.18 \) cm and the Reynolds number \( R_j \) about 2.04 \( \times 10^4 \). Measurements were made at a streamwise distance \( x \) from the nozzle exit plane equal to 50 \( d \) for the 18 cm diameter circular jet. For the 2.54 cm circular jet and plane jet, measurements of \( x/d \) were made over the range 70–120 and 60–140 respectively.

The atmospheric data were obtained at the Bungendore field site, CSIRO Division of Environmental Mechanics, which was planted with wheat. The wheat crop was about 0.12 m high and the fetch over wheat was between 300 and 400 m depending on wind direction while the surrounding area was grazing land of similar surface roughness. A detailed description of the site, instrumentation and experimental conditions is given in Bradley et al. (1980). The measurements were made at a height \( z \) of 4 m under stability conditions which range from near-neutral to slightly unstable. Values of \( z/L \) (\( L \) is the Monin–Obukhov length) and turbulence Reynolds number \( R_\lambda \) are given in Table 1.

<table>
<thead>
<tr>
<th>TABLE 1. COMPARISON BETWEEN KOLMOGOROV CONSTANTS</th>
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<tbody>
<tr>
<td>Flow description</td>
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<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>( x/d )</td>
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<tr>
<td>Circular jet (( d = 18 ) cm)</td>
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<tr>
<td>Circular jet (( d = 2.54 ) cm)</td>
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<tr>
<td>Plane jet</td>
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<tr>
<td>Run No.*</td>
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<tr>
<td>Atmospheric boundary layer</td>
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<td>15</td>
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<td>28</td>
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* These run numbers are the same as those referred to in Bradley et al. (1980). Detailed experimental conditions for these runs are given in that paper.

Both laboratory and atmospheric measurements of the velocity fluctuations were made with a hot wire, of diameter \( d_w \approx 2.5 \) \( \mu \)m, operated by a DISA 55M10 constant temperature anemometer. The anemometer signal \( e \) was passed through a signal conditioner before analogue differentiation (the differentiator has a gain of unity at 100 Hz and a linear frequency response up to 10 kHz). Signals proportional to \( e \) and \( \partial e/\partial t \) were recorded on a 4-track FM tape recorder at a speed of 38.1 cm s\(^{-1}\). The tapes were later played back in the
laboratory and the signals digitized on a PDP 11/20 computer to obtain various statistics of \( u, \Delta u \) and \( \partial u/\partial x \). Note that only the temporal derivative was computed and statistics of \( \partial u/\partial x \) were inferred from those of \( \partial u/\partial t \) by using Taylor’s hypothesis \( \partial u/\partial x = -U^{-1} \partial u/\partial t \), where \( U \) is the local mean velocity in the streamwise direction. The statistics of the difference \( u(t) - u(t+\tau) \), where \( \tau \) is the time delay, were computed and those of \( \Delta u = u(x+r) - u(x) \) inferred by assuming Taylor’s hypothesis, viz. \( \Delta u \equiv u(t) - u(t+\tau) \). Prior to digitization, \( \partial u/\partial t \) was passed through a low-pass filter with sharp cutoff \( f_c \) set at \( 1.75 f_k \) (\( f_k \) is the Kolmogorov frequency equal to \( U/2\eta \)) for both laboratory and atmospheric data. To obtain \( \Delta u, e \) was digitized with \( f_c \) equal to \( 1.75 f_k \) for the laboratory data and \( 0.1 f_k \) for the atmospheric data. The sampling frequency used was equal to \( 2f_c \). The length \( l_w \) of the hot wire was such that \( l_w/\eta \) was less than unity for the atmospheric data. Although the ratio \( l_w/\eta \) was in the range \( 1.3 - 2.4 \) for the laboratory data, no length correction was applied to these data. An estimate of the effect of this correction on the value of \( \langle e \rangle \) is given by Antonia et al. (1980). Values of \( \langle e \rangle \) equal to \( 15 \nu U^{-2} \langle (\partial u/\partial t)^2 \rangle \) were not corrected for the effect of turbulence intensity on Taylor’s hypothesis. Values of the Taylor microscale \( \lambda \) and Kolmogorov microscale \( \eta \), inferred from the measurements of \( \langle e \rangle \) are shown in Table 1. The effect of parasitic sensitivity to temperature of the hot wire on the third-order moment of \( \Delta u \) and \( \partial u/\partial t \) was found to be small by Park and Van Atta (1980) and no correction for this effect was applied to the presented data.

3. RESULTS AND DISCUSSION

Second-order velocity structure functions for the atmospheric and laboratory shear flows are plotted, normalized by \( \nu_k \) and \( \eta \), in Fig. 1 and 2 respectively. The data in Fig. 1 follow a linear trend over a significant range of \( r/\eta \), given approximately by \( 30 < (r/\eta)^4 < 100 \) or \( 160 < r/\eta < 1000 \). The lower bound of this range can be identified with the ratio \( \lambda/\eta \) (values of \( \lambda \) and \( \eta \) are given in Table 1). The factor of 2 has been included in the normalization of \( \langle (\Delta u)^2 \rangle \) to conform with the presentation of the structure function results of Van Atta and Chen (1970), Dickey and Mellor (1979) and others. With this normalization, the slope of the straight lines (fitted to the inertial sub-range data by eye) in Fig. 1 and 2 is approximately equal to \( 2\alpha_1 \). Values of \( \alpha_1 \) calculated from these slopes are shown in Table 1. It is obvious from Fig. 1 and 2 and the Table that the scatter in \( C \) is large for both atmospheric and laboratory data. The scatter seems more pronounced for the atmospheric data. Table 1 indicates that there is no detectable dependence of \( C \), or \( \alpha_1 \) estimated from (4), on \( R_e \). For the laboratory data, no obvious dependence of \( C \) on \( x/d \) is observed. There are insufficient atmospheric data to establish any possible dependence of \( C \) on \( x/L \).

Dickey and Mellor (1979) suggested that the \( r^3 \) law is improved by addition of a constant \( D \) such that (1) is replaced by

\[
\langle (\Delta u)^2 \rangle/2\nu_k^2 = \frac{1}{4} C (r/\eta)^3 + D,
\]

where \( D \) is given by

\[
D = 1.41 - 6.9 \alpha_1.
\]

The data in Fig. 1 and 2 do not support (8) since linear functional least squares fits to the data in Fig. 1 (\( 29 < (r/\eta)^4 < 100 \)) and Fig. 2 (\( 10 < (r/\eta)^4 < 45 \)) give values of \( D \) in the range \(-3.3 \) to \( 0.2 \) (mean equals \(-1.3 \)) and \(-0.9 \) to \( 1.1 \) (mean equals \( 0.1 \)) for the atmospheric and jet data respectively. The standard deviation of the difference between the least
squares fits and the measured values of \( \langle (\Delta u)^2 \rangle / 2v_K^2 \) was 0.5 for both the atmospheric and jet data. Van Atta and Chen inferred, as noted earlier, that \( x_1 \approx 0.58 \) from a log–log plot of \( \langle (\Delta u)^2 \rangle / 2v_K^2 \) versus \( r/\eta \) by assuming that \( D = 0 \). Figures 1 and 2 suggest that the inertial subrange does not extend to values of \( (r/\eta)^4 \) less than about 30 in the atmosphere (Fig. 1) and about 10 in the plane jet (Fig. 2). As noted earlier the lower bound, in terms of \( r \), for the inertial sub-range is approximately given by \( r = \lambda \). Since \( (\lambda/\eta) \approx 2R_3^\frac{1}{3} \) (isotropic turbulence) and there is one order of magnitude difference in \( R_3 \) between the atmosphere and the plane jet, the decrease in the lower bound of \( (r/\eta)^4 \) from 30 to 10 with increase in \( R_3 \) appears reasonable. Van Atta and Chen’s data were obtained at a value of \( R_3 \) comparable with the values for the present atmospheric data and the deviation of \( \langle (\Delta u)^2 \rangle \) from an \( r^4 \) behaviour, as inferred from Fig. 2 of Van Atta and Chen’s paper, occurs at \( (r/\eta)^4 \approx 30 \) in good agreement with the present observations. Although relatively few data points were obtained by Van Atta and Chen at small \( r/\eta \) (due to the low sampling frequency used during digitization), the value of \( C = 2.3 \) \( (x_1 \approx 0.58) \) obtained by these authors (with \( D \) zero) is not unreasonable in the light of the present data and those of Paquin and Pond (1971).

As small values of \( r \) are approached, viscous effects become important. The structure function is related to the derivative by

\[
\langle (\Delta u)^2 \rangle / 2v_K^2 = C (r/\eta)^{\alpha_1}
\]
\[
\lim_{r \to 0} (\Delta u/r) = \partial u/\partial x
\]

or, more generally,
\[
\lim_{r \to 0} \langle (\Delta u)^2 \rangle = r^n \langle (\partial u/\partial x)^n \rangle
\]  

(9)

Of present interest is the departure of \(\langle (\Delta u)^2 \rangle\) from the inertial sub-range expression and its approach to the behaviour appropriate to the viscous range. As \(r \to 0\), it follows from (9) and the isotropic expression for \(\langle \phi \rangle\) that \(\langle (\Delta u)^2 \rangle / 2v_k^2 \to 30^{-1} (r/\eta)^2\). The plane jet data at \(x/d = 80\) closely follow this viscous relation when \((r/\eta)^4\) is less than about 6. The behaviour of the data shown in the inset of Fig. 2 suggests that the transition from viscous to inertial ranges occurs for \((r/\eta)^4 \geq 10\).

Third-order structure functions are plotted in log-log coordinates in Fig. 3. For the sake of clarity, only a few experimental runs are shown in the figure. An estimate of the convergence of third-order (and second-order) structure functions was made by computing running means of these structure functions as a function of record duration. As a criterion for convergence, the time required for the mean to reach its final value to within 5% was determined. A similar procedure was used by Antonia and Van Atta (1978) to calculate convergence times for temperature structure functions in the atmospheric boundary layer. The present results, like those of Antonia and Van Atta, indicate that second-order moments
converge faster than third-order moments but, for every run examined, the convergence time for $(\Delta u)^3$ was comfortably smaller than the experimental record duration available. Typical figures for convergence times of $(\Delta u)^3$ were 6·5 min for the atmosphere and 1·7 min for the laboratory. The results in Fig. 3 show that the laboratory values of $\langle (\Delta u)^3 \rangle$ obtained at the largest Reynolds number ($R_z$ about 966 for the 18 cm circular jet) follow relation (5b) quite closely over the range $20 < r/\eta < 200$. The plane jet data all exhibit a linear behaviour over a significant part of the inertial sub-range but they lie slightly above the line represented by (5b). The atmospheric data follow (5b) reasonably well.

Since the verification of (1) or (5a) requires that $\langle \varepsilon \rangle$ is known accurately, it seems reasonable to comment on the present estimates of $\langle \varepsilon \rangle$, especially in view of the isotropic assumption made in obtaining $\langle \varepsilon \rangle$. It was shown by Antonia et al. (1980) that values of $\langle \varepsilon \rangle$ measured on the axes of circular and plane jets were in good agreement with universal relations, derived from requirements of self-preservation, between $\langle \varepsilon \rangle$ and $x$. Isotropic values of $\langle \varepsilon \rangle$ in the atmospheric surface layer (Bradley et al. 1980) indicated that, within the stability range $-0·4 < z/L < 0$, the dissipation term is very nearly balanced by the total production due to shear and buoyancy. While this result cannot provide unequivocal support for the measurement of $\langle \varepsilon \rangle$ (no attempt was made to measure the contribution to the diffusion term by pressure fluctuations so that a complete balance of the turbulent energy budget was not possible), it does provide some measure of confidence in the present isotropic estimates of $\langle \varepsilon \rangle$. 
Figure 4. Skewness coefficient of velocity structure function. Circular jet (d = 2.54 cm): X, x/d = 70; ○, 80; △, 90; □, 120. Circular jet (d = 18 cm): +, 50. Atmospheric surface layer: △, run 15; □, 16; ○, 19; ▽, 35. ▼, Park (1976).

The skewness $S$, shown in Fig. 4, is constant over an appreciable range of $r/\eta$ for the laboratory data. Only the circular jet data for $d$ of 2.54 cm are shown here, but the plane jet and the circular jet data for $d$ of 18 cm also exhibit a wide plateau where $S$ is constant, equal to approximately $-0.20$. While the jet data of Fig. 4 exhibit reasonable similarity at different values of $x/d$, the atmospheric data, shown for different values of $z/L$, exhibit greater scatter than the laboratory data but the inertial sub-range value of $S$ is not significantly different from $-0.2$. For $r/\eta > 1000$, atmospheric values of $-S$ show either an increase (the values of Park and Van Atta exhibit such a trend) or decrease with increasing $r/\eta$. Inertial sub-range values of $S$ were used in conjunction with (6b) to obtain estimates of $\alpha_1$. These values are shown in Table 1 and are generally in reasonable agreement with estimates from (4) except for the 2.54 cm jet data where the average value of $\alpha_1$, as inferred from (6b) is 37% larger than the average estimate obtained from (4) but in reasonable agreement with the average spectral estimate. There is little variation in the average values of $\alpha_1$, inferred from (6b), obtained in the different flows considered here.

Spectral estimates of $\alpha_1$ were obtained by plotting the product $(k_1^2)\phi$ as a function of $k_1\eta$ and identifying a plateau region which corresponds approximately with the inertial sub-range. Typical spectra for the different flows are presented in Fig. 5 and the plateau extends over values of $k_1\eta$ between 0.01 and 0.07 in the laboratory, and between 0.005 and 0.06 in the atmosphere. The scatter, in the inertial sub-range, of $\alpha_1$ with $k_1\eta$, is slightly larger for the atmospheric runs (standard deviation about 0.035) than for the laboratory (0.023) runs. The scatter in the spectral estimates of $\alpha_1$ for the different runs given in Table 1 is large for both atmospheric and laboratory data. There is no detectable $R_\lambda$ dependence
for this estimate of $z_1$. Further, there does not appear to be a dependence of $z_1$ on $x/d$ for the laboratory data or $z/L$ for the atmospheric data.

4. CONCLUDING REMARKS

Measurements of second-order velocity structure functions in both laboratory and atmosphere indicate good support for the $r^4$ law. The agreement between the third-order velocity structure functions and relation (5b) can be considered as satisfactory. This relation is a consequence of the Navier-Stokes equations with the assumption of local isotropy and, since it contains no unknown constants to be determined empirically, it provides an important, although perhaps stringent, test for the data. The importance of this test is further emphasized when it is realized that this relation is not affected by fluctuations in the instantaneous dissipation of turbulent energy. The additive constant in the $r^4$ law is approximately zero for the laboratory data. The atmospheric data indicate an average value for this constant of $-1.3 \pm 0.5$. Atmospheric values of $z_1$ obtained using the structure function are, on average, smaller by about 17% than estimates of $z_1$ inferred from the velocity spectrum. A similar difference was noted by Van Atta and Chen (1970). The present atmospheric average value of $C$ is in good agreement with that obtained by Van Atta and Chen, and Paquin and Pond (1971). It is not in agreement with the estimate of Dickey and Mellor (1979) from a reassessment of Van Atta and Chen’s data. The laboratory results indicate a difference between structure function and spectral estimates of $z_1$ of similar magnitude to that found in the atmosphere.

If a ‘universal’ value of $z_1$ is to be used, as is often the case in atmospheric investigations, to infer the mean dissipation rate $\langle \epsilon \rangle$, it is important that (1) is used when $\langle (\Delta u)^2 \rangle$ is
measured, with the appropriate value of $C$. Alternatively, when the spectrum is measured, it is important that the 'spectral' estimate of $\alpha_1$ is used. It would be incorrect to estimate $\langle \varepsilon \rangle$ from (1) using 'spectral' estimates of $\alpha_1$ via relation (4).

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