An economical method for computing the radiative energy transfer in circulation models

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SUMMARY

Based on the methodology of a two-stream approximation (Kerschgens et al. 1978) the transfer of radiative energy in model atmospheres can now be calculated with high economy and sufficient accuracy. In this model the number of spectral intervals has been minimized to four (solar spectrum) and six (infrared spectrum) for which new effective transmission functions have been computed, where standard aerosol profiles and absorption and scattering coefficients are incorporated. The concentrations of major atmospheric gases (water vapour, carbon dioxide, ozone), the cloud cover and cloud liquid water content of each layer can be changed freely. Thus this model could be used in any numerical circulation model.

In the first part of this paper we discuss the basic principles of the method and results obtained for cloudless model atmospheres. The accuracy of computed flux densities is better than 5% and of flux divergences better than 20% in the worst cases.

In the second part this method will be applied to total and partial cloud cover in each layer.

1. INTRODUCTION

A new radiation routine has been developed for use in the new general circulation model (GCM) being developed by the research staff of the German Weather Service on the basis of the older model of Tiedtke and Geleyn (1975). This routine must fulfil three major conditions:

(a) Input data are solely atmospheric (including cloudiness) and ground properties, as generated by a GCM.

(b) The computational time for each profile should not exceed 15 ms on a CDC 7600 Cyber computer for a ten layer model.

(c) The accuracy should be of the same order as that of more sophisticated multispectral models.

Many methods to simulate accurately the transfer of solar and infrared radiation in realistic model atmospheres have already been developed. Those which divide the spectrum into many intervals require too much computer time. Other more empirical formulations, e.g. Möller’s parametrization of radiative heating (Mügge and Möller 1932) are considered to be too inaccurate and inflexible with respect to all possible states of the atmosphere created by a GCM. Other radiation models used in GCM’s (e.g. Somerville et al. 1974) treat only water vapour and ozone absorption with additionally, in the infrared, carbon dioxide absorption, but they neither contain an aerosol model in the solar region nor treat water vapour dimer absorption in the atmospheric window region, effects which have proven to be a major radiative heat source in a clear moist atmosphere (e.g. Grassl 1973). A summary of such models has been given by Cox (1978). This new routine is in principle a two-stream approximation, as discussed by Kerschgens et al. (1978). It uses new transmission functions representative of only ten broad spectral intervals. This method of integrating the radiative transfer equation directly is of course not as economical as the highly parametrized methods, but it is much more flexible in the sense that it allows any stratifications occurring within a GCM to be taken into account. Its accuracy has been checked by intercomparisons with results calculated using high spectral resolution methods (Raschke 1972; Jung 1975).

In the first part of this paper we describe the method and present results obtained with cloudless model atmospheres, which contain aerosols (absorbing and scattering) and the water vapour dimers, as well as the major absorbing gases. Since clouds overwhelmingly
dominate the radiative energy budget of the earth-atmosphere-system, their inclusion in the algorithm – in particular for cases of broken or partial cloudiness within a grid field – will be discussed in the second part.

2. Theory

All calculations use a plane-parallel geometry and may be applied to an arbitrary number of layers.

(a) Solar spectral range (0.2 to 4.0 μm)

In this spectral range it has been assumed that Eq. (1) the thermal emission by the atmosphere is negligible, Eq. (2) all diffuse fluxes exhibit axial symmetry, and Eq. (3) the integration of radiances over a hemisphere can be estimated by an empirical relation. Then, one can obtain from the well-known set of differential equations for the upward and downward radiances (e.g. Chandrasekhar 1950) a set of coupled linear differential equations of the first order for the upward and downward radiation flux densities \( M^+ \) and \( M^- \)

\[
\frac{dM^+(\delta)}{d\delta} = \left(1 - \tilde{\omega}(1 - \beta) \right) \frac{M^+(\delta)}{\bar{\mu}^+} - \tilde{\omega} \beta \frac{M^-(\delta)}{\bar{\mu}^-} - \tilde{\omega} \beta_0 S_0 \exp(-\delta/\mu_0) \\
\frac{dM^-(\delta)}{d\delta} = \tilde{\omega} \beta \frac{M^+(\delta)}{\bar{\mu}^+} - \left(1 - \tilde{\omega}(1 - \beta) \right) \frac{M^-(\delta)}{\bar{\mu}^-} + (1 - \beta_0) S_0 \exp(-\delta/\mu_0)
\]

where
\( \delta \) = optical depth in the spectral interval
\( \tilde{\omega} \) = single scattering albedo
\( \beta, \beta_0 \) = fractional backward scattering for diffuse and direct solar radiation
\( \bar{\mu}^+, \bar{\mu}^- \) = mean effective cosine of the zenith angle of the upward and downward radiation fluxes, as determined by integration of radiances over a hemisphere
\( S_0 \) = solar irradiance

The derivation of Eq. (1) and (2) and the evaluation of the backscattering parameters \( \beta_0, \beta, \mu^+, \mu^- \) is discussed by Kerschgens et al. (1978); they form the basis of the two-stream method which will be used below.

(b) Terrestrial radiation

Neglecting scattering and the azimuthal dependence of the radiation fields, and using again an empirical integration of all radiances within one hemisphere, the flux density of thermal radiation \( M^+ \) and \( M^- \) can be determined from Eq. (3):

\[
\frac{dM^\pm(\delta)}{d\delta} = (M^\pm(\delta) + \pi B(T))/\bar{\mu} \quad . \quad \quad \quad .
\]

Assuming the Planck function to be described by \( B(T) = B_0 + B_1 \delta \) within a single atmospheric layer, where \( B_0 \) is a boundary value, and \( 1/\bar{\mu} = \pm 1.66 \) (e.g. Rodgers and Walshaw 1966) one obtains for the fluxes:

\[
M^-(\delta) = \pi B_0 + \pi B_1 (1.66 \delta - 1)/1.66 + C^- \exp(-1.66 \delta) \quad . \quad \quad \quad (3a)
\]

\[
M^+(\delta) = \pi B_0^+ + \pi B_1 (1.66 \delta - 1)/1.66 + C^+ \exp(1.66 \delta) \quad . \quad \quad \quad (3b)
\]

The values of the constants \( C^+ \) and \( C^- \) can be determined for each layer within the model from the boundary conditions:

(a) at the top of the atmosphere \( M^-(\delta = 0) = 0 \)
(b) between two layers \( M^\pm(\delta_i) = M^\pm(\delta_{i+1}) = 0 \)
(c) at the ground (or cloud surface) \( M(\delta) = \pi \epsilon B(T_B) \)

with \( \epsilon = \) emittance and
\( T_B \) = boundary temperature.

3. Parametrization

The spectral transmission functions for all atmospheric gases participating in radiative
transfer processes reported by Selby et al. (1975) have been divided into only four intervals in the solar and six intervals in the infrared region. In principle, the parametrization of the new transmission functions or relation between the optical depth \( \delta_i \) of the \( i \)-th layer and the absorber mass follows the same scheme. First the upward and downward fluxes for the spectral intervals given in Tables 1 and 2 have been calculated with two-stream approximations using high spectral resolution. These flux values are the basis for the inversion of the equations to determine average values of \( \delta_i \) for each layer.

**TABLE 1. Sub-intervals in the thermal radiation region**

<table>
<thead>
<tr>
<th>Wave number interval (cm(^{-1}))</th>
<th>Absorber</th>
<th>Number of intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 25 – 500</td>
<td>H(_2)O</td>
<td>95</td>
</tr>
<tr>
<td>2 500 – 830</td>
<td>H(_2)O, CO(_2)</td>
<td>66</td>
</tr>
<tr>
<td>3 830 – 1000</td>
<td>H(_2)O, dimer</td>
<td>14</td>
</tr>
<tr>
<td>4 1000 – 1100</td>
<td>O(_a), dimer</td>
<td>22</td>
</tr>
<tr>
<td>5 1100 – 1250</td>
<td>H(_2)O, dimer</td>
<td>28</td>
</tr>
<tr>
<td>6 1250 – 2525</td>
<td>H(_2)O, CO(_2), NO</td>
<td>255</td>
</tr>
</tbody>
</table>

**TABLE 2. Sub-intervals in the solar radiation region**

<table>
<thead>
<tr>
<th>Sub-ranges ((\mu m))</th>
<th>Absorbing gases</th>
<th>Number of spectral intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.215 – 0.685</td>
<td>O(_a)</td>
<td>86</td>
</tr>
<tr>
<td>0.685 – 0.891</td>
<td>H(_2)O</td>
<td>540</td>
</tr>
<tr>
<td>0.891 – 1.273</td>
<td>H(_2)O</td>
<td>525</td>
</tr>
<tr>
<td>1.273 – 3.58</td>
<td>H(_2)O, CO(_2)</td>
<td>1020</td>
</tr>
</tbody>
</table>

(a) *Terrestrial radiation*

The absorber limits and numbers of intervals in the tables of Selby et al. (1975) are listed in Table 1 for each of the six intervals:

The additional absorber in the window region, which causes major cooling in a clear moist boundary layer and which is commonly regarded as water vapour dimer or polymer absorption, is treated in a somewhat crude, but efficient way with, following Kuhn (1972), a constant absorption coefficient of 0.2 m\(^2\) g\(^{-1}\) between 830 and 1100 cm\(^{-1}\) and 0.1 m\(^2\) g\(^{-1}\) between 1100 and 1250 cm\(^{-1}\). The mean integrated transmission functions for each of the six spectral ranges were determined from fluxes calculated with an exact spectral two-stream model for a variety of model atmospheres using an inversion of Eq. (3). Since these calculations give directly the flux densities at each of the \( i \) boundary layers, the value of \( C_i \) can be obtained from

\[
C_i^+ = M_i^+ - \pi B_i + \pi (B_{i+1} - B_i)/(1.66\delta_i) 
\]

where \( M_i, B \) and \( \delta_i \) are now averaged values over the wavelength intervals. Inserting Eq. (4) into Eq. (3) one obtains, with

\[
A1 = M_{i+1} - \pi B_{i+1}, \quad A2 = \pi (B_{i+1} - B_i)/1.66, \quad A3 = M_i - \pi B_i
\]

an equation for the mean optical depth \( \delta_i \):

\[
A1 = A2/\delta_i + (A3 + A2/\delta_i)\exp(-1.66\delta_i). 
\]

Eq. (5) can be solved numerically to obtain generalized values for \( \delta_i \), which are different for the upward and downward radiation fluxes. They depend (through the Planck function) on the temperature profiles and (through the mean fluxes \( M \)) on the absorber amount per layer. A functional relationship between all values of \( \delta \) and the absorber amount was obtained with a fit according to Eq. (6):

\[
\log_{10} \delta(u) = \sum_{j=0}^N a_j (\log_{10} u)^j
\]

where \( u \) is the pressure-reduced absorber amount and \( N \) is the number of required terms. The pressure reduction is made to \( P_0 \) of 1013 mb with the exponents suggested by Selby et al.
Different relations for $\delta(u)$ were derived for all layers below and above a major inversion, such as the tropopause. They are available on request. An example of a fit according to Eq. (6) is shown in Fig. 1. If the temperature dependence is modelled correctly the sensitivity of the fluxes and divergences to changes of the fitted functions at small path-lengths is of the same order as the sensitivity of the exact fluxes when changing the spectral input data of the high resolution calculations. Therefore, for computational efficiency functions are chosen which are as simple as possible.

(b) Solar spectral region

The spectral region between 0.2 and 3.58 $\mu$m has been divided into four spectral sub-intervals, listed in Table 2, with the number of intervals in the Tables of Selby et al. (1975):

Absorption and scattering by aerosols are also taken into account. As in (a) for each layer of the model atmosphere mean values for the optical depth $\delta_m$, and also for the single scattering albedo $\omega_m$, have to be determined for each spectral sub-range from fluxes calculated with very high spectral resolution (last column in Table 2). $\omega$ and $\delta_m$ are determined by inversion of Eq. (1) as described briefly below.
For the $i$-th layer of the model atmosphere the following homogeneous solutions of Eq. (1) and (2) for the flux densities are obtained:

\[
M^+_{i}(\delta_i) = C_1, i \exp(r_i \delta_i) + C_2, i \exp(- r_i \delta_i) \\
M^-_{i}(\delta_i) = C_3, i \exp(r_i \delta_i) + C_4, i \exp(- r_i \delta_i)
\]  
(7)

with

\[
r_i = (1/\mu)(1 - 2\bar{\omega}_i(1 - \beta) + \bar{\omega}_i^2 (1 - 2\beta)^2)^{1/2}
\]  
(8)

and

\[
C_{3, i} = C_{2, i} (\tilde{\omega}_i/\beta) (1 - \bar{\omega}_i (1 - \beta) + r_i \mu)^{-1} \\
C_{4, i} = C_{2, i} (\tilde{\omega}_i/\beta) (1 - \bar{\omega}_i (1 - \beta) - r_i \mu)^{-1}
\]  
(9)

Continuity of flux densities must exist at each boundary: hence

\[
M^+_{i+1}(\delta_{i+1}) = 0 = M^+_{i}(\delta_i) \\
M^-_{i+1}(\delta_{i+1}) = 0 = M^-_{i}(\delta_i)
\]  
(10)

The aim now is to form from Eq. (7) a set of two equations for two unknowns, $\delta_m$ and $\omega_m$, by using relations Eq. (8), (9) and (10) and the results of the high resolution calculation. This can be done by the following procedure:

(a) $M^+_{i+1}$ and $M^-_{i+1}$ on the left hand side of Eq. (10) are calculated by averaging the respective fluxes of the high resolution calculation

(b) Since the accurate calculations give $C_{1, i}$ and $C_{2, i}$, the averaging is also applied to these and the resulting mean values are inserted into Eq. (9)

(c) A mean backscattering parameter $\beta$ is prescribed externally and inserted in Eq. (8) and (9).

If now Eqs. (8), (9) and (10) are combined with Eq. (7) the two equations for $\delta_m$ and $\omega_m$ are found, and are solved numerically with a two-dimensional Newton-iteration scheme (Selder, 1973). The mean spectral absorption $\sigma_{a, m}$ and scattering $\sigma_{s, m}$ coefficients can now be obtained for each layer:

\[
\sigma_{a, m} = (1 - \tilde{\omega}_m) \delta_m/\Delta Z \\
\sigma_{s, m} = \tilde{\omega}_m \delta_m/\Delta Z
\]  
(11)

Since different mean photon path lengths are involved in the multiple scattering, which generates the diffuse radiation, and in the single scattering of the direct radiation, separate parametrization procedures have to be developed for the diffuse and direct components.

\[c)\text{ Diffuse radiation}\]

The parametrization scheme requires several steps:

(1) The absorption by each individual gas in a pure molecular atmosphere has been calculated first for a set of model atmospheres. So each point in Figs. 2, 3 and 7 represents a single atmospheric layer of a single model atmosphere. The overlap of the absorption bands of two gases (H$_2$O and CO$_2$ in the fourth sub-range) is treated as a functional relationship between the sum of the absorption by each individual component and the absorption computed when both gases are considered together. This relationship is shown in Fig. 3. It allows a flexible choice of the amount of CO$_2$ for further experiments with the GCM.

(2) Aerosol scattering and absorption has been computed with the spectral data of Shettle and Fenn (1973), using their stratospheric aerosol for all layers above 100 mb. Their mid-tropospheric aerosol model is taken for the upper troposphere ($p$ between 100 and 500 mb), and a rural model (or maritime for ocean areas) in all layers below. These values already enter the accurate calculations as constants in each spectral sub-interval. The inversion method discussed above was used to derive mean scattering and absorption optical depths for the combined action of aerosol and gases. For the mean scattering and absorption coefficients a single relation of the form:

\[
\sigma_{m(a,s)} = A \delta_{m, a}
\]  
(12)
Figure 2. Relation between the optical depth due to water vapour absorption $\delta_w$ and water vapour path length of diffuse solar radiation in the spectral range 0.685 – 0.894 $\mu$m.

Figure 3. Relation between the optical depth due to absorption of diffuse radiation and the sum of optical depths due to water vapour and carbon dioxide absorption in the spectral interval 1.273 – 3.580 $\mu$m.
Figure 4. Relations between the effective optical depth due to scattering and the sum of Rayleigh and aerosol scattering optical depths.

can be obtained where $\delta_{m,x}$ may be the sum of either all gaseous absorption ($\delta_{m}(\text{H}_2\text{O}) + \delta_{m}(\text{CO}_2)$) (Fig. 3), or gaseous and aerosol absorption (Fig. 5), or molecular and aerosol scattering coefficients (Fig. 4). Values for $A$ and $B$ are from a least squares fit.

Aerosols are included in the solar model, because they contribute a significant part of the heating rate in the near surface layers. The heating rates, which arise not only by pure aerosol absorption but also by feedbacks between multiple scattering, surface albedo and gaseous absorption can reach values of 1 to 1.5 K d$^{-1}$ (e.g. Grassl 1974). The profiles of the aerosol models mentioned above are fixed for land or ocean surfaces. This generalization is necessary since, at present, no general circulation model predicts them. Nevertheless, if the distribution function and chemical composition do not change, some interaction is still possible by changing the number density of the aerosol content, normally chosen to give a visibility of 23 km. A suitable parametrization has to be found for this, of course.

(d) Direct radiation

For the downward beam irradiance the computation of mean optical depth $\delta_{i,m}$ for the $i$-th layer follows an integration of the exponential extinction law:

$$\delta_{i,m} = \int \delta(\lambda) \exp \frac{-\delta(\lambda) / \mu_0}{\int \exp \left( \frac{-\delta(\lambda) / \mu_0}{\mu_0} \right) d\lambda} d\lambda$$

with

$$\delta_x = (d\delta/dz) \Delta z_i$$

in the $i$-th layer, $\mu_0 = \cos \theta_0$. 

$$\delta_{i,m} = \int \delta(\lambda) \exp \frac{-\delta(\lambda) / \mu_0}{\int \exp \left( \frac{-\delta(\lambda) / \mu_0}{\mu_0} \right) d\lambda} d\lambda$$

with

$$\delta_x = (d\delta/dz) \Delta z_i$$

in the $i$-th layer, $\mu_0 = \cos \theta_0$. 

Figure 5. Relations between the effective optical depth due to absorption and the sum of gas and aerosol absorption optical depths.

Figure 6. Dependence of the optical depth $\delta_{c,m}$ of the direct solar radiation on the cosine of solar zenith angle $\mu_0$. 
Figure 7. Relation between the optical depth due to water vapour absorption and water vapour path length of the direct solar radiation in the spectral range 0.685 - 0.894 μm

(a) \( \delta_\theta(\text{direct, } 35^\circ) \)

(b) \( \delta_\theta(\text{direct, } 75^\circ) \)

- a: \( \theta_\theta = 35^\circ \)
- b: \( \theta_\theta = 75^\circ \)
The optical path of the direct beam in each layer depends on the zenith angle $\theta_0$ of incidence. For the range of $\mu_0$ between 0.17 and 1.0, and of $\theta_0$ between 80° and 0°, we found a linear relation between $\delta_{l,m}$ and $\mu_0$, shown in Fig. 6. The optical depths $\delta_{l,m}$ could therefore be determined from the calculations for only two solar zenith angles (35°, 75°). Subtracting the Rayleigh-scattering coefficients from $\delta_{l,m}$ one obtains the absorption coefficients for direct radiation as a function of the absorber amount in each layer. Results are shown in Fig. 7. They are based on the same data as were used to determine absorption and scattering coefficients for downward and upward diffuse radiation. At an arbitrary zenith angle the direct absorption coefficient $\sigma_0$ can then be evaluated by a linear interpolation in $\mu_0$ between the parametrized values at the above two zenith angles. From Eq. (13) it follows immediately that an additional extinction, which like the aerosol extinction is constant across the wavelength interval, results in an additive extinction in both the high resolution and parametrized calculations.

The sensitivity to changes in the fit of the diffuse and direct parametrizations is different. For the diffuse parametrization the situation is the same as indicated above in (a) for terrestrial radiation. Again parametrizing functions, which are as simple as possible, are chosen. This is not true for the direct beam parametrization where different parametrizations are needed for short and long paths.

4. TEST CALCULATIONS

Tests to verify the accuracy of this parametrized two-stream approximation were made by comparing a large number of model calculations with results obtained from the method with high spectral resolution. Fig. 8 shows examples of infrared heating rate profiles for subtropical (STS) and sub-arctic (SAS) model atmospheres (Valley 1965) above a wind-roughened ocean surface. The vertical resolution is 1 km from the ground to 30 km. In the lower troposphere deviations occur in the cooling rates of around 10 to 15%, while near the tropopause a very small cooling is predicted, where the exact method gives a small heating. The flux densities of atmospheric radiation to the ground and to space were computed and agree within 11 to 18 W m$^{-2}$ or 4 to 6%. The total flux divergences agree even better, as shown in Table 3.

<table>
<thead>
<tr>
<th>Model atmosphere</th>
<th>Sub-tropical</th>
<th>Sub-arctic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation flux to space</td>
<td>exact 285.0</td>
<td>approx. 296.2</td>
</tr>
<tr>
<td>Radiation flux to ground</td>
<td>exact 397.0</td>
<td>approx. 381.7</td>
</tr>
<tr>
<td>Flux divergence in troposphere</td>
<td>exact -201.2</td>
<td>approx. -202.8</td>
</tr>
<tr>
<td>Flux divergence in atmosphere</td>
<td>exact -213.9</td>
<td>approx. -212.6</td>
</tr>
</tbody>
</table>

Vertical profiles of solar heating are compared in Fig. 9. The vertical resolution in the test cases is 1 km between ground and a height of 10 km, and 10 km between 10 km and 70 km. They show the largest deviations in the lower troposphere and upper stratosphere. But the total flux density at the ground responsible for ground heating, and subsequent effects in the planetary boundary layer and to space, again show a closer agreement with differences of 6-12 W m$^{-2}$. In Table 4 values of the global radiation at ground and upward radiation at the top of the atmosphere are compared at different solar zenith angles.
Figure 8. Longwave heating of the atmosphere compared with the exact (-----) and approximative (-----) methods for (a) the subtropical summer (STS) and (b) the subarctic summer (SAS) model atmosphere.

<p>| TABLE 4. COMPARISON OF EXACT AND APPROXIMATE CALCULATIONS OF SOLAR RADIATION FLUXES (Units: W m⁻²) AND PLANETARY ALBEDO (%) FOR A CLOUDLESS SUBTROPICAL MODEL ATMOSPHERE ABOVE AN OCEAN |</p>
<table>
<thead>
<tr>
<th>Zenith Angle</th>
<th>75°</th>
<th>50°</th>
<th>35°</th>
<th>5°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downward flux at ground</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exact</td>
<td>155.9</td>
<td>566.5</td>
<td>770.1</td>
<td>977.2</td>
</tr>
<tr>
<td>approx.</td>
<td>169.9</td>
<td>571.8</td>
<td>767.3</td>
<td>965.1</td>
</tr>
<tr>
<td>Upward flux at the top</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exact</td>
<td>95.7</td>
<td>103.6</td>
<td>95.0</td>
<td>87.7</td>
</tr>
<tr>
<td>approx.</td>
<td>105.8</td>
<td>110.6</td>
<td>102.1</td>
<td>94.5</td>
</tr>
<tr>
<td>Flux divergence in atmosphere</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exact</td>
<td>109.4</td>
<td>217.6</td>
<td>262.0</td>
<td>304.6</td>
</tr>
<tr>
<td>approx.</td>
<td>83.0</td>
<td>201.1</td>
<td>253.0</td>
<td>303.3</td>
</tr>
<tr>
<td>Flux divergence in troposphere</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exact</td>
<td>76.6</td>
<td>152.5</td>
<td>183.3</td>
<td>212.7</td>
</tr>
<tr>
<td>approx.</td>
<td>61.3</td>
<td>143.9</td>
<td>182.8</td>
<td>221.1</td>
</tr>
<tr>
<td>Planetary albedo (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exact</td>
<td>27.4</td>
<td>11.9</td>
<td>8.6</td>
<td>6.5</td>
</tr>
<tr>
<td>approx.</td>
<td>30.3</td>
<td>12.8</td>
<td>9.2</td>
<td>7.0</td>
</tr>
</tbody>
</table>
Figure 9. Solar heating of the atmosphere computed with the exact (- - - -) and approximate (---) method for different solar zenith angles \( \theta_o \) in a subtropical summer (STS). a: \( \theta_o = 35^\circ \); b: \( \theta_o = 50^\circ \); c: \( \theta_o = 75^\circ \).
5. CLOUDS IN THE ATMOSPHERE

(a) General remarks

Clouds are the most effective radiation modulators in the earth-atmosphere system. In parametrizing their radiative effects, they have generally been treated as internal boundaries; for example, in the infrared as black or grey emitting bodies (c.f. Manabe and Strickler 1964) and in the solar region characteristics are prescribed depending on height and cloud cover (c.f. Somerville et al. 1974). More recently some modellers have used the cloud liquid water content (diagnosed from a computed humidity field) to vary the prescribed albedo values or to incorporate a zenith angle dependence (Washington et al. 1979). Geleyn and Hollingsworth (1979) use a prescribed single scattering albedo for cloud droplets and a cloud optical depth which depends on the liquid water content. Broken cloud in all models is treated by varying the prescribed values with cloud amount in the infrared as well as in the solar region.

In this paper we present a different approach. It does not significantly increase the computing time nor the computer storage. The verification of the parametrization of broken cloud is difficult because it requires a large quantity of observations of the vertical and horizontal cloud structure together with measurements of the radiation fields above and below cloud sheets; for example as obtained during field experiments such as GATE or JASIN. An attempt to verify the parametrization using JASIN data is given below.
(b) Treatment of complete cloud decks

**Infrared region (4 to 100 μm)**

The basic feature of the infrared model is the neglect of scattering. The clouds are treated as additional emitting boundaries within the atmosphere. The clear sky fluxes are replaced by grey body fluxes, where the emittance depends on the cloud liquid water path, following Stephens (1978). To test this parametrization a comparison is made with a model which includes scattering (Schmetz and Raschke 1980a) for a variety of liquid water contents. In tests, shown in Fig. 10, the model atmosphere consisted of 40 layers equally spaced in $\sigma = p/p_0$ (Δ$\sigma = 0.025$). The cloud was placed in the layer between 836 to 861 mb (about 1350 to 1600 m). Profiles of temperature, pressure and ozone concentration were taken from the US Standard Atmosphere, the relative humidity $\nu$ in the troposphere being chosen equal to $\sigma$; in the stratosphere the mixing ratio was $2.0 \times 10^{-6}$ (g per g).

Between the models the agreement is generally reasonable.

**Solar region (0.2 to 4 μm)**

In the solar spectral region clouds are treated as an additional scattering and absorbing medium. The parametrization of their integral radiative transfer properties again requires calculations with high spectral resolution, following the procedure for the clear atmosphere. This was done using the model described in (3b) inserting for the clouds a prescribed single scattering albedo for cloud water droplets $\bar{\omega}^*$ and a cloud optical thickness parametrized in terms of the liquid water content again following Stephens (1978). Some values are given in Table 5. The backscattering parameters for cloud were taken from Kerschgens et al. (1978).

Our calculations show that the integral optical depth of the cloud layer is equal to the sum of cloud optical depth and the respective clear sky (gaseous and aerosol) optical thickness. This is not a trivial result, since we obtain an entirely different relation for the aerosol-

**TABLE 5. SINGLE SCATTERING ALBEDOS OF CLOUD DROPLETS ($\bar{\omega}^*$) AND PARAMETRIZATION OF CLOUD OPTICAL THICKNESS ($\delta_c$) (Stephens 1978)**

<table>
<thead>
<tr>
<th>Spectral interval (μm)</th>
<th>0.215 - 0.685</th>
<th>0.685 - 0.894</th>
<th>0.894 - 1.273</th>
<th>1.273 - 3.58</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\omega}^*$</td>
<td>0.999998</td>
<td>0.999985</td>
<td>0.9996</td>
<td>0.9973</td>
</tr>
<tr>
<td>$\log_{10}\delta_c$</td>
<td>0.2633 + $\ln(\log_{10}(LWC,\Delta Z))$</td>
<td>0.3492 + 1.6518 $\ln(\log_{10}(LWC,\Delta Z))$</td>
<td>0.9973</td>
<td></td>
</tr>
</tbody>
</table>
gas mixture in 3(c) Eq. (12). The mean single scattering albedo is parametrized in terms of the inverse optical depth (Fig. 11).

\[ \tilde{\omega} = (A - B/\delta_w)\tilde{\omega}^* \]  

(14)

\( \tilde{\omega}^* \) is the prescribed single scattering albedo of cloud water droplets (Table 5) and the parameters \( A, B \) depend mainly on the height of the cloud.

We found it necessary to introduce the Delta-Two-Stream Approximation for the cloudy atmosphere as suggested for example by Schaller (1979) or in a modified way by Zdunkowski et al. (1980). We compared our model with another two-stream approximation code, whose results have reproduced detailed cloud radiation measurements (Schmetz and Raschke 1980b), and which also agrees quite well with other radiation models (e.g. Slingo 1980, private communication). A sample result of the comparison is presented in Fig. 12. The vertical resolution and the physical input data are the same as described above 5(c). The absolute as well as the relative differences between computed results are well within the variations arising from the variability of parameters, such as the droplet distribution, which cannot be predicted in GCM's.

(c) Treatment of broken cloud decks

Infrared region

The inclusion of broken cloud in the infrared part of the model is straightforward. The upward or downward fluxes at each boundary are calculated simply as the weighted average of the clear sky fluxes and the grey body fluxes of the cloud layer (see above), using the fractional cloud cover as the weighting factor. As a test we compare the model with a realistic situation, given by data obtained on 29 July 1978, during the JASIN experiment over the Northern Atlantic Ocean. Cloud cover observations were both synoptic and photo-
Figure 12. Solar fluxes downward at the surface (a, b, e) and upward at the top of the atmosphere (d, c, f) as a function of cloud liquid water content.

- a, d: high standard model, small droplets $r_m = 5.5\, \mu m$
- b, c: high standard model, large droplets $r_m = 10\, \mu m$
- e, f: parametrized model

Figure 13. Comparison of hourly averaged measurements and calculations of the downward infrared flux at the surface for 29 July 1978 (JASIN experiment).
graphic, and radiative flux measurements were made on board the research vessel ‘METEOR’ (Gube 1979). Temperature, pressure and humidity were taken from radiosonde data for the same day, matched with the observed sea surface temperature. Low clouds had a liquid water content of 0.2 g m\(^{-3}\), as measured from aircraft nearby; medium and high clouds 0.05 and 0.008 g m\(^{-3}\), respectively. The bases of the lower cloud sheets were given by observations, whereas the heights of medium and high clouds were fixed at 4 and 7 km. The calculated fluxes and the observations, averaged hourly, are compared in Fig. 13. The daily mean is indicated by an arrow. The agreement between the time series is remarkably good for this special study except for the last 3 to 4 hours during the night, when only synoptic cloud observations were used.

Solar region

Scattering in the shortwave part of the spectrum prohibits the same solution for incorporating broken cloud as used in the infrared. But the inversion technique described in 3(b) provides a useful tool for treating partial cloudiness in a relatively simple and straightforward manner. Again for partial cloudiness the parametrization starts with computations for the clear sky and the totally cloud-covered sky. A mean flux for each value of fractional cloud cover can be defined by averaging linearly the two values above. This average flux enters the inversion method (as described in 3(b)) in just the same manner as the spectrally averaged fluxes in the clear sky case. As result a mean optical depth of the cloudy layer as a function of cloud cover and optical thickness of the clear atmosphere and the cloud is derived (Fig. 14). The analytical fit gives an exponential increase of optical depth with cloud cover, but for cloud values less than 10% a linear relation holds (Fig. 15). There is some observational evidence that the optical thickness derived from bulk transmissivities of observed broken cloud field follow a similar relation (Schmetz 1980, private communication).

The single scattering albedo is calculated from Eq. (14) using the mean optical depth. For partial cloudiness the downward solar flux at the surface is a function of cloudiness, where values must be interpreted to be representative of an area equal to the GCM grid square. The general shape of the curve (Fig. 16) showing this dependence is remarkably similar to the observations given by Kasten and Czeplak (1980). This seems an astonishing result remembering that the cloud cover parametrization was derived from a linear flux average as described above. The reasons for the deviation of the calculated fluxes from a linear relation and for their approach to the observed results are twofold:

(a) Although only the homogeneous solution of the two-stream approximation can be used in calculating mean optical depths and single scattering albedos, in the computation of the solar flux the inhomogeneous terms, which generate the diffuse radiation, are included.

(b) The downward direct irradiance does not vary linearly with cloud cover. For the calculation of its extinction the average optical depth of the cloudy layer has been used.

Together, (a) and (b) mean that the cloud, which covers only part of the grid square, is smeared out over the entire grid area and this ‘new’ cloud stimulates some observational features.

This parametrization was also tested with the JASIN data set and results are shown in Fig. 17. The differences between the calculated and measured daily mean is 7.5 W m\(^{-2}\) (i.e. 2.5%), which is well within the observational errors of the surface solar radiation budget recommended for climate measurements (GARP Report No. 16). We suggest further comparisons with data from experiments with greater variation of cloudiness than observed during JASIN.

6. Concluding Remarks

The fast method for computing radiative energy transfer in general circulation models
Figure 14. Mean optical depth of a cloud layer as a function of cloud cover $N$ ($0.1 \leq N \leq 1$). Numbers indicate the optical depth of the cloudy layer for $N = 1$ (Spectral range $0.894 - 1.273 \mu m$).

Figure 15. Same as Fig. 14 but $0 \leq N \leq 0.1$. 
Figure 16. Downward solar flux at the surface as a function of cloud cover for different solar elevations.
(a) --- calculated, ---- linear decrease
(b) observations after Kasten and Czeplak (1980).
Figure 17. Same as Fig. 13, but for solar flux at the surface.

(GCM), discussed here, is in principle a delta-two-stream approximation with transmission functions parametrized for very broad spectral ranges. Due to the lack of a sufficient knowledge of their interaction with the dynamics, aerosol effects on radiative transfer are included using standard profiles and coefficients. But the concentrations of water vapour, ozone and carbon dioxide as well as the cloud cover and the cloud liquid water content are treated as variable parameters to be provided by the GCM. In a cloud-free atmosphere the method produces results on the vertical flux divergence and the radiative flux densities, which agree well with those computed with more accurate methods. The same can be said about the treatment of cloud decks. But in all these cases it is clear that errors arise from unknown cloud parameters such as the size distribution of droplets. This may introduce uncertainties of up to 30 W m$^{-2}$ for the global radiation. In the infrared, clouds are treated as grey emitting boundaries in the atmosphere. The effects of broken cloud fields in the solar region are computed with a cloud-cover-dependent optical depth. This treatment of solid and broken cloud decks is straightforward and does not require much additional computing time. In Table 6 some CPU-times in ms are listed; we expect to reduce them with more efficient programming efforts.

<table>
<thead>
<tr>
<th>Number of layers</th>
<th>CPU-time (ms, approx.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>13 ms</td>
</tr>
<tr>
<td></td>
<td>CDC 7600 (University Cologne)</td>
</tr>
<tr>
<td>11</td>
<td>15 ms</td>
</tr>
<tr>
<td></td>
<td>CRAY 1, scalar (NCAR)</td>
</tr>
<tr>
<td>10</td>
<td>15 ms</td>
</tr>
<tr>
<td></td>
<td>CDC 7600 (DWD, University Cologne)</td>
</tr>
<tr>
<td>15</td>
<td>15 ms</td>
</tr>
<tr>
<td></td>
<td>CRAY 1, scalar (ECMWF)</td>
</tr>
<tr>
<td>40</td>
<td>70 ms</td>
</tr>
<tr>
<td></td>
<td>CDC 7600 (University Cologne)</td>
</tr>
</tbody>
</table>
This parametrization scheme has been tested for fractional covered sky by comparison with measurements obtained over the Northern Atlantic Ocean during the JASIN experiment. We intend to perform more elaborate tests with the data from GATE and from an experiment to be conducted in autumn 1981 over the North Sea. Tests of the radiation model in a General Circulation Model and in the forecast model of the European Centre of Medium Range Weather Forecasts (ECMWF) (Burridge and Hasler 1977) are under way. The simulated fluxes at the top of the atmosphere will be compared with satellite measurements.

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