An Analysis of the Large-scale Features of the Upper Troposphere and the Stratosphere in a Global, Three-Dimensional, General Circulation Model

By A. O'NEILL, R. L. NEWSON*, R. J. MURGATROYD
Meteorological Office, Bracknell

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SUMMARY

The formulation of a 13-level general circulation model of the troposphere and stratosphere is described and an account given of the climatology produced from an annual integration. Emphasis is placed on the interhemispheric differences in the stratosphere and the seasonal variability exhibited by its circulation. Overall, a realistic simulation of various features of the large-scale tropospheric and stratospheric flow patterns was achieved, although a number of shortcomings are noted.

The atmospheric state of the model is resolved into zonal mean and eddy components by Fourier analysis around latitude circles, while the latter are further divided into time-averaged and transient modes. The structure of these aspects of the flow is described and the influence of the eddies on the mean flow in the stratosphere is discussed from a dynamical point of view by considering momentum and heat budgets. An attempt is made to identify the various factors governing the seasonal cycle of zonal mean wind and temperature in the stratosphere, and it is suggested that wave transience initiated in the troposphere may play a role in this respect, in addition to the evolving field of net radiative heating.

The apparent success of the model in spontaneously generating events which resemble stratospheric sudden warmings is also reported.

1. INTRODUCTION

It is well known from observations that the average winds in extra-tropical latitudes of the stratosphere are moderate easterly and rather steady in summer but strong westerly with planetary-scale disturbances in winter. These conditions exist in both hemispheres but the Northern Hemisphere stratosphere is much more disturbed in winter than that of the Southern Hemisphere. There have been many theoretical and diagnostic studies of this seasonal evolution (see, e.g., Holton, 1975; Labitzke and van Loon, 1972) and several attempts to gain insight into the processes involved by using numerical models (e.g., Manabe and Hunt, 1968; Cunnold, et al., 1975; Manabe and Mahlman, 1976). The present paper describes some results which were obtained during an investigation using data acquired from an integration of a three-dimensional, general circulation model of the troposphere and stratosphere. Details of the model formulation are included and the account concentrates on the seasonal evolution of the large-scale circulation in the stratosphere and upper troposphere, emphasizing the interhemispheric differences shown by these regions of the atmosphere.

The model was originally developed in the Meteorological Office to be used in studies of the possible meteorological effects of aircraft flights in the stratosphere, particularly of potential effects on ozone amounts (COMESA, 1975). It has also provided data on other important features of the simulated general circulation. An analysis of the mechanism within the model which produced several apparently successful simulations of stratospheric sudden warmings has been made by O'Neill (1980; denoted as Paper I). In addition, Allam et al. (1981) have used the fields of water vapour and ozone in their investigation into the distribution of the chemical species OH in the atmosphere.

2. THE MODEL

Corby et al. (1972) described a general circulation model suitable for long period integrations. A developed and extended version of this model was constructed by Newson for the investigations described in this paper.

*Present affiliation: World Meteorological Organisation, Geneva
The primary purpose of this work being to study the stratosphere, a number of levels were added above those in the original model. At the same time other changes and additions were made, either to deal adequately with the stratospheric conditions or to improve specific features of the tropospheric representation. These include extension of the model to a global domain, the use of a variable resolution lateral grid, the introduction of seasonal variation and changes to the physical parametrizations such as the surface transfer process, the radiation scheme and the representation of deep convection. The latter are improvements or empirical developments of the original formulations and are considered to have contributed to making the model suitable for its primary purpose, as well as being economic computationally.

Other versions of the original model have also been developed which have some features in common with that of Newson, for example Francis (1975) and Corby et al. (1977). In particular the computing grid and finite difference approximations in Newson's version are essentially as described by Francis (1975). That is the grid consists of points along lines of latitude, is irregular, non-staggered and designed to provide a quasi-constant grid-length of about 330 km; a minimum number of 16 points per line of latitude is used so that the effective grid-length is smaller near the poles and a reduced time-step is used there to maintain linear stability. The time-step for the dynamical calculation is centred with a weak time-filter to prevent separation of the grids at successive time-levels. For physical processes, including dissipation, a forward step is used.

As in the basic model, the sigma-coordinate system introduced by Phillips (1957) is used in the vertical. There are 13 atmospheric levels, placed as in Table 1.

<table>
<thead>
<tr>
<th>Level</th>
<th>$\sigma$</th>
<th>Approximate height in km, if surface pressure is 1000 mb.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0015</td>
<td>44</td>
</tr>
<tr>
<td>2</td>
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<td>35</td>
</tr>
<tr>
<td>3</td>
<td>0.011</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>0.020</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>0.032</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
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<tr>
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<td>10</td>
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<tr>
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<td>3</td>
</tr>
<tr>
<td>13</td>
<td>0.896</td>
<td>1</td>
</tr>
</tbody>
</table>

The scheme to represent the transfer of heat and moisture by sub grid-scale convection is based on that of Corby et al. (1972). Some minor changes were necessary because of the unequal sigma intervals in the vertical, and condensation appearing as a result of convective interchange between two contiguous layers is evaporated into the lower layer (rather than being treated immediately as precipitation at the ground) unless the relative humidity there is 80% or more. The latter change provides a better simulation of upper tropospheric lapse rates than the original scheme (Gilchrist et al. 1973). A further addition is that the convective interchange mechanism is allowed to operate on momentum, as well as on heat and moisture.

Surface transfer processes were modified from those of the original model more substantially than other sub grid-scale parametrizations. The original proposals were based on a bulk treatment of the atmospheric boundary layer and there were difficulties in estimating quantities dependent upon atmospheric properties close to the earth's surface. Although
the calculation of the momentum flux was identical to that of Corby et al. (1972), using four drag coefficients corresponding to land/sea, stable/unstable conditions, significant changes were made to the treatment of the sensible and latent heat fluxes. The upward fluxes of these quantities at the surface were taken to be proportional to wind-speed and temperature or moisture gradient as appropriate, and did not include the additional \((\Delta \Theta / \Theta_g)\) dependence used by Corby et al. (symbols used in the text are defined in Appendix 1). The temperature and moisture gradients were derived from the differences between values at the surface and those estimated empirically at a near-surface level. The definition of neutral conditions separating stable and unstable categories in the atmospheric boundary layer was also altered. Over the sea, the 'neutral' temperature difference was taken to be that given by the moist adiabat from the sea surface. Over land, the neutral condition was taken as the dry adiabat from \((T_s + 2 K)\), where \(T_s\) is the model's predicted surface temperature. In general the wind-speed used in the calculation of the surface fluxes was that of the lowest model level. However, a minimum value of 3 m s\(^{-1}\), or 5 m s\(^{-1}\) in unstable conditions, was imposed. This was to avoid the development of excessively low temperatures in polar regions in anticyclonic inversions, or high temperatures in the tropics in calm conditions. In the calculation of the latent heat flux, a factor was introduced to convert potential to actual evaporation. Over the sea and in stable conditions elsewhere the factor was assumed to be 1·0, but in unstable conditions over land and ice points it was reduced to 0·25.

The radiation scheme for the experiments described here was particularly simple. The infra-red cooling of the atmosphere was linearly dependent on the temperature. The constants varied with latitude, height and season, and were derived by comparing calculated cooling rates with the appropriate zonally averaged mean temperature at a number of points at a given height in 45° latitude bands. The rates computed by Doplick (1970) were used up to 30 mb and those derived by Murgatroyd and Goody (1958), and Kuhn and London (1969) were used above this. Figure 1 shows the long-wave cooling resulting from the radiation calculation when applied to the zonally averaged mean temperature field for January. For direct heating of the atmosphere by absorption of shortwave radiation, rates varying with latitude, height and season were used. Figure 2 shows the heating field used for January.

Seasonal variation was simulated by taking the values shown for January and applying a sinusoidal variation with time with a yearly cycle. A cut-off in the solar heating was applied to take account of high latitude winter conditions when the sun is constantly below the horizon.

The incoming short-wave component of radiation at the surface was based on the figures of Doplick; they do not differ much from those used by Corby et al. (1972). The values of emissivity of the upward long-wave radiation from the surface, taken as function of latitude, were also fixed at values based on Doplick's figures.

As in the model of Corby et al. (1972), each grid point was designated as land, sea or sea-ice, with each land point being given an appropriate topographic height. The designation of a point as sea or sea-ice was amended at 30 model day intervals according to climatological statistics. The variation of the surface temperature at a grid point depended on the nature of the surface at that point, distinguishing the three cases: sea, sea-ice and land. The sea-surface temperature was not interactive with the model variables but was imposed at its climatological value. To account for seasonal variation, the values of the sea surface temperature were changed after each five days, the change being derived from climatological, monthly sea-surface temperatures. The prediction of the surface temperature at sea-ice and land points employs the methods that have also been used by Corby et al. (1977). For ice points, account was taken of heat conduction through the ice, but the surface temperature was not allowed to exceed 273 K. For land points the surface temperature was allowed to vary freely according to radiative and heat exchanges with the atmosphere. In computing the radiative fluxes, the albedo of ice surfaces, or land where the temperature is less than 263 K was assumed to be 0·8; otherwise for land, a value of 0·25 was taken.
The model also includes a simulation of large-scale precipitation and latent heating as described by Corby et al. (1972). A lateral eddy viscosity term of the form $K \nabla \cdot p_\ast \nabla \chi$ was used; $K$ is proportional to $|\nabla \cdot p_\ast \nabla \chi|$ for $\chi = u, v, T$ and constant ($5 \times 10^5 \text{ m}^2 \text{ s}^{-1}$) for $\chi = q$.

The treatment of negative water vapour mixing ratios, which can develop on account of errors generated by the finite difference equivalent of the continuity equation, was the same as that described by Corby et al. (1977). That is, if a negative value arose at a grid point, any advective transfer (horizontal or vertical) from that grid point was set to zero if this would have made the existing value more negative.

3. Model Experiment and Analysis Procedures

In this paper, results obtained from a seasonal integration with a global version of the model will be presented. The model was integrated for 454 days from January 1, with seasonal variation being achieved by prescribing radiation conditions and sea-surface temperatures. The integration was initialized using fields obtained on day 60 of a simulation in which radiation conditions and sea-surface temperatures were held fixed at values appropriate to January.

Zonal and meridional wind components and temperature at all grid points were output
from the model at 6-hourly intervals. Geopotential heights and vertical velocities were recovered at the sigma levels using the same computing algorithms as in the model formulation. These fields were selected at 24 h intervals and linearly interpolated, with respect to pressure, to pressure surfaces at 2, 5, 10, 20, 30, 50, 70, 100, 200, 300, 500, 700, and 900 mb.

The fields thus obtained were decomposed into Fourier components around latitude circles to produce zonal means, and amplitudes and phases for wave numbers 1 to 12 for each variable. These components were further resolved into time-averaged and transient contributions by computing monthly averages and departures therefrom. Expressions in general form for quantities to be discussed in subsequent sections are given in Appendix 2.

4. Model Results: Mean Fields

(a) Zonal Mean Temperatures and Winds

Figure 3 shows the distributions of zonal mean temperatures and winds for the second February and July of the seasonal integration. In all the discussion below of phenomena in the Northern Hemisphere winter, conditions during the second February of the integration rather than the first are described, since then the model has attained a state more representative of the winter stratosphere when the circulation is comparatively undisturbed by stratospheric warmings.
Figure 3. (a) Zonal mean wind (m s$^{-1}$) and (b) zonal mean temperature (K) for the second February of the integration; (c) and (d), the same but for July.
The temperature fields produced by the model are generally in reasonable agreement with mean climatological values (see, e.g., Newell et al., 1969) with a good representation of the seasonal and interhemispheric differences. Other well-known features, including the sudden warming phenomenon discussed in I and the occurrence of maximum temperatures in the lower stratosphere at middle latitudes in winter were well simulated. The main discrepancies with observations occur at high latitudes of the stratosphere in winter, particularly in the Southern Hemisphere where the model temperatures are 10–20 K too low. The reproduction of Northern Hemisphere winter conditions is rather better in the middle stratosphere, although the lower stratosphere near the pole is about 10 K too cold. In summer, temperatures in the Northern Hemisphere are quite accurately reproduced, but polar temperatures in the Southern Hemisphere are too cold. The cross-sections of zonal wind presented in Fig. 3 are also in reasonable accord with climatological values (e.g., Newell et al., 1969), but there are more obvious discrepancies than in the temperature comparisons. Thus, the zonal wind speeds in the winter extra-tropical stratosphere are too strong in the Southern Hemisphere by about 30 m s\(^{-1}\) at the top model level when compared with the late-winter average derived by Hartmann (1976a, Fig. 4). The intensity of the polar-night jet is also too strong during the first winter of the Northern Hemisphere (not shown), and in neither hemisphere are the polar westerlies sufficiently separated from the tropospheric, subtropical jet streams. These failings are consistent with the unrealistically large meridional temperature gradients found at these times. The polar-night jet was of a more realistic strength once it was established during the second Northern Hemisphere winter. This improvement may perhaps be explained by the establishment of a quasi-equilibrium state between the mean flow and the planetary-wave perturbations well after the start of the integration which never seems to occur during the first winter. On the other hand, the summertime easterlies in the model are generally too weak at mid-latitudes in the Northern Hemisphere stratosphere, and never replace westerlies at high latitudes in the Southern Hemisphere as they do in the real atmosphere. The simplified treatment of radiation in the model might account for these discrepancies. The easterlies are also too weak in the equatorial upper troposphere.

The seasonal variation of zonal mean temperature in the model's middle stratosphere is displayed in Fig. 4(a), which shows the distribution of the temperature deviation at each latitude from the April-March annual mean at that latitude. The amplitude of the seasonal temperature cycle in the stratosphere is a maximum in the polar regions, as in the atmosphere, and has about the correct value in the Northern Hemisphere but is rather too large in the Southern Hemisphere. The temperature cycle has an asymmetrical behaviour about the equator at 5 mb, in that, in the tropics of both hemispheres, the temperatures vary in phase, with the Northern Hemisphere cycle encroaching well into the Southern Hemisphere. This reflects differences in orography between hemispheres (Reed and Vlcek, 1969). Departures from a smooth variation are evident near the North Pole during the first and second winters around February and January respectively. These are related to large temperature perturbations in the stratosphere associated with the growth of planetary waves, and resemble observed stratospheric warmings as discussed in I.

Figure 4(b) shows the seasonal evolution of zonal mean wind at 5 mb. The replacement of winter westerlies by summer easterlies is successfully simulated in the Northern Hemisphere, but not in the Southern Hemisphere at high latitudes (the origin of this discrepancy is unknown), and the easterlies are of about the correct duration (cf. Richards, 1967). The seasonal variation of the easterly flow at low latitudes is reproduced, with its core switching from one hemisphere to the other between summer and winter, thus producing a semi-annual component in the cycle of zonal wind speeds at equatorial latitudes. The westerly phase of this cycle, observed by Reed (1966) in the real stratosphere, is not, however, reproduced.
Figure 4. Latitude-time sections at 5 mb of (a) the temperature deviation (°C) from the April-March mean for each latitude, and (b) the seasonal variation of zonal mean wind (m s⁻¹).

(b) Mean Meridional Circulations

Figure 5 presents stream-functions of the simulated mean meridional circulations during the four seasons of the year. In the troposphere, the Hadley, Ferrel and polar mean circulations are evident in each season, and all penetrate the tropopause, extending well into the stratosphere. Near the solstices a weak meridional circulation occupies most of the equatorial stratosphere and extends up to mid-latitudes in the winter hemisphere. In the Summer Hemisphere at polar latitudes, the tropospheric polar cell extends up to 2 mb and there is very weak subsidence near the pole throughout the model atmosphere.
Figure 5. Stream functions of the simulated, monthly-mean meridional circulations for February, April, July and October. The difference between two contour values (units: $10^{10}$ kg s$^{-1}$) gives the meridional mass flux between those contours (positive values denote northward flux). Stippled and clear areas denote circulations in the clockwise and anti-clockwise senses, respectively.

For the real atmosphere, determination of the meridional circulation for the summer stratosphere is subject to considerable uncertainty owing to its weakness and the importance for the circulation of the field of net radiative heating, which can only be approximately calculated. Newell et al. (1974) found weak ascent near the summer pole in the middle stratosphere and, if this finding is generally valid for the real atmosphere, the model is deficient in this respect. This could possibly be accounted for by the simplified treatment of radiation.

At high latitudes in the winter stratosphere, an extension of the tropospheric Ferrel cell dominates, as observed by Vincent (1968) for the actual stratosphere, with ascent at polar latitudes and descent at mid-latitudes. The circulation is thermally indirect with the region of subsidence in the lower stratosphere coinciding with a local temperature maximum. The corresponding indirect cell during the Southern Hemisphere winter is much weaker and is replaced by weak descent in the zonal mean at lower stratospheric levels in the south polar region. Thus there is a three-cell meridional circulation in this hemisphere. Such a structure has been determined for the actual Southern Hemisphere stratosphere by Adler (1975) and Hartmann (1975), and has also been simulated by Manabe and Mahlman (1976). A possible explanation is that the mean circulation of the Southern Hemisphere stratosphere in winter is, in comparison with its Northern Hemisphere counterpart, only weakly disturbed by planetary-scale eddies. Consequently, their effect in forcing an indirect circulation is not sufficient to prevent the establishment of a single direct cell in the strato-
sphere by the field of net radiative heating, except at mid-latitudes where the eddy fluxes of momentum and heat are largest.

5. Model fields of geopotential height and temperature

The simulated monthly-mean distributions of geopotential height and temperature at 5 mb, and geopotential height at 300 mb are shown in Fig. 6 for the Northern Hemisphere. At 5 mb in winter, an intense cold vortex prevails, displaced from the pole, with well-marked troughs extending over North America and Northern Asia. It is accompanied by an anticyclone over the North Pacific Ocean – the 'Aleutian High'. During spring, the polar vortex is replaced by an anticyclone which persists during summer and, being undisturbed by planetary-wave perturbations, is almost concentric with the North Pole. These features are in qualitative agreement with observed conditions (e.g. Labitzke et al., 1977), although the winter vortex is too strong, especially during the first winter of the integration, and the summer anticyclone is less developed than is typically observed.

At 300 mb, the simulated winter circulation includes well-marked troughs over the Canadian Arctic and Northeast Asia, and weak ridges in the North Atlantic and Pacific areas, but again the polar vortex is too intense. With the approach of summer, the flow becomes more zonal as the polar vortex fills. Long-wave features are re-established during autumn.

The corresponding patterns for the Southern Hemisphere are shown in Fig. 7. The winter vortex at 5 mb is more symmetric relative to the pole than that of the Northern Hemisphere and is more intense. There are also inter-hemispheric differences in the upper troposphere, with the time-mean flow in the Southern Hemisphere being typically more zonal than that of the Northern Hemisphere.

6. Eddy components of the simulated circulation

In the Northern Hemisphere winter stratosphere, planetary waves 1 and 2 account for most of the variance of geopotential height (Hare and Boville, 1965; van Loon et al., 1973). In the middle stratosphere, wave 1 amplitudes are typically an order of magnitude larger than those of wave 3 in the middle stratosphere, and waves of higher wave number are of progressively smaller amplitude on average. These observations support the theoretical work of Charney and Drazin (1961), Matsuno (1970) and others, and are reproduced by the present model. In the Southern Hemisphere stratosphere, planetary wave 1 is much weaker during winter than in the Northern Hemisphere and wave numbers higher than 2 have relatively greater importance. Nevertheless, results from the model indicate that most of the variance in geopotential height is again contained in waves 1 and 2, in agreement with the observational findings of Hartmann (1976a). Consequently, the discussion below of planetary-wave amplitudes and their associated eddy momentum and heat fluxes will be limited to waves 1 and 2. Data are presented for the middle stratosphere and also for the upper troposphere to illustrate connections between these regions of the atmosphere.

(a) Time-Averaged Waves

The seasonal variation of time-averaged waves 1 and 2 at 5 and 300 mb are shown in Fig. 8. The amplitudes were calculated from the Fourier coefficients derived from the monthly-mean fields of geopotential height (see Appendix 2). At stratospheric levels, they provide a measure of the response to steady forcing in the troposphere.

In the Northern Hemisphere, there is a pronounced seasonal cycle in wave 1 at 5 mb with maximum amplitude in winter and minimum in summer (Fig. 8(a)). This is an outstanding feature of the seasonal variability of the real stratosphere, and is consistent with the theoretical result of Charney and Drazin (1961) that the easterly wind régime of the summer


Figure 6. (a) and (b): Geopotential height (dam) at 5 mb for the Northern Hemisphere during winter (February 2) and summer (July). (c) and (d): The same but for temperature (K). (e) and (f) Geopotential height (dam) at 300 mb during winter and summer.
Figure 7. As for Fig. 6, but for the Southern Hemisphere during winter (July) and summer (February 2).
Figure 8. Latitude-time sections of time-averaged amplitudes of geopotential height (m) for (a) wave 1 at 5 mb, (b) wave 1 at 300 mb, (c) wave 2 at 5 mb, (d) wave 2 at 300 mb.
stratosphere inhibits upward penetration of wave disturbances generated in the troposphere. As stratospheric planetary waves are believed to be forced by large-scale disturbances in the troposphere (see Holton, 1975, for a review of the evidence) the findings of Charney and Drazin are generally taken to explain the contrasting characteristics of the stratospheric circulation in winter and summer. However, Fig. 8(b) shows that in the model there is a further important seasonal variation likely to influence stratospheric wave amplitudes, namely the seasonal variation in the forcing due to the cycle of wave amplitudes in the upper troposphere. This cycle is approximately in phase with that of the stratosphere and must be ultimately related to the evolving field of solar radiative heating.

In the Southern Hemisphere stratosphere, the simulated seasonal variation of wave 1 amplitude is smaller, in agreement with observations (Labitzke and van Loon, 1972). During the simulated winter, the amplitude at 5 mb is up to one order of magnitude smaller than that of the Northern Hemisphere winter. (At lower levels in the middle stratosphere, a ratio of amplitudes in mid winter of about one third is more representative both for the model and for the real atmosphere). There is a seasonal cycle in the upper troposphere, although a significant difference from the Northern Hemisphere is that large amplitudes are sustained over a much shorter period. As linear wave calculations (e.g. those of Matsuno, 1970) indicate that wave 1 will propagate vertically in the polar westerly jet more readily than shorter waves, this inter-hemispheric difference is likely to be important in understanding the pronounced differences in the stratosphere. This hypothesis is supported by the numerical experiments of Kasahara et al. (1973), and Manabe and Terpstra (1974) who demonstrated that the large scale mountain ranges in the Northern Hemisphere play an important part in maintaining the time-averaged planetary waves.

The seasonal cycle of wave number 2 in the Northern Hemisphere stratosphere (Fig. 8(c)) has peak amplitude considerably smaller than that of wave 1, but the amplitudes are roughly comparable in the Southern Hemisphere. Shorter waves in the time-averaged circulations have progressively smaller amplitude at 5 mb but still exhibit a seasonal cycle in both hemispheres with maxima in winter. In the upper troposphere, wave 2 amplitudes are comparable with and weaker than those of wave 1 in the Northern and Southern Hemispheres respectively (Figs. 8(b) and (d)). These findings are again in qualitative agreement with observations.

Figure 9 shows the vertical structure of time-averaged waves 1 and 2 in both hemispheres during winter. Generally, there appears to be reasonable agreement between the simulated and observed values (e.g. van Loon et al., 1973; Hartmann, 1977). As would be expected from the theoretical work of Simmons (1974, 1978), the latitudinal distribution of wave amplitude broadly follows that of the zonal wind in the winter stratosphere, and the asymmetry in structure between the hemispheres is evident. In the Northern Hemisphere it can be seen that, whereas near the tropopause, the amplitudes of wave 1 and 2 are similar, the wave 1 amplitude increases with height much more than that of wave 2, indicating there is limited vertical penetration of wave 2 compared with wave 1. Although in the middle stratosphere time-averaged wave amplitudes are quite close to typically observed values, the amplitudes in the upper troposphere are too weak when compared with the observational results of van Loon et al. for the Northern Hemisphere and Hartmann for the Southern Hemisphere (op. cit.). Assuming a tropospheric origin for the stratospheric waves, these results appear contradictory at first sight. The explanation may lie in the presence of excessively strong westerlies in the winter stratosphere and their influence on vertical wave structure. Also, the waves in the model are likely to be less damped than in the real atmosphere as no provision is made in the radiation scheme for the coupling between ozone concentration and temperature. This coupling increases the thermal damping rate, especially above 30 km (Blake and Lindzen, 1973).

Wave 1 in the Northern Hemisphere exhibits a pronounced westward tilt with height up to the top level (not shown) so that the stratospheric vortex may be associated with the East Asian 'low' in the upper troposphere. The wave 2 field shows a much smaller westward tilt in the vertical. From the position of the troughs and ridges in the wave 1
and wave 2 patterns at the 5 mb level, the 'Aleutian High' is found to arise predominantly from constructive interference between these waves. In the Southern Hemisphere, the vertical tilt with height is very small for wave 1, but wave 2 shows more pronounced westward tilt. The observational study of Hartmann (1977) indicates more westward tilt for time-averaged wave 1 than found from the model's fields.

(b) Transient Waves

The seasonal evolution of transient wave amplitudes for waves 1 and 2 is shown in Fig. 10. These components were calculated as deviations from the monthly-averaged fields (Appendix 2) and, therefore, include travelling waves and fluctuations in the amplitudes of standing waves.

In general, the transient wave patterns are similar to their stationary counterparts in Fig. 8. For both wave numbers in both hemispheres, the amplitudes are large in winter and small in summer at all levels, with broad agreement in the phase of this seasonal cycle between the upper troposphere and stratosphere. At 5 mb in the Northern Hemisphere, amplitudes are considerably smaller than those of time-averaged waves, particularly for wave 1. This provides some justification for the commonly used steady-state approximation in theoretical analyses of the structure of stratospheric planetary waves, although the importance of the contribution of transient waves is recognised. During the Southern Hemisphere winter at 5 mb, on the other hand, amplitudes are comparable with their time-averaged counterparts. The importance of transient waves in the real stratosphere of the
Figure 10. As for Fig. 8 but for transient wave 1 ((a) and (b)) and transient wave 2 ((c) and (d))
Southern Hemisphere has been noted by Deland (1973), Hartmann (1976b), and Harwood (1975).

At 300 mb, the amplitudes of the transient and time-averaged wave components are comparable, but there are significant differences in their latitudinal distribution in the Northern Hemisphere. Whereas Figs. 8(b) and (d) show that the time-averaged waves are largest in a relatively low latitude band in mid-winter (approximately 30-45°N), the transient waves are largest at 300 mb near 60°N (Figs. 10(b) and (d)).

It is demonstrated in I that the high-latitude, transient perturbations are of prime importance in initiating the stratospheric warmings simulated by the model.

7. EDDY FLUXES IN THE MODEL STRATOSPHERE

(a) Time-Averaged Wave Contributions

Eddy momentum and heat fluxes associated with upward-propagating, quasi-stationary planetary waves play an important part in determining the circulation of the winter stratosphere, particularly in the Northern Hemisphere. Figure 11 shows latitude-time sections of the meridional component of these fluxes at 5 mb for the dominant waves 1 and 2. Corresponding vertical fluxes were found to play a comparatively minor role in the dynamics of the extra-tropical stratosphere, as would be expected from a quasi-geostrophic scale analysis, and they will not be considered.

Momentum and heat fluxes are generally directed poleward in both winter hemispheres. Their distribution in latitude and height, together with that of the net heating field, determines the mean meridional circulation (Holton, 1972, pp. 228 et seq.). In the Northern Hemisphere during winter, their net effect is sufficient to drive a well-marked, indirect mean meridional circulation at high latitudes, described above, against the opposing effect of net cooling in the polar night. Eddy fluxes associated with wave 1 dominate in the Northern Hemisphere, whereas, although much weaker, the largest contribution in the Southern Hemisphere is from wave 2. It is of interest to note that the wave 2 eddy momentum flux is directed equatorwards at high latitudes during both Northern Hemispheres of the integration, (Fig. 11(b)), for reasons which are unknown at present.

Vertical sections of the above quantities during the Northern Hemisphere winter are in good qualitative agreement with those derived from observational data. Quantitatively, eddy fluxes in the model's middle stratosphere are typical of observed values. In the lower stratosphere, however, they appear to be too small, as noted above for the wave amplitudes. It seems likely that the anomalous structure of the mean zonal wind and temperature fields in the lower stratosphere, referred to previously, is linked to this deficiency.

(b) Transient Wave Contributions

The meridional fluxes of momentum and heat associated with transient waves 1 and 2 at 5 mb (not shown) are substantially less than the time-averaged contributions during the Northern Hemisphere winter. During winter in the Southern Hemisphere stratosphere, transient eddy fluxes are dominant, particularly the contributions from wave 2, but nevertheless they are small compared with those of the Northern Hemisphere. One striking feature is that the largest transient eddy momentum fluxes for both waves 1 and 2 are directed equatorwards during both Northern Hemispheric winters of the simulation. This observation is relevant to the dynamical mechanism proposed in I for the simulated stratospheric sudden warmings.

8. MOMENTUM AND HEAT BUDGETS AND THE SEASONAL CYCLE IN THE STRATOSPHERE

As described above, the numerical model employed reproduced a seasonal cycle of zonal wind and temperature in the stratosphere. In this section, this cycle is discussed from a dynamical standpoint and an attempt is made to explain it. Momentum and heat budgets
Figure 11. Latitude-time sections at 5 mb of (a) $\bar{u} \bar{v}'$ (1), (b) $u' \bar{v}'$ (2) (c) $v' \bar{T}'$ (1), (d) $v' \bar{T}'$ (2). Units: m$^3$s$^{-1}$ for momentum fluxes, mKs$^{-1}$ for heat fluxes.
were calculated at all model levels throughout the period of the integration. Basically, the procedure involved expressing the zonal mean wind and temperature changes in terms of the effects of meridional eddy fluxes of momentum and heat, the mean meridional circulation and the field of net radiative heating.

It was found by analysis of the model output, and is suggested by a quasi-geostrophic scale-analysis of the wave motions, that the stratospheric momentum and heat budgets can be adequately represented by these quantities; for the most part vertical eddy fluxes could be neglected in comparison with the horizontal fluxes. Neglecting some additional terms also found to be generally small, the approximate budget equations below may be derived:

\[
\frac{\partial \bar{u}}{\partial t} = f\bar{v} - \frac{1}{\alpha \cos^3 \phi} \frac{\partial}{\partial \phi} \left( u' \bar{v}' \cos^2 \phi \right) .
\]  

(1)

Figure 12. Terms in the momentum budget at the 5 mb level for the Northern Hemisphere (left) and Southern Hemisphere (right). Units: 10^{-5} \text{ m s}^{-2}.
Figure 13. Terms in the heat budget at the 5 mb level for the Northern Hemisphere (left) and Southern Hemisphere (right). Units: $10^{-5}$ K s$^{-1}$.

\[
\frac{\partial \bar{T}}{\partial t} = \bar{Q}/c_p - \bar{\omega} \frac{\partial (\ln \bar{\theta})}{\partial p} - \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (\bar{\nu} \bar{u} \cos \phi)
\]

where the overbar represents a zonal average on a constant pressure surface, a prime a departure from that average, and the symbols have their customary meanings as explained in Appendix 1. All terms in equations (1) and (2) were calculated independently from the numerical output of the model and are plotted in Figs. 12 to 14 for the 5 mb level which is taken to be dynamically representative of the model’s middle stratosphere.

(a) Results for the Northern Hemisphere

Figure 12 shows that in the Northern Hemisphere winter nearly complete compensation occurs between the contributions to the momentum budget of large-scale eddies and the
induced mean meridional circulation; zonal wind changes occur primarily as a result of small imbalances between these contributions. Similarly, temperature changes at extratropical latitudes in winter are the result of diabatic heating and small imbalances between opposing contributions of eddy heat flux convergence and adiabatic effects accompanying the mean meridional circulation (Fig. 13). This compensation is well known and has been reproduced in many numerical model simulations of the stratospheric general circulation (e.g. Manabe and Mahlman, 1976).

Theoretical studies have established the precise conditions for this dynamical cancellation between the eddies and the mean meridional circulation (Andrews and McIntyre, 1978). It is found that the cancellation is exact when the waves are steady and conservative and if there are no critical surfaces (where the zonal phase-speed of a wave component equals that of the zonal wind); this is sometimes referred to as the Charney-Drazin (1961) non-acceleration theorem. The compensation does not occur during periods of substantial planetary wave development in the troposphere, which was the situation during the stratospheric sudden warmings simulated by the model (I). Dramatic warmings during both Northern Hemisphere winters account for some specific features of Fig. 13, e.g. the rapid easterly acceleration of zonal wind during the first February.

Broadly, the seasonal cycle of zonal wind in the model's stratosphere must be related to the evolving field of net radiative heating through the Coriolis force which acts on the component of the mean meridional circulation driven diabatically. Owing to the dominant role of eddies in determining mean meridional circulations in the Northern Hemisphere winter stratosphere, this component does not usually correspond to the actual meridional flow (at high latitudes in winter the actual circulation and the diabatically-driven component have opposite senses). But to the extent that wave transience and departures from conservative motion can be neglected, the Charney-Drazin non-acceleration theorem implies that the difference between these mean circulations merely serves to cancel the dynamical effect of the eddies. Consequently, although in the stratosphere \( Q/c_p \) may be the smallest individual term in Eq. 2, it represents the underlying drive for the seasonal wind reversal in the region.

However, wave transience in the stratosphere may lead to departures from this simple picture of a radiatively-driven annual wind cycle. Such transience is illustrated in Fig. 10 and by month-to-month variations in the amplitudes of the monthly-averaged waves shown in Fig. 8. The transience may arise, for example, as a result of planetary wave variability in the troposphere related to its annual cycle, or as a result of changing vertical-propagation characteristics for the waves in the varying zonal wind field of the stratosphere. Thus, in the Northern Hemisphere especially, there is not always agreement between the zonal wind acceleration shown in Fig. 12 and that which would be expected from the net heating field in Fig. 14.

As illustration, in the Northern Hemisphere stratosphere Fig. 14 shows that during the first March of the integration a particularly strong latitudinal gradient of net radiative heating is present with maximum cooling rates at polar latitudes. This would be expected to contribute to mean polar cooling and to drive a component of mean circulation with northward movement of air, subsidence over the pole (Holton, 1972, pp. 228–234) and hence westerly acceleration of the zonal wind; in fact, pronounced easterly acceleration occurs in conjunction with polar warmings (Figs. 12 and 13). These developments mark the transition to an easterly wind régime in the stratosphere (or 'final warming') and are accompanied by enhanced eddy momentum and heat flux convergence and a northward movement of the region of maximum convergence to very high latitudes during March. Similar behaviour is present at other model levels down to and including the upper troposphere. The resulting enhanced mean meridional circulation is such that the region of most rapid zonal mean ascent moves similarly northward.

In this context, it may be significant for time-averaged wave 1 that a northward movement in the latitude of maximum tropospheric amplitudes occurs from middle to high
Figure 14. Radiative heating and cooling contributions at the 5 mb level for the Northern Hemisphere (left) and Southern Hemisphere (right). Units: 10^{-4} K s^{-1}. Stippled areas denote regions of cooling.

latitudes in the Northern Hemisphere during the final warming period (Fig. 8(b)). Synoptically, the behaviour in the troposphere corresponds to a marked weakening of the East Asian 'low' during late winter, in conjunction with retrogression of wave 1 by about 90° of longitude in the upper troposphere and stratosphere. Subsequently, the tropospheric large-scale flow becomes considerably more zonal in agreement with the climatological data. These observations are in general agreement with those made by O'Neill (1979) in a study of a final warming in the real stratosphere, and suggest that the annual cycle of the stratospheric circulation can be strongly influenced by seasonal variation of tropospheric long-wave activity as well as by the seasonally evolving field of stratospheric heating.

In the simulated summer stratosphere, the magnitudes of the momentum and heat contributions of both large-scale eddies and meridional circulation are very small. The transition to weak westerly acceleration at high latitudes around mid-summer occurs in
the presence of weak poleward meridional flow (Fig. 12), appearing as the upward extension of the tropospheric polar cell (Fig. 5, July). In the equatorial stratosphere, the appearance of weak equatorward flow contributes to easterly zonal wind accelerations around midsummer in the Northern Hemisphere. In view of the weak meridional gradient of net heating (Fig. 14), the origin of the meridional flow in the stratosphere is not obvious. It is possible that at high latitudes and in relative absence of stratospheric eddy activity, the stratospheric circulation is forced by tropospheric sources of momentum and heat associated with the polar cell. At equatorial latitudes, the flow appears as an upward extension of the tropospheric Hadley cell, but the cross-equatorial flow may also be influenced by stratospheric eddy activity in the winter hemisphere. Indeed, Fig. 4(b) shows that maximum easterlies occur in the equatorial stratosphere during the Northern Hemisphere winter when eddy activity in the stratosphere is most pronounced.

(b) Results for the Southern Hemisphere

Figures 12 to 14 also show the corresponding terms in the momentum and heat budgets for the Southern Hemisphere at 5 mb. The pronounced inter-hemispheric differences in the terms are clearly shown. The momentum and heat contributions of both large-scale eddies and meridional circulation during winter are individually much larger in the Northern Hemisphere than in the Southern, and the compensating relationship between them is less evident for the latter. In particular, a comparison of each term in Eq. (2) shows that net radiative cooling during the Southern Hemisphere winter provides a proportionally much greater contribution to the heat budget than during the Northern Hemisphere winter.

Concerning the momentum budget (Fig. 12), the zonal wind acceleration at middle and high latitudes in the stratosphere is approximately in phase with the acceleration due to Coriolis forces acting on the mean meridional circulation. This is in marked contrast to the situation in the Northern Hemisphere. The growth of a strong westerly jet is augmented by weak eddy momentum flux convergence at mid-latitudes, although Coriolis forces dominate.

Temperature variations in the Southern Hemisphere are also far less irregular. Unlike the situation in the Northern Hemisphere, the temperature cycle at high latitudes is in approximate phase with the cycle of net radiative heating near the pole (Figs. 13 and 14), which consists of maximum net heating and cooling near the equinoxes so that maximum and minimum temperatures are attained near the solstices.

These interhemispheric differences are again presumably related to the differing extents to which tropospherically-generated planetary waves influence the stratospheric circulation.

9. Summary and discussion

It has been demonstrated that the global general circulation model of the troposphere and stratosphere described here is capable of simulating many features of the observed stratosphere of both a synoptic and dynamical nature. The seasonal cycle of the stratospheric circulation is well reproduced and it has been shown that, at extra-tropical latitudes in the Northern Hemisphere, the evolution of the zonal mean flow results from small imbalances between the opposing dynamical effects of large-scale eddies and the induced mean meridional circulation. In the light of the Charney-Drazin non-acceleration theorem, it was suggested that the origin of the imbalances is the presence of a diabatic heating field, or fluctuations in the planetary-wave amplitudes.

In comparison with that of the Northern Hemisphere, the Southern Hemisphere stratosphere is far less disturbed by planetary waves in winter, and is closer to radiative equilibrium as indicated by the smaller net heating rates (Fig. 14). Its mean temperature distribution, the attendant zonal mean flow and their seasonal development are governed
to a greater extent by the balance between solar heating and temperature-dependent thermal cooling.

In the Northern Hemisphere, it was shown that the transition from the winter to summer circulation regimes, i.e. the so-called final warming, is dynamically driven by the eddies rather than by net radiative heating in the example studied (although the non-recovery of the westerly jet must be a consequence of the approach of summer radiation conditions). In fact, this final warming occurs in the presence of a peak in net radiative cooling at polar latitudes. It was proposed that seasonal changes in tropospheric long-wave activity, specifically that of wave 1, are an integral part of this rapid final warming, which would mean that the tropospheric seasonal cycle can have a significant effect on that of the stratosphere.

There is a pronounced seasonal cycle in the amplitudes of planetary wave disturbances and their associated eddy fluxes of momentum and heat in the stratosphere. Maximum values are attained in winter and minimum in summer. Thus in summer, the forcing by the planetary waves of the mean meridional circulations is correspondingly weak and in the stratosphere, these circulations become upward extensions of tropospheric cells.

Although it is frequently argued that the seasonal cycle in amplitudes can be accounted for by the different characteristics of vertical wave propagation expected within westerly and easterly zonal wind régimes, it was pointed out that a marked cycle is also exhibited by the forcing due to planetary waves in the model troposphere which is in phase with that of the stratosphere. Inter-hemispheric asymmetries in the model stratosphere appear to be related to differences in planetary wave behaviour in the troposphere. In particular, the relative absence of time-averaged planetary wave 1 in the Southern Hemisphere troposphere may account for the fact that the stratospheric polar vortex is more zonally symmetric than its Northern Hemisphere counterpart, and for the absence of major warmings in the Southern Hemisphere stratosphere during the course of the integration. In the Southern Hemisphere stratosphere time-averaged and transient wave amplitudes are of comparable magnitude in contrast to the Northern Hemisphere where the former component dominates.

However, a number of deficiencies are present in the model simulation of the stratospheric circulation. The most pronounced is that the winter westerlies are generally too strong, this being related to excessively low polar temperatures. The second Northern Hemisphere winter shows an improvement in this respect, and therefore it is possible that, in the early stages of the integration, dynamical equilibrium had not been achieved between the zonal flow and the eddies. This shortcoming in simulating the zonal wind field is common to a number of general circulation models (e.g. Cunnold et al., 1975; Manabe and Mahlman, 1976). It has not been possible to draw any firm conclusion as to the cause and further investigation is needed. The results suggest that it may be related at least in part to planetary-wave amplitudes being generally underestimated in the upper troposphere as compared with the real atmosphere, possibly on account of misrepresentation of the processes by which planetary waves are generated in the troposphere. Because radiative damping of upward-propagating planetary waves acts to decelerate the mean flow, waves which in the model are too weak in the upper troposphere would be accompanied by winds that are too strong in the stratosphere. This is most easily seen by considering Fig. 2 of Holton (1980). Referring to that figure, it can be deduced that the average zonal wind deceleration in the layer will decrease with the disturbance amplitude at the lower surface, if the layer is of sufficient depth that the radiation stress on its upper surface can be neglected. This picture is consistent with both the observed and modelled Southern Hemisphere having weaker tropospheric waves and stronger stratospheric winds than the Northern Hemisphere during winter.

Other possible causes of the excessively strong polar-night jets are limited vertical resolution in the model's stratosphere and the imposition of a 'rigid-lid' upper boundary condition, both of which could affect planetary-wave structure and hence the simulation of the mean zonal wind fields. Lindzen et al. (1968) and Kirkwood and Derome (1977) have
considered the effect of this boundary condition on wave structure by using finite difference models of varying vertical resolution to conduct experiments under conditions for which analytical solutions were obtainable in equivalent continuous models. They showed that the boundary condition \( \omega = 0 \) at \( \rho = 0 \) in a finite-difference model is equivalent to applying a rigid top to the continuous atmosphere at some finite pressure which is resolution dependent. However, it was noted for the present model that the long waves maintain a marked westward tilt with height in the Northern Hemisphere winter even at upper levels. This indicates that the effect is not important as, in the monthly mean, wave reflections would produce a reduced phase tilt towards upper levels because waves with approximately equal but oppositely directed energy fluxes would destructively interfere.

The westward tilt observed may be maintained by strong thermal damping present at the model's upper levels, as this would act to damp the waves to some extent before they reach the upper boundary and hence to suppress the formation of a reflected wave. In addition, the planetary waves in the model have their energy fluxes directed toward the equatorial zero-wind line separating the westerly and easterly wind regimes, and this is likely to be especially important in reducing the amount of mechanical energy reaching the upper boundary and hence in further inhibiting the formation of a reflected wave. This latter effect was absent in the work of Lindzen et al., and in that of Kirkwood and Derome (op.cit.) because of simplifications made in their analyses.

Three stratospheric phenomena that have attracted particular attention from dynamical meteorologists are the semi-annual and quasi-biennial oscillations of zonal wind, observed primarily in the equatorial stratosphere, and the sudden warming in the polar winter stratosphere. With regard to the semi-annual oscillation, while there is evidence for this periodicity in the strength of the zonal easterlies in the equatorial stratosphere of the model, the annual mean wind is such that the westerly phase of the observed cycle is absent. As for the quasi-biennial oscillation (QBO), the model integration was of insufficient duration to determine whether this phenomenon was reproduced.

In any case, limitations in the formulation of this general circulation model (and others of its type) do not favour the successful simulation of either the semi-annual oscillation or the QBO. The westerly phases of both are believed to be driven by upward-propagating, dissipating Kelvin waves generated in the troposphere. Although no attempt was made to isolate any waves of this type, the model would not be expected to describe them adequately as they have vertical wavelengths not much greater than the separation between model levels (Wallace, 1973). Moreover, Plumb and McEwan (1978) found that the mechanism for the QBO is rather easily destroyed by viscous diffusion of the mean flow. If this were too large in the model, the phenomenon would not be simulated.

One of the principal successes of the present general circulation model was its spontaneous production of sudden warmings in the Northern Hemisphere winter stratosphere. These warmings are described and analysed dynamically in I.

### Appendix 1

Symbols used in the text

- \( a \) radius of the earth
- \( c_s \) specific heat at constant pressure
- \( f \) Coriolis parameter \(( = 2\Omega \sin \phi)\)
- \( p \) pressure
- \( p_s \) surface pressure
- \( q \) mixing ratio of water vapour
- \( t \) time
- \( u \) eastward component of velocity
- \( v \) northward component of velocity
- \( z \) geopotential height
Mathematical Basis for the Procedures Employed in Analyzing Model Data

Letting $f$ represent an arbitrary meteorological field variable, we may write:

$$f(\lambda) = \sum_{n=0}^{\infty} F(n) e^{-in\lambda} \quad . \quad (A2.1)$$

where $\lambda$ represents longitude, $n$ the zonal wave number, and $F(n)$ is the complex Fourier coefficient for wave number $n$. Standard techniques of Fourier analysis lead to the following expressions.

The amplitude $A(n)$ of the wave number $n$ component of $f$ is given by

$$A(n) = 2 \{ F(n) F(-n) \}^\dagger \quad . \quad (A2.2)$$

with $F(-n) = F(n)^\ast$, the complex conjugate of $F(n)$.

Quantities of the form $\bar{f}'g'$, where the overbars represent a zonal average and primes departures therefrom, may be expressed as

$$\bar{f}'g' = \sum_{n=1}^{\infty} \left\{ F(-n)G(n) + F(n)G(-n) \right\} \quad . \quad (A2.3)$$

where $(f,F)$ and $(g,G)$ are Fourier transform pairs.

From this, the contribution of wave number $n$ to the total eddy quantity $\bar{f}'g'$, abbreviated as $\bar{f}'g'(n)$, may be identified with $\{ F(-n)G(n) + F(n)G(-n) \}$

The Fourier coefficient $F(n)$ may be decomposed as

$$F(n) = [F(n)] + [F(n)]' \quad . \quad (A2.4)$$

where $[\ ]$ represents a monthly mean and the superfix, $t$, denotes the instantaneous departure of $F(n)$ from its monthly-mean value. Expanding Eq. (A2.3),

$$[\bar{f}'g'] = \sum_{n=1}^{\infty} \left\{ [F(n)][G(-n)] + [F(-n)][G(n)] \right\} +$$

$$+ \sum_{n=1}^{\infty} \left\{ [F(n)'G(-n)'] + [F(-n)G(n)'] \right\} \quad . \quad (A2.5)$$

Equation (A2.5) can therefore be interpreted to mean that the term $\{ [F(n)][G(-n)] + [F(-n)][G(n)] \}$ provides the wave number $n$ contribution due to time-averaged eddies while the term $\{ [F(n)'G(-n)'] + [F(-n)G(n)'] \}$ is the corresponding contribution due to transient eddies:
Equation (A2.2) can be written as:
\[
[A(n)^2] = 4[F(n)]^2[F(-n)] + 4[F(n)F(-n)]'
\]  \hspace{1cm} (A2.6)

The quantity $2\{[F(n)][F(-n)]\}^\frac{1}{2}$ is taken to be the amplitude of time-averaged wave number $n$, and $2\{[F(n)F(-n)]\}^\frac{1}{2}$ that of the transient wave.

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