A non-linear atmospheric long wave model incorporating parametrizations of transient baroclinic eddies

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SUMMARY

Numerical experiments with a simple spectral model of tropospheric long waves are described. The model is based on the 2-parameter, β-plane, quasi-geostrophic equations and a coarse truncation is applied. Transient baroclinic eddy motion is represented parametrically using transfer coefficient expressions which are extensions of those proposed by Green in 1970 for zonal average formulations. The long waves are forced directly by externally specified diabatic and orographic functions. Zonal average diabatic forcing is also applied. When the eddy flux parametrizations act on the zonal fields only, realistic steady states are obtained if a coarse meridional truncation is applied, but explicit baroclinic instability occurs if higher meridional modes are included. Explicit baroclinic instability is suppressed if the parametrizations act also on the long wave fields; and in this case it is found that diabatic forcing of the largest scales of motion is the most important generator of stationary long waves in the model. Various reformulations are considered, and the problem of devising better transient eddy parametrizations is discussed.

1. INTRODUCTION

One of the main objectives of climate research is the construction of reliable numerical models of the global atmospheric circulation. Most of the activity in this field is at present being channelled into the development of general circulation models which attempt to resolve the transient large-scale eddies in sufficient detail (both temporally and spatially) to reproduce their transfer properties adequately.

Parametrized models, in which the transfers carried out by the transient large-scale eddies are represented in terms of the mean fields, are less developed, and nearly all deal explicitly with the zonal average fields only. (For reviews, see Schneider and Dickinson (1974), and GARP (1975).) These zonal average models are extremely useful in the development of rational theories of the general circulation, and for testing parametrization schemes (see, for example, White and Green (1982)) but it is difficult to represent in such formulations the effects of the quasi-stationary zonal asymmetries which arise in the real atmosphere because of the distribution of thermal properties and the elevation of the continents. In any case, to rival the general circulation models, parametrized models should surely attempt to reproduce the three-dimensional time-averaged circulation, and not just the zonal average fields.

In this paper we explore the possibility of using a three-dimensional time-dependent model to represent the largest scales of atmospheric motion, whilst applying parametric expressions to represent the effects of all smaller scales. The explicit part of the model is composed of the zonal average fields and the first three zonal wavenumbers. We imagine the zonally asymmetric part of the model to represent the slowly-evolving long wave pattern which dominates the circulation averaged over periods of a month or more (Eliasen and Machenhauer 1965; Blackmon 1976) and whose ultimate origin is the zonally asymmetric diabatic and orographic forcing provided by the oceans and continents. The parametrized motion is imagined to be the transient baroclinic eddies with zonal wavenumbers of about 6 and more. (It is necessary, as usual, to parametrize small-scale turbulent motion in the boundary layer.)

Long wave models incorporating parametrizations of transient baroclinic eddies have been discussed by Green (1970), Simmons and Hoskins (1976) and Frederiksen (1979). Adem (see GARP 1975, p. 170, for a full list of references) has used three-dimensional energy balance models extensively, and Trenberth (1973) and Yao (1980) have used heavily

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truncated spectral quasi-geostrophic models to represent both long waves and transient baroclinic eddies. However, we are not aware of any previous attempt to operate a three-dimensional dynamical model with parametric representation of transient eddies.

Our long wave model is based on the familiar $\beta$-channel formulation. Field variations in the vertical are represented by a 2-parameter specification which is formally very similar to the customary 2-level specification. Field variations in the horizontal are represented spectrally. These, and other fundamental aspects of the model, are outlined in Sections 2 and 3; greater detail is given in a report by White (1978) (which will be referred to here as SQ). The adopted parametric representation of the transient eddy fluxes is described in Section 4. The scheme is based on extensions of Green’s (1970) transfer coefficient theory and is closely related to Scheme 1 of White and Green (1982), the simplest of the three zonal average schemes examined in that paper (which will be referred to as WG2). Sections 5 and 6 cover the representation of diabatic heating and orographic effects, and the integration procedure. Results of model integrations are presented in Sections 7 and 8. In the integrations described in Section 7, the transient eddy parametrizations act only on the zonal average fields (the ‘intermediate’ long wave model). In the integrations described in Section 8, the parametrizations act also on the long wave fields (the ‘full’ long wave model). The results are discussed in the concluding Section 9.

### 2. The 2-Parameter Model

Eady (1952) derived 2-parameter equations for free quasi-geostrophic motion in the troposphere. The equations to be used here are extensions of Eady’s forms to include diabatic and orographic forcing, Ekman layer effects, and arbitrary vorticity and heat sources.

With altitude $z$ as vertical coordinate, and static compressibility neglected, the Type I quasi-geostrophic vorticity and thermodynamic equations for motion on a mid-latitude $\beta$-plane are

$$\begin{align*}
\frac{\partial}{\partial t} \mathbf{v}^2 \psi + J(\psi, \mathbf{v}^2 \psi) + \beta \frac{\partial \psi}{\partial x} &= f_0 \frac{\partial w}{\partial z} + F \\
\frac{\partial}{\partial t} \frac{\partial \psi}{\partial z} + J \left( \psi, \frac{\partial \psi}{\partial z} \right) + \frac{N^2}{f_0} w &= \frac{g}{f_0} (S + G)
\end{align*}$$

(1) (2)

See Phillips (1963) and Pedlosky (1964). In Eqs. (1) and (2), $J(\psi, \lambda)$ denotes $(\partial \psi/\partial x \partial \lambda/\partial y - \partial \psi/\partial y \partial \lambda/\partial x)$, the Jacobian of $\psi$ and $\lambda$ with respect to $x$ and $y$, the eastward and northward coordinates (in a Cartesian system); $t = \text{time}; \psi$ is the stream function of the non-divergent part of the horizontal flow; $\mathbf{V}$ is the horizontal gradient operator $i\partial/\partial x + j\partial/\partial y$, $i$ and $j$ being unit vectors in the $x$ and $y$ directions respectively; $f_0$ is a constant mean value of the Coriolis parameter $f$, and $\beta$ is a constant mean value of $df/\partial y$; $N^2 = (g/\theta_0)(\partial \theta_0/\partial z)$ is the square of the buoyancy frequency, $g$ being gravity and $\theta_0 = \theta_0(z)$ a reference state profile of potential temperature $\theta$; $S$ is the diabatic heating function; and $F$ and $G$ are respectively source functions of vorticity $\nabla^2 \psi$ and entropy $(f_0 g)(\partial \psi/\partial z) = \delta \theta/\theta_0$ when $\delta \theta = \theta - \theta_0$. $F$ and $G$ represent in our long wave model the flux convergences by transient baroclinic eddies, to be expressed in terms of $\psi$ using appropriate parametrizations.

Equations (1) and (2) are applied to motion which is cyclic in the $x$ direction with repeat distance $L_x$, and confined by frictionless rigid walls at $y = 0$, $L_y$, and a frictionless rigid surface at $z = \frac{1}{2}H$. At $z = -\frac{1}{2}H$, $w = w_0$ is related to the stream function at that level via the familiar Ekman pumping and orographic ascent expressions, whose particular forms for the 2-parameter model are given below. Figure 1 illustrates the model domain.

Using the procedure outlined in the Appendix, the functional representations

$$\begin{align*}
\psi(x, y, z, t) &= \psi_0(x, y, t) + \left( \frac{z}{H} \right) \psi_1(x, y, t) \\
w(x, y, z, t) &= w_0(x, y, t) + \left( \frac{z}{H} \right) w_1(x, y, t) + \left( \frac{z}{H} \right)^2 w_2(x, y, t) \\
F(x, y, z, t) &= F_0(x, y, t) + \left( \frac{z}{H} \right) F_1(x, y, t)
\end{align*}$$

(3) (4) (5)
\[ S(x, y, z, t) = S_0(x, y, t) \]  \hspace{1cm} (6)
\[ G(x, y, z, t) = G_0(x, y, t) \]  \hspace{1cm} (7)

may be applied to derive the '2-parameter' equations for the time evolution of \( \psi_0 \) and \( \psi_1 \):

\[ \frac{\partial}{\partial t} \nabla^2 \psi_0 + \beta \frac{\partial \psi_0}{\partial x} = -\frac{f_0}{H} w_B + F_0 - J(\psi_0, \nabla^2 \psi_0) - J(\psi_1, \nabla^2 \psi_1)/12 = A_0. \]  \hspace{1cm} (8)

\[ \frac{\partial}{\partial t} \nabla^2 \psi_1 - \frac{12f_0}{N^2 H^2} \frac{\partial \psi_1}{\partial x} + \beta \frac{\partial \psi_1}{\partial x} = \frac{6f_0}{H} w_B + F_1 + \]

\[ + \frac{12f_0}{N^2 H^2} \left\{ J(\psi_0, \psi_1) - \frac{gH}{f_0} (S_0 + G_0) \right\} \]

\[ - J(\psi_0, \nabla^2 \psi_1) - J(\psi_1, \nabla^2 \psi_0) \]

\[ = A_1. \]  \hspace{1cm} (9)

\( N^2 \) is assumed independent of height \( z \) in the derivation. The vertical velocity at \( z = -\frac{1}{2}H \), \( w_B \), is given by

\[ w_B = \frac{k_f}{f_0} \nabla^2 (\psi_0 - \frac{1}{2} \psi_1) + J(\psi_0 - \frac{1}{2} \psi_1, h) \]  \hspace{1cm} (10)

where \( k_f \) is a (constant) friction velocity parameter and \( h = h(x, y) \) is a mountain height function. Because of the forms of the representations (3), (5), (6) and (7), and the choice of origin of \( z \), the quantities \( \psi_0, F_0, S_0 \) and \( G_0 \) are proportional to the height averages of the corresponding variables. \( \psi_0 \) and \( \psi_1 \) may be considered as the barotropic and baroclinic stream functions respectively.

In SQ it was shown that the 2-parameter equations (8) and (9) are formally very similar to the 2-level equations which can be derived from Eqs. (1) and (2) by familiar methods. Each term in Eqs. (8) and (9) has its counterpart in the 2-level equations written in terms of

\( \tilde{\psi}_0 = \frac{1}{2} \{ \psi(z = \frac{1}{2}) + \psi(z = -\frac{1}{2}) \} \) and \( \tilde{\psi}_1 = 2 \{ \psi(z = \frac{1}{2}) - \psi(z = -\frac{1}{2}) \}, \)

but some of the constant factors are different. Like their 2-level analogues, the 2-parameter equations can also be cast into potential vorticity forms (relating, in fact, to advection with the horizontal flow at \( z = \pm 1/\sqrt{12} \)). When applied to simple linearized forced long wave problems the two sets give results that are almost identical and very similar to those obtained with continuous field variations in the vertical. In the baroclinic instability analysis of the zonal flow \( \bar{U} = \bar{U} + (z/H) \Delta U \), the 2-parameter model generally overestimates growth rates, while the 2-level model slightly underestimates growth rates over a considerable range of wavenumbers (see SQ).
Apart from the attraction of using a largely neglected formulation, the 2-parameter model is employed here because its specification in terms of height average fields and the deviations from them is especially well-suited to the application of the chosen transient eddy parametrizations (see Section 4). Another advantage of the 2-parameter model is its superior energetics. Because the stream function $\psi$ is represented at all heights $-\frac{1}{2}H \leq z \leq \frac{1}{2}H$, the global effect of Ekman layer pumping and suction is always to give positive dissipation of kinetic energy, and the global effect of mountain forcing on the energy budget is identically zero. These characteristics of the continuous equations (1) and (2) are not exhibited by the 2-level equations when, as is usual, the flow at $z = -\frac{1}{2}H$ is represented by an extrapolation based on values at the two dynamical levels $z = \pm \frac{1}{2}H$. (See Sela and Wiin-Nielsen (1971), p. 463.)

Figure 2 shows some important features of the behaviour of the 2-parameter model in problems of linearized waves on the basic flow $U = U + (z/H)\Delta U$. The ordinate is the non-dimensional $\beta$ effect

$$\beta_* = N^2 H^2 \beta f_0^2 \Delta U$$

and the abscissa is the non-dimensional total wavenumber $p$ given by

$$p^2 = N^2 H^2 (q^2 + r^2) f_0^2$$

Figure 2. Diagram summarizing some important properties of the 2-parameter model in linearized problems. Abscissa and ordinate are the non-dimensional total wavenumber $p$ and $\beta$ parameter $\beta_*$ defined by Eqs. (11) and (12). Full lines are contours of the growth rate measure $(\rho c_1/\Delta U)$ for baroclinic waves on the basic flow $U = U + (z/H)\Delta U$. Broken lines give the resonant wavenumbers for orographic or diabatic forcing when $\sigma (= \Delta U/2U)$ takes the values indicated. For further details see text. Wave modes $(m, n)$ on the abscissa are located assuming the parameter values given in section 5.
where \(q\) and \(r\) are the zonal and meridional wavenumbers of the relevant motion. In the absence of all forcing and dissipative effects \((k_F = F_0 = F_1 = G_0 = S_0 = h = 0)\) a conventional baroclinic instability analysis for wave modes of the form

\[
\psi_j = \mathcal{R}(D,e^{iq(x-ct)}) \sin ry,
\]

where \(j = 0,1\) and \(c = c_r + ic_t\), reveals that

\[
\left( c - U \frac{\partial}{\partial U} \right) = \left[ \frac{-\beta(p^2 + 6) \pm i\{p^4(144 - p^2)/12 - 36\beta^2\}^{1/2}}{p^2(p^2 + 12)} \right].
\]

The full lines in Fig. 2 are contours of the growth rate measure \((pc_j/\Delta U)\). The zero contour indicates the boundary of the region outside which no unstable wave modes exist.

The broken lines in Fig. 2 are the loci of the resonant wavenumber \(p_{res}(\beta_*, \sigma)\) for stationary orographic or diabatic forcing having the horizontal variation \(\exp(\ell \sigma) \sin ry\), when \(\sigma = (\Delta U/2U)\) takes the values 0-5 and 1-0. (No dissipative processes are included.) \(p_{res}\) is the positive real root of the biquadratic equation

\[
b(\beta_*, \sigma, p) = \frac{1}{2}a^2p^2(12 - p^2) + (2a\beta_* - p^2)(2a\beta_* - p^2 - 12).
\]

The response

\[
\psi_0 = \mathcal{R}(\hat{\psi}_0 e^{i\xi x} \sin ry)
\]

\[
\psi_1 = \mathcal{R}(\hat{\psi}_1 e^{i\xi x} \sin ry)
\]

to orographic forcing

\[
h = \left( \frac{f_0}{N^2 H} \right) \mathcal{R}(\hat{h} e^{i\xi x} \sin ry)
\]

has

\[
\hat{\psi}_0 = \{p^2(1 + \sigma) + 12 - 2a\beta_*)\}((1 - \sigma)\hat{h}/b)\ .
\]

\[
\hat{\psi}_1 = \{6(2a\beta_* - p^2) + 2a(12 - p^2)\}((1 - \sigma)\hat{h}/b)\ .
\]

(For \(\sigma = 1-0\) the response vanishes because there is then no zonal flow at \(z = -\frac{1}{2}H\); however, for values close to unity, resonance will occur close to the value of \(p_{res}\) given by Eq. (14) for \(\sigma = 1-0\).) Figure 3(a) shows \(\hat{\psi}_0\) and \(\hat{\psi}_1\) in units of \(\hat{h}\) for \(0 \leq p \leq 3\), when \(\sigma = 0-5\).

When diabatic forcing

\[
S_0 = \frac{f_0}{gH} \mathcal{R}(i\hat{S} e^{i\xi x} \sin ry)
\]

is applied, the response has

\[
\hat{\psi}_0 = -2a\beta^2 \hat{S}/qb\ .
\]

\[
\hat{\psi}_1 = -12(2a\beta_* - p^2)\hat{S}/qb\ .
\]

Figure 3(b) shows the functions \(q\hat{\psi}_0/p\) and \(q\hat{\psi}_1/p\) in units of \(\hat{S}\) for \(0 \leq p \leq 3\), when \(\sigma = 0-5\). These functions are very similar in form to \(\hat{\psi}_0\) and \(\hat{\psi}_1\); \(q/p\) introduces a factor \(q/(q^2 + r^2)^{1/2}\) which is always less than unity, but noticeably so only when \(q \lesssim r\).

Although it must be borne in mind that the results illustrated in Figs. 2, 3(a) and 3(b) apply to linearized waves on a basic flow with no lateral shear, and in the absence of dissipative or diffusive processes, the following aspects are relevant to the integrations described in Sections 6 and 7:

(a) The 2-parameter model (in common with the 2-level model) does not reproduce the long wave instabilities found when continuous vertical structure is assumed, and \(\beta_* > 0\) (Green 1960). However, this should not be considered as a shortcoming: the long wave instabilities are in any case weak, and the consequent contribution to tropospheric eddy fluxes is reasonably supposed to be small.

(b) For typical values of \(\beta_* (\approx 1)\) the wavelength of the most unstable baroclinic wave is about a factor of 2 smaller than the resonant wavelength for stationary orographic or diabatic forcing, when \(\sigma \lesssim 1\). This scale separation is the main justification for our strategy of representing the long waves explicitly but the transient baroclinic waves parametrically. Nevertheless, a practical difficulty arises because the resonant wavelength is only slightly greater than the long wave cut-off to instability; furthermore,
Figure 3. (a) Response functions, as a function of non-dimensional total wavenumber \( p \), describing the stationary waves forced by sinusoidal orography acting on the basic flow \( U = \bar{U}(1 + Z/H) \). Full line: barotropic response function \( \hat{\psi}_0 \); broken line: baroclinic response function \( \hat{\psi}_1 \).

(b) As (a) but for stationary waves forced by sinusoidal diabatic heating/cooling and showing \( (q/p)\hat{\psi}_0 \) and \( (q/p)\hat{\psi}_1 \) rather than \( \hat{\psi}_0 \) and \( \hat{\psi}_1 \) themselves.
growth rates increase very rapidly on the short wave side of the long wave cut-off. Thus a model which is able to represent the resonant wavelengths (around which the stationary response to forcing is expected to be large – see Figs. 3(a) and 3(b)) is likely also to contain scales which are quite strongly baroclinically unstable. The occurrence of such instabilities would vitiate the application of the transient eddy parametrizations, since it is partly these instabilities which the parametrizations are intended to represent.

(c) The baroclinic response to diabatic forcing (see Fig. 3(b)), although exhibiting a resonance, is also large for smaller values of the wavenumber \( p \). Thus very large scale diabatic forcing will be effective at producing a long wave response. Mountain forcing, in contrast, becomes progressively less effective at very large scales. Our modelling strategy seems well-fitted to the situation in which the long wave motion is dominated by the response to very large scale diabatic forcing. In this case the long wave scale is far removed from the scales at which baroclinic instability occurs.

The proximity of the resonant wavenumber to the region of strong baroclinic instability is easily understood in terms of well-known properties of the external Rossby mode. The resonant wavenumber corresponds to the scale at which the external Rossby mode has zero phase speed relative to the surface. On the other hand, strong baroclinic instability (the ‘Charney branch’) sets in at wavenumbers for which the external Rossby mode has a phase speed close to the surface flow speed \( U_1 \) in a current with positive vertical shear. Thus it follows that resonance and the onset of strong baroclinic instability will occur at about the same wavenumber if \( |U_1| \ll U \).

The difference in the scale dependence of the response to orographic and diabatic forcing reflects a fundamental difference in the nature of the two forcing mechanisms. Suppose that a rigid lid upper boundary condition is applied (as in the present 2-parameter model). When a vortex tube traverses the up-slope of a mountain, the total vortex compression due directly to the forced ascent depends only on the height of the mountain crest above the valley trough, and not on the horizontal distance between trough and crest. However, in the case of diabatic forcing, the total vortex stretching or compression due directly to the forced vertical motion depends on the heating integrated over the horizontal distance travelled; thus the amplitude of the direct response depends on the product of the amplitude and the wavelength of the forcing. This picture is essentially correct in spite of the complications introduced by the \( \beta \)-effect and the possibility of resonance with the external Rossby mode.

3. SPECTRAL REPRESENTATION

Because we wish to describe forced long waves satisfactorily and yet, as far as possible, to exclude baroclinically unstable scales from the explicit part of the model, spectral expansions are more suitable than grid-point values for the representation of horizontal field variations in our model.

An appropriate spectral representation of \( \psi_0 \) and \( \psi_1 \) in the \( \beta \)-channel depicted in Fig. 1 is that used by Lorenz (1963), Quinet (1973), Hartjenstein and Egger (1979), Charney and Straus (1980) and Yao (1980) in 2-level quasi-geostrophic models:

\[
\psi_j = \sum_{n=1}^{N} \bar{\psi}_n^j \cos n \alpha k y + \sum_{n=1}^{N} \sum_{m=1}^{M} (\psi_{mn}^j \sin m \alpha x + \psi_{mn}^{jc} \cos m \alpha x) \sin n \alpha k y, \quad (j = 0, 1). \tag{19}
\]

Here \( k = 2\pi/L_x \) and \( x = L_x/2L_y \). Similar expressions are adopted for the right-hand sides of Eqs. (8) and (9). Thus

\[
A_j = \sum_{n=1}^{N} \bar{A}_n^j \cos n \alpha k y + \sum_{n=1}^{N} \sum_{m=1}^{M} (A_{mn}^j \sin m \alpha x + A_{mn}^{jc} \cos m \alpha x) \sin n \alpha k y, \quad (j = 0, 1). \tag{20}
\]
\(A_0\) and \(A_1\) contain contributions from \(F_0, F_1, S_0, G_0\) and the various Jacobian terms. Some of these quantities will be pre-specified forms (see §5), while others must be expressed in terms of \(\psi^l, \psi^m, \text{ and } \psi^mc\). Conspicuous amongst the latter class are the Jacobians, whose evaluation in the required terms is algebraically laborious. The details are given in SQ for truncations \(M = 4, N = 5\) and will not be revived here. \(F_0, F_1\) and \(G_0\) are expressed in terms of the spectral amplitudes of \(\psi_j\) using the parametrizations discussed in Section 4.

We shall refer to the zonal average and zonally asymmetric parts of \(\psi_j\) and \(A_j\) (as given by Eqs. (19) and (20)) as the 'zonal' and 'wave' parts respectively. The spectral representations of \(\psi_j\) and \(A_j\) may be considered as being composed of zonal components \((n)(n = 1, \ldots N)\) and wave components \((m, n)(m = 1, \ldots M; n = 1, \ldots N)\).

Substitution of Eqs. (19) and (20) into Eqs. (8) and (9) leads to \(2N(2M + 1)\) equations governing the time evolution of the spectral amplitudes of \(\psi_j\):

\[
\begin{align*}
\frac{d\bar{\psi}^0}{dt} &= -\frac{A^0}{n^2\alpha^2k^2} \\
\frac{d\psi^0}{dt} &= -\frac{(A^0 + m\beta k\psi^c)}{(m^2 + n^2\alpha^2)k^2} \\
\frac{d\psi^0}{dt} &= -\frac{(A^0 - m\beta k\psi^c)}{(m^2 + n^2\alpha^2)k^2} \\
\frac{d\bar{\psi}^1}{dt} &= -\frac{A^1}{n^2\alpha^2 + \nu^2k^2} \\
\frac{d\psi^1}{dt} &= -\frac{(A^1 + m\beta k\psi^c)}{(m^2 + n^2\alpha^2 + \nu^2)k^2} \\
\frac{d\psi^1}{dt} &= -\frac{(A^1 - m\beta k\psi^c)}{(m^2 + n^2\alpha^2 + \nu^2)k^2}
\end{align*}
\]

\(m = 1, \ldots M; n = 1, \ldots N, \nu^2 = 12f_0^2/N^2H^2k^2\).

In the limit as \(M, N\) tend to infinity, the representation (19) (or (20)) constitutes a complete orthonormal set for functions \(\lambda\) satisfying \(\lambda(x, y) = \lambda(x + L_v, y), \partial\lambda/\partial x = 0\) on \(y = 0, L_v\) and \(\partial^2\bar{\lambda}/\partial y = 0\) on \(y = 0, L_v\). Such lateral conditions on \(\psi_0\) and \(\psi_1\) ensure consistent energetics and other desirable properties. Some interesting repercussions of the appearance of \(\sin mzky\) in the wave part of (19) and (20) but \(\cos mzky\) in the accompanying zonal parts are discussed in SQ. Veronis (1963) describes a similar mathematical situation regarding the inclusion of the \(\beta\)-effect in a model of quasi-geostrophic flow in a closed ocean basin.

Because of the character of the representation (19), Eqs. (21) give consistent energetics for any truncation \(M, N\). Energy conservation is also retained if the Jacobian interactions of all pairs of wave components to produce other wave components are ignored. The computer program for the time integration of the \(2N(2M + 1)\) equations (21) was constructed so that all such (wave + wave) \(\rightarrow\) (wave) interactions could be omitted, leaving only (wave + wave) \(\rightarrow\) (zonal flow), and (zonal flow + wave) \(\rightarrow\) (wave) interactions. In this simplified version, substantial improvements in computational efficiency were achieved. In spite of their good energy conserving properties, Eqs. (21) do not conserve angular momentum (J. Egger, private communication).

The zonal average model used in WG2 is formally identical to the zonal part of the present formulation. In the zonal average model, however, truncations up to \(N = 9\) are applied, while \(N = 5\) is the highest truncation used in the long wave model (and most integrations have been carried out with a form having \(N = 3\)). The zonal wavenumber truncation is \(M = 3\) in all the integrations described in this paper. For brevity, a form of the long wave model having truncation \(M, N\) will be referred to as the \('M, N\) model'.
On the abscissa of Fig. 2 are shown the locations of some wave components \((m,n)\), assuming the parameter values listed in Section 5 (which were applied in all the integrations described in later Sections). For typical values of \(\beta_s (\simeq 1)\), some of the higher wavenumber components are within the region of strong baroclinic instability. Thus, dissipative effects apart, difficulties with explicit instabilities may be expected, according to the discussion given in Section 2.

4. THE TRANSIENT EDDY FLUX PARAMETRIZATIONS

The vorticity and entropy \((\equiv \log \text{potential temperature})\) source functions \(F\) and \(G\) in Eqs. (1) and (2) are regarded as representing the flux convergences due to transient baroclinic eddies. Thus

\[
F = -\nabla \cdot \left( \mathbf{v}_\psi \mathbf{\phi}^t \right) \quad G = -\nabla \cdot \left( \mathbf{v}_\phi \mathbf{\phi}^t \right)
\]  

(22)

where \(\mathbf{v}_\psi = k \times \nabla \psi\) (\(k\) denotes unit vector in the vertical) is the rotational non-divergent horizontal flow, \(\zeta = \nabla^2 \psi\), and \(\phi = \delta \phi / \delta \theta\). The primes indicate deviations from a time average; e.g. \(\zeta' = \zeta - \overline{\zeta}'\). An averaging period of a month or so is envisaged. The left hand sides of Eqs. (1) and (2) are considered to apply to the time-averaged fields which, according to observation, are dominated by the first few zonal wavenumbers.

To express \(F\) and \(G\) in terms of the slowly-varying stream function \(\overline{\psi}'\), we use extensions of the zonal average parametrizations applied by Green (1970) to the eddy fluxes of quantities closely conserved during the lifetime of a baroclinic eddy. For the eddy flux of \(\phi\) the expressions

\[
\overline{u'} \phi' = -K_{ux} \frac{\partial \phi'}{\partial x} - K_{uy} \frac{\partial \phi'}{\partial y} - K_{uz} \frac{\partial \phi'}{\partial z}
\]  

(23)

\[
\overline{v'} \phi' = -K_{vx} \frac{\partial \phi'}{\partial x} - K_{vy} \frac{\partial \phi'}{\partial y} - K_{vz} \frac{\partial \phi'}{\partial z}
\]  

(24)

in terms of the transfer coefficients \(K_{ux}\), etc., and the average gradients of the \(\phi\) field, are obvious modifications of Green's (1970) form for the meridional eddy flux of \(\phi\). We shall use simplified versions of Eqs. (23) and (24), in which \(K_{ux} = K_{uy} = K_h\) and \(K_{uv} = K_{ux} = 0\):

\[
\overline{u'} \phi' = -K_h \frac{\partial \phi'}{\partial x} - K_{uz} \frac{\partial \phi'}{\partial z}
\]  

(25)

\[
\overline{v'} \phi' = -K_h \frac{\partial \phi'}{\partial y} - K_{vz} \frac{\partial \phi'}{\partial z}
\]  

(26)

For the eddy flux of potential vorticity

\[Q = f + \zeta + f \frac{\partial}{\partial z} \left( \frac{1}{N^2} \frac{\partial \psi}{\partial z} \right)\]

(27)

the corresponding parametrization is simply

\[
\overline{v'} Q' = -K_h \overline{Q}'
\]  

(28)

We do not believe that the essentially diffusive parametrizations (23)–(26) and (28) are ideal. As discussed in Section 9, an increasing amount of theoretical and observational evidence suggests that transient eddy fluxes sometimes exhibit non-diffusive behaviour. However, modified parametrization schemes of workable simplicity have not, to our knowledge, been put forward. (See Section 9 for discussion of Stone's (1978) 'baroclinic adjustment' scheme.) We choose to use transfer coefficient schemes because they are well-defined, mathematically tractable, and have some rational basis.

From Eq. (27) it is clear that
\[ \vec{v}_\phi Q' = \vec{v}_\phi \zeta' + g f_0 \left( \frac{\partial}{\partial z} \left( \frac{\phi'}{N^2} \right) - \frac{\phi'}{N^2} \frac{\partial v_\phi'}{\partial z} \right) \]  

(29)

Since the last term in Eq. (29) is non-divergent (by the thermal wind equation), it follows that

\[ F = -\nabla \cdot (\vec{v}_\phi \zeta') = g f_0 \left( \frac{\partial}{\partial z} \left( \frac{\partial (\vec{v}_\phi \phi')}{\partial z} \right) - \frac{\phi'}{N^2} \frac{\partial \vec{v}_\phi'}{\partial z} \right) \]  

(30)

Equation (30) is the key relation which enables the eddy vorticity flux convergence \( F \) to be represented in terms of the mean fields and transfer coefficients, via Eqs. (25), (26) and (28). It differs from the corresponding form for eddies defined as deviations from the zonal average (Green 1970, Eq. (11)) in relating the eddy flux convergences of \( \zeta, Q \) and \( \phi \), rather than the eddy fluxes themselves. (Compare also Eq. (29) with Green’s Eq. (11).) Note that Eq. (30) itself is a general property of the Type 1 quasi-geostrophic formulation, independent of any eddy flux parametrization scheme.

Using Eqs. (25)–(28), Eq. (30) becomes

\[ F = \frac{\partial}{\partial y} \left\{ K_h \left( \beta + \frac{\partial \zeta'}{\partial y} \right) \right\} - \frac{g f_0}{N^2} \frac{\partial K_h}{\partial z} \frac{\partial \zeta'}{\partial y} - f_0 \frac{\partial K}{\partial z} + \frac{\partial}{\partial x} \left\{ K_h \frac{\partial \zeta'}{\partial x} - \frac{g f_0}{N^2} \frac{\partial K_h}{\partial z} \frac{\partial \zeta'}{\partial x} - f_0 \frac{\partial K}{\partial z} \right\} \]  

(31)

Here it has been assumed that \( \zeta' \approx N^2/g \) (which follows from the conditions under which the standard quasi-geostrophic equations are derived: see White 1977).

Integration of Eq. (31) over \( z \) (assuming that \( K_{ux} \) and \( K_{uz} \) are negligible at \( z = \pm \frac{1}{2} H \); see Green 1970) gives

\[ \int_{-\frac{1}{2}H}^{+\frac{1}{2}H} F dz = \frac{\partial}{\partial y} \int_{-\frac{1}{2}H}^{+\frac{1}{2}H} \left\{ K_h \left( \beta + \frac{\partial \zeta'}{\partial y} \right) - \frac{g f_0}{N^2} \frac{\partial K_h}{\partial z} \frac{\partial \zeta'}{\partial y} \right\} dz + \frac{\partial}{\partial x} \int_{-\frac{1}{2}H}^{+\frac{1}{2}H} \left\{ K_h \frac{\partial \zeta'}{\partial x} - \frac{g f_0}{N^2} \frac{\partial K_h}{\partial z} \frac{\partial \zeta'}{\partial x} \right\} dz \]  

(32)

The application of Eq. (32) to the 2-parameter model follows closely the procedure used in the zonal average scheme 1 of WG2. It is assumed that: (i) the horizontal variation of \( K_h \) is separable from its vertical variation, i.e., \( K_h(x,y,z) = K(x,y) K(z) \); (ii) the contribution of the baroclinic streamfunction \( \psi \), to the mean absolute vorticity gradients can be neglected. Eq. (32) now becomes

\[ \int_{-\frac{1}{2}H}^{+\frac{1}{2}H} F dz = \frac{\partial}{\partial y} \int_{-\frac{1}{2}H}^{+\frac{1}{2}H} \left( \beta + \frac{\partial \nabla^2 \psi_0 + \frac{\gamma f_0}{N^2 H^2} \frac{\partial \psi_0}{\partial y} }{g f_0} \right) K_h dz + \frac{\partial}{\partial x} \int_{-\frac{1}{2}H}^{+\frac{1}{2}H} \left( \frac{\partial \nabla^2 \psi_0 + \frac{\gamma f_0}{N^2 H^2} \frac{\partial \psi_0}{\partial x} }{g f_0} \right) K_h dz \]  

(33)

where \( \gamma = -H \frac{\partial K_h}{\partial z} \) is independent of \( x \) and \( y \). In the zonal average model, \( \gamma \) is determined from the constraint that the average vorticity flux \( \bar{\psi}_x \bar{\psi}_y = 0 \) at all times. To find analogous constraints in the present case we consider Eq. (29). From the thermal wind equation it follows that

\[ \frac{2 f_0}{g} \int \int \phi' \frac{\partial \psi_0}{\partial z} dx dy = \int \int \left( \frac{\partial}{\partial x} \left( \frac{\phi'}{\partial x} \right) - i \frac{\partial}{\partial y} \left( \frac{\phi'}{\partial y} \right) \right) dx dy = 0 \]  

(35)

given that \( \phi' = 0 \) on the lateral boundaries. (The horizontal integration is taken over the domain \( 0 \leq x \leq L_x, 0 \leq y \leq L_y ) \). Also,

\[ \int \int \vec{v}_\phi \zeta' dx dy = \int \int \left( i \frac{\partial}{\partial x} \left( u_\phi \nu_\phi' \right) - \frac{1}{2} \frac{\partial}{\partial y} \left( \nu_\phi' \right) \right) dx dy - \int \int \left( i \frac{\partial}{\partial x} \left( u_\phi \nu_\phi' \right) - \frac{1}{2} \frac{\partial}{\partial y} \left( \nu_\phi' \phi' \right) \right) dx dy \]
\[-j \left[ \frac{\partial}{\partial y} (u_\phi \psi_\phi) + \frac{1}{2} \frac{\partial}{\partial x} (|u_\phi|^2 - |\psi_\phi|^2) \right] \, dx \, dy = 0 \tag{36}\]

given that \(v_\phi\) and \(u_\phi\) vanish on the lateral boundaries. \(\phi' = 0\) and \(v_\phi' = 0\) are uncontroversial lateral boundary conditions, but \(u_\phi = 0\) requires some comment: it is not an identical property of the standard quasi-geostrophic model given the conditions on \(v_\phi\). It can be justified in that baroclinic eddy activity should be weak in extreme latitudes because average temperature gradients there will be small. Accepting the result (36) as well as (35), it follows from Eq. (29) that
\[
\int \left( \frac{\partial}{\partial x} (\psi_\phi \phi' i) \right) \, dx \, dy = 0
\tag{37}
\]
(Equation (37) in fact applies to the \(y\) component of the integrand irrespective of the behaviour of \(u_\phi\) on the lateral boundaries.) Integrating over \(z\), applying Eqs. (25), (26) and (28), incorporating the approximations made in deriving Eq. (33), and separating \(x\) and \(y\) components, Eq. (37) yields
\[
\int \left( \beta + \frac{\partial}{\partial y} \nabla^2 \psi_0 + \frac{\gamma f_0^2}{N^2 H^2} \frac{\partial \psi_1}{\partial y} \right) K_h^* \, dx \, dy = 0 \tag{38}
\]
\[
\int \left( \frac{\partial}{\partial x} \nabla^2 \psi_0 + \frac{\gamma f_0^2}{N^2 H^2} \frac{\partial \psi_1}{\partial x} \right) K_h^* \, dx \, dy = 0 \tag{39}
\]
Equations (38) and (39) constitute two relations which \(K_h^*\) and the quantity \(\gamma\) must obey. We now make the assumption that the height-average transfer coefficient \(K_h^*\) is independent of \(\text{longitude}\). This is evidently consistent with earlier assumptions and restrictions since it ensures that Eq. (39) is satisfied whatever the value of \(\gamma\). Equation (38) reduces to
\[
\int_0^{1.5} \left( \beta + \frac{\partial^3 \psi_0}{\partial y^3} + \frac{\gamma f_0^2}{N^2 H^2} \frac{\partial \phi'}{\partial y} \right) K_h^* \, dy = 0 \tag{40}
\]
which can be used to evaluate \(\gamma\) at any time, given the latitude variation of \(K_h^*\). We assume
\[
K_h^* = K \sin^2 \alpha k y \tag{41}
\]
as in the zonal average scheme 1 of WG2.

Equation (40) is formally identical to the relation used to evaluate \(\gamma\) at each time step in the zonal average scheme 1. The neglect of longitude variations in \(K_h^*\) is the crucial simplifying step; note, however, that the fluxes themselves will still vary with longitude if the mean fields vary with longitude. It would be interesting to examine more elaborate formulations in which longitudinal variations of \(K_h^*\) were permitted, with two independent constraints (similar to Eqs. (38) (39)) still to be satisfied. Different latitude variations of \(K_h^*\) (as in schemes 2 and 3 of WG2) would also be of interest.

The value of \(K\) is calculated using Eq. (44) below. When \(\gamma\) has been determined from Eq. (40), Eq. (33) can be applied to give the height-average vorticity flux convergence \(F_0\) required in Eq. (8). The parametrization of \(F_1\) and \(G_0\) (see Eqs. (8) and (9)) proceeds analogously to Scheme 1 of WG2; we give only essential details here. \(F_1\) is specified as
\[
F_1 = 2 F_0 \tag{42}
\]
so that \(F\) increases from zero at \(z = -\frac{1}{2} H\) to a maximum absolute value \(2 F_0\) at \(z = \frac{1}{2} H\), in keeping with the gross height variation of observed eddy momentum and vorticity fluxes. Equation (42) also ensures that Eq. (36) is obeyed by the height-varying part of the vorticity flux \(\psi_\phi \phi'\). For \(G_0\) the simple form
\[
G_0 = -K \delta \nabla^2 \phi' \tag{43}
\]
is used. The factor \(\delta\) is introduced to account for the omission of the terms in \(K_{\psi_\phi}\) and \(K_{\phi\phi}\) from Eqs. (25) and (26); throughout, we take \(\delta = \frac{1}{2}\). The use of Eq. (43) (rather than a form involving \(K_{\phi\phi} = K \sin^2 \alpha k y\)) enables a reasonable zonal mean heat balance to be achieved near
the lateral boundaries. This replacement is interpreted and discussed at length in WG2. $\mathcal{R}$ is determined from the relation

$$
\mathcal{R} = 4 \times 10^7 \Delta \phi \text{ m}^2 \text{s}^{-1}
$$

where $\Delta \phi$ is the zonal average difference in log potential temperature between the lateral boundaries at $y = 0, L_y$. Equation (44) was obtained by Green (1970) by considering the gross energetics and dissipation of baroclinic eddies in the troposphere. A different dependence would be obtained by applying Held's (1978) arguments, which are based essentially on linear theory.

Finally, we note that the 2-parameter equations, if written in potential vorticity form, can be integrated in time using parametric expressions for the eddy flux convergence of potential vorticity flux only. Marshall (1980) uses this procedure in a 2-layer ocean circulation model. See also WG2 for further discussion.

Nearly all the terms in Eqs. (33) and (43) are easily expressed in the required spectral form using the representations (19) and (20) and trigonometric identities. For the term in Eq. (33) involving $\beta \sin^2 kx$, suitably truncated forms of the half-range Fourier sine series for $\sin^2 kx$ are applied.

In Section 8 we shall describe model results obtained with long wave forcing (see Section 5) in conjunction with parametrizations (33), (42) and (43). First, in Section 7, we present results obtained when the parametrizations act on the zonal average fields only: terms in Eqs. (33) and (43) which vanish in the zonal average are omitted. The parametrizations are then identical to those applied (under Scheme 1) in WG2's zonal average formulation. The composition of the long wave model in this case is of special interest, as the parametrizations drive and regulate the zonal flow but do not directly affect the long waves. Otherwise expressed, the parametrizations represent the effects of the transient eddies on the zonal flow, but not the consequences of longitudinal inhomogeneities in the generation of transient eddies. The model thus appears as a natural intermediate between the zonal average and the full long wave formulation examined in Section 8. For completeness of interpretation we note that the long wave fields themselves modify the zonal fields to some extent, and thus exert some indirect control over the parametrized fluxes. Also, the long wave response to forcing is affected indirectly by the (parametrized) transient eddies through their action on the zonal fields. The physical interactions allowed in the zonal average, intermediate long wave, and full long wave models are shown diagrammatically in Fig. 4.

5. REPRESENTATION OF DIABATIC AND OROGRAPHIC FORCING

In this study, the diabatic heating $S_0$ and the orography function $h$ (see Eqs. (9) and (10)) are represented in all cases by externally specified constant functions, as described in detail below. Zonal average diabatic heating/cooling is applied in order to generate a pole/equator temperature gradient, but the orography function consists only of wave components. The use of externally specified functions to represent the diabatic heating simplifies interpretation of the results and enables the essentially novel aspects of the model to be evaluated easily. In an operative climate model the diabatic forcing would, of course, be partially dependent on the model fields: the development of the appropriate functional relations is comparable in difficulty to the eddy flux parametrization problem (Arakawa 1975).

Three separate patterns of externally specified constant diabatic and orographic forcing are applied in the integrations described in Sections 7 and 8. Pattern D1 has $h = 0$ and

$$
S_0 = 0.6 \cos kx \cos kx + 0.5 \sin kx \sin kx \quad \text{Cd}^{-1}
$$

Pattern D2 has $h = 0$ and, in $\text{Cd}^{-1}$,

$$
S_0 = 0.6 \cos kx \cos kx - 0.05 \cos 2kx + 0.2 \cos 3kx + \\
+ \{0.15(\cos kx \sin kx) + 0.375 \sin 2kx\} \{\sin 3kx \sin kx\}
$$

(46)
Figure 4. Box diagrams showing schematically the structure of (a) the zonal average model used by White and Green (1982), and (b) the long wave models used in the present paper. Circles represent the zonal average and long wave fields while the rectangles represent various forcing effects. The arrows indicate influence in the appropriate sense: thus, for example, in Fig. 4(a) the transient eddy parametrizations act on the zonal average fields and *vice versa:* the diabatic forcing, however, acts on the zonal average fields, but is not affected by them. In Fig. 4(b) the dotted two-headed arrow represents influences that are included in the 'full' long wave model but not in the 'intermediate' long wave model.
Figure 5. (a) Diabatic forcing pattern D1 (Units: °C d⁻¹).

(b) Diabatic forcing pattern D2 (Units: °C d⁻¹).

Pattern M has

\[ S_0 \equiv 0.6 \cos \pi ky \quad \text{°C d}^{-1} \]

and

\[ h \equiv 0.5 (\sin kx + \sin 2kx + \sin 3kx) \quad \text{km} \]

The patterns D1 and D2 are shown in Fig. 5(a) and (b). They have been chosen as examples of spectrally confined and spectrally widespread diabatic forcing; the wave part of D2 bears some resemblance to the expected variation in the Earth’s troposphere, but we do not wish to push this comparison too far. Some variants of D1 and D2 are also applied in the runs described in Sections 7 and 8. The mountain forcing pattern M has been chosen particularly to emphasize the scale dependence of the model’s response to orographic forcing.

6. Integration procedure

The 2N(2M + 1) equations (21) for the time evolution \( \tilde{\psi}_0, \psi_{mn}, \psi'_{mn} \) \( (j = 0, 1) \) are integrated using the simple leap-frog scheme with a step of 6 hours. To start each integration the Miyakoda method is used (see Hoskins and Simmons 1975). The initial condition in all runs is \( \psi_0 = \psi_{1} = 0 \) everywhere, corresponding to no flow or temperature gradients anywhere. All eddy flux parametrizations are expressed in terms of the dependent variables one time-step in arrears, in order to avert a computational instability. Ekman pumping is treated similarly. For the first few steps of each run (when horizontal temperature gradients are always small) the eddy flux parametrizations are omitted.

Integrations are continued for 600 or 800 steps, by which time an effectively steady state has been reached in all the runs to be described.

Only two horizontal fields are needed to define the response of the 2-parameter model to
steady forcing. We have chosen to examine the surface pressure field $\rho_s(x, y)$ viz. $\{\rho_s + \rho_s f_0 (\psi_0 - \frac{1}{2} \psi_1)\}$ and the potential temperature deviation field $\delta \theta(x, y)$ viz. $(\bar{\theta}_0 f_0 \psi_1 / gh)$. Here $\rho_s$ is a horizontal average surface pressure, $\bar{\theta}_0$ is a volume average potential temperature, and $\rho_s$ is a standard surface density. $\delta \theta(x, y)$ is dominated by the meridional variation of the zonal part $\delta \theta^x(y)$: in order to show the long wave components clearly, the wave field $\delta \theta = \delta \theta - \delta \theta^x$ is presented instead of $\delta \theta$ itself.

The following parameter values are adopted in all runs:

$L_x = 3 \times 10^7$ m; $L_y = 10^7$ m; $H = 10^4$ m; $f_0 = 10^{-4}$ s$^{-1}$; $\beta = 1.5 \times 10^{-11}$ m$^{-1}$ s$^{-1}$; $N^2 H/g = 0.13$; $k_F = 2 \times 10^{-2}$ m s$^{-1}$; $g = 10$ m s$^{-2}$; $\bar{\rho}_s = 1012$ mb; $\bar{\theta}_0 = 300$ K; $\rho_s = 1$ kg m$^{-3}$.

7. Numerical integrations with steady long wave forcing and transient eddy parametrizations acting on the zonal average fields only (intermediate long wave model)

(a) Preliminary investigations

Before considering integrations with long wave forcing, we examine the behaviour of the model when only zonal average (diabatic) forcing is applied. Figures 6(a) and (b) show the steady state surface zonal flow $U_s(y)$, surface pressure deviation $\delta p_s(y)$ and temperature deviation $\delta \theta(y)$ generated in the 3,3 model by the zonal part $S^*_{0}$ of $D1$ (or $M$) forcing acting alone (see Eqs. (45) and (47)). $S^*_{0}$ is shown in Fig. 6(b). Figures 6(c) and (d) show the corresponding variations for the zonal average part of $D2$ forcing acting alone in the 3,3 model. There is no explicit wave response in either integration because there is no wave forcing. The variations shown in Fig. 6 are identical to those obtainable (using the same diabatic forcing and Scheme 1 parametrizations) with the zonal average model used in
WG2; indeed, the run whose results are shown in Fig. 6(a) and (b) is the same as one of the runs mentioned in that study (Table 1). All the variations shown in Fig. 6 are qualitatively realistic.

When subjected to the same zonal average forcing, the 3,5 model gives very similar results to those obtained with the 3,3 model. Steady-state flow fields differ by less than 0.2 m s\(^{-1}\) everywhere, and the potential temperature fields by less than 0.04 °C. Because of these small differences the 3,5 results are not shown (but see Table 1 of WG2).

(b) 3,3 model results

Figure 7 shows the steady state fields of \(p_s\) and \(\delta \theta'\) obtained with \(D2\) forcing. The zonal average fields (not shown) are very similar to those obtained without the wave part of the forcing (Fig. 6(a) and (b)): \(\bar{U}^s\) differs everywhere by less than 0.1 m s\(^{-1}\) and \(\delta \theta^s\) by less than 0.05 °C. Consistently, diagnosis shows that the poleward fluxes of heat and momentum by the explicit long waves are small compared with the parametrized fluxes (typically about 1%).

The integration has been repeated with the exclusion of those interactions involving only wave components (see Section 3). The steady state results (not shown) are almost the same as those obtained with full non-linearity: zonal average fields are effectively unchanged, \(p_s\) differs everywhere by less than 0.1 mb, and \(\delta \theta'\) by less than 0.07 °C. Further evidence of the quasi-linearity of the steady state was obtained by repeating the fully non-linear run with the same zonal average forcing, but with all wave forcing amplitudes doubled. The response was an approximate doubling of the directly forced components, though changes were discernible in the zonal average fields – up to 0.24 m s\(^{-1}\) in \(\bar{U}^s\) and 0.15 °C in \(\delta \theta^s\). We conclude that wave/wave interactions are unimportant here but that wave/zonal flow interactions are somewhat more significant.

Other aspects of Fig. 7 also suggest that the steady-state fields are consistent with simple linear theories. As shown in Fig. 12, the two main forced waves (1,1) and (2,1) bear the
expected phase relation to the forcing, the maxima of the surface pressure responses being about 90° (intrinsic phase) upstream of the maximum heating. Also, the amplitudes of the \((1,1)\) \(p_s\) and \(\delta\theta\) responses are somewhat larger than those of their \((2,1)\) counterparts, although the amplitude of the \((1,1)\) forcing is only about half that of the \((2,1)\) forcing. This bears out the increased linear response to diabatic forcing at the largest scales, which was discussed in Section 2.

It is important to note the crucial rôles played by the parametrized heat and vorticity fluxes in our long wave model. Figure 2, although based on analysis for a baroclinic flow with no lateral shear and in the absence of dissipative processes, suggests that explicit baroclinic instability will not occur in the model unless the vertical shear exceeds a certain critical value (depending on the truncation \(M, N\), the \(\beta\)-effect and other parameters). Because the steady-state mean meridional circulation in this quasi-geostrophic model is governed by the parametrized vorticity fluxes, and the zonal average diabatic heating/cooling is a constant function of latitude, the result of omitting the parametrized eddy fluxes is that the pole-equator temperature gradient rises to a value at which explicit baroclinic instability occurs with sufficient intensity to carry heat fluxes to balance the diabatic heating and cooling. An integration with \(D2\) forcing and without parametrized heat or vorticity fluxes clearly exhibited such behaviour. The quasi-steady state was achieved when the pole-equator temperature difference was about 72°C in the time average.

Generally, of course, we wish to avoid explicit baroclinic instability because the parametrizations are intended to account for the effects of baroclinic eddies. In any case,
explicit baroclinic instability is likely to be very badly described at the coarse truncations used in the present study.

If the heat flux parametrization is retained but the vorticity flux parametrization omitted, the steady-state surface zonal flow is very weak (in the 3,3 model) because explicit baroclinic instability does not occur and so there is no effective agency for meridional transfer of momentum. (The explicit long wave fluxes are generally small.) Figure 8 shows the steady-state fields obtained when D2 forcing is applied and only the heat flux parametrization is included. The surface zonal flow is indeed weak: $p_s(x, y)$ has a cellular form. Both $\delta \theta'$ and $p_s$ are more intense than when the vorticity flux parametrization is retained (Fig. 7). Clearly this is related to the reduction in the mid-latitude zonal flow, which reflects the weakness of the surface flow and the nearly unchanged zonal average vertical shear. Thus, in mid-latitudes (where the wave diabatic forcing is greatest) parcels of air remain subject to heating or cooling for longer than before and so the response, in terms of either vortex stretching or temperature change, is increased. It appears that the vorticity flux parametrization plays an important role in our model, albeit less crucial than the heat flux parametrization; note, however, that forcing by orography depends essentially on the magnitude of the surface flow, and hence the vorticity flux parametrization is crucial for a realistic response to orographic forcing in our model.

Figure 9 shows the steady-state $p_s$ and $\delta \theta'$ response of the model to D1 forcing. The (1,1) wave bears approximately the same phase relationship with the (1,1) forcing as that derived from linear theory (see Fig. 14), and other investigations (similar to those described above for the D2 case) also suggest that the steady-state was quasi-linear. However, it is noticeable – especially in the $\delta \theta'$ field – that the higher meridional mode (1,3) is more marked than in the D2 case (Fig. 7). The (1,3) and (1,1) waves are indeed of comparable amplitude.
A NON-LINEAR LONG WAVE MODEL

Figure 9. Steady-state fields of $\rho_s$ (a) and $\delta \theta'$ (b) obtained with $D_1$ forcing (see Fig. 5(a)) in the 3,3 intermediate long wave model. Units: $\mu$mb, $\delta \theta'$ in °C.

(see Fig. 14). Explicit long wave fluxes are also more important than in the $D_2$ run, and the zonal average fields are perceptibly altered from the values obtained in the absence of long wave forcing (Fig. 6(a), (b)). Thus the explicit momentum flux is about 10% of the parameterized flux at $y = \frac{1}{4} L_y$, $y = \frac{3}{4} L_y$, although the explicit heat flux is less important. Changes in $\overline{U^x}$ range up to 0.2 m s$^{-1}$ at the surface and up to 1.6 m s$^{-1}$ at the upper lid; changes in $\overline{\delta \theta^x}$ range up to 0.45 °C.

Figure 10 shows the steady-state $\rho_s$ and $\delta \theta'$ fields obtained with $M$ forcing (Eq. (47)). Both fields are dominated by $m = 3$ components, although all three zonal wavenumbers are of equal amplitude in the forcing. This result is in accord with the scale dependence derived from linear theory (see section 2): on the long wave side of resonance the response to orographic forcing becomes progressively less marked, while the response to diabatic forcing at first decreases, and then increases as very large scales are approached (Fig. 3(a), (b)).

From Fig. 10(a) and (b) it is clear that wave components having $n = 3$, and especially the (3,3) components, are very well developed in the steady-state obtained with $M$ forcing. Because of the importance of $n = 3$ meridional modes in the runs carried out with $D_1$ and $M$ forcing (and in many others not described here, in which the forcing has $n = 1$ structure) we suspected that our results might be severely dependent on the applied $n = 3$ truncation. Some of the integrations were therefore repeated with increased meridional resolution.

(c) 3,5 model results

The 3,5 integrations with $D_1$, $D_2$ and $M$ long wave forcing were all accompanied by strong explicit baroclinic instability. In the $D_2$ run the proportions of the total kinetic
energy in modes (1,5), (2,5) and (3,5) after 400 time steps were 7%, 37% and 43% respectively. Qualitatively similar effects occurred in the other two cases.

Attempts were made to suppress the instability by excluding modes (2,5), (3,4) and (3,5) (by the inefficient but convenient method of putting the appropriate spectral amplitudes to zero after every time step). In each case baroclinic instability still occurred, though less spectacularly than before. For example, with D2 forcing the (1,5) and (2,4) modes accounted for 6% and 7% of the total kinetic energy after 600 steps. Similar appreciable instabilities were encountered in a 4,3 form of the model; this time it was the (4,3) mode which exhibited instability.

We conclude that the 3,3 model contains very nearly all the scales which are baroclinically stable under the conditions assumed for the present integrations. This is consistent with the discussion in Section 2, which was based on the results of a simple baroclinic instability analysis. It seems that the investigation of meridional truncation effects in the 3,3 model by including more meridional modes is likely to be confounded by the occurrence of explicit baroclinic instability – with the present combination of realistic parameter values, at least.

The order of presentation in this section roughly matches the order in which the work was actually carried out. We had considered that the intermediate long wave model was a theoretically promising formulation which would be capable of producing significant results. The 3,3 model results were satisfactory, superficially at any rate, but the prevalence of baroclinic instability in the 3,5 model was discouraging. It was decided to investigate the full long wave model to see if more reliable results might be obtainable.
8. NUMERICAL INTEGRATIONS WITH STEADY LONG WAVE FORCING AND TRANSIENT EDDY PARAMETRIZATIONS ACTING ON THE LONG WAVE FIELDS AS WELL AS THE ZONAL AVERAGE FIELDS (FULL LONG WAVE MODEL)

(a) 3,3 model results

Figure 11 shows the steady-state fields $p_s(x, y)$ and $\delta \theta'(x, y)$ resulting when $D2$ forcing is applied. They are generally similar to those obtained with the same forcing in the intermediate long wave model (Fig. 7). Differences in the zonal average flow range up to 0.05 m s$^{-1}$ at the surface and up to 0.32 m s$^{-1}$ at the top level; the zonal average potential temperature differs by less than 0.12°C everywhere. Changes in the wave fields are somewhat larger, ranging up to 0.8 mb in $p_s$ and 1.5°C in $\delta \theta'$. These changes are due mainly to a general westward phase shift: from Fig. 12 it is apparent that the (1,1) and (2,1) surface pressure and potential temperature components all exhibit such a shift, while all amplitudes are slightly reduced.

An integration with the wave part of the forcing doubled showed an approximate doubling of all wave amplitudes, thus indicating the quasi-linear character of the response. Consistently, explicit long wave fluxes were insignificant in both runs, and the steady-state fields were only slightly changed when interactions involving only wave components were excluded.

Application of $D1$ forcing reveals much more marked differences from the corresponding run with the intermediate long wave model (Fig. 9). Figure 13 shows the steady-state fields of $p_s$ and $\delta \theta'$ in the full long wave model. Changes in the zonal average fields were as
Figure 12. (a) Amplitude/phase diagram for the (1,1) wave components of the steady-state surface pressure $p_s$ and potential temperature $\theta$ fields produced by $D2$ forcing in the 3,3 intermediate long wave ($\rightarrow$) and 3,3 full long wave ($\leftrightarrow$) models. The amplitude and phase of the (1,1) component of the forcing are also indicated.

Amplitude unit = $1$ °C d$^{-1}$ (heating), 4.77 mb ($p_s$) and 4.96 °C ($\theta$).

(b) As (a) but for the (2,1) wave components of $p_s$, $\theta$ and the forcing.
Figure 13. Steady-state fields of $p_s$ (a) and $\delta \theta'$ (b) obtained with D1 forcing in the 3,3 full long wave model. Units: $p_s$ in mb, $\delta \theta'$ in °C.

Figure 14. Amplitude/phase diagrams for the (1,1) and (1,3) wave components of the steady-state $p_s$ (full lines) and $\delta \theta'$ (broken lines) fields produced by D1 forcing in the 3,3 intermediate long wave (→) and 3,3 full long wave (↔) models. The amplitude and phase of the (1,1) component of the forcing are also indicated. Amplitude unit as for Fig. 12 (but note different scale).
follows: in the surface zonal flow, up to 0.2 m s$^{-1}$; in the top level zonal flow, up to 1.6 m s$^{-1}$; in the potential temperature, up to 0.5°C. Changes in $p'_s$ and $\delta \theta'$ are considerable, ranging up to 2.6 mb and 3.4°C respectively. They reflect mainly the drastic reduction in the amplitude of the (1,3) components, which is most clearly demonstrated in Fig. 14. The (1,1) components suffer slight diminutions in amplitude, and opposite changes in phase.

![Diagram](image1)

Figure 15. Steady-state fields of $p_s$ (a) and $\delta \theta'$ (b) obtained with $M$ forcing in the 3,3 full long wave model. Units: $p_s$ in mb, $\delta \theta'$ in °C.

Figure 15 shows the steady-state fields of $p_s$ and $\delta \theta'$ produced when $M$ forcing is applied. Comparison with the corresponding intermediate model run (see Fig. 10) reveals a large reduction of wave amplitudes. All the $n = 3$ meridional modes are particularly enfeebled.

We conclude that the net effect of the parametrizations on the long waves is a reduction in wave amplitude which increases with the total wavenumber of the component. The largest scales, (1,1) and (2,1), are only slightly weakened, whilst the $n = 3$ meridional modes are heavily damped. The difference in the behaviour of the full and intermediate long wave models is most marked when higher zonal wavenumbers and/or meridional modes are strongly developed in the intermediate model.

(b) 3,5 model results

In noticeable contrast to the intermediate model integrations described in Section 7(c), explicit baroclinic instability did not occur in the full long wave model with either $D1$, $D2$ or $M$ forcing. The response to $D2$ forcing is very similar to that obtained with the 3,3 model (see Fig. 11), and is not shown. Wave amplitudes are almost identical to those found in the
3,3 model and the \( n = 4 \) and 5 meridional modes are extremely weak. The only perceptible differences which do arise occur in the zonal average fields, and these are largely the changes which result when the meridional truncation is increased from \( N = 3 \) to \( N = 5 \) even in the absence of long wave forcing (see Section 7(a)).

With \( D1 \) forcing the steady-state fields shown in Fig. 16 result. They are very similar to those obtained with the 3,3 model (Fig. 13). Inspection of the spectral amplitudes reveals hardly any changes in the \( n = 1 \) modes, slight changes in the \( n = 3 \) modes, and negligible amplitudes for the \( n = 5 \) modes. Differences in the zonal average fields are perceptible but small. The steady-state fields obtained with \( M \) forcing also differ only slightly from those found in the corresponding run with the 3,3 model.

The response is evidently quasi-linear in all cases. For example, repetition of the \( D1 \) integration with the wave forcing doubled gave an approximate doubling of the major wave amplitudes.

The \( D1 \) run was repeated with the vorticity flux parametrization acting on the zonal average fields only, but with no other change. The \( n = 3 \) modes were even weaker in this run. A similar result was obtained with \( D2 \) forcing. Evidently the heat and vorticity flux parametrizations have opposite effects on the long wave fields. The former tends to weaken the baroclinic fields, particularly the smaller scale components, while the latter tends to intensify the barotropic fields. The damping effect of the heat flux parametrization exceeds the intensifying effect of the vorticity flux parametrization in all the cases examined; hence the weakening of the higher meridional modes.
9. Conclusions and suggestions for future work

In this paper we have described the structure and behaviour of a simple general circulation model in which the quasi-stationary long waves are represented explicitly but the transient baroclinic waves parametrically.

A major requirement in a long wave model of this type is that transient baroclinic waves should not develop appreciably in the explicit part of the model, for the parametrizations are intended to represent the fluxes carried by such systems. From the outset we faced the problem that the long wave response to forcing in the model was likely to be strong (because of resonance effects) at scales close to those at which fairly strong baroclinic instabilities occur: although the most unstable baroclinic waves are about half the scale of resonant stationary waves, growth rates increase rapidly on the short-wave side of the long-wave cut-off to instability (see Fig. 2). Explicit baroclinic instability has been encountered in some integrations when the transient eddy parametrizations act only on the zonal fields, but not when the parametrizations act on the long wave fields too. The diffusive character of the heat flux parametrization is evidently responsible for the suppression of explicit baroclinic instability. (Linearized analyses suggest that the presence of Ekman layer dissipation is crucial if the heat flux parametrization is to have this marked damping effect. We are currently investigating the surprising qualitative differences which exist between the separate and combined effects of lateral heat diffusion and Ekman dissipation on baroclinic instability.)

The heat flux parametrization also brings about a drastic reduction in long-wave resonance effects. The importance of orographic forcing in the model is thereby greatly reduced, since resonance with the external Rossby mode accounts for the only maxima in the response functions. On the other hand, diabatic forcing retains its importance at very large scales, where the baroclinic response function has a secondary maximum. In our model, therefore, diabatic forcing is a more important agency than orographic forcing in determining the configuration of the quasi-stationary long waves. This aspect is evidently dependent on the adopted parametrization scheme, and in particular on the diffusive character of the heat flux parametrization.

Observational studies by Clapp (1970), Savijärvi (1978) and others indicate that relationships between transient eddy fluxes and mean gradients in the real atmosphere are essentially more complicated than our diffusive-type relations. Theoretical investigations (Rhines and Holland 1979) have also revealed conditions under which different behaviour occurs. Our intention here has been to avoid complicated parametrizations, for it would be all too easy to erode the structural and conceptual simplicity of the model by applying ever more elaborate schemes. Note also that Marshall and Shutts (1981) conclude that the case against diffusive parametrizations has been widely overstated: the dynamically significant quantities are the flux divergences, not the fluxes themselves. Nevertheless, several modifications, or replacements, of our parametrization schemes clearly deserve study.

Within the framework of the transfer coefficient approach it is desirable that the simplifying assumptions made in Section 4 be relaxed, and that WG2's zonal average schemes 2 and 3 be extended to the three dimensional case, ideally with longitudinal (as well as latitudinal) variation of the transfer coefficients. Clapp's study suggests that relations between eddy fluxes and gradients upstream may be more reliable than purely local parametrizations; this possibility should also be explored.

Stone (1978) and others have noted that the zonal mean temperature structure of the troposphere is consistent with the hypothesis that baroclinic instability acts to maintain the horizontal temperature gradient just above the threshold value for instability in a 2-level quasi-geostrophic model. Held (1978) gave a partial justification of this 'baroclinic adjustment' hypothesis, in terms of Charney's linear baroclinic instability problem. If Stone's interpretation is correct, the use of analytic flux parametrizations of the type applied in the present study would appear to be unnecessary and even inappropriate. The baroclinic adjustment approach is clearly a promising line for further study; though in its present form
it makes no statement about vorticity fluxes, only the temperature field being 'adjusted'.
Our transfer coefficient parametrizations could be modified to take account of Held's reasoning: one result would be a different dependence of the height-averaged transfer coefficient $R_c^h$ on the pole/equator potential temperature difference (Eq. (44)). We note, however, that Held's argument is based on linear theory and includes a closure statement (similar to that used by Stone (1972)) whereby the typical eddy velocity is set equal to the mean vertical shear multiplied by a scale height. Green's (1970) relation (here Eq. (44)), on the other hand, is not dependent on linear theory, and follows from a closure statement which we consider to be no more arbitrary than that applied by Stone and Held. Saltzmann (1968) has derived a relation similar to Green's but by a different argument (based on idealized eddy energetics).

All the above parametrization schemes clearly depend on an assumption that the transient eddies are of a substantially smaller scale than the background long wave fields. With truncation at $M = 3$, say, in our model, the parametrizations are likely to represent very poorly the effects of any transient or quasi-stationary motion with zonal wavenumber $m = 4$ or 5. When significant quasi-stationary motion is present at these scales, it should be included explicitly. The parametrizations should then represent only the comparatively small baroclinic disturbances which grow in the quasi-stationary baroclinic zones. This procedure is foreshadowed in the blocking case-study described by Green (1977) – in which local baroclinic activity is visualized as a very anomalous phenomenon. When transient motion with $m = 4$ or 5 is significant, the difficulties are evidently even greater. In reality, such motion seems likely to interact with the longer waves in a complicated way that would be ill-represented by any known parametrizations. Once again, incorporation into the explicit part of the model would appear to be the best course.

The difficulty of developing good transient eddy parametrizations for long wave climate models can be traced to one fundamental deficiency in meteorological knowledge: in spite of well-established forecasting rules, not nearly enough is known about the development of transient disturbances on baroclinic long wave fields. A similar point is made by Charney and De Vore (1979). Theoretical analyses have been limited to the linear stability of long wave fields which are steady – either because they are exact solutions of the governing equations or because forcing terms have been added to ensure steadiness (Frederiksen 1979, 1980). More general problems, in which the long wave field is evolving slowly because of dynamical self-interactions, are probably beyond the scope of semi-analytical treatment. These problems would be best examined using time-dependent numerical models: development of disturbances to finite amplitude could then be investigated. Experiments with simple long wave fields would enable the transfer coefficient and baroclinic adjustment theories to be compared, and might suggest improved parametrization schemes. Such studies with conventional (eddy resolving) models are, in our opinion, essential for the rational development of the approach to climate modelling which has been initiated in this paper.

Various changes in the formulation of the explicit long wave part of our model should be considered. It may be desirable to limit the explicit part of the model to the very large scales (wave components (1,1) and (2,1), say) at which diabatic effects are almost certainly dominant in reality. The problem would then be to parametrize the effects of stationary quasi-resonant motion as well as transient baroclinic waves. Instead of Eqs. (8) and (9), the most convenient governing equations would be

$$\beta \frac{\partial \psi_0}{\partial x} = - \frac{f_0}{H} w_8 + F_0, \quad (48)$$

$$- \frac{12 f_0^{\frac{3}{2}}}{N^2 H^2} \frac{\partial \psi_1}{\partial t} + \beta \frac{\partial \psi_1}{\partial x} = 6 \frac{f_0}{H} w_8 + F_1 +$$

$$+ \frac{12 f_0}{N^2 H^2} \left\{ J(\psi_0, \psi_1) - \frac{g N^2}{f_0} (S_0 + G_0) \right\} \quad (49)$$
Equations (48) and (49) are a 2-parameter, \( \beta \)-plane version of the Type 2 quasi-geostrophic equations proposed by Phillips (1963) to describe planetary-scale stationary waves. In SQ it was shown that the external Rossby mode and baroclinic instability are not implied by Eqs. (48) and (49). External mode resonance phenomena do not occur, and so the linear nondissipative response to orographic forcing is generally small. The corresponding response to diabatic forcing is, however, large for the very large (planetary) scales, and effectively the same as that exhibited by Eqs. (8) and (9). Equations (48) and (49) are therefore appropriate, both theoretically and practically, for modelling the very long waves forced by diabatic heating and cooling. (In SQ two other semi-stationary systems like Eqs. (48) and (49) were noted. Subsequent investigation has revealed that they were incompletely defined. The revised versions, and the associated theoretical considerations, will be described in a future paper. Note also that the continuous vertical structure analogue of Eqs. (48) and (49) does imply baroclinic instability.)

The integrations described in Sections 7 and 8 are quasi-linear in the sense that interactions not involving the zonal flow are unimportant. A model in which only wave/zonal flow interactions are described would be computationally and conceptually simpler than the one used in the present study. It would be a useful adjunct to work with modified versions that involve increases in complexity: these include reformulations in spherical geometry and with greater vertical resolution. A marked feature of the integrations described in Sections 7 and 8 is the general insignificance of the explicit long wave fluxes. It is not clear to what extent this is a consequence of the limited vertical domain and resolution. Examination of a stratosphere/troposphere modification and of the feasibility of energy-transmitting upper boundary conditions (see Shutts 1978) are therefore suggested. It may simply be that the conditions applied in the integrations described here are unsuitable for the production of significant stationary long wave fluxes. Yao (1980), in his 2-level \( \beta \)-plane quasi-geostrophic model, found such fluxes to be important in a certain regime with orographic wave forcing and high thermal forcing of the zonal mean flow. Further experiments with our existing long wave model are therefore suggested. These might also investigate the multiple flow equilibria described by Charney and De Vore (1979) and Charney and Straus (1980); such phenomena were not encountered in the work on which the present paper is based.

Consideration should also be given to the problem of representing diabatic effects realistically by expressions involving the long wave fields. The use of externally specified functions in this study has nonetheless simplified interpretation of the results, and so would be a useful technique in future work where the emphasis is on long wave modelling or eddy flux parametrization.

**APPENDIX**

Outline of the derivation of the 2-parameter equations from the continuous quasi-geostrophic vorticity and thermodynamic equations

Substitution of the representations (3), (4) and (5) into the vorticity equation (1) gives a part which is independent of \( z \), a part which is a linear function of \( z \), and a single term which is quadratic in \( z \): \( (z/H)^2 J(\psi, \nabla \cdot \psi) \). The term in \( z^2 \) is not neglected, but is re-expressed in terms of the least squares fit \( a_0 + a_1 (z/H) \) to \((z/H)^2 \) over the interval \(-\frac{1}{2} \leq z/H \leq \frac{1}{2}\), and then incorporated into the two equations representing the parts of Eq. (1) which are independent of \( z \) and linear in \( z \). The least squares procedure is an essential feature of the formulation of the 2-parameter equations, but it is not apparent in the derivation given by Eady (1952).

If the functions \( z^n (n = 0, 1, 2, \ldots) \) were orthogonal on the interval \(-\frac{1}{2} \leq z/H \leq \frac{1}{2}\) (which, of course, they are not) the least squares fit of \((z/H)^2 \) in terms of \( a_0 + a_1 (z/H) \) would have \( a_0 = a_1 = 0 \). The least squares re-expression procedure is therefore entirely analogous to the truncation of higher order terms when a functional representation by a limited number of orthogonal functions is used.
Since
\[ \frac{\partial}{\partial t} \nabla^2 \psi_0 + J(\psi_0, \nabla^2 \psi_0) + \frac{1}{12} J(\psi_1, \nabla^2 \psi_1) + \beta \frac{\partial \psi_0}{\partial x} = \frac{f_0}{H} w_1 + F_0 \] (A1)
and
\[ \frac{\partial}{\partial t} \nabla^2 \psi_1 + J(\psi_0, \nabla^2 \psi_1) + J(\psi_1, \nabla^2 \psi_0) + \beta \frac{\partial \psi_1}{\partial x} = \frac{g f_0}{H} w_2 + F_1 \] (A2)

\( w_1 \) and \( w_2 \) are now expressed in terms of \( \psi_0 \) and \( \psi_1 \) by using the thermodynamic equation (2) and the chosen boundary conditions on \( w \) at \( z = \pm \frac{1}{2} H \). Using the representations (3), (4), (6) and (7), and applying the least squares procedure to \( w(z) \) (assuming \( N^2 \) to be independent of height) we obtain from Eq. (2)
\[ \frac{\partial \psi_1}{\partial t} + J(\psi_0, \psi_1) + \frac{N^2 H}{f_0} \left( w_0 + \frac{1}{12} w_2 \right) = \frac{g H}{f_0} (S_0 + G_0) \] (A3)

Because of the rigid lid at \( z = \frac{1}{2} H \) the representation (5) must be such that
\[ w_0 + \frac{1}{2} w_1 + \frac{1}{4} w_2 = 0 \] (A4)

At \( z = -\frac{1}{2} H \), \( w = w_B \neq 0 \) (because of Ekman layer and orographic effects). Thus
\[ w_0 - \frac{1}{2} w_1 + \frac{1}{4} w_2 = w_B \] (A5)

Using Eqs. (A3), (A4) and (A5), \( w_1 \) and \( w_2 \) can be eliminated from Eqs. (A1) and (A2) in favour of \( w_B \), \( S_0 \), \( G_0 \), \( \psi_0 \) and \( \psi_1 \) to give the 2-parameter equations (8) and (9).

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