Simulations of an observed stratospheric warming with quasigeostrophic refractive index as a model diagnostic

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SUMMARY

A three-dimensional primitive equation model of the stratosphere and mesosphere is described. The model was forced at its lower boundary by observed 100 mb height fields for the wavenumber 2 stratospheric warming period of February 1979, and correctly simulated the reversal of the high latitude circulation. This behaviour contrasts with earlier model simulations of wavenumber 2 warmings in which forcing of a climatological zonally symmetric initial circulation by a stationary wave perturbation led to an initial reversal of the circulation in low latitudes. In a series of idealized experiments we show that wave-wave interaction played no essential role in this simulation and that the ingredients leading to its success were firstly the non-climatological initial wind structure and secondly an imposed longitudinal phase speed for the upward propagating planetary wave at the lower boundary. These studies also demonstrate that in the pre-warming period the direction of propagation of planetary wave activity, as represented by integral curves of the so-called Eliassen-Palm flux, may be qualitatively described by the WKB limit of the quasi-geostrophic potential vorticity equation. In this limit, the Eliassen-Palm flux is simply related to the zonal mean refractive index, leading to two complementary diagnostics for studying wave, mean-flow interaction.

1. INTRODUCTION

The stratospheric sudden warming is a dramatic geophysical event during which temperatures in the high latitude stratosphere can rise by as much as $50^\circ C$ in a few days. Associated with these temperature changes the stratospheric westerly circumpolar vortex breaks down and is often replaced by a strong easterly circulation. Originally, observation of these phenomena was limited by the availability of radiosonde and rocketsonde ascents. In recent years, however, remote sensing of stratospheric temperatures by satellite radiometry has enabled both the spatial and temporal characteristics of warmings to be studied in detail. These studies have demonstrated the association between warmings and planetary scale wave motions and a theory has evolved which accounts for the stratospheric sudden warming in terms of the interaction of upward propagating, transient planetary scale waves with the stratospheric circumpolar circulation. The waves originate in the troposphere through the effects of orography and land/sea temperature contrasts on the Earth’s surface; (see Holton 1980 for a review of the dynamics of stratospheric warmings).

The possibility that the interaction between forced upward propagating Rossby waves and the circumpolar circulation is capable of generating a sudden warming was first demonstrated by Matsuno (1971) using a quasi-geostrophic model of the stratosphere and mesosphere with lower boundary at a level corresponding roughly to the tropopause. The lower boundary condition was specified by an idealized geopotential height field expressed as a simple algebraic function of latitude and time with a single wavenumber in the longitudinal direction. Initial conditions corresponded to a typical zonally-averaged winter climatology. Furthermore Matsuno assumed that the effect of wave, wave interaction on the zonal mean circulation could be neglected. Holton (1976) showed that many of the features of Matsuno’s simulations are still valid in a spectrally truncated primitive equation model with similar model domain.
The success of these models in reproducing the qualitative features of a sudden warming, together with the improved data coverage that satellite instrumentation affords, suggests that it is now feasible to attempt a numerical simulation of an observed warming. Such a simulation is of considerable interest as a test of whether a forced model integration can stand up to detailed quantitative comparison with observation. In order to make such detailed comparison, zonally symmetric initial conditions and simple algebraic functions for the lower boundary condition are no longer adequate; the model must be initialized and forced with observed height fields.

In this paper we describe such an experiment and compare the results with observations. The wavenumber 2 sudden warming of February 1979 was chosen for study because it had been found (Palmer 1981a) to evolve very differently from the model integrations of Matsuno (1971) and Holton (1976). Most importantly, in the observed warming an easterly circulation first appeared in high latitudes and spread downward and equatorward, while the models first developed low latitude easterlies which then spread polewards. Understanding this disparity was the prime motive for attempting both the simulation and subsequent experiments.

The initial conditions for the model simulation were taken from Stratospheric Sounding Unit (SSU) (Miller et al. 1980) observations of the pre-warming period and 100 mb height fields from European Centre for Medium Range Weather Forecasts (ECMWF) series IIIB FGGE analyses were used for lower boundary data. The model integrations reproduced both the timing and position of the easterly jet well, thus demonstrating its ability to follow developments over several days. This success led us to attempt a series of idealized experiments to isolate the mechanisms at play in our simulation. These experiments were designed to test the effects of various components abstracted from the simulation in order to deduce which were necessary and sufficient to reproduce the chosen principal features. Since the simulation compared well with the atmosphere, we infer that these components were crucial in producing the observed warming.

The results of the idealized experiments showed that the timing and location of reversal could in this case be adequately represented in terms of the interaction of a single forcing wave with the zonal mean flow, given the appropriate specification of each. In particular, two ingredients were essential to the high latitude reversal – firstly the non-climatological zonal wind profile prior to the warming, and secondly the fact that before the warming wavenumber 2 at 100 mb was not stationary but progressive with a phase speed of about 10 m s\(^{-1}\).

The possible importance of the zonal wind profile in the pre-warming phase has already been stressed by Palmer (1981b) and Dunkerton et al. (1981). In particular, these authors have suggested that if the polar night jet is displaced towards the pole, the associated distribution of quasi-geostrophic refractive index might focus wave activity into high latitudes. Although our results confirm this conjecture, we have shown that it is important to consider the effect of subsequent wave, mean-flow interaction in changing this distribution of refractive index. For example, with stationary forcing and the pre-warming zonal wind, we found that wave activity initially propagated into high latitudes and decelerated the high latitude jet weakly, though sufficiently to direct subsequent wave activity towards low latitudes. When the model was forced with a height field which had a progressive phase speed, a similar initial propagation into high latitudes was found, but in this case the magnitude of high latitude deceleration was sufficient to completely reverse the high latitude circulation.

In Section 2 we describe the model used in these experiments. Section 3 contains a review of some recent developments in the theory of wave, mean-flow interaction with particular emphasis placed on the transformed Eulerian-mean formalism of Andrews and McIntyre (1976, 1978), and its WKBJ limit. Results of the simulation of the February 1979 warming are discussed in Section 4, and the idealized experiments are then described in Section 5. In Section 6 we study the evolution of the refractive index for these idealized experiments, showing that an explanation of much of their behaviour can be given in terms of the effect
of the refractive index upon the quasi-geostrophic Eliassen-Palm flux. Some concluding remarks are made in Section 7.

2. Model formulation

The model described here is a development of the global three-dimensional primitive equation model originally formulated by Murgatroyd (1971). The vertical coordinate is defined as $z = -H \ln(p/p_0)$ where $H$ is a constant scale height and $p$ a constant standard pressure. In ‘flux form’ the horizontal momentum equations, the thermodynamic equation, continuity equation and hydrostatic approximation may be expressed in spherical coordinates (Holton, 1975) as follows:

$$\frac{\partial u}{\partial t} + \frac{1}{a \cos \phi} \frac{\partial u^2}{\partial \lambda} + \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (uv \cos \phi) + \frac{1}{\rho_o} \frac{\partial}{\partial z} (\rho_o \mu w) -$$

$$- v \left( f + \frac{u \tan \phi}{a} \right) + \frac{g}{a \cos \phi} \frac{\partial \Phi}{\partial \lambda} = - F_z (u - \bar{u})$$

$$\frac{\partial v}{\partial t} + \frac{1}{a \cos \phi} \frac{\partial (uv)}{\partial \lambda} + \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (v^2 \cos \phi) + \frac{1}{\rho_o} \frac{\partial}{\partial z} (\rho_o \nu w) +$$

$$+ u \left( f + \frac{u \tan \phi}{a} \right) + \frac{g}{a \cos \phi} \frac{\partial \Phi}{\partial \phi} = - F_z (v - \bar{v})$$

$$\frac{\partial T}{\partial t} + \frac{1}{a \cos \phi} \frac{\partial}{\partial \lambda} (Tu) + \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (Tv \cos \phi) + \frac{1}{\rho_o} \frac{\partial}{\partial z} (\rho_o Tw) + \frac{w_k T}{H} = \frac{\dot{Q}}{c_p}$$

$$\frac{1}{a \cos \phi} \frac{\partial u}{\partial \lambda} + \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (v \cos \phi) + \frac{1}{\rho_o} \frac{\partial}{\partial z} (\rho_o w) = 0$$

$$\frac{\partial \Phi}{\partial z} = \frac{RT}{Hg}$$

(1)

(see Appendix A for a glossary of all mathematical symbols).

Newtonian cooling was used to parametrize atmospheric radiation in the form:

$$\dot{Q}/c_p = - \alpha(z) \{T - T^*(\phi, z)\}$$

(2)

$T^*$ is a zonally symmetric equilibrium temperature, chosen for simplicity to be the initial zonal mean temperature, and $\alpha(z)$ is the Newtonian cooling coefficient. The values of $\alpha$ used were given by (Holton 1976):

$$\alpha(z) = [1.5 + \tanh((z - 35)/7)] \times 10^{-6} \text{s}^{-1}$$

(3)

where $z$ is in km. It may be noted that relaxation to an observed temperature distribution in this manner underestimates the radiative cooling in the polar night region, and so may allow a sudden warming too readily (Hsu 1980). However, this method has been adopted both for simplicity and comparability with other work. As in Holton (1976) Rayleigh friction was used to damp local departures from the zonal mean velocity. The height variation of the friction coefficient is given by:

$$F_z(z) = \begin{cases} 
10^{-7}s^{-1}, & z \leq 50 \text{ km} \\
10^{-7} + 5 \times 10^{-6}[1 - \exp((35 - z)/40)] \text{s}^{-1}, & z > 50 \text{ km}
\end{cases}$$

(4)

In this work two different model domains were used, both extending from pole to pole. The first (version A) has a 36 × 16 regular latitude-longitude grid and 33 levels in the vertical
while the other (version B) has a $36 \times 36$ horizontal grid and 17 levels in the vertical. Version A was used for the idealized experiments described in section 5 where azimuthal resolution was not crucial and the height of the lower boundary corresponded to that used in the other mechanistic models (see Introduction). On the other hand it was found that a more accurate simulation was possible with improved azimuthal resolution. Further the lower boundary was placed at 100 mb, at which level observational data were readily available. Better overall resolution could not be obtained because of computer storage constraints.

The levels are spaced at equal increments of $z$. In the 17-level version $z$ varies from 16 km to 80 km so that $\Delta z = 4$ km and for the 33-level model $z$ ranges from 8 to 93$\frac{1}{2}$ km with $\Delta z = 2\frac{1}{2}$ km. Values of the model variables (apart from vertical velocity which is kept at mid-level) are carried at each grid point, and represent the state of the surrounding regions with boundaries mid-way between grid points. Fluxes of heat and momentum across these boundaries are determined by a spatial finite difference scheme with the property that fluxes through the pole are zero. The form of the spatial finite difference equations given in Appendix B conserves global kinetic and potential energy.

A leap-frog time differencing scheme was used, but to avoid decoupling of odd and even time-steps a forward step was performed every 90 time-steps. To avoid linear instability a time-step of 8 min was used, together with Fourier truncation down to wavenumber 2 in polar latitudes. In order to suppress non-linear instability all the fields were filtered at every time step. Filtering was carried out separately in the zonal and meridional directions using the eighth order filter of Shapiro (1971).

The upper boundary condition was such that the vertical velocity was set to zero half a level above the model’s top level while the lower boundary condition was determined by specification of geopotential heights; temperatures and winds at that level being obtained by linear downward extrapolation. Height data were obtained either from series IIIb FGGE analyses produced by the ECMWF or from idealized forcing functions.

For the real data simulation (Section 4) stratospheric thickness fields were derived by linear regression on radiance data from the SSU on board the Tiros N satellite (Miller et al. 1980). By adding 100 mb heights to these thicknesses a set of stratospheric height fields was obtained and interpolated by cubic splines to the model levels below 1 mb. Above 1 mb, zonal heights were constructed from the hydrostatic approximation by using a climatological global static stability to integrate temperatures upward from the 1 mb level. Wave components were added by extrapolating amplitudes, phases and phase gradients from the levels below. By this means reasonable values of static stability were ensured and discontinuities in vertical gradients of eddy fluxes at the 1 mb level avoided. The hydrostatic and geostrophic relationships were used to generate initial temperatures and winds at model levels from the height fields. The use of the geostrophic approximation produced unrealistic features at grid points adjacent to the equator, but these were found to disappear quickly during integrations. For the idealized integrations initial winds were either similar to those used by Holton (1976), or extracted from atmospheric data for 16 February 1979 (see Sections 4 and 5).

3. THEORY AND DIAGNOSTIC FORMALISM

The stratospheric sudden warming is one of the many geophysical phenomena that involve the interaction of forced waves with a mean flow. In fact the results presented in this paper confirm, at least for the February 1979 warming, that wave, mean-flow interaction dominated the reversal of the high latitude jet, with wave-wave interactions and other processes playing only a secondary role. Recent diagnostic studies of actual (Palmer 1981a, b) and model (Dunkerton, et al. 1981) warmings have suggested that the transformed Eulerian-mean formalism (Andrews and McIntyre 1976, 1978) provides a useful framework in which to examine the effect of waves on a mean flow. This formalism will be adopted here.
The transformed Eulerian equations are obtained by defining a ‘residual meridional circulation’ \( \tilde{e}_* \) (Andrews and McIntyre 1976, 1978)

\[
\tilde{e}_* = \ddot{v} - \frac{1}{\rho_0} \frac{\partial}{\partial z} \left( \rho_0 \overset{\cdot}{v} \frac{\partial}{\partial \phi} \frac{\partial}{\partial \theta} \right) \tag{5a}
\]

\[
\tilde{w}_* = \ddot{w} - \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left( \cos \phi \overset{\cdot}{v} \frac{\partial}{\partial \phi} \right) \tag{5b}
\]

The zonally averaged zonal momentum equation then becomes (Dunkerton, et al. 1981)

\[
\ddot{u}_z + \ddot{e}_* [ (\tilde{u} \cos \phi) \frac{\partial}{\partial \phi} (\cos \phi) \overset{\cdot}{f} + \tilde{w}_* \tilde{u}_z ] = (\rho_0 a \cos \phi)^{-1} \nabla \cdot \mathbf{F} \equiv \mathcal{Q}_\mathbf{F} \tag{6}
\]

The direct wave forcing on the mean zonal circulation is due to the term

\[
\mathcal{Q}_\mathbf{F} \equiv (\rho_0 a \cos \phi)^{-1} \nabla \cdot \mathbf{F}
\]

where \( \mathbf{F} \) is the so-called Eliassen-Palm (EP) flux (Edmon et al. 1980) defined by

\[
\mathbf{F}^{(\phi)} = \rho_0 a \cos \phi \left( \tilde{u}_z \overset{\cdot}{v} \frac{\partial}{\partial \theta} \right) \tag{7a}
\]

\[
\mathbf{F}^{(z)} = \rho_0 a \cos \phi \left[ (\overset{\cdot}{u} \cos \phi) \frac{\partial}{\partial \phi} (\overset{\cdot}{v} \frac{\partial}{\partial \phi} \overset{\cdot}{f} + \overset{\cdot}{w}_* \overset{\cdot}{u}_z) \right] \tag{7b}
\]

\[
\nabla \cdot \mathbf{F} = (a \cos \phi)^{-1} \partial (\mathbf{F}^{(\phi)} \cos \phi) / \partial \phi + \partial (\mathbf{F}^{(z)}) / \partial z \tag{8}
\]

\( \mathbf{F} \) has the important property that for steady, linear conservative waves it is non-divergent, and from Eq. (6) there can be no wave induced change in the zonal mean wind under these ‘non-acceleration’ conditions. Latitude height cross-sections of \( \nabla \cdot \mathbf{F} \) then display precisely where non-acceleration conditions are being violated by the waves. The factor \((\rho_0 a \cos \phi)^{-1}\) arises because the true flux divergence relates to changes in angular momentum.

The graphical convention used throughout this paper is essentially the same as that adopted by other authors (see Dunkerton et al. 1981 for example). The volume element for a zonally symmetric portion of the atmosphere is

\[
dV = 2\pi a^2 \cos \phi \, d\phi \, dz \tag{9}
\]

From Eqs. (8) and (9)

\[
\nabla \cdot \mathbf{F} \, dV = B \, dy \, dz \tag{10}
\]

where

\[
B = \partial (\mathbf{F}^{(z)}) / \partial y + \partial (\mathbf{F}^{(y)}) / \partial z \tag{11}
\]

and

\[
\mathbf{F} = (\mathbf{F}^{(\phi)}, \mathbf{F}^{(z)}) = 2\pi a \cos \phi \mathbf{F} \tag{12}
\]

\( B \) is the natural form of the divergence of \( \mathbf{F} \) in the \((y,z)\) plane and hence the appropriate pattern of arrows representing \( \mathbf{F} \) will look non-divergent if and only if \( \nabla \cdot \mathbf{F} = 0 \). A further point requiring care in illustrating the direction of \( \mathbf{F} \) in a \((y,z)\) plane arises because of the difference in scales of coordinates \( y \) and \( z \). This requires a rescaling operation

\[
\mathbf{F} \rightarrow (c \mathbf{F}^{(\phi)}, \mathbf{F}^{(z)}) 
\]

where \( c \) is a constant such that the coordinate difference representing a 1 km change in \( c \phi \) is equal to \( c \) times the coordinate difference representing a 1 km change in \( z \). Throughout this paper the flux \( \mathbf{F} \) given in Eq. (13) will be used in illustrations of the EP flux.
The significance of $F$ as a diagnostic for studying wave propagation arises from its appearance in a conservation equation valid for linear conservative waves. This 'generalized Eliassen-Palm' relation takes the form (Andrews and McIntyre 1976, 1978)

$$\frac{\partial A}{\partial t} + \nabla \cdot F = 0 \quad \text{for linear conservative waves.}$$

$A$ can be regarded as a local density of wave activity and $F$ can then be thought of as the net rate of transfer of wave activity from one latitude and height to another (Edmon et al. 1980). In the quasi-geostrophic approximation $F$ takes the form

$$F = (-\rho_0 a \cos \phi \overline{u'v'}, \rho_0 af \cos \phi \overline{v'\theta'/\theta_z}) \quad \text{and $A$ reduces to the usual quasi-geostrophic potential enstrophy divided by twice the zonal mean gradient of potential vorticity i.e.}$$

$$A = \frac{1}{2} \left( \overline{(q^2)} / q \right)_r \quad \text{with $q$ being the quasi-geostrophic potential vorticity}$$

$$q = f + (a \cos \phi)^{-1} v_a - (u \cos \phi) \frac{\partial}{\partial \lambda} + (gf \rho_0) \frac{\partial}{\partial \theta} (\rho_0 \overline{\theta^2 \theta_z}) / \overline{\theta_z} \quad \text{and $\delta \theta$ is defined as the departure of potential temperature $\theta$ from a global horizontal average $\overline{\theta}$. The components of $F$ are now proportional to the usual meridional eddy momentum and heat fluxes. Furthermore, when quasi-geostrophic theory holds, it can be shown (Edmon et al. 1980) that $\nabla \cdot F$ measures the departure from non-acceleration conditions for finite amplitude waves.}$$

With quasi-geostrophic assumptions and linearization about a zonal mean state the essential dynamics may be expressed in terms of a single equation for a conservative eddy geopotential height field $\Phi'$. In spherical coordinates $(\lambda, \phi, z)$ this equation is

$$\left( \frac{\partial}{\partial t} + \frac{u}{a \cos \phi} \frac{\partial}{\partial \lambda} \right) \left( \frac{1}{a^2 \cos^2 \phi} \frac{\partial^2}{\partial \lambda^2} + f^2 \frac{\partial}{\partial \phi} \frac{\partial}{\partial \phi} + f \frac{\partial}{\partial \theta} \frac{\partial}{\partial \phi} + \frac{\rho_0}{\overline{\theta_z}} \frac{\partial}{\partial \phi} \frac{\partial}{\partial \phi} \right) \Phi' +$$

$$+ \frac{\overline{\theta_z}}{a^2 \cos \phi} \frac{\partial^2 \Phi'}{\partial \lambda^2} = 0 \quad \text{where $N^2(z)$ is defined in terms of the global average $\overline{\theta_z}(z)$. If $\Phi'$ is a steady linear wave of single zonal wavenumber $k$ i.e.}$$

$$\Phi' = \frac{1}{k} e^{i \lambda z} (\psi_k e^{i k \lambda - \sigma t}) + \text{C.C.} \quad \text{gives the equation}$$

$$\left( \frac{f}{a^2 \cos \phi} \frac{\partial}{\partial \phi} \left( \cos \phi \frac{\partial}{\partial \phi} \right) + f^2 \frac{\partial}{\partial z} \left( \frac{1}{N^2} \frac{\partial}{\partial z} \right) \right) \psi_k + \frac{Q_k}{a^2} \psi_k = 0 \quad \text{where}$$

$$Q_k = aq f (\overline{u - a \sigma \cos \phi}) - k^2 \overline{\cos^2 \phi} - a^2 f^2 / 4 N^2 H^2$$

Equation (20) is a two-dimensional wave equation describing the propagation of a wave in the $(\phi, z)$ plane; the quantity $\sqrt{Q_k}$ is the 'refractive index' (Matsuno 1970) and represents the influence of the mean wind pattern on the propagation of a steady wave. This 'refractive index theory' of wave propagation may be linked to the use of EP fluxes in describing the
transfer of disturbances in the meridional plane by assuming WKBJ theory to be applicable (see Lighthill 1978). The wave is then assumed steady to a first approximation, as well as propagating in a particular direction. Group velocity concepts then become meaningful and it can be shown (Edmon et al. 1980) that the meridional projection of the group velocity is everywhere parallel to $\mathbf{F}$ so that

$$\mathbf{F} = CA$$

(Of course $\mathbf{F}$ is always well-defined, even when WKBJ theory is not valid).

Using WKBJ theory it is possible further to find how the local magnitude and direction of $\mathbf{F}$ is governed by the corresponding value of $\mathcal{Q}_k$ for a single propagating wave (Palmer 1981b). This is best seen by defining a new coordinate system $(\tilde{y}, \tilde{z})$ such that

$$d\tilde{y} = f dy/\Omega; \quad d\tilde{z} = N dz/\Omega . \quad . \quad . \quad . \quad (22)$$

Then if the perturbation geopotential height field (Eq. (19)) obeys the geostrophic and hydrostatic relations $\mathbf{F}$ takes the form

$$\mathbf{F} = (F(\tilde{y}), F(\tilde{z})) = (F(\tilde{y})d\tilde{y}/\Omega, F(\tilde{z})d\tilde{z}/\Omega) = \frac{1}{2} \rho_0 k (|\psi_k|^2 \tilde{\psi}_k)/\Omega^2 \quad . \quad . \quad . \quad (23)$$

where

$$\psi_k = |\psi_k| \exp i\tilde{\xi}_k$$

with norm

$$||\mathbf{F}|| = [(F(\tilde{y}))^2 + (F(\tilde{z}))^2]^{1/2} \quad . \quad . \quad . \quad (24)$$

The dispersion relation for WKBJ waves can now be written as

$$||\mathbf{F}|| = \frac{1}{2} k |\psi_k|^2 (\tilde{\mathcal{Q}}_k)^4 \rho_0/a \Omega^2 \quad . \quad . \quad . \quad (25)$$

where

$$\tilde{\mathcal{Q}}_k = \mathcal{Q}_k / \sin^2 \phi \quad . \quad . \quad . \quad (26)$$

Differentiating Eq. (25) it can be shown that trajectories of $\mathbf{F}$ are curved up the gradient of $\tilde{\mathcal{Q}}_k$, and in particular will tend to be channelled along ridges of $\tilde{\mathcal{Q}}_k$. A similar result has been obtained by Karoly and Hoskins (1982) using a ray tracing approach. Equation (25) is essentially equivalent to Charney and Drazin's (1961) results that wave propagation (here defined by $||\mathbf{F}|| > 0$) is not possible in regions of imaginary refractive index. For given wave amplitude $|\psi_k|$, e.g. at the forcing level of the model, larger fluxes should be obtained in regions of high refractive index. Although these results are only strictly applicable for conservative, linear, steady WKBJ waves the diagnostics presented in section 6 show that they give a good qualitative picture of wave propagation during the build up to a stratospheric warming.

4. Simulation of the February 1979 Warming

The wavenumber 2 stratospheric warming of February 1979 has been described by Quiroz (1979), and Palmer (1981a). It is clear from the diagnostics discussed by these authors that the stratospheric warming could be regarded as a response to large wavenumber 2 geopotential amplitudes in the troposphere.
The geopotential amplitudes of wavenumbers 1 to 3 at 100 mb and 62.5°N are illustrated in Fig. 1a for the period of the warming. (Note: Observational diagnostics in this paper are not identical to those in Palmer (1981a) since different tropospheric analyses have been used.) It is seen that wavenumber 2 reaches a peak amplitude of almost 600 m on 21 February, a value 4 times greater than the stationary wave climatology of Van Loon et al. (1973). By contrast, wavenumbers 1 and 3 had smaller amplitudes during the warming, with wavenumber 1 only becoming larger than wavenumber 2 during the decay phase of the warming. The longitudinal phase of wavenumber 2 at 100 mb is illustrated in Fig. 1b for latitudes 57.5°N and 67.5°N. Two features of this diagram are worthy of comment. Firstly, in the pre-warming period, wavenumber 2 was progressing at about 10° d⁻¹, becoming stationary during the warming. Secondly, it can be seen that the latitudinal phase gradient reversed several times. From Eqns (15) and (23) such a reversal corresponds to a switching in the direction of the meridional component of the quasi-geostrophic EP flux. This switching in direction has been discussed in Palmer (1981a) where it was remarked that zonal deceleration was strongest during periods when the EP flux had poleward tilt.

The model was forced at its lower boundary with the 100 mb height fields from which Fig. 1 was derived. The simulation was initialized using data from 16 February which was considered to be a convenient date, representing part of the pre-warming period before
substantial growth of wavenumber 2 in the mid- and upper stratosphere, but after the major decay phases of earlier wavenumber 1 activity (see Quiroz 1979).

The zonal mean wind, \( \bar{u} \), for 16 February 1979 is illustrated in Fig. 2a. It is seen that the polar night jet is displaced north of its typical climatological position equatorward of 60°N (Murgatroyd 1970) and there is a region of weak westerlies around 45°N in the mid-stratosphere. This non-climatological zonal wind structure appears to result from earlier wave activity in the stratosphere. EP fluxes for the 16th are illustrated in Fig. 2b. The wavenumber 2 flux is directed up and equatorwards from the mid-latitude tropopause with regions of divergence in the high latitude upper stratosphere (due to wavenumber 1 only) and also in mid- to low latitudes.

In Fig. 3 the simulated zonal mean winds are compared with observed values (geostrophic winds) for several days. By 17 February (Fig. 3a) the observations show that in high latitudes the upper stratospheric circulation had accelerated, whilst in the lower stratosphere the zonal wind had changed very little. The mid- and low latitude winds remained steady. The model shows some acceleration in the higher levels, though the simulated jet is by now somewhat weaker than observed at all levels.
Figure 3. Mean zonal wind (m s⁻¹) for observed data (left hand diagram) and model simulation (right hand diagram) (a) 17 February, (b) 19 February, (c) 22 February, (d) 24 February.
Figure 4. EP flux $\vec{F}$ and contours of $B$ labelled in units of $10^4 \text{kg s}^{-2}$ for observed data (left hand diagram) and model simulation (right hand diagram) (a) 17 February, (b) 19 February, (c) 21 February, (d) 24 February. The value of $c$ is 0.0072.
By 19 February (Fig. 3b) the circulation in high latitudes had decelerated significantly, while in mid- and low latitudes there had been slight acceleration. The model integration for 19 February has captured this deceleration quite well, but because of its underestimation of the acceleration for the previous days the model has produced a region of weak easterlies in the high stratosphere over the pole. The model’s low latitude winds remain fairly steady up to the 19th with only very weak acceleration in the lower stratosphere. By 22 February (Fig. 3c) both the observations and the model integration show a significant region of high latitude easterlies. Once more the model easterlies are stronger, though the actual position of the easterly jet core compares well with observations. The model also fails to produce easterly winds in the mid- and lower stratosphere in polar latitudes.

On 24 February (Fig. 3d) both the model and atmospheric winds have accelerated in high latitudes though by 27 February (not shown) the atmospheric winds were undergoing a second period of deceleration which the model did not reproduce. Instead the model circulation continued to recover back to a westerly flow.

The EP fluxes for the model and atmosphere are compared in Fig. 4, together with contours of Ψ (c.f. Eq. (11)). In Fig. 4a it is seen that the model fluxes by 17 February are larger and directed more vertically than the initial fluxes of 16 February. Furthermore, it can be seen that convergence of the upward directed model fluxes is larger than the corresponding atmospheric fluxes. Similarly, the region of high latitude EP flux divergence in the upper stratosphere is not quite as extensive in the model as in the atmosphere. This general overestimation of EP flux convergence is consistent with the overestimation of zonal deceleration reported above. On 19 February (Fig. 4b) the direction of EP fluxes again agrees well with the observations; in particular both model and atmosphere show a noticeable poleward tilt which is greatest in the middle stratosphere. Once again, the magnitude of the convergence is greater for the model fluxes. By 21 February, as Fig. 4c shows, the model and atmospheric EP fluxes are generally in good agreement in high latitudes, although there is now a small region of divergence in the model polar latitudes consistent with the limited encroachment of easterlies over the model pole. However, whilst the EP fluxes at 40°N in both model and atmosphere are upward and equatorward, the model shows a significant region of divergence not apparent in the observed field. By 24 February (Fig. 4d) EP flux divergence obtains throughout the model’s lower stratosphere in high latitudes with downward directed fluxes at the lower boundary. The atmospheric fluxes, on the other hand, continue to be upward directed throughout most of the stratosphere, though in the high latitude middle stratosphere there is a weak region of flux divergence: as commented previously, both atmosphere and model showed acceleration of the zonal flow. On the 27th (not illustrated), model and atmospheric fluxes were qualitatively different, again consistent with the different evolution in $\vec{u}$.

The model’s qualitatively correct positioning of the easterly jet in high latitudes on 22 February and its ability to simulate the general direction and size of the EP fluxes both contrast with the previous model integrations of idealized wavenumber 2 warmings (see Dunkerton et al. 1981). In such integrations easterlies first appear in low latitudes, and evolve via poleward migration of the zero wind line. The corresponding EP fluxes are strongly convergent in low latitudes. In the next section we shall describe some idealized experiments designed to identify the mechanisms present in our model which led to the reproduction of the first period of deceleration in the February 1979 warming.

5. IDEALIZED EXPERIMENTS

In this section we shall attempt to understand why easterlies first appeared at high latitudes in our simulation. To do this it is useful to enumerate the differences between our integration and the model integrations of Matsuno (1971) and Holton (1976). These differences fall broadly into three categories:

(a) in model formulation;
SIMULATIONS OF AN OBSERVED STRATOSPHERIC WARMING

(b) in initial data;
(c) in lower boundary data.

Included in (a) is the fact that we are dealing with a three-dimensional fully non-linear primitive equation model (albeit with limited zonal resolution), whereas Matsuno, for example, used a quasi-geostrophic model and Holton used a truncated semi-spectral model, both retaining only one zonal wavenumber. Included in (b) are the facts that the initial zonal mean wind is not climatological and that the initial state is not zonally symmetric. Included in (c) are the facts that the lower boundary data contains many wave components, all with complicated growth or decay functions, and in general non-zero phase speeds and latitudinal phase gradients. (c.f. Fig. 1b).

Out of all these factors we have attempted to isolate the crucial ingredients responsible for the difference in evolution between our simulation of the February 1979 warming and the evolution of the model wavenumber 2 warmings. To do this we have performed a series of idealized experiments to test the importance of the differences described above. Our aim at this stage is to provide a recipe whereby a mechanistic model with simple boundary and initial conditions can, at least qualitatively, reproduce our results. The first two experiments to be described indicate whether this goal should, in principle, be achievable.

For experiment (I) Fourier analysis was performed on the initial and boundary data sets for the simulation of section 4, and all components apart from wavenumber 2 and the zonal mean were removed. This provided initial and boundary conditions for a ‘wavenumber 2 only’ integration. Since the amplitudes of wavenumbers 1 and 3 were identically zero initially, symmetry will require that they remain zero throughout the integration, while higher even wavenumbers attain only small amplitudes. The results of this experiment (which were essentially positive) then enable us to assert whether the warming simulated in the previous section was largely the outcome of the interaction of wavenumber 2 with the mean flow.

Experiment (II) was essentially similar to those of Matsuno (1971) and Holton (1976). This was found to confirm that the evolutionary behaviour of earlier wavenumber 2 model integrations (zonally symmetric initial winds corresponding to a climatological circulation and idealized lower boundary forcing) was not a consequence of model formulation (cf (a)). The detailed results of experiments (I) and (II) are outlined below.

Experiment (I)

With Day 0 denoting the initial day, the zonal mean winds for several days are illustrated in Fig. 5 (ū at Day 0 is as on 16 February illustrated in Fig. 2a). Figure 5a shows that at Day 3 the mid-latitude zonal wind has decelerated, and the polar night jet has migrated northwards. Comparison with the simulation of 19 February (Fig. 3b) shows that in the mid- and upper stratosphere there is less deceleration in experiment (I). Figure 5b for Day 6 shows that a band of easterlies has appeared in high latitudes, though the winds have remained westerly north of about 75°N. Again this compares well with the simulation of 22 February, though the simulated easterly jet core in Fig. 3c is stronger, and extends further towards the pole. Figure 5c for Day 8 (c.f. Fig. 3d) shows the easterly jet core to have intensified, while the region of westerlies in polar latitudes has diminished. After Day 10 (not illustrated) the zonal mean wind accelerated steadily back to a westerly circulation.

The EP fluxes for Day 0 are illustrated in Fig. 6a. With only wavenumber 2 and its harmonics present, it is seen that the convergence of the upward directed wave flux is a little larger than that of the EP fluxes given in Fig. 2b. The EP fluxes for Day 5 (corresponding to 21 February) are illustrated in Fig. 6b. It is seen that the general size and direction of the fluxes and their convergence are similar to Fig. 4c, though it is noticeable that the fluxes for this experiment do not have as large a poleward tilt as the simulated fluxes for 21 February.

The results of experiment (I) demonstrate conclusively that wavenumber 2 activity was primarily responsible for producing high latitude easterlies, and that neither wave, wave interaction nor the influence of wavenumber 1 was a crucial ingredient in the simulation.
Figure 5. Mean zonal wind (m s⁻¹) for Experiment I (a) Day 3, (b) Day 6, (c) Day 8.

Figure 6. FP flux (W m⁻²) and contours of B labelled in units of 10⁻⁴ kg m⁻² s⁻¹ for Experiment (I) (a) Day 0, (b) Day 5, (c) Day 8.
The only region of the stratosphere in which wave, wave interactions did appear to play a role was in the polar latitudes; experiment (I) illustrates the fact that wavenumber 2 is unable to propagate to these highest latitudes and thereby decelerate the zonal circulation (see section 6). This is in good agreement with observation; a study of the atmospheric EP fluxes associated with individual wave components (not illustrated) shows convergence in the polar upper stratosphere due mainly to wavenumber 1.

Experiment (II)

In this integration the initial data was zonally symmetric, typical of a climatological

![Figure 7. As Fig. 5 but for Experiment (II) (a) Day 0, (b) Day 16. (Note the different vertical scale to Fig. 5.)](image)

zonal wind (see Fig. 7a). The wavenumber 2 forcing at the lower boundary (which in all the following runs was put at 8 km) followed a simple idealized scheme viz.

\[
\Phi(\lambda, \phi, t) = 430 \, g(\phi) \left\{1 - \exp(-t/t_0)\right\} \sin 2\lambda,
\]

\[
g(\phi) = \begin{cases} 
0 & \text{if } 90^\circ S \leq \phi \leq 30^\circ N \\
\sin^2[(\phi - 30^\circ)\pi/(60^\circ - (\phi - 60^\circ)/3)] & 30^\circ N < \phi < 82.5^\circ N \\
0 & 82.5^\circ N \leq \phi \leq 90^\circ N
\end{cases}
\]

where \( \Phi \) is in m and \( t_0 = 2.5 \times 10^5 \) s.

This represents a growing stationary wave with peak amplitude of 430 m; this value is larger than that used by Dunkerton et al. (1981), but was close to the observed maximum in wavenumber 2 geopotential amplitude at 8 km during February 1979. The latitude dependence expressed by \( g(\phi) \) was chosen to resemble that of Holton (1976) with slight
modification such that the finite difference approximation to the first derivative is zero in the
neighbourhood of the pole.

The evolution of zonal wind is illustrated in Fig. 7. The appearance of a detached region
of easterlies in low latitudes on Day 14 and their subsequent poleward encroachment follow
the evolution of the zonal wind described by Dunkerton et al. (1981). This is qualitatively
very different from both the evolutionary behaviour of the February 1979 simulation and
experiment (I). From Day 0 to the appearance of easterlies the high latitude zonal wind
did not decelerate, and at most levels accelerated (see also Fig. 7b); by Day 16 no warming
had occurred.

![Day 6](image)

![Day 16](image)

**EXPERIMENT II**

Figure 8. As Fig. 6 but for Experiment (II) (a) Day 6; (b) Day 16; \( \alpha = 0.0096 \). (Note the different vertical scale from Fig. 6.)

The EP fluxes for experiment (II) are illustrated in Fig. 8 for Day 6 and Day 16. Both
diagrams show the fluxes directed upward and equatorward. Note that on Day 16 the region
of maximum EP flux convergence occurs in lower latitudes, approximately at the latitude
where the easterly circulation first appears. At no time up to Day 16 did the fluxes switch to
a poleward direction.

Experiment (II) effectively eliminates the model formulation (category a) as being a
cause of the different evolutionary behaviour of the mechanistic models. Our problem now is
to find out why the ‘wavenumber 2 only’ experiments (I) and (II) exhibited such widely
different responses to the upward propagating wavenumber 2. Why is it that the EP fluxes
are channelled into high latitudes in experiment (I) whilst in experiment (II) they are
directed equatorwards? There are a number of possibilities. Perhaps it is simply a result of
their different direction at the lower boundary (category c). In their study of the 1976–77
warming O’Neill and Taylor (1979) observed that at the 10 km level in the atmosphere
latitudinal phase gradients are often retrogressive during stratospheric warmings (see Fig.
1b), in contrast to the simple boundary condition Eq. (27). Alternatively, the different mean wind structure and in particular different initial winds (category b) might be responsible for actually focussing the waves into regions of convergence. This focussing might be modified by a forcing wave of non-zero phase speed (category c). From Fig. 1b we see that in the period prior to the warming wavenumber 2 at the lower boundary of experiment (I) was progressing by about $10^7$ d$^{-1}$; on the other hand stationary wave forcing was used throughout experiment (II). Further differences in the lower boundary condition (category c) arise from the shape of the transient envelope for wavenumber 2; there is almost linear growth of the 100 mb wavenumber 2 geopotential height before the February 1979 warming (see Fig. 1a) and hence at the lower boundary of experiment (I). On the other hand the idealized forcing used for experiment (II) had growth exponentially damped with time constant around 2-5 d. The possible importance of the different growth functions arises from the appearance of the EP flux in the conservation equation (14); a change in the wave transience $\partial A/\partial t$ will affect the flux divergence and hence the wave forcing of the mean flow (c.f. Eq. (6)).

From the above discussion it is clear that the differences between experiments (I) and (II) can arise in a variety of ways. The most practical way to find out which, if any, of these mechanisms is necessary for changing the evolutionary behaviour of a wavenumber 2 warming is to carry out a series of idealized model integrations. This we have done. Starting from the basic model of experiment (II) we tested various different initial and boundary conditions until we were able to obtain an idealized model integration qualitatively similar to the February 1979 warming. In reaching this goal we found that neither the form of the wave transience (at least whether growth should be linear or exponentially damped) nor the imposing of a negative phase gradient with latitude on the wavenumber 2 lower boundary forcing, made any significant difference to the evolution of the zonal mean wind. This second result has also been obtained by Hsu (see McIntyre (1982)) using Holton’s truncated semi-spectral model. The lower boundary phase gradients give a poleward EP flux though when the waves have propagated vertically by about a scale height, the EP flux returns to its normal equatorward direction, and the evolution of the zonal wind is very similar to that in experiment (II). Another experiment, identical to (II) apart from the addition of a $10^6$ d$^{-1}$ progressive phase speed to the forcing field, also exhibited similar behaviour to experiment (II). The easterlies first appear in mid-latitudes, further north of the position where they first appear in (II), but the high latitude circulation does not decelerate. As in experiment (II) there is subsequent poleward encroachment of easterlies. It is shown below that one essential to obtaining a high latitude reversal was a non-climatological zonally symmetric initial circulation (experiment (III)). However, this condition alone was not sufficient, for after an initial period of high latitude deceleration the EP flux switches to an equatorward direction and the integration evolves in a similar manner to experiment (II). In order to obtain a complete reversal of the high latitude circulation it was found necessary also to impose a progressive phase speed on the forcing wave (experiment (IV)). As we shall show this greatly enhanced wave propagation into the upper stratosphere, apparently for reasons well explained by linear planetary wave theory.

**Experiment (III)**

In this experiment the idealized forcing function Eq. (27) was used for the lower boundary, but instead of the climatological zonal wind (Fig. 7a), a zonally symmetric wind extracted from the atmospheric data for 16 February (Fig. 2a) was used to provide the initial conditions.

Figure 9a shows the zonal mean wind at Day 4 of the integration. The displaced polar night jet has started to decelerate at 75°N, whereas the low latitude wind has remained fairly constant. Figure 9b, for Day 10, shows the jet to have decelerated further, and high latitude easterlies are starting to descend from the mesosphere. However, this behaviour does not continue. By Day 18 (Fig. 9c) the mesospheric easterlies have receded, and a small region
EXPERIMENT III

Figure 9. As Fig. 7 but for Experiment (III) (a) Day 4, (b) Day 10, (c) Day 18.

EXPERIMENT III

Figure 10. As Fig. 8 but for Experiment (III) (a) Day 4, (b) Day 10, Day 18.
of easterly circulation has appeared in low latitudes. After Day 18 the evolution of zonal wind is qualitatively similar to that described in experiment (II), with low latitude easterlies spreading poleward. To re-emphasize, however, the evolution of zonal wind, $\bar{u}$, before Day 18 is quite different from that described by experiment (II), where deceleration of the high latitude zonal flow did not occur at all.

The EP fluxes for Days 4, 10 and 18 of this integration are illustrated in Fig. 10. Day 4 shows EP fluxes north of 60°N directed upward and poleward. This is quite unlike the behaviour of experiment (II), and similar to the observed fluxes in Fig. 4b. By Day 10 this poleward tilt has virtually disappeared, and the region of flux convergence is largely dissociated from the lower boundary. By Day 18 the EP fluxes are almost all directed upward and equatorward, and a significant region of flux convergence occurs in low latitudes. Figure 10c is quite similar to Fig. 8b of experiment (II).

Experiment (III) represents an important stage in producing a synthesis of the February 1979 warming by demonstrating the importance of the initial zonal wind profile. However, it is clear that another element is necessary to completely account for the success of the full simulation.

**Experiment (IV)**

The initial data for this experiment also corresponds to the zonally symmetric circulation given by the zonal wind of 16 February. The lower boundary forcing function is given by

$$\Phi(\lambda, \phi, t) = 430 \{1 - \exp(-t/t_0)\} g(\phi) \sin\{2(\lambda - \sigma t)\}$$

(28)

![Figure 11](image-url)

Figure 11. As Fig. 7 but for Experiment (IV) (a) Day 4, (b) Day 7, (c) Day 10, (d) Day 16. The broken contour shows the critical line for a wave with a $10^\circ$ d$^{-1}$ progressive phase speed.
in m, with $\sigma = 2 \times 10^{-6} \text{s}^{-1}$. This is identical to the forcing function of experiment (III), except that the forcing represents a progressive wave with phase speed equivalent to $10^7 \text{d}^{-1}$.

The evolution of the zonal mean wind is illustrated in Fig. 11. (Because of the non-zero phase speed of the forcing we have marked on the diagrams the position of the critical line for a wave with a $10^7 \text{d}^{-1}$ phase speed). On Day 4 (Fig. 11a) the zonal wind is similar to that shown for Day 4 of experiment (III) though for this experiment there exists a critical line near $45^\circ \text{N}$. By Day 7 (Fig. 11b) the high latitude jet has moved northward and strengthened while the middle latitude winds are reduced. Indeed the wind structure in the stratosphere strongly resembles Day 3 of experiment I (Fig. 5a). Three days later Fig. 11c for Day 10 shows easterlies in high latitudes after they have descended from the mesosphere. Low latitude winds show weak acceleration. At Day 16, (Fig. 11d) there is continued deceleration in the high latitudes, the easterly jet has intensified, and the westerly jet in polar latitudes has

![Figure 12. As Fig. 8 but for Experiment (IV) (a) Day 4, (b) Day 10, (c) Day 16. The broken contour in (c) shows the position of the critical line.](image-url)
noticeably decreased. Again low latitude winds remain relatively constant throughout this period.

Figure 12 illustrates the EP fluxes at Days 4, 10 and 16 of this integration. Day 4 shows fluxes propagating vertically and poleward in high latitudes. In this respect the fluxes are similar to Day 4 of experiment (III) (Fig. 10a), but their magnitude and convergence are larger. By Day 10 the flux has similar direction to that of Day 10 in experiment (III) but its convergence is considerably stronger in mid- and high latitudes (see Fig. 10b). By Day 16 a region of divergence appears near the critical line for the forcing wave.

The different evolutionary behaviour of experiments (II), (III) and (IV) is clearly summarized in Fig. 13, which shows the development of the zonal wind at 32 km averaged

$$Z = 32 \text{ km}$$

$$\int \frac{\zeta_1^2 \cos (\zeta_1) \cos (\zeta_2) d\zeta_1}{\int \zeta_1^2 \cos (\zeta_1) d\zeta_1}$$

Figure 13. Mean zonal wind (m s\(^{-1}\)) weighted with cosine of latitude between 60-90\(^\circ\)N and 25-45\(^\circ\)N for Experiments (II) to (IV) at z = 32 km (6.5 mb).

(with cosine of latitude weighting) over the two latitude bands 90-60\(^\circ\)N and 45-25\(^\circ\)N. It may be seen how in experiment (II) zonal deceleration is primarily concentrated in low latitudes, whereas in experiment (IV) zonal deceleration occurs mostly in high latitudes. Experiment (III) starts to evolve like experiment (IV), but then reverts to the behaviour found in experiment (II).

Hence we have been able to simulate by means of an idealized set of initial and boundary data, the essential features of the simulation of the February 1979 warming. We have reproduced the pronounced vertical propagation of wave activity without imposing latitudinal phase gradients at the lower boundary, and we have simulated the appearance of high latitude easterlies, without having first produced the poleward migrating zero wind line necessary in earlier studies (Dunkerton et al. 1981). The two features required were the non-climatological initial zonal wind profile, and the progressive wave at the lower boundary.
In the next section we attempt to give a dynamical explanation for the differing behaviour of these idealized experiments.

6. REFRACTIVE INDEX

In this section we examine the evolution of quasi-geostrophic refractive index in some of the idealized experiments described above. Strictly speaking, refractive index theory is not applicable to the conditions present at times during stratospheric warmings when waves are transient and may not satisfy the WKBJ requirement. Nevertheless, as our results show, the theory does give a very clear and simple qualitative picture of wave propagation preceding the simulated warmings. The results are presented in the form of latitude-height cross sections of $\tilde{Q}_2$ (see Eqs. (21) and (26)) upon which integral curves of the EP flux (curves which are everywhere parallel to $\tilde{F}$) are superimposed. We consider, in particular, the results taken from the integrations of experiments (II)–(IV). Although each of these experiments had identical geopotential forcing amplitudes, their evolutionary behaviour was quite distinct, as a result of differing wave propagation characteristics.

Figure 14 illustrates the refractive index patterns in two stages of each of the three experiments. The horizontal scale in Fig. 14 is linear in latitude and is therefore not identical to the transformed coordinate $\tilde{y}$ defined in Eq. (22). In mid-latitudes, however, the ratio $f/N$ is well approximated by the ratio of vertical and horizontal scales used in Fig. 14. Furthermore, since we shall only use Eq. (25) in a qualitative sense, we felt it unnecessary to present the results on a diagram scaled precisely as the transformed coordinate pair ($\tilde{y}, \tilde{Z}$). Common to all the diagrams is a band of negative $\tilde{Q}_2$ (arising from the term $-k^2/\cos^2\phi\sin^2\phi$) north of about 65°N. Although wave propagation in this region is not completely suppressed, as would be expected if Eq. (25) held exactly, it is severely restricted. In fact these restrictions are such that wavenumber 2, by itself, is not sufficient to reverse the circulation poleward of about 80°N, a result confirmed by a comparison of the results of experiment (I) with the simulation of Section 4 in which all components are present (c.f. Figs. 3 and 5).

The refractive indices for Days 4 and 12 of experiment (II) are shown in Figs. 14a and 14b. Day 4 (Fig. 14a) is typical of the early days of the integration—gradients of $\tilde{Q}_2$ being generally equatorward though fairly weak in middle latitudes. In accord with the discussion of Section 3 the curves of $\tilde{F}$ show the waves being refracted equatorward by this gradient. For example, the curve originating from the lower boundary at 57°N propagates up to 35 km before attaining a horizontal direction. After Day 8 the region of slow gradient disappears and by Day 12 (Fig. 14b) there is a uniformly strong horizontal gradient through the middle and low latitudes. This gradient arises from the fact that in these latitudes $\tilde{u}$ is increasing monotonically with latitude (whereas on Day 4 there was a mid-latitude maximum in $\tilde{u}$). The integral curves of $\tilde{F}$ now display a pronounced curvature into the low latitude region where the reversal eventually occurs. On Day 12 the integral curve from 57°N reaches a height of only 27 km.

The refractive index distribution at Day 4 of experiment (III) (Fig. 14c) is appreciably different from either of the previous two diagrams. In particular the high latitude maximum and mid-latitude minimum in $\tilde{u}$ (see Fig. 9a) gives rise to a region of negative $\tilde{Q}_2$ centred on 45°N and 20 mb, and a vertical ridge at 60°N extending into the upper stratosphere. This configuration has the effect of channelling the waves through to the upper stratosphere. The diagram shows that integral curves of $\tilde{F}$ in middle latitudes are directed vertically up to a height of 45 km (c.f. the maximum of 35 km in experiment (II) where the gradient in $\tilde{Q}_2$ refractions them equatorward). The curve that originates at 50°N, for example, is clearly affected by the poleward gradient of $\tilde{Q}_2$ in the middle stratosphere. By Day 12 (Fig. 14d), however, deceleration in the middle stratosphere has sufficiently changed the horizontal wind shears that the cross-section of $\tilde{Q}_2$ in the low to middle stratosphere has become quite similar to Day 12 of experiment (II) (Fig. 14b), apart from local differences associated with mesospheric easterlies. (The mesospheric zero wind line produces singular refractive index values and since this singularity did not appear to be relevant to the discussion, the region
Figure 14. Some integral curves of EP flux and contours of $\bar{Q}_s$ (see Eq. (26)). (a) Experiment (II) Day 4, (b) Experiment (II) Day 12, (c) Experiment (III) Day 4, (d) Experiment (III) Day 12, (e) Experiment (IV) Day 4, (f) Experiment (IV) Day 6. Contours of $\bar{Q}_s$, greater than 180 or less than zero are not plotted. Heavy contours denote singular regions and the blanked out area in (d) represents a region of rapidly fluctuating values.
around it has been blanked out of Fig. 14d). The integral curves of $\mathbf{F}$ now turn equatorward because of this refractive index gradient, and there is little wave propagation above 35 km. The curves are very similar to those shown in Fig. 14b for Day 12 of experiment (II) and indeed, as has been mentioned in Section 5, the dynamics of the two experiments evolve in a similar manner with eventual reversal in low latitudes.

Experiment (III) clearly demonstrates the ability of non-climatological initial winds to channel wave activity into the upper stratosphere. However, before wave flux convergence was sufficient to produce easterlies, the refractive index had evolved far enough to guide further wave flux equatorward. Experiment (IV) overcame this difficulty through larger EP flux convergence in the early stages of the integration (c.f. Figs. 10 and 12). In terms of the refractive index, the magnitude of $\mathbf{F}$ is given by Eq. (25) viz.

$$||\mathbf{F}|| = \frac{1}{k} |\psi_k|^2 (\tilde{Q}_k)^3 \rho_v/a\Omega^2$$

Since experiments (II)-(IV) had identical forcing amplitudes, $||\mathbf{F}||$ at the models lower boundary will vary as $(\tilde{Q}_k)^3$. Also, recalling that

$$\tilde{Q}_k = (a\tilde{q}_k)/(\tilde{u} - \sigma \cos \phi) - k^2/\cos^2 \phi - a^2 f^2/4N^2 H^2$$

then $\tilde{Q}_k$ varies inversely as $(\tilde{u} - \sigma \cos \phi)$. Hence, if $(\tilde{u} - \sigma \cos \phi)$ is small near the forcing level, then, with positive $\tilde{q}_k$, the magnitude of $\mathbf{F}$ at that level should be comparatively large.

The cross sections of $\tilde{Q}_2$ for Day 4 of experiment (IV) are shown in Fig. 14e. Once again there is a region of negative $\tilde{Q}_2$ centred on 45°N and 20 mb and a vertical ridge at 60°N. The progressive phase speed of $10^3$ d⁻¹ has led to the appearance of a critical line near 45°N. However, as in Fig. 14c, the poleward gradient in refractive index south of the high latitude ridge arises from the rapid decrease in $\tilde{q}_k$ away from the maximum associated with the westerly jet core. From Fig. 14e it can be seen that the values of $\tilde{Q}_2$ at 60°N in the lower stratosphere are roughly two to three times greater than corresponding values for experiment (III) (see Fig. 14c); a clear indication of the importance of the progressive phase speed in this region of weak winds. The EP fluxes shown in Figs. 10a and 12a confirm that the increased refractive index does indeed lead to increased wave propagation, and thereby promotes larger EP flux convergence. By Day 6 (Fig. 14f), the earlier configuration has evolved appreciably; nevertheless, there remains a strong vertical gradient of $\tilde{Q}_2$ in the path of the wave which maintains its upward direction until, on Day 9, an easterly circulation has been established in the high latitude stratosphere (see Fig. 11b). Further wave propagation in middle latitudes is then prevented by negative $\tilde{Q}_2$ (see Fig. 12c).

The results presented above demonstrate the importance of the feedback interaction between wave propagation and the zonal mean circulation. This feedback has been vividly illustrated by means of integral curves of the EP flux and the refractive index, the latter apparently highlighting the essential dynamical features contained in the zonal wind field.

7. Discussion and Conclusions

The results of the model studies described above clearly demonstrate that a numerical model can realistically describe many aspects of an actual stratospheric warming and, further, can be used to experimentally develop detailed mechanistic theories by isolating essential elements of the dynamics. Directly using observational data in a combination of simulation and idealized experimentation thus provides a means of bridging the gap between theory and observation.

Because of the central role of the numerical model in this process the limitations shown in the comparisons of Section 4 call for further comment before discussing our results from a dynamical point of view. Although the model described the onset of warming reasonably well some deficiencies were apparent both before and after the reversal of the flow. From
Figs. 3a and 3b it can be seen that the model does not maintain the westerly jet in the pre-warming period as well as was observed, and deviates markedly from the observations after the initial warming pulse. As discussed in Section 2 the weak circulation prior to the warming may be a consequence of the elementary radiation scheme used, though differences may also arise from the definition of the initial state, the boundary conditions or the numerical formulation of the model.

The definition of an initial state from the available temperature information required several approximations. Since long waves are of prime interest a simple geostrophic initialization was used. More importantly, perhaps, the initial data used in the model are derived ultimately from very little information, the 100mb geopotential and radiances of SSU channels 25 and 26 (near 15 and 5mb respectively); the lower stratosphere region which, as discussed below, is crucial to wave propagation is thus only coarsely resolved. The model's overestimation of EP fluxes at the lower boundary in the early stages of the simulation (Fig. 4a, b) could well be a consequence of this coarse resolution, though the elementary nature of the lower boundary condition is likely to be a more important source of error in the overall integration. By specifying only the 100mb geopotential we are probably providing insufficient information to constrain the model's evolution in the lowest levels to parallel that of the atmosphere, thus the interactions between the troposphere and stratosphere are not completely represented. The limitations caused by this lack of information are likely to become more apparent as the integration progresses. Once the circulation has reversed the EP flux might have a downward component due to wave decay or reflection from critical lines. In these circumstances resonance becomes a possibility, and indeed such behaviour has been observed in a similar forced model under certain conditions (McIntyre 1982).

Apart from the shortcomings of initial and boundary data the limited accuracy and resolution of the model itself must be considered. Kalnay-Rivas (1976) has demonstrated that, in 'box models' of this type, energy conservation is achieved at the expense of large truncation errors in pressure gradient terms near the pole. Deficiencies in the conservation of Ertel potential vorticity near the pole in some of our earlier experiments lead us to consider this and other numerical sources of error appreciable, and are to be subject to further research. Despite these shortcomings, however, the overall validation of the model was certainly sufficient to justify its application to a study of the dynamics of an atmospheric warming.

Let us now consider what the simulation and idealized experiments allow us to conclude about the dynamics of the wavenumber 2 stratospheric warming of February 1979. First of all, the results of experiment (I) demonstrate that non-linear wave-wave interactions did not affect the qualitative evolution of the warming; the high-latitude circulation reversal arises purely from the interaction of wavenumber 2 with the mean flow, except in a very small region near the pole.

Secondly, the diagnostic relation (Eq. (25)) between the magnitude of the EP flux and the quasi-geostrophic refractive index provides an understanding of the nature of this wave, mean-flow interaction in the pre-warming period. In deriving Eq. (25) we have specifically neglected non-WKB terms, the results justifying this approximation a posteriori. We have shown that the direction of EP fluxes within the model domain depends only very weakly upon the direction of fluxes at the lower boundary, and that P changes direction, or 'switches' in direct response to evolution in the in situ gradient of refractive index.

Since the quasi-geostrophic refractive index depends sensitively on both the zonal mean wind and its gradients, we can expect that the evolution of a stratospheric warming will depend strongly upon the zonal mean wind profile prior to the warming – indeed this was demonstrated in Section 5: with initial zonal winds in experiments (III) and (IV) differing by an angular speed of 10°d⁻¹, relative to the forcing wave, a qualitatively different zonal mean evolution obtains. This sensitivity also suggests that the refractive index may be more suitable as a model diagnostic than as a diagnostic of atmospheric behaviour with attendant errors in observation. Furthermore, it should be noted that the value of refractive index as a diagnostic of wave propagation has only been demonstrated.
here for a wavenumber 2 event. Some support for the usefulness of the refractive index theory in the study of a wavenumber 1 warming is given by O'Neill and Youngblut's (1982) application of ray tracing techniques to wave propagation in the 1976–77 winter.

The results of experiments (III) and (IV) give some indication of the part played by critical lines in the evolution of the warming. In experiment (IV) we noted that a critical line existed in the initial data at around 30°N (see Fig. 2a). However, it is clear from Figs. 14c, and e, that on Day 4 the integral curves of Φ have the same direction as those in experiment (III) (in which no critical line is present in the initial data). The implication is, therefore, that the upward propagating waves are refracted by the poleward gradient in q, north of the critical line; there does not appear to be any northward reflection of wave activity by the critical line in the early stages of the integration (c.f. Dunkerton et al. 1981). Of course it is possible that as the integration progressed the role of critical layer dynamics became important and may have aided both the persistence of focussing and the strength of convergence of wave activity into high latitudes. Certainly when the circulation was fully reversed the simulation was deficient (see e.g. Fig. 4d). This deficiency may be explained in terms of data or model errors, or in the uncertainties of accurately resolving the dynamics of a non-linear critical line.

The prime motivation of this study has been to understand the major stratospheric warming of February 1979. In using our particular type of model we have been able to do so without knowledge of earlier stratospheric behaviour or concurrent events in the troposphere. To obtain a complete picture of the warming requires an explanation of the causes of both the large amplitude wavenumber 2 disturbance in the troposphere and the ambient stratospheric circulation.

The last reported wavenumber 2 major warming prior to 1979 occurred in January–February 1963 (Finger and Tewes 1964). The tropospheric flow during this period was abnormally disturbed; Rowntree (1976) showed that the Atlantic surface pressure anomaly pattern could be associated with anomalous tropical Atlantic sea surface temperatures. Sea surface temperatures for the month of February 1979 show anomalies similar in size, though not in areal extent, to those reported by Rowntree. Further work may clarify the possible relationship between stratospheric warmings and such anomalies.

It remains to explain how the non-climatological zonal circulation arose in the pre-warming period. Previous studies (Palmer 1981b; Dunkerton et al. 1981) have suggested that the circulation has been preconditioned by earlier minor warmings and in future work we will describe a simulation of the wavenumber-1 minor warming of January 1979. The aim of these studies will be to provide mechanistic models which can describe the essential aspects of both the pre-conditioning phase and the eventual high latitude reversal of the wavenumber 2 warming of 1979.

Acknowledgments

The authors wish to thank Dr M. E. McIntyre for several useful discussions.

Appendix A
Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>longitude</td>
</tr>
<tr>
<td>φ</td>
<td>latitude</td>
</tr>
<tr>
<td>a</td>
<td>radius of the earth</td>
</tr>
<tr>
<td>y</td>
<td>aφ</td>
</tr>
<tr>
<td>ρ</td>
<td>pressure</td>
</tr>
<tr>
<td>(u, v, w)</td>
<td>components of velocity</td>
</tr>
<tr>
<td>Φ</td>
<td>geopotential height (m)</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>θ</td>
<td>potential temperature</td>
</tr>
<tr>
<td>H</td>
<td>scale height (6950 m)</td>
</tr>
</tbody>
</table>
\[ p_s = 1000 \text{ mb} \]
\[ \rho_s = 1 \text{ kg m}^{-3} \]
\[ \rho_0 \exp(-z/H) \]
\[ \Omega \]
\[ f \]
\[ c_p \]
\[ R \]
\[ \kappa \]
\[ g \]
\[ N \]
\[ t_0 \]
\[ \bar{X} \]
\[ X' \]
\[ \delta X \]

**APPENDIX B**

The finite difference equations of the model.

\[ \delta F_{ij}^{\prime} = -D(u)_{ij} + v_{ij}\{(u_{i,j,k}/a)\tan \phi_j + f_j\} - g(\Phi_{i+1,j,k} - \Phi_{i-1,j,k})/2a \Delta \lambda \cos \phi_j . \]  

\[ \delta F_{ijk}^{\prime} = -D(v)_{ijk} - u_{ijk}\{(u_{ijk}/a)\tan \phi_j + f_j\} - 
- (g/2a \Delta \phi \cos \phi_j)\{(\phi_{i,j-1,k} - \Phi_{ijk})\cos \phi_{j-1/2} - 
(\phi_{i+1,j,k} - \Phi_{ijk})\cos \phi_{j+1/2}\} . \]  

\[ \delta T^{\prime} = -D(T)_{ijk} - \frac{1}{2}(\kappa/H \rho_0 k)T_k(\rho_{0,1/2w_{i,j,k}} - \rho_{0,1/2w_{i,j,k}} + \rho_{0,1/2w_{i,j,k}+1/2}) + Q_{ijk}/c_p . \]  

\[ \rho_{o,k-1/2w_{i,j,k}+1} = \rho_{o,k+1/2w_{i,j,k}+1/2} + (\rho_{o,k} \Delta z/a) \Delta \lambda \cos \phi_j\{u_{i+1,j,k} - u_{i-1,j,k}\} + 
+ (\rho_{o,k} \Delta z/a) \Delta \phi \cos \phi_j\{(v_{i,j-1,k} - v_{i+1,j,k}) + v_{ijk}\} \cos \phi_{j-1/2} - 
(\phi_{i,j+1,k} - \phi_{ijk})\cos \phi_{j-1/2} \]  

where \( X_{ijk} = X \left( (i-1)\Delta \lambda, \frac{\pi}{2} - (j-\frac{1}{2})\Delta \phi, 8(16)\text{km} + k \Delta z \right) \) and the grid spacing (\( \Delta \lambda, \Delta \phi \), \( \Delta z \)) and time step \( \delta t \) are as described in section 2 and

\[ D(X)_{ijk} = (4a \Delta \lambda \cos \phi_j)^{-1}\{u_{i+1,j,k} + u_{ijk}\}X_{i+1,j,k} - (u_{i-1,j,k} + u_{ijk})X_{i-1,j,k}\} + 
+ (4a \Delta \phi \cos \phi_j)^{-1}\{v_{i,j-1,k} + v_{ijk}\} \cos \phi_{j-1/2}X_{i,j-1,k} - 
(v_{i+1,j,k} + v_{ijk}) \cos \phi_{j-1/2}X_{i,j+1,k}\} + 
+ (2\rho_{o,k} \Delta z)^{-1}\{\rho_{o,k+1/2w_{i,j,k}+1/2}X_{i,j,k+1} - \rho_{o,k+1/2w_{i,j,k}+1/2}X_{i,j,k-1}\} \]

**REFERENCES**

Andrews, D. G. and McIntyre, M. E. 1976  

Charney, J. G. and Drazin, P. G. 1961  

Dunkerton, T., Hsu, C.-P. F. and McIntyre, M. E. 1981  

Edmon, H. J., Hoskins, B. J. and McIntyre, M. E. 1980  


Rowntree, P. R. 1976 Response of the atmosphere to a tropical Atlantic ocean temperature anomaly, Quart. J. R. Met. Soc., 102, 607–625.

