Profile relationships in the superadiabatic surface layer

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SUMMARY

By analyses of profile data for the superadiabatic surface layer from the Johns Hopkins group (U.S.A.) in 1953 and from Swinbank and co-workers (Australia) in 1962–64, mean profile forms which give a close representation of the data are adopted.

For the profiles of $U$, $\theta$, and $r$ the gradients initially pass from $z^{-1}$ dependence through a smooth transition at $z/|L| \approx 0.03$ towards $z^{-4/3}$ dependence. The corresponding ‘Im4/3’ form adopted is broadly similar to the O’KEYPS, Businger-Dyer, and BWB (Kansas) $U$ profiles, but its change of gradient with increasing instability is a little more rapid than for these three. This form is maintained, with $K_H/K_M$ remaining constant, up to $z/|L| \approx 0.27$.

For larger $z/|L|$, $K_H/K_M$ increases, $|\partial \theta/\partial z|$ and $|\partial r/\partial z|$ falling relatively more rapidly and $\partial U/\partial z$ relatively less rapidly. Intermittent near-vanishing of $|\partial \theta/\partial z|$ is observed over intervals of a few minutes. Approaching $z/|L|$ values around 1 (the data limit) the profiles, while not tightly defined by the data, are suitably represented by a continuing $z^{-4/3}$ dependence with $K_H/K_M$ nearly twice its near-neutral value.

The analyses and results are independent of the von Kármán constants $k_U$, $k_\theta$, and $k_r$ (a possible dependence on the ratio $k_\theta/k_U$ in relation to the stability parameter being avoided by use of a modified form of Obukhov’s length). They do not provide values of these three constants and the adopted forms remain unaffected by any future assignment of these values.

1. INTRODUCTION

Formulation of mean profile relationships from high quality measurements in the atmospheric surface layer has been a subject of much investigation. A study in which the log-linear range and the stable regime were considered has been reported earlier (Webb 1970, here ‘LLS’); the present paper discusses the extension over the unstable (superadiabatic) regime based again on the same general principles of analysis, leading to the adoption of profile forms which give a close representation of the data.

Currently there are several alternative profile forms proposed for the unstable case, as reviewed by Monin and Yaglom (1971), Businger (1973), Dyer (1974), and Yaglom (1977). While the respective forms reflect a rough concordance which has been reached, the differences between them are nevertheless significant, as will be seen in the ensuing analyses.

The data examined here are from the same sources as in LLS, being from observations by the Johns Hopkins group at O’Neill, USA in 1953 and by Swinbank and co-workers in four series at Kerang and Hay, Australia during 1962–64. Though other data have since become available, notably from the comprehensive series of measurements by the AFCRCL group at the Kansas site in 1968 (Businger et al. 1971; Izumi 1971), these are not included in the scope of the present paper.

As in LLS, attention is confined to profile observations alone, no appeal being made to any measured fluxes or drag or to the drag coefficient of the surface. Treatment of the data is kept as simple and direct as possible; the quantities examined being simply the interheight differences $\Delta$ of mean wind speed $U$, potential temperature $\theta$, and humidity mixing ratio $r$, and various ratios of these differences; these quantities come directly and unequivocally from the experimental data. The mixing ratio $r$ is taken rather than specific humidity $q$ because the mean gradient of the former rather than of the latter is basically related to the water vapour transfer (Webb et al. 1980, Eqs. (27) and (32)), though this does not essentially affect the analyses in LLS and in this paper.

The limitation to profile data is not at all meant to detract from the great importance of flux measurements and analysis. But the independent profile analysis does afford some advantages.
One of these is that the analysis is made completely independent of the surface roughness and of the values of the von Kármán constants $k_u, k_\theta, k_r$ in the $U, \theta$, and $r$ profiles; the results therefore remain valid irrespective of uncertainties in any of these quantities. In particular they are not affected by the current question as to whether the value of $k_u$ is close to 0.41 or to 0.35 with the corresponding ratio $k_\theta/k_u$ close to 1.0 or to 1.35 (Businger et al. 1971; Wieringa 1980, 1982; Wyngaard et al. 1982).

Another advantage is that the profile forms obtained can then be subjected to independent test from mean gradient and flux measurements. This is pursued in a subsequent paper (in preparation), where the implied fluxes of heat and latent heat calculated from the measured mean gradients are compared with the independently measured sum of these fluxes from the heat budget.

In the present paper section 2 outlines some relevant background on profile forms and analysis, and section 3 gives details of the data. The analysis is described for $U$ in section 4 and for $\theta$ in section 5. Information from comparisons of $U$ and $\theta$ data is presented in section 6 and from the behaviour of inter-height difference ratios for $U$, $\theta$, and $r$ in section 7.

2. PRELIMINARY CONSIDERATIONS

(a) Perspective on profile forms

In the profile forms to be adopted in this paper, the gradients $\partial U/\partial z$ etc. behave as $z^{-1}$ at small $z/L$, and $z^{-4/3}$ at larger $z/L$ up to moderate values ($z$ is height and $L$ is an Obukhov length), with smooth merging between the two across an interface at height $z_m$, e.g. Eqs. (10a, b) for the wind profile. At an earlier stage of this work such a form was found to give a good representation of data up to moderate values of $z/L$ (as quoted without analysis details – Webb 1960, 1965); here it will be referred to as the ‘lm4/3’ form, signifying the two indices with smooth matching. Further analyses have continued to support this form and have provided a suitable extension to cover the range from moderate to large $z/L$. All of the relevant analyses are presented in this paper.

The wind profile behaviour represented by the lm4/3 form is broadly similar to that of the well known O’KEYPS and Businger-Dyer relationships and the modified form of the latter by Businger et al. (1971, here ‘BW1B’), but its change of gradient with increasing instability is a little more rapid than for these three. The O’KEYPS form also approaches $z^{-4/3}$ dependence at large $z/L$.

A $z^{-4/3}$ dependence was originally proposed for the limiting condition of free convection, independently by Prandtl (1932), Obukhov (1946), and Priestley (1954) – see also Monin and Obukhov (1954) and Priestley (1959). From experimental heat flux-gradient data Priestley (1955) found that a transition between $z^{-1}$ and $z^{-4/3}$ forms for $\partial U/\partial z$ occurs at a Richardson number around $-0.03$, and subsequent analyses of data for the $\theta$ profile itself (Webb 1958) and also for the $U$ profile (Taylor 1960) supported the same conclusion for both profiles. This observed onset of a $z^{-4/3}$ form in conditions far short of free convection, i.e. in a kind of composite convection with wind shear still playing an important part, has been discussed by Townsend (1962), Priestley (1962), Webb (1962, 1977), Betchov and Yaglom (1971), and Zilitinkevich (1973).

As a modification of Priestley’s (1955) result, a two-segment representation with the gradient having a transition from the Monin-Obukhov form $z^{-1}(1+az/L)$ to the $z^{-4/3}$ form was adopted by Kazanskii and Monin (1958), Taylor (1960), Pandolfo (1966), and Zilitinkevich and Chalikov (1968). This was further developed in Priestley’s (1960) application of the Monin-Obukhov power series $z^{-1}(1+z/L+az(L)^2+\ldots)$ to match the limiting form $z^{-4/3}$ at large $z/L$. The lm4/3 form (Webb 1960, 1965) introduced two-sided matching with the ‘mirror-image’ factors $(1-z^2/z_m)$ (equivalent to $1+az/L$) below the transition and $(1-z_m/z)$ above it (Eqs. (10a, b) of the present paper).

Obukhov (1946), Ellison (1957) and others introduced a single function with limiting $z^{-1}$ and $z^{-4/3}$ dependences: the ‘KEYPS’ or ‘O’KEYPS’ relationship (Panofsky 1963;
Businger and Yaglom (1971) – Eq. (15) of this paper. The Businger-Dyer (‘BD’) form was introduced (Businger 1966; Businger et al. 1967) as a related representation with $z/L$ appearing explicitly rather than the (flux) Richardson number as in the O’KEYPS function – Eqs. (16a, b). The ‘BWIB’ relationship (Businger et al. 1971) is similar to that of BD but it allows the parameter $\gamma$ (coefficient of $z/L$) to be different for the $U$ and $\theta$ profiles – Eqs. (17a, b).

(b) Analysis preliminaries

The usual profile shape functions $\phi_U, \phi_\theta, \phi_r$, which depend on the stability parameter and approach unity in the near-neutral limit, are defined by

\[ \frac{\partial U}{\partial z} = (u_*/k_U z) \phi_U \quad \ldots \quad (1a) \]
\[ \frac{\partial \theta}{\partial z} = -(\theta_*k_\theta z) \phi_\theta \quad \ldots \quad (1b) \]
\[ \frac{\partial r}{\partial z} = -(r_*k_r z) \phi_r \quad \ldots \quad (1c) \]

where $u_*$ is the friction velocity, and the temperature and humidity scales $\theta_*$ and $r_*$ are defined by $H = c_p \rho_w \theta_*$ and $E = \rho_v \rho_w r_*$, with $H$ and $E$ the vertical fluxes of heat and water vapour, $\rho$ and $\rho_v$ the density of the air and of the dry air constituent, and $c_p$ the specific heat of the air at constant pressure.

Obukhov’s length $L$ is

\[ L = -u_*^3(k_U gH/c_p \rho \theta)^{-1} = -u_*^3(k_U g \theta_*/\theta)^{-1} \quad \ldots \quad (2a) \]

where $g$ is the acceleration of gravity and $\theta$ is the absolute temperature. In the context of profile analysis it is useful to introduce a modified Obukhov length with a constant factor $k_\theta'/k_U$ applied; this is denoted here by $L$ and defined by

\[ L = -(k_\theta'/k_U)u_*^3(k_U gH/c_p \rho \theta)^{-1} = -(u_*k_U)^3(g \theta_*/k_\theta \theta)^{-1} \quad \ldots \quad (2b) \]

This modified $L$ retains its relationship with Richardson number $Ri$ in the near-neutral limit, i.e. $Ri \sim z/L$, irrespective of the values of $k_\theta'$ and $k_U$ (as does also a version of $L$ with a factor $K_M/K_M$, the ratio of diffusivities for heat and momentum, introduced by Panofsky et al. 1960).

In LLS and in the present paper the analyses are completely independent of the values of $k_U, k_\theta$, and $k_r$; a dependence on $k_\theta'/k_U$ in relation to the stability parameter is avoided by use of the modified Obukhov length $L$ throughout. Subsequently, when a value is assigned to the ratio $k_\theta'/k_U$ (equal to the near-neutral limit of $K_M$) then all the relationships involving $L$ can readily be expressed in terms of $L$ if desired.

The Richardson number $Ri$ is

\[ Ri = (g/\theta) \frac{\partial \theta}{\partial z} (\partial U/\partial z)^{-2} \quad \ldots \quad (3a) \]

At fairly strong instability $Ri$ becomes invalid as a parameter on account of the behaviour of $\partial \theta/\partial z$, as discussed at the end of section 8. Therefore in this paper we adopt a ‘robust’ form of $Ri$, denoted here by $Ri$, and defined at height $z$ in terms of measured differences between heights $a$ and $b$ say, according to

\[ Ri_z = z \ln(b/a) (g/\theta) (\theta_b - \theta_a) (U_b - U_a)^{-2} \quad \ldots \quad (3b) \]

in which we shall take $z = \sqrt{(ab)}$. This parameter is equal to $Ri_z$ in the near neutral limit and the two remain approximately equal up to moderate degrees of instability. At strong instability the ‘robust’ $Ri$ continues to be a suitable parameter.

A common practice for convenient data analysis has been to apply Eq. (3b) but to take the value obtained to represent approximately the conventional gradient $Ri$ of Eq. (3a). In this paper however $Ri$ is consistently recognized as the robust form defined by Eq. (3b).

For dealing with interheight differences rather than gradients let us introduce integrated
profile functions \( S_U, S_\theta, S_r \) and write the integrated forms of Eqs. (1a, b, c) as

\[
U_b - U_a = (u_b/k_U) S_U, \quad \theta_b - \theta_a = (\theta_b/k_\theta) S_\theta, \quad r_b - r_a = -(r_b/k_r) S_r. \tag{4a, 4b, 4c}
\]

where \( S_U = \int_a^b \phi_U dz/z = \int_{z=a}^b \phi_U d\ln z \) , \( S_\theta = \int_a^b \phi_\theta dz/z \), and \( S_r = \int_a^b \phi_r dz/z \). While \( S_U, S_\theta, S_r \) are convenient for practical calculations, we may also introduce shape functions \( s_U, s_\theta, s_r \) with \( S_U = s_U \ln(b/a) \) etc., so that we have the following integrated forms as more direct counterparts of Eqs. (1a, b, c):

\[
U_b - U_a = (u_b/k_U) \ln(b/a) s_U, \tag{5a}
\]

\[
\theta_b - \theta_a = -(\theta_b/k_\theta) \ln(b/a) s_\theta, \tag{5b}
\]

\[
r_b - r_a = -(r_b/k_r) \ln(b/a) s_r. \tag{5c}
\]

Here \( s_U = (\ln(b/a))^{-1} \int_z^b \phi_U d\ln z \), \( s_\theta = (\ln(b/a))^{-1} \int_z^b \phi_\theta d\ln z \), \( s_r = (\ln(b/a))^{-1} \int_z^b \phi_r d\ln z \), and we see that \( s_U, s_\theta, s_r \) are the averages of \( \phi_U, \phi_\theta, \phi_r \) with respect to \( \ln z \) over the height range concerned; in near-neutral conditions \( s_U, s_\theta, s_r \) approach unity, as do the \( \phi \)'s.

In terms of gradients we have from Eqs. (3a) and (1a, b) the relationship

\[
R_l = (\phi^2/\phi_U^2)(z/L), \tag{6a}
\]

while the counterpart of this in terms of interheight differences, from Eqs. (3b) and (5a, b) is

\[
R_l = (s_\theta/s_U^2)(z/L). \tag{6b}
\]

In each case \( L \) is the modified Obukhov length, Eq. (2b).

(c) Near-neutral relationships

In LLS it was concluded that in the unstable case for small values of \( z/L \) up to about 0.03 the profiles of \( U, \theta, \) and \( r \) all follow a similar log-linear form, expressed by

\[
\frac{\partial U}{\partial z} = (u_b/k_U z)(1 + \alpha z/L) \tag{7}
\]

\[
\frac{\partial \theta}{\partial z} = -(\theta_b/k_\theta z)(1 + \alpha z/L) \tag{8}
\]

\[
\frac{\partial r}{\partial z} = -(r_b/k_r z)(1 + \alpha z/L), \tag{9}
\]

with the Monin-Obukhov coefficient \( \alpha \) approximately 4.5 (within about 10\%\). The present paper considers the extension of these up to the strongest instability available in the data.

3. Data

The data examined (for fuller discussion see LLS) are from the Johns Hopkins (J.H.) group at O'Neill, USA in 1953 (Johns Hopkins University 1953; Lettau and Davidson 1957), and from four Australian expeditions, at Kerang site 1 in February 1962, at Kerang site 2 in October 1963 and September 1964, and at Hay in March 1964 (Swinbank 1964; Swinbank and Dyer 1968). In the Australian measurements there were two independent wind masts, except in February 1962. The J.H. runs were of one hour duration with heights 0.4 to 6.4 m, and the Australian runs 30 minutes with heights 1 to 16 m (0.5 to 16 m in February 1962). Some supplementary information on the data is given in Appendix 1, including details of five runs rejected on account of obvious anomalies or atypical behaviour.

To avoid unsteady conditions, only observations between 1000h and 1710h (all times local standard) are accepted, and occasions with amounts of low cloud greater than 4/8 are excluded. A few runs in extremely light winds (less than 2 m s\(^{-1}\)) at any measurement
height) are excluded, as are also a few runs in conditions close to neutral with very small and therefore relatively uncertain $\Delta \theta$ (|$\Delta \theta$| < 0·12°C, also $|RI|$ < 0·008 at the reference height 1·6 or 2·0 m in the case of $\theta$ or RI profile analyses).

Compensation is applied to the J.H. data for zero-plane displacement, taken to be 5 cm before the grass was mowed on 12 August and 2 cm after that date with increase to 3 cm on and after 25 August. These values have been determined as the most suitable from the near-neutral wind data by the procedure described in LLS, supported by the overall evidence in Table 1 hereof (section 4(a)) that for the whole of the unstable wind data after compensation no detectable zero-plane effect remains. For the Australian data as in LLS no zero-plane compensation is applied, the instrument heights having been measured from an assumed zero plane at about half the average grass height.

As stability parameter for the J.H. data the robust $RI_{1·6}$ is evaluated from the differences 0·8 to 3·2 m, and for the Australian data $RI_{1·2}$ from differences 1 to 4 m. To include the buoyancy effect of water vapour in Eq. (3b), $\theta_u - \theta_a = \Delta \theta$ is replaced by the virtual potential temperature difference $(\Delta \theta + 0·61 \; \theta \; \Delta q)$.

4. Wind profile

(a) Johns Hopkins data

The J.H. observations cover the range from small to moderate instability, and serve to indicate the initial extension of wind profile form as $z/|L|$ increases above 0·03.

Figure 1 shows the J.H. wind differences $\Delta U = U_b - U_a$ for all available pairs of adjacent heights ($b/a = 2$) plotted against the geometric mid-heights $z = \sqrt{(ab)}$, both being taken relative to their values at the height $(z)_{0·03}$ at which $-z/L = 0·03$ in each run. The reference height $(z)_{0·03}$ has been evaluated from $RI_{1·6}$ via the profile forms adopted, by

![Figure 1: Johns Hopkins 1953 data. Wind differences $\Delta U$ between adjacent heights (height ratio 2) plotted against geometric mid-height $z$, both taken relative to their values at the height where $-z/L = 0·03$. Reference value $(\Delta U)_{0·03}$ is found by linear interpolation with respect to In $z$ for each run. Number of values and standard error of each logarithmic average are shown.

--- Im4/3, Eqs. (10a, b). --- O'KEYPS, Eq. (15), $\gamma = 18$.

--- BD, Eqs. (16a,b), $\gamma = 18$. --- BW, Eqs. (17a,b).

Inversion of Eq. (6b) (calculation by successive approximation – the relationship is also plotted in Fig. 16); there is no risk of circularity in this step since the evaluation of $(z)_{0·03}$ is negligibly affected by the choice of profile form within the extremes allowed by the data. The reference value $(\Delta U)_{0·03}$ at height $(z)_{0·03}$ in each run has then been found objectively by linear interpolation with respect to In $z$ between the two $\Delta U$ values for the height intervals centred immediately above and below.

The heavy curve in Fig. 1 represents the Im4/3 relationship, given by

\[
\frac{\partial U}{\partial z} = \left( \frac{u_0}{k_v} \right) z^{-1} \left( 1 - \frac{z}{7z_m} \right) \quad \text{for } z \leq z_m \quad \text{(10a)}
\]

\[
\frac{\partial U}{\partial z} = \left( \frac{u_0}{k_v} \right) z_m^{1/3} z^{-4/3} \left( 1 - \frac{z_m}{7z} \right) \quad \text{for } z_m \leq z \leq z_1 \quad \text{(10b)}
\]
where $z_1$ is the uppermost height of validity of Eq. (10b) (discussed later). As well as $\partial U/\partial z$ being continuous at height $z_m$, the coefficient in the matching factor in each equation is set at the value $1/7$ which makes $\partial^2 U/\partial z^2$ continuous, and it then turns out that $\partial^3 U/\partial z^3$ is also continuous, giving a very smooth transition at height $z_m$. The one adjustable parameter $z_m/L$ is related to $z$ by

$$-z_m/L = (7z)^{-1}, \quad \text{(11a)}$$

which follows since Eq. (10a) must be identical with Eq. (7). Having adopted $z = 4.5$ as found in LLS, we then have from Eq. (11a)

$$z_m/L = 31.5^{-1} = 0.03175. \quad \text{(11b)}$$

The curve in Fig. 1 is plotted with this value from the integrated form of Eqs. (10a, b), which is given later in Eqs. (18a, b).

It seems obligatory to compare other profile forms. The O'KEYPS and BD profiles, Eqs. (15a) and (16a, b) with $\gamma = 18$ corresponding to $z = 4.5$, and the BWIB profile, Eqs. (17a, b) with its original parameter values, give (from their integrated forms) the curves shown in Fig. 1, which lie somewhat above the Im4/3 curve and the data points.

(The O'KEYPS and BD values become slightly higher still if $\gamma$ is taken at the common alternative value of 16.)

From the evidence of Fig. 1 we next take the Im4/3 form as a provisional basis for further analysis. It is suggested that the comparatively high ordinate value of the point at abseissa value just below 2 $(z/L)$ just below 0.06) in Fig. 1 arises fortuitously from experimental scatter (as mentioned below in relation to Fig. 2) – it is unlikely that such a sharp localized departure would be realistic and also no such bulge is evident in the Australian data.

The departures of the other points from the curve in Fig. 1 are small, ranging up to about 2%. We note also that any effect of the limited fetch over the mown O'Neill site would be to cause the points towards the right-hand side of Fig. 1 (which represent almost exclusively the uppermost height interval) to read slightly too high, by up to 1% or so as estimated in LLS.

A more comprehensive analysis using test evaluations of $u_w/k_U$ is shown in Fig. 2. First $L$ or $z_m$ is calculated from the measured $R_1$ for each run by inversion of Eq. (6b) using the adopted profile forms; again this step is not unduly sensitive to the detailed profile forms since the $|R_1|$ values are not large. Then, from the observed $\Delta U$ for each of the four adjacent-height intervals, $u_w/k_U$ is evaluated using the integrated form of the adopted wind profile (Eqs. 18a, b), and for each run the four values of $u_w/k_U$ are taken relative to their mean. After grouping the runs in ranges of $L$, these relative $u_w/k_U$ values are logarithmically averaged for each of the respective height intervals, giving four values as plotted against $z/L$ for each group in Fig. 2(a) ($z$ being the geometric mid-height of each interval and $L$ the logarithmic mean $L$ for the group).

Ideally the apparent $u_w/k_U$ should be constant with height (if we ignore the slight planetary boundary layer decrease) and the actual behaviour gives a sensitive indication of both the quality of the data and the validity of the adopted $U$ profile form. Data imperfections from site effects or instrument errors will cause variations with height $z$ and a corresponding pattern over the four points, while an incorrect profile form will show up as a systematic variation with $z/L$, i.e. related to the vertical scale.

The values plotted in Fig. 2(a) show departures ranging up to several per cent on either side of unity. The height pattern tends to persist within each group (this is evident as the largest deviations still have comparatively small standard errors) and to some extent to persist in the different groups through each day.

But no consistent variation with $z/L$ is apparent in Fig. 2(a) – generally the larger departures are counterbalanced by others in the opposite sense at about the same $z/L$ values.

However, it does appear at least plausible that a chance lack of counterbalance at
Figure 2. Johns Hopkins 1953 data. (a) Relative $u_z/k_u$ evaluated from $\Delta U$ for adjacent heights (height ratio 2) in individual runs by the integrated lm4/3 relationship Eqs. (18a, b), plotted against $z/L$ with $z$ the geometric midheight for each interval. Runs are grouped in ranges of $L$. Number $n$ of runs in each group and standard errors of logarithmic averages are shown. (b) Logarithmic means of the mean values from (a) with all days combined, after grouping in ranges of $z/L$. Standard error and number $n$ of values in each group are shown. (c) Same analysis applied to hypothetical data following other profile forms, O'KEYPS and BD with $\gamma = 18$, and BWIB.
$z/L \simeq 0.06$, including the unusually large departure of 11% seen in Fig. 2(a) for one run on 22 August, gives rise to the high-ordinate point in Fig. 1.

From a practical point of view it is evident that to make measurements of adequate quality requires very high standards of experimental accuracy and site uniformity. The J.H. 3.2–6.4 m $\Delta U$ values in the upper part of the $z/L$ range are typically 0.07 to 0.08 times $U$ itself, therefore an anomaly of say 2% in $\Delta U$ or $u_*/k_U$ can arise from errors of only 0.075% in $U_{3.2}$ and $U_{6.4}$. Again, the uncharacteristic anomaly of 11% in $\Delta U$ as seen on 22 August near $z/L = 0.06$ in Fig. 2(a) could arise from an anomalous wind shear of only about 1% within the height interval 1.6–3.2 m.

Figure 2(b) shows an analysis covering all the days, wherein the mean values plotted in Fig. 2(a) are themselves logarithmically averaged after grouping in ranges of $z/L$. (Averaging of means rather than of individual values avoids the weighting of observational anomalies when these persist through several runs.) The departures range up to about 2% and do not suggest significant deviation from unity.

To compare other profile forms (without repeating the whole analysis adapted to each form) we can calculate how the corresponding data would behave in Fig. 2 over some chosen span of $z/L$. If the data were to conform exactly to the O'KEYPS or BD relationship with $\gamma = 18$, or to the BWB relationship, then the same analysis as in Fig. 2(a) would give apparent relative $u_*/k_U$ values as plotted in Fig. 2(c). We see that the variation of these values with $z/L$ in each of the three cases is somewhat too large to be supported by the plotted data in Figs. (2a, b).

As a check on overall anomalies of $\Delta U$ associated with the different height intervals the logarithmic means of relative $u_*/k_U$ over all runs ($n = 43$) are given in Table 1. For the upper two intervals the values are very close to unity. The lower two intervals show opposite departures of 2%, which is compatible with the greater number of occurrences of this pattern than of the reverse as seen in Fig. 2(a). The absence of any systematically increasing deviation at decreasing heights supports the suitability of the allocated zero-plane displacement.

### Table 1. Overall Relative $u_*/k_U$ for Each Height Interval

<table>
<thead>
<tr>
<th>Height interval (m)</th>
<th>Log. mean of rel. $u_*/k_U$</th>
<th>log$_{10}$ LM</th>
<th>Standard error of log$_{10}$ LM</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2–6.4</td>
<td>0.994</td>
<td>−0.0027</td>
<td>0.0031</td>
</tr>
<tr>
<td>1.6–3.2</td>
<td>1.001</td>
<td>0.0005</td>
<td>0.0022</td>
</tr>
<tr>
<td>0.8–1.6</td>
<td>0.982</td>
<td>−0.0077</td>
<td>0.0024</td>
</tr>
<tr>
<td>0.4–0.8</td>
<td>1.020</td>
<td>+0.0086</td>
<td>0.0021</td>
</tr>
</tbody>
</table>

(b) Australian data

By the same type of relative $u_*/k_U$ analysis it is found that the Australian wind data are well fitted by the lm4/3 form for $z/L$ ranging up to about 0.27, beyond which the measured $\Delta U$ values become comparatively too large. By trial of alternative power law forms for the larger $z/L$ the following representation is found to be suitable:

$$
\frac{\partial U}{\partial z} = (u_*/k_U)Pz^{-p}(1 - z_m/7z) \quad \text{for} \quad z_1 \leq z \leq z_2 \quad \text{. (10c)}
$$

$$
\frac{\partial U}{\partial z} = (u_*/k_U)Sz^{-s}(1 - z_m/7z) \quad \text{for} \quad z \geq z_2 \quad \text{. (10d)}
$$

where $P = (z_m/z_1)^{1/3}z_1^{p-1}$ and $S = (z_m/z_1)^{1/3}(z_1/z_2)^{s-1}z_2^{s-1}$. Parameter values which give a suitable fit to the data are

$$
p = 1.1, \quad s = 4/3 \quad \text{. (12a)}
$$

with the transition heights given by

$$
z_1/L = 0.27, \quad z_2/L = 0.54 \quad \text{. (12b)}
$$
In this formulation the index of $z$ changes from $-4/3$ to $-1.1$ and then reverts to $-4/3$. \( \partial U/\partial z \) being continuous throughout.

Figures 3 to 6 present analyses of the Australian data, as discussed in turn below. Here $u_*/k_U$ is evaluated by using the complete extended – Im4/3 profile given by Eqs. (10a, b, c, d) and in integrated form by Eqs. (18a, b, c, d), with the parameter values specified by Eqs. (11b) and (12a, b). As a result of these analyses, this 'Im4/3E' form is adopted for the mean wind profile.

For certain ranges of wind direction at the different sites a systematic increase of the evaluated $u_*/k_U$ with height is found, and this is suggestive of increased surface roughness at some distance upwind. The runs are therefore classified in accordance with ranges of wind direction, as category A or B for sectors which are, respectively, typically free from or typically subject to this increase of $u_*/k_U$. It is emphasized that the adopted profile forms are based entirely on the category A data.

(i) Kerang site 1, February 1962. For February 1962 three days with winds from the easterly sector are rated category A. Figure 3(a) shows for the separate days the calculated relative $u_*/k_U$ values, which exhibit scatter but no systematic variation with $z/L$.

![Figure 3](image)

Figure 3. Kerang site 1, Feb. 1962 data. Relative $u_*/k_U$ evaluated from $\Delta U$ for adjacent heights (height ratio 2) in individual runs by the integrated extended – Im4/3 relationship Eqs. (18a, b, c, d), plotted against $z/L$ with $z$ the geometric midheight for each interval. Runs are grouped in ranges of $L$. Number $n$ of runs in each group and standard errors of logarithmic averages are shown. (a) Category A runs, (b) category B runs (see text). (c) Same analysis applied to hypothetical data following other profile forms, O'KEYPS and BD with $\gamma = 18$, and BWIB, over two spans of $z/L$ for comparison with Figs. 3 to 6.

The remaining day 20 February 1962 with winds from W and WSW, Fig. 3(b), is rated category B; for this day both sets of plotted points show a swing to smaller values of the calculated $u_*/k_U$ at the lowest heights. We are forced to regard this protrusion as anomalous – in the log-linear range the departure is far beyond accept-
able limits (to force the log-linear form to fit would require $\alpha \approx -6$ rather than the adopted $\alpha \approx +4.5$), and also the comparative behaviour of the two groups in Fig. 3(b) suggests a dependence on $z$ rather than on $z/L$.

No conclusive explanation for this anomaly is at hand. It may well be relevant that at Kerang site 1 the ground was rougher with taller grass beyond a fetch of some 300 m to the west of the wind measurement mast; but the effect of this would be expected to extend through the middle and upper parts of the profile rather than to be confined to heights below 2 m as in Fig. 3(b).

The behaviour of other profile forms with the same analysis is shown over two spans of $z/L$ in Fig. 3(c), for comparison with the data in Figs. 3 to 6. This will be considered after presentation of these Figures.

![Diagrams](image)

Figure 4. Hay, March 1964 data. Relative $u_{3}/k_{H}$ plotted against $z/L$ as in Fig. 3, but with all days combined and masts 1 and 2 treated separately.

(ii) Hay, March 1964. Figure 4(a) shows the analysis for Hay March 1964 category A, which includes all wind directions except those assigned to category B below. There are separate analyses for the two masts, with all relevant days included together. No consistent variation of the calculated $u_{3}/k_{H}$ with $z/L$ is evident in Fig. 4(a), especially when considered in conjunction with Fig. 3(a).

Category B includes winds from NW for mast 1 and from NW and also NNW
(one run) for mast 2 and for the temperature mast; these directions prevailed throughout the last three days 16-18 March 1964. The calculated $u_i/k_U$ values, Fig. 4(b), show a trend to larger values with increasing height (though there is also a persistent irregular pattern which happens to have reversed shape from one mast to the other). A re-analysis combining data from both masts is also shown in Fig. 4(b), and here the increasing trend of $u_i/k_U$ is more clearly evident.

Undoubtedly the cause of this trend is a patch of bushes about 40–45 cm high which were densely spread over an area centred north-west of mast 1, spanning the distance range 0.3–0.7 mi (0.48–1.13 km) from the mast with a total azimuthal span of about 30°. These were 'cotton bush' (*Kochia aphylla*) of the kind illustrated by Clarke *et al*. (1971, Fig. 4). The observed anomaly at mast 2 with the wind from NNW as well as NW is consistent with the airflow being over the bushes for both these directions -- mast 2 was located 200 m south-southwest from mast 1.

(iii) *Kerang site 2, October 1963*. The relative $u_i/k_U$ analyses for October 1963 are shown in Fig. 5(a) and (b).

![Figure 5: Kerang site 2, Oct. 1963 data. Relative $u_i/k_U$ plotted against $z/L$, as in Fig. 4.](image)

Assignment of directional sectors is common to the October 1963 and September 1964 data from the same site. Category A includes winds from the eastern sector NE to ESE, also from a western sector which narrows at measuring points further towards the west; SW to NW for wind mast 1, WSW to NW for the temperature and humidity masts ($\theta$, $r$, and $R_I$ analyses in Section 7), and no westerly directions for wind mast 2. (Mast 2 was 200 m west of mast 1, with the $\theta$ and $r$ masts midway between the two.) Category B includes the other directions -- northerly and southerly sectors and for mast 2 the westerly sector.
In category A (Fig. 5(a)), while there are fairly pronounced patterns of variation with \( z \) which are different for the two masts, there is no evidence of consistent variation with \( z/L \). In category B (Fig. 5(b)) the increase of \( u_*/k_U \) with \( z \) is generally evident.

The occurrence of categories A and B does in fact correspond broadly with the pattern of surface cover at Kerang site 2, where the instrument masts were located in a strip of uniform short grass running east-west with its southern edge about 600 m south of the masts – in most of the category B runs the wind was from the southern sector. Unfortunately no comprehensive information on the surface immediately south of the strip is available; however, two features observed at the time were a field of dry wheat stubble about 40 cm high situated with its near edge about 700 m SSW from mast 1, and a marshy area with tall reeds about 650 m SSE from mast 1. No information is available on surface characteristics to the west which might have affected mast 2 in the few runs with westerly winds.

(iv) Kerang site 2, September 1964. This location was the same as in October 1963, and categories A and B are again defined by the same wind directions as in (iii).

The analyses for September 1964 are shown in Figs. 6(a) and (b). Again no systematic variation of \( u_*/k_U \) with \( z/L \) is evident in category A. In category B the increase of \( u_*/k_U \) with \( z \) is comparatively mild, and in a few cases, mainly with winds from the north, is absent altogether as in the plot at lowest \( z/L \) for mast 1 in Fig. 6(b).

(v) Comparison with other profile forms. In Fig. 3(c) there are two features which might be distinguishable against the experimental scatter in Figs. 3 to 6, if the data did indeed follow the respective profile form: the rise of apparent \( u_*/k_U \) from \( z/L \) = 0.04 to 0.16, and (O'KEYPS only) the larger \( u_*/k_U \) for \( z/L \) between 0.15 and

Figure 6. Kerang site 2, Sept. 1964 data. Relative \( u_*/k_U \) plotted against \( z/L \), as in Fig. 4. For mast 2 all runs are in category B.
0.3 than for \( z/|L| \) between 0.6 and 1.2. In fact no systematic behaviour of either of these types is evident in the category A data of Figs. 3 to 6.

5. Potential temperature profile

The Johns Hopkins \( \theta \) profile data have appreciable random sampling scatter, due to the measurement of temperatures rather than of inter-height temperature differences (which have much smaller natural fluctuations) at the different heights in turn. Therefore the J.H. data are not applied directly here for \( \theta \) profile analysis.

But the J.H. data are adequate for an examination of the ratios \( \Delta U/\Delta \theta \), as presented in the next section, Fig. 9. There it is concluded that the profiles of \( \theta \) and \( U \) are similar in form for \( z/|L| \) ranging up to at least 0.1. We therefore carry over the \( 1m4/3 \) form, Eqs. (10a, b), as

\[
\frac{\partial \theta}{\partial z} = -\left(\frac{\theta}{k}\right) z^{-1} (1 - z/|z_m|) \quad \text{for } z \leq z_m. \quad (13a)
\]

\[
\frac{\partial \theta}{\partial z} = -\left(\frac{\theta}{k}\right) z_m^{1/3} z^{-4/3} (1 - z_m/|z|) \quad \text{for } z_m \leq z \leq z_1. \quad (13b)
\]

where \( z_m \) is still given by Eqs. (11a, b); the upper limiting height \( z_1 \) is discussed below.

In the Australian \( \theta \) profile data, illustrated in Fig. 7 by examples from February 1962,

\[\text{Figure 7. Kerang site 1, Feb. 1962 data. (a), (b), (c) Examples of potential temperature profiles averaged over subintervals to show the intermittent near-vanishing of } \left| \frac{\partial \theta}{\partial z} \right|. \text{ Full curves fitted to the data at } 1 \text{ and } 4 \text{ m are plotted from the integrated } 1m4/3 \text{ form Eq. (19a, b); extended to Eq. (19c) for the first profile in (b), shown by pecked line.}\]
it is evident that the \( \text{Im4/3} \) form provides a good fit, but that in the high \( z/L \) range there is an intermittent breakdown when \( \partial \theta / \partial z \) drops to small or near-zero values for time intervals of up to several minutes. In Fig. 7 the three runs are subdivided into intervals chosen to correspond with the intermittent behaviour. The curves fitted to the measurements at 1 and 4 m represent the \( \text{Im4/3} \) form (Eq. 13a, b) with \( L \) assigned the same value as for the whole run (in the absence of mean wind data and hence \( L \) for the individual subintervals). Apparently when the near-vanished condition of \( \partial \theta / \partial z \) is present it sets in sharply at heights somewhat below \( L \).

Evidently it should be suitable to represent \( \partial \theta / \partial z \) by the \( \text{Im4/3} \) form with a reduction factor in the upper region to allow for the mean intermittency. It is found by analysis of the type shown in Fig. 8 that the data do support such a form for a certain range of \( z/L \), beyond which a further reduction is found to be necessary; in Fig. 8 potential temperature difference ratios for February 1962 category A runs are plotted against \( z/L \), where \( z \) is in each case the lower height in the numerator of the ratio. Thus, the extension of Eqs. (13a, b) introducing reduction factors \( a_1 \) and \( a_2 \) is represented by

\[
\begin{align*}
\partial \theta / \partial z &= - (\theta_4 / k_0) a_1 z_m^{1/3} z^{-4/3} (1 - z_m / 7z) \quad \text{for} \quad z_1 < z \leq z_2. \\
\partial \theta / \partial z &= - (\theta_4 / k_0) a_2 z_m^{1/3} z^{-4/3} (1 - z_m / 7z) \quad \text{for} \quad z > z_2.
\end{align*}
\]

Figure 8. Kerang site 1, Feb. 1962 category A data. Potential temperature difference ratios for the different height intervals relative to 1–4 m, plotted against \(- z/L \) with \( z \) the lower of the numerator heights. Black and open symbols represent two separate groups of runs according to range of \( L \), with 11 and 10 runs, respectively. Standard error of each logarithmic mean is shown. Plotted straight lines show the values corresponding to \( \partial \theta / \partial z \propto z^{-1} \) and \( \propto z^{-4/3} \). The three sections for the three different ratios all have the same ordinate scale but with relative vertical shifts to align the values corresponding to the \( z^{-4/3} \) dependence.

--- Im4/3, Eqs (13a, b). --- Im4/3E, Eqs. (13a, b, c, d)

--- BD, Eqs. (16a, b), \( y = 18 \). --- BWIB, Eqs. (17a, b).
The parameter values adopted to give optimum agreement by eye with the data plotted in Fig. 8 are

\[ a_1 = 0.84, \quad a_2 = 0.64 \]  \hspace{1cm} (14a)

\[ z_{i1}/|L| = 0.27, \quad z_{i2}/|L| = 0.54 \]  \hspace{1cm} (14b)

The \text{l}m4/3E curves plotted in Fig. 8 represent Eqs. (13a, b, c, d), of which the integrated forms are given in Eqs. (19a, b, c, d), with the values given by Eqs. (14a, b). The need for a two-stage reduction in \( |\delta \theta/\delta z| \) is understandable since there is some variation in the height at which the intermittent drop in \( |\delta \theta/\delta z| \) occurs, as seen in Fig. 7.

Owing to the sharp changes of \( \partial \theta/\partial z \) at \( z_1 \) and \( z_2 \) the values of \( z_{i1}/|L| \) and \( z_{i2}/|L| \) are rather closely constrained by the fitting in Fig. 8. On the other hand the behaviour of \( \partial U/\partial z \) is far more gentle and the heights \( z_1 \) and \( z_2 \) for the \( U \) profile are open to wider choice; they have been chosen in Eq. (12b) to be the same as those for the \( \theta \) profile in Eq. (14b).

For comparison in Fig. 8 the BD \((\gamma = 18)\) and BWIB relationships are plotted from the integrated forms of Eqs. (16a, b) and (17a, b). At abscissa values up to about 0-1 the BWIB curve lies close to the \text{l}m4/3E curve and the data points; the BD curve lies somewhat lower (and alternatively with \( \gamma = 16 \) is slightly lifted, about one-fifth of the way towards the BWIB curve). At abscissa around 0-5 both BD and BWIB lie well above the data.

The Australian measurements other than from February 1962 are unfortunately not suitable for the above type of analysis, as the \( \theta \) differences were measured between only three heights, 1, 4, and 16 m.

6. \textbf{Comparison of }\( U \text{ and } \theta \text{ : behaviour of } K_H/K_M \)

\textbf{(a) Johns Hopkins data}

A direct comparison of the observed \( U \) and \( \theta \) profile forms can be made by examining the stability dependence of the ratio \( \Delta U/\Delta \theta \) from pairs of adjacent heights. Provided the heights are below \( z_1 \) of Eqs. (10c) and (13c), this ratio closely represents \( (\partial U/\partial z)/(\partial \theta/\partial z) \) which in turn, assuming the fluxes to be constant with height, is proportional to \( K_H/K_M \).

![Figure 9. Johns Hopkins 1953 data. Ratio of \( U \) and \( \theta \) differences for adjacent heights (height ratio 2), with \( z \) at geometric midheight for each interval, relative to reference value at the height where \( -z/L = 0.03 \). Reference value \((\Delta U/\Delta \theta)_{0.03}\) is found by linear interpolation with respect to \( \ln z \) for each run. All points are relative to the reference point (black square symbol); vertical scale shows relative values only and has no absolute significance. Number of values and standard error of logarithmic average for each group are shown. --- BD, Eqs. (16a, b), \( \gamma = 18 \); ---- BWIB, Eqs. (17a, b); both set at ordinate 1 in near-neutral limit.]

Such an analysis for the J.H. data is presented in Fig. 9; it is similar to the analysis in Fig. 9(a) of LLS with extension to the full available instability range. The procedure (similar to that used in Fig. 1) is first to estimate for each of the J.H. runs the 'reference height' \((z)_{0.03}\) at which \(-z/L = 0.03\), using the adopted profile forms with the measured
RI_{1.6} - Eq. (6b) and Fig. 16. The reference value \((\Delta U/\Delta \theta)_{0.0.3}\) at height \((z)_{0.0.3}\) in each run is then found objectively by linear interpolation with respect to \(ln z\) between the two \((\Delta U/\Delta \theta)\) values for the height intervals centred immediately above and below \((z\) being the geometric midheight for each interval). All available ratios \((\Delta U/\Delta \theta)/(\Delta U/\Delta \theta)_{0.0.3}\) are grouped in ranges of \(z/L\) and logarithmically averaged, the results being plotted in Fig. 9. (Two runs with excessive scatter of \(\Delta \theta\) resulting in a spread of \(\Delta U/\Delta \theta\) values greater than 3:1 over the different height intervals are excluded – 13 August 1100–1200 and 25 August 1300–1400.)

We conclude from Fig. 9 that it is unlikely there is any appreciable variation of \(\Delta U/\Delta \theta\), nor, therefore, of \(K_h/K_M\) – at least there is no suggestion of any systematic increase – as \(z/L\) increases up to at least 0.1. It is emphasized that the vertical scale has no absolute significance, the values plotted being all relative to the reference value at \((z)_{0.0.3}\) (black square symbol) which is arbitrarily placed at ordinate value 1; the true relationship is not required to pass through the reference point.

Curves for the same analysis applied to the BD (\(y = 18\)) and BWIB profiles, Eqs. (16a, b) and (17a, b), are also shown in Fig. 9. Allowing for vertical shifts to give the best fit, we see that the rising trend of the curves is too great (marginally so for BWIB) compared with the data.

The greater part of the experimental scatter in Fig. 9 is due to the poor temperature sampling. It is apparent that, given observations of comparably high quality but with good sampling, this type of analysis could provide a well-defined indication of the behaviour of \(K_h/K_M\).

(b) Australian data

The behaviour of \(\Delta U/\Delta \theta\) for the Australian February 1962 category A data is shown in Fig. 10. For each run the ratio \((\Delta U/\Delta \theta)/(\Delta U/\Delta \theta)_{1-4}\) is taken with denominator reference

\[
\begin{align*}
\Delta U/\Delta \theta \\
(\Delta U/\Delta \theta)_{1-4}
\end{align*}
\]

Figure 10. Kerang site 1, Feb. 1962 category A data. Ratios of \(U\) and \(\theta\) differences for different adjacent-height intervals relative to 1–4 m. \(z\) is geometric midheight for the numerator. Black and open symbols represent two separate groups of runs according to range of \(L\), with 11 and 10 runs, respectively. Values plotted are relative to respective reference point (square) which is set on the plotted curve representing adopted profile forms, plotted from Eqs. (18a, b, c, d) and (19a, b, c, d). ●, ○, 0.5 to 1 m; ●, ○, 4 to 8 m; ▲, Δ, 8 to 16 m; ■, □, 1 to 4 m, reference value.

value for the height interval 1–4 m and numerator for the three other height intervals. In \(z/L\) of the abscissa; \(z\) is the geometric mid-height for the numerator and \(L\) is evaluated via the adopted profile forms from the measured \(RI_2\) for each run. The curve segments representing the adopted \(Im4/3E\) forms are plotted from Eqs. (18a, b, c, d) and (19a, b, c, d).
In Fig. 10 the black and the open symbols for the two groups of runs are entirely independent of each other. For each group the values plotted are relative to the reference point (square) which is set on the curve representing the adopted profile forms. The vertical scale shows only relative values and has no absolute significance.

We see in Fig. 10 that (with one exception) the data points are consistent with the plotted curve, which is to be expected since the curve is based on the preceding analyses of the same data. The exceptional high ordinate of the point at \(-z/L = 0.064\) must be regarded as unrepresentative, being due to the apparently anomalous large \(U_1 - U_{0.5}\) for some of the runs on 18 February 1962, as seen in Fig. 3(a); it is not likely that \(K_H/K_M\) would decrease as \(-z/L\) increases from 0.064 to 0.18 as indicated by this point!

According to the adopted profile forms and as illustrated in Fig. 10, the behaviour of \(K_H/K_M\) is to maintain its near-neutral value as \(z/|L|\) ranges up to 0.27 and then to rise by a factor which finally reaches \(2^{4/5} - 11/10^4 - 0.64 = 1.84\) for \(z/|L| > 0.54\). (This numerical result comes from Eqs. (13c) and (10c) with their different indices \((-4/3)\) and \((-11/10)\) over a 2-fold height range \(z_1\) to \(z_2\) and Eq. (13d) having the numerical factor 0.64.)

7. INTER-HEIGHT DIFFERENCE RATIOS

Finally we examine the observed inter-height difference ratios plotted against Richardson number. This type of presentation gives a compact overview without the full detail provided by the foregoing analyses. For the wind profile it gives no further independent information, while for the \(\theta\) and \(r\) profiles it expands coverage to the whole of the Australian data.

\(\text{(a) Johns Hopkins data}\)

Figure 11 shows \((U_{0.4} - U_{1.6})/(U_{1.6} - U_{0.4})\) plotted against \(-RI_{1.6}\) for the J.H. data. The data points lie within a few per cent of the curve representing the \(\ln 4/3\) form.

![Figure 11. Johns Hopkins 1953 data. \((U_{0.4} - U_{1.6})/(U_{1.6} - U_{0.4})\) plotted against \(-RI_{1.6}\). Number of runs and standard error are shown for each logarithmic mean.](image-url)

- \(\ln 4/3\), Eqs. (10a, b), (13a, b).
- O'KEYPS, Eq. (15), \(y = 18\).
- BD, Eqs. (16a, b), \(y = 18\).
- BWIB, Eqs. (17a, b).
For comparison in Fig. 11 are shown curves for the O'KEYPS and BD forms, Eqs. (15) and (16a, b) with $\gamma = 18$ (with $\gamma = 16$ they would be raised slightly) and the BWIB form, Eqs. (17a, b). These curves lie somewhat too high compared with the data points.

(b) Australian data

Difference ratios for the Australian data are plotted in Figs. 12–15, where (a) and (b) show categories A and B, respectively. The height intervals are 4–16 m for the numerator and 1–4 m for the denominator.

For the wind profile, Fig. 12, as would be anticipated the observed ratios are in reasonable agreement with the adopted form (Im4/3E curve) for category A but are

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Figure 12. Kerang and Hay 1962–1964 data, $(U_{16} - U_4)/(U_4 - U_1)$ plotted against $-RI_2$. Number of runs and standard error are shown for each logarithmic mean.

- Im4/3, Eqs. (10a, b), (13a, b).
- O'KEYPS, Eq. (15), $\gamma = 18$.
- BD, Eqs. (16a, b), $\gamma = 18$.
- BWIB, Eqs. (17a, b).

(a) Category A runs, (b) Category B runs.
significantly larger for category B. In category B (Fig. 12(b)) there is a tendency for these larger ratios to converge towards the adopted form at increasing \(|RI|\) values, and this would be qualitatively consistent with enhanced vertical coupling to the more immediate surface in the stronger degrees of instability.

In Fig. 12(a) curves for the O'KEYPS and BD forms \((\gamma = 18)\) and the BWIB form are again shown for comparison. At \(-RI_2\) around 0.07 these lie too high compared with the data (marginally so for the O'KEYPS). For \(-RI_2\) between 0.1 and 0.2 the BD and BWIB values are somewhat too high, while between 0.2 and 0.4 the O'KEYPS values are too low, compared with the data.

Figure 13 shows the potential temperature difference ratios, the whole of the Australian data being compared with the Im4/3E form adopted solely by reference to the February 1962 category A data (section 5). For category A the observed ratios are

![Diagram](image-url)

Figure 13. Kerang and Hay 1962–1964 data \((\theta_{16} - \theta_4)/(\theta_4 - \theta_1)\) plotted against \(-RI_2\). Details as for Fig. 12.
in reasonable agreement with the adopted form (though the two October 1963 values are comparatively low and values at largest \( R_{12} \) are somewhat high). But for category B (Fig. 13(b)) they are significantly smaller, i.e. the disparity is in the opposite sense from that of the wind profile in Fig. 12(b). This is consistent with rougher surface upwind affecting the upper observation levels to give relatively larger \( \Delta U \), larger turbulent diffusivity, and hence smaller \(|\Delta \theta|\) for a given heat flux.

In Fig. 13(a) the curve for BD (Eqs. 16a, b; \( \gamma = 18 \)) lies a little above, and that for BW1B (Eqs 17a, b) more significantly above the data points for \( R_{12} > 0.1 \).

---

**Figure 14.** Kerang and Hay 1962–1964 data \((r_6-r_4)/(r_4-r_1)\) plotted against \(-R_{12}\). Black and open symbols – measurements by crystal hygrometer and by psychrometer, respectively. Other details as for Fig. 12.
Figure 14 shows the difference ratios for humidity mixing ratio \( r \). The experimental scatter is larger for the crystal-hygrometer \( r \) than for \( U \) and \( \theta \) (Figs. 12 and 13), and this is likely to be because \( r \) itself rather than the inter-height difference \( \Delta r \) was measured, with sampling at the different heights in turn. There is a tendency in the case of Kerang site 2 for the values to lie above the curve in Fig. 14(a) (where we again discount the two low values for October 1963) and to lie near the curve rather than below it in Fig. 14(b). These larger ratios for \( r \) than for \( \theta \) are plausibly explained if we suppose the surface to have been moister at some distance upwind. (This would similarly explain the difference of behaviour between \( \Delta U/\Delta \theta \) and \( \Delta U/\Delta q \) noted previously in Fig. 10(a) of LLS, where also category B runs were included which would give increased values of both \( \Delta U/\Delta \theta \) and \( \Delta U/\Delta q \).) Thus the data are arguably consistent with the generally accepted similarity of \( \theta \) and \( q \) (or \( r \)) profile forms (Crawford 1965; Dyer 1967; Denmead and McClroy 1970; Monin and Yaglom 1971, pp. 501–502). We therefore carry over Eqs. (13a, b, c, d) to represent the

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Figure 15. Kerang and Hay 1962–1964 data and (in (a) only) Johns Hopkins 1953 data. Ratio of robust Richardson numbers \( R1_{4z}/R1_z \) with \( z = 2 \) m for Kerang and Hay data and 0.8 m for J.H. data. This ratio with divisor 4 would remain at unity if \( R1 \propto z/L \). Other details as for Fig. 12.
profile of $r$ (or other constituents), with $\theta_0$, $\theta_s$, and $k_{s0}$ replaced by $r$, $r_s$, and $k_{s0}$, respectively.

In Fig. 15 the ratio of robust Richardson numbers, $R_{1_{0.8}}/4R_{I_2}$ (with $R_{1_{0.8}}$ and $R_{I_2}$ evaluated from differences 4–16 m and 1–4 m, respectively), is plotted against $-R_{I_2}$ for the Australian data, and also included in Fig. 15(a) is the comparable ratio $R_{1_{3.2}}/4R_{I_0.8}$ plotted against $-R_{I_{0.8}}$ for the J.H. data. These ratios including the divisor 4 would remain very near unity if $R_i$ were proportional to $z/L$; however, such proportionality is not indicated by the data. The plotted Im4/3E curve shows an upward departure towards the value $4^{1/3} = 1.587$ which would correspond to the $-4/3$ profiles (Eqs. (10b), (13b)), followed by further variations occasioned by Eqs. (10c, d) and (13c, d).

The observations plotted in Fig. 15 behave as expected from Figs. 12 and 13. For category A the data points are essentially concordant with the Im4/3E curve (though values for October 1963 tend to be comparatively low, as in Fig. 13(a)), while for category B the points follow the general form of the curve, but are displaced significantly below it.

In Fig. 15 (a) the BD ($y = 18$) curve lies very close to the unity axis, the BW1B curve lying a little above.

**Figure 16.** Robust $-RI_2$ for height ratios $b/a = 4$ (full line) and 2 (broken line) plotted against $-z/L$ with $z = \sqrt{(ab)}$. Curves represent the Im4/3E relationship, plotted from Eqs. (6b), (18a, b, c, d), and (19a, b, c, d) with $S_{0, \beta} = \frac{S_{0, \beta}}{b/a \ln(b/a)}$. Equality $RI = z/L$ is also shown for comparison (pecked line). Robust $RI$ is almost identical with actual $R_i$ for $z/L$ up to 0.14 if $b/a = 4$ or up to 0.2 if $b/a = 2$. 
8. Discussion

(a) Analysis methods

The methods employed here do not depend on flux measurements. They are therefore free from possible confusion in circumstances when \( u_* \) or \( \theta_* \) may have a different value in its two roles of scaling for a mean profile and for a flux. For example with the variability of wind strength which is typical in convective conditions the \( u_* \) which scales the mean wind gradient may well differ from the root-mean-square \( u_* \) obtained from measured mean shearing stress \( pu_*^2 \).

Again, if free convection were to develop during intervals of extremely light wind then the heat flux as measured would perhaps differ from that represented by the mean gradients on a conventional \( z/L \) or \( Ri \) similarity basis. In fact, evidence for some influence of free convection during runs in the lightest winds is found in the flux-gradient relationship for heat and latent heat fluxes, as described in the subsequent paper (in preparation).

No account is taken of a possible need for corrections for cup anemometer response in fluctuating winds (see reviews by Wieringa 1980 Appendix, and Wyngaard 1981). Thus the results are directly applicable to measurements made with Thornthwaite or Sheppard type sensitive cup anemometers, and would still be applicable to the actual wind if a correction approximately constant with height were found to be necessary. If a constant fractional correction to the wind strength were to be required then the results would be affected to a limited extent – the same fractional correction would be applicable to \( \Delta U \), which would affect the evaluation of \( Ri \) and hence \( L \), but otherwise the analyses would remain unaffected.

(b) Form of profiles

The O'KEYPS form is given by

\[
\phi_u = (1 - \gamma Rf)^{-1/4}
\]

(15)

where \( Rf \) is the flux Richardson number, \( Rf = (K u/K_M) Ri \), and \( \gamma = 4\pi \) in view of the near-neutral limit. If \( K u/K_M \) remains constant, as found to be the case for \( z/L \leq 0.27 \) in the present paper, then \( Rf \) can be replaced by \( Ri \) (the same value of \( \gamma \) being retained if \( K u/K_M = 1 \)) and \( \phi_\theta \) has the same form as \( \phi_u \). This is assumed to be the case in this paper, and the value of \( \gamma \), if taken as the coefficient of \( Ri \), not \( Rf \), is independent of the value of \( K u/K_M \).

The Businger-Dyer (BD) forms of \( U \) and \( \theta \) profiles are

\[
\phi_u = (1 - \gamma_U z/L)^{-1/4}
\]

\[
\phi_\theta = (1 - \gamma_\theta z/L)^{-1/2}
\]

(16a)

(16b)

The BWIB forms are

\[
\phi_u = (1 - \gamma_U z/L)^{-1/4}
\]

\[
\phi_\theta = (1 - \gamma_\theta z/L)^{-1/2}
\]

(17a)

(17b)

where \( L \) is introduced in place of the original BWIB \( L \). The \( \gamma_U \) and \( \gamma_\theta \) values given by BWIB as coefficients of \( z/L \) are 15 and 9, based on \( k_\theta/k_U = 0.74^{-1} = 1.35 \) which implies \( L = 0.74L \); we therefore take \( \gamma_U = 15/0.74 = 20.27 \), \( \gamma_\theta = 9/0.74 = 12.16 \), and these values are then independent of \( k_\theta \) and \( k_U \).

Integrated forms of Eq. (15) and Eqs. (16a, b) or (17a, b) have been given by Paulson (1970).

Some profile forms, e.g. BD and Pandolfo (1966), have assumed \( \text{Ri} = z/L \) and accordingly \( \phi_\theta = \phi_u^2 \). However, the equality \( \text{Ri} = z/L \) is not supported by the Kansas data, as pointed out by BWIB, nor by the present results, which show [Ri] increasing more rapidly than \( z/[L] \) in the Im4/3 range (Eqs. (3a), (10a, b), (13a, b), and Fig. 15). But we note that the trend of \( \text{Ri} \) or \( \text{RI} \) away from \( z/L \) is comparatively shallow (Figs. 15 and 16) and may often be difficult to identify from data in the presence of experimental scatter and possible confusion from fetch effects as in the category B runs (Fig. 15(b)); there is also the
downturn of $|\Omega|$ at moderately high values of $z/L$ due to the drop in $|\partial \theta/\partial z|$ (Eq. (13c, d)).

The Im4/3 profile, on account of its very smooth matching across the interface at $z_m$, is very similar in behaviour to the O'KEYPS, BD, and BWIB wind profile forms. However, as indicated in conjunction with Fig. 1, the Im4/3 departure from near-neutral form is a little more rapid than the O'KEYPS, which in turn is slightly more rapid than the BD and BWIB.

At increasing $z/L$ the $\theta_\text{profile}$ departure with intermittent near-vanishing of $\partial \theta/\partial z$ sets in at $z/L \approx 0.27$ (Eq. (13b)) when $L$ is evaluated over reasonably long runs, e.g. 30 min. On the other hand for short 5-min runs the vanishing of $\partial \theta/\partial z$ has been found to set in at around $z/L \approx 1$ (Webb 1958). This implies that during intervals when $\partial \theta/\partial z \approx 0$ the wind must be weaker than average giving a smaller magnitude of $L$.

On account of the drop in $|\Omega|$ associated with that in $|\partial \theta/\partial z|$ for $z/L \gtrsim 0.27$, the actual $\Omega$ does not remain suitable as a stability parameter at the larger negative $z/L$. But the robust $\Omega$ as defined in Eq. (3b) is suitable throughout (for sufficiently large height interval, say at least 2:1, and reasonably long runs, e.g. 30 min) since its relationship with $z/L$ according to the adopted profile forms remains monotonically increasing – see Fig. 16. In addition the evaluation of robust $\Omega$ from observations is independent of profile forms and is simpler and more direct than that of the gradient $\Omega$.

9. CONCLUSIONS

Profile forms adopted for the superadiabatic case are of the extended – Im4/3 ('Im4/3E') type given for the wind profile by Eqs. (10a, b, c, d) and for potential temperature and similarly water vapour or other constituents by Eqs. (13a, b, c, d). The analyses do not evaluate von Karman's constants $k_b$, $k_v$, or the ratio $k_b/k_v$ (near-neutral ($K_u/K_M$)). The adopted forms remain unaffected by the values of the $k$'s assigned, a dependence on $k_b/k_v$ being avoided by use of the modified Obukhov length $L$ defined by Eq. (2b).

The Im4/3E relationship is specified in four separate ranges of $z/L$ to secure a good representation of the data, though for practical applications this is less convenient than having a single overall function. On the other hand each of the four functions is simply the sum of two power laws and is easy to evaluate and to integrate in each range.

Integration of Eqs. (10a, b, c, d) and (13a, b, c, d) with the parameter values adopted in Eqs. (11b), (12a, b), and (14a, b) gives the following expressions. For two heights $a$ and $b$ we write $U_a - U_b = (u_b/k_v)S_{uv}$ etc. as in Eqs. (4a, b, c), and we let subscripts 1, 2, 3, 4 signify the four ranges. With $z_m = |L|/7\pi = |L|/31.5$ we write $A = a/z_m$ and $B = b/z_m$.

For the wind profile:

$$S_{u1} = \ln(B/A) - (A/7)(B/A - 1)$$
for $A, B \leq 1$ (18a)

$$S_{u2} = 3A^{-1/3}[\{1 - (A/B)^{1/3}\} - (28A)^{-1}\{1 - (A/B)^{4/3}\}]$$
for $1 \leq A, B \leq 8.505$ (18b)

$$S_{u3} = 10 \times 8.505^{-7/30}A^{-1/10}\{\{1 - (A/B)^{1/10}\} - (77A)^{-1}\{1 - (A/B)^{11/10}\}\}$$
for $8.505 \leq A, B \leq 17.01$ (18d)

$$S_{u4} = 3 \times 27^{1/30}A^{-1/3}\{\{1 - (A/B)^{1/3}\} - (28A)^{-1}\{1 - (A/B)^{4/3}\}\}$$
for $A, B \geq 17.01$ (18d)

For the potential temperature profile:

$$S_{\theta 1} = \ln(B/A) - (A/7)(B/A - 1)$$
for $A, B \leq 1$ (19a)

$$S_{\theta 2} = 3A^{-1/3}[\{1 - (A/B)^{1/3}\} - (28A)^{-1}\{1 - (A/B)^{4/3}\}]$$
for $1 \leq A, B \leq 8.505$ (19b)

$$S_{\theta 3} = 3 \times 0.84A^{-1/3}\{\{1 - (A/B)^{1/3}\} - (28A)^{-1}\{1 - (A/B)^{4/3}\}\}$$
for $8.505 \leq A, B \leq 17.01$ (19c)

$$S_{\theta 4} = 3 \times 0.64A^{-1/3}\{\{1 - (A/B)^{1/3}\} - (28A)^{-1}\{1 - (A/B)^{4/3}\}\}$$
for $A, B \geq 17.01$. (19d)

For water vapour, etc.: $S_v$ is the same as $S_\theta$.

The total $S_{uv}$, $S_{u\theta}$, or $S_v$ is of course the sum of the contributions from the different ranges spanned by the total height interval.

In Fig. 16 the 'robust' Richardson number $\Omega_{\text{r}}$ defined in Eq. (3b) is plotted against
z/L according to the Im4/3E forms, by use of Eqs. (6b), (18a, b, c, d), and (19a, b, c, d). For practical evaluation of fluxes from profile measurements, first \( L \) can be determined from the measured RI via Fig. 16, and then \( u_0/k_0, \theta_0/k_0, \) and \( r_0/k_0 \) (and thus the vertical fluxes) can be evaluated from Eqs. (18a, b, c, d) and (19a, b, c, d).

**ACKNOWLEDGMENTS**

It is again a pleasure to acknowledge the high quality of the experimental data on which this investigation depends, and I am grateful to the late Dr W. C. Swinbank and to Dr A. J. Dyer and Mr B. B. Hicks for making available the detailed field records of the Australian observations. Computer processing for this work was programmed by Mr G. F. Rutter, to whom I am particularly grateful for this substantial contribution. I thank Dr R. J. Francey and Dr K. T. Spillane for their constructive comments on a draft of the paper.

**APPENDIX 1**

**SUPPLEMENTARY INFORMATION ON THE DATA**

(a) **Additional Australian data**

The Australian data examined in this paper include two runs which do not appear in the publications of Swinbank (1964) and Swinbank and Dyer (1968), but for which the profile observations are complete and acceptable (W. C. Swinbank, private communication). The data, kindly provided by the late Dr Swinbank, some years ago, are reproduced for reference in Table A1.

<table>
<thead>
<tr>
<th>Table A1. Two additional runs at Kerang (site 1), Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data of W. C. Swinbank and co-workers: 19 February 1962</td>
</tr>
<tr>
<td>Time of start (30-min runs)</td>
</tr>
<tr>
<td>1130</td>
</tr>
<tr>
<td>1201</td>
</tr>
<tr>
<td>Wind direction Cloud</td>
</tr>
<tr>
<td>Wind speed (cm s(^{-1}))</td>
</tr>
<tr>
<td>( U_{14} )</td>
</tr>
<tr>
<td>( U_0 )</td>
</tr>
<tr>
<td>( U_0 )</td>
</tr>
<tr>
<td>( U_0 )</td>
</tr>
<tr>
<td>( U_{14} )</td>
</tr>
<tr>
<td>Temperature (°C)</td>
</tr>
<tr>
<td>( \theta_{1} )</td>
</tr>
<tr>
<td>( \theta_{14} - \theta_{1} )</td>
</tr>
<tr>
<td>( \theta_{8} - \theta_{1} )</td>
</tr>
<tr>
<td>( \theta_{8} - \theta_{14} )</td>
</tr>
<tr>
<td>( \theta_{8} - \theta_{1} )</td>
</tr>
<tr>
<td>Pot. temp. diff. (°C)</td>
</tr>
<tr>
<td>( \theta_{8} - \theta_{14} )</td>
</tr>
<tr>
<td>( \theta_{8} - \theta_{1} )</td>
</tr>
<tr>
<td>( \theta_{8} - \theta_{14} )</td>
</tr>
<tr>
<td>NE TrCI, 1/8 Cu</td>
</tr>
<tr>
<td>658</td>
</tr>
<tr>
<td>625</td>
</tr>
<tr>
<td>597</td>
</tr>
<tr>
<td>556</td>
</tr>
<tr>
<td>503</td>
</tr>
<tr>
<td>442</td>
</tr>
<tr>
<td>28.7</td>
</tr>
<tr>
<td>29.2</td>
</tr>
<tr>
<td>0.24</td>
</tr>
<tr>
<td>0.42</td>
</tr>
<tr>
<td>1.27</td>
</tr>
<tr>
<td>0.92</td>
</tr>
<tr>
<td>600</td>
</tr>
<tr>
<td>570</td>
</tr>
<tr>
<td>544</td>
</tr>
<tr>
<td>511</td>
</tr>
<tr>
<td>464</td>
</tr>
<tr>
<td>412</td>
</tr>
<tr>
<td>TrCl, TrAs</td>
</tr>
</tbody>
</table>

(b) **Corrections**

A few items in the Australian data found to require correction are listed in Table A2.

<table>
<thead>
<tr>
<th>Location</th>
<th>Date</th>
<th>Run</th>
<th>Time</th>
<th>Item</th>
<th>Corrected Value</th>
<th>Source of original error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ker. site 1</td>
<td>19.2.62</td>
<td>23</td>
<td>1544</td>
<td>( \theta_{a} - \theta_{14} )</td>
<td>0.08</td>
<td>Conversion from field records</td>
</tr>
<tr>
<td>20.2.62</td>
<td>32</td>
<td>1425</td>
<td></td>
<td>( \theta_{a} - \theta_{14} )</td>
<td>0.30</td>
<td>( \theta_{a} - \theta_{14} ) had been quoted (also error in run 32)</td>
</tr>
<tr>
<td>33</td>
<td>1457</td>
<td></td>
<td></td>
<td>( \theta_{a} - \theta_{14} )</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>1530</td>
<td></td>
<td></td>
<td>( \theta_{a} - \theta_{14} )</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>1537</td>
<td></td>
<td></td>
<td>( \theta_{a} - \theta_{14} )</td>
<td>1.13</td>
<td>Adiabatic correction had been omitted</td>
</tr>
<tr>
<td>36</td>
<td>1537</td>
<td></td>
<td></td>
<td>( \theta_{a} - \theta_{14} )</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>Hay</td>
<td>18.3.64</td>
<td>51</td>
<td>1502</td>
<td>( \theta_{a} - \theta_{14} )</td>
<td>1.13</td>
<td>Adiabatic correction had been omitted</td>
</tr>
<tr>
<td>52</td>
<td>1537</td>
<td></td>
<td></td>
<td>( \theta_{a} - \theta_{14} )</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>Ker. site 2</td>
<td>16.9.64</td>
<td>32</td>
<td>1123</td>
<td>( q_{a} - q_{14} )</td>
<td>Delete</td>
<td>Hygrometer values not acceptable due to inadequate sampling</td>
</tr>
<tr>
<td>33</td>
<td>1457</td>
<td></td>
<td></td>
<td>( q_{a} - q_{14} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(c) Additional humidity measurements

In the Australian observations of October 1963 the primary measurement of humidity difference was by crystal hygrometer; psychrometer data were also obtained but not published. For runs with no hygrometer values the available psychrometer data, which have been used here in the evaluation of Richardson numbers, are reproduced in Table A3.

<table>
<thead>
<tr>
<th>Date</th>
<th>Run</th>
<th>Time</th>
<th>$q_e - q_s$ (g kg$^{-1}$)</th>
<th>Date</th>
<th>Run</th>
<th>Time</th>
<th>$q_e - q_s$ (g kg$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.10.63</td>
<td>1</td>
<td>1156</td>
<td>0.320</td>
<td>12.10.63</td>
<td>7</td>
<td>1148</td>
<td>0.256</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1232</td>
<td>0.355</td>
<td></td>
<td>19.10.63</td>
<td>38</td>
<td>0.457</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1329</td>
<td>0.346</td>
<td></td>
<td>1544</td>
<td>39</td>
<td>0.314</td>
</tr>
<tr>
<td>11.10.63</td>
<td>5</td>
<td>1425</td>
<td>0.450</td>
<td>22.10.63</td>
<td>50</td>
<td>1226</td>
<td>0.317</td>
</tr>
</tbody>
</table>

(d) Specification of wind direction in Australian data

In some cases the observed wind directions are simplified in the published Australian data, and generally this has no practical effect. However, for Hay 18 March 1964, runs 47, 48, 49 at (local standard) times 1137, 1210, 1256, the field records give the wind direction for each run as NNW--NW and this is simplified in the publication to NNW. In view of the special significance of the direction NW at this site (section 4) these runs being affected by NW winds for part of the time are counted in the same category as runs with solely NW wind. Again, for Kerang September 1964 run 3 at 1533 the direction SW to SxW in the field records is simplified to SW in the data publication. This run is counted as category B for all masts.

(e) Excluded data

In the J.H. data runs with an ‘interpolated value’ of wind speed are excluded from the analyses.

Three runs are excluded on account of evident profile anomalies. For J.H. 7 September 1953, run 1500–1600, $U_{0.4}$ appears too large, giving far too small a value of $u_\infty/k$ derived from $U_{0.8} - U_{0.4}$; the possibility of an instrument fault is suggested by the listing of $U_{0.4}$ as an ‘interpolated value’ in the immediately preceding run. For Kerang site 1, 18 February 1962 run 18 at 1617, and again for Kerang site 2, 21 October 1963 run 40 at 1025, $U_{16}$ appears too small, giving far too small a value of $u_\infty/k$ derived from $U_{16} - U_8$.

In addition two Australian runs are excluded as being atypical, possibly in association with a temporary change of wind direction; each is the first run of a series and has different direction category from the subsequent runs. For Kerang site 2, 10 October 1963 run 1 at 1156, the wind direction is ENE–NE from the field records, marginally category A, while for the subsequent three runs the direction is NNE, marginally category B; in fact the $U$ and $\theta$ profiles for this run are markedly abnormal. For Hay, 11 March 1964 run 1 at 1212, the field records give the wind direction as ‘NW, variable at beginning’, while for the three subsequent runs the direction is W. Thus run 1 is an isolated category B case, but in fact its $U$ and $\theta$ profile behaviour is found to be typical of category A.

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