Notes and Correspondence

Note on the paper 'Radiation conditions for the lateral boundaries of limited-area numerical models' by M. J. Miller and A. J. Thorpe (Q. J., 107, 615–628)

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Miller and Thorpe (1981) have discussed and illustrated the use of radiation conditions for the lateral boundaries of limited-area numerical models. They draw a sharp distinction between nested models, where time-dependent boundary conditions can be taken from a larger scale model, and models that are used in circumstances where the only requirement is to allow disturbances to propagate smoothly out through the boundaries. In fact, of course, the situation is far less clear-cut than they imply. Even for idealized or theoretical studies one may wish to 'impose' some time dependency in the 'large scale' environment, and nested models are prone to boundary reflection however well matched the models involved are. Fortunately it may well be relatively easy to modify the boundary conditions described by Miller and Thorpe so that changes in the large scale environment can be accommodated. If the modification described below is practicable, Miller and Thorpe's work could be very relevant to the more complex nested forecasting models that they appear to disown!

In order to see how to generalize the Sommerfeld radiation condition

$$\frac{\partial \phi}{\partial t} = -\hat{e} \frac{\partial \phi}{\partial x},$$

(1)

where $\hat{e}$ is a velocity that includes wave propagation and advection, we must return to its original derivation. This was in the context of the solution of the equations for the perturbation of a field $\phi$ by decomposition into Fourier modes (Sommerfeld 1964). The modes of $\phi$ can be separated into those moving* outwards, or away from the source of the perturbation, and those moving inwards. The radiation boundary condition corresponds to stating that there is no physical cause for inward moving modes, and therefore they vanish. If all the outward moving modes have the same phase velocity, this assumption gives Eq. (1), where $\hat{e}$ is phase velocity. In general, Eq. (1) is an approximation obtained either by supposing that all the important outward moving modes have about the same phase velocity ($\hat{e}$) or by supposing that there is a group of outward moving modes, which forms a well defined and localized entity in physical space, with group velocity $\hat{e}$. The versions of Eq. (1) used in limited area atmospheric modelling involve further approximations due to the numerical (finite difference) implementation of Eq. (1) and the need to calculate $\hat{e}$ when there is no single theoretically correct value.

The assumption that there is no physical cause for the inward moving modes can be modified without affecting the other approximations and assumptions involved in using a radiation boundary condition. In general, we can write a field $\phi$ in terms of its outward moving part $\phi_+$ and its inward moving part $\phi_-$.

$$\phi = \phi_+ + \phi_-$$

(2)

$\phi_+$ satisfies the same equation as before, i.e.

$$\frac{\partial \phi_+}{\partial t} = -\hat{e} \frac{\partial \phi_+}{\partial x},$$

(3)

but $\phi_-$ must be given by the large scale environment. We can postulate two solutions to the governing equations, viz $\phi^m$, which is the solution that would exist in the absence of the local perturbation and describes the large scale environment, and $\phi^m$, which is the correct solution and is, in practice, only known in the interior of the model domain. In these terms

*More accurately, carrying energy or information outwards. It is possible for individual Fourier modes to have phase velocity in the opposite direction, e.g. gravity waves.
\[ \phi_m^n = \phi^s \]  
\[ \frac{\partial \phi_m}{\partial t} = - \dot{c} \frac{\partial \phi_m}{\partial x} \]  
and 
\[ \frac{\partial \phi^s}{\partial t} = - \dot{c} \frac{\partial \phi^s}{\partial x} \]  

(Eq. (6) hides a further assumption that it is reasonable to use the same velocity \( \dot{c} \) for \( \phi^s \)). From (4),
\[ \phi_m^n - \phi^s = \phi^s - \phi^s, \]
and thus, using (5) and (6)
\[ \frac{\partial \phi_m^n}{\partial t} - \frac{\partial \phi^s}{\partial t} = - \dot{c} \left( \frac{\partial \phi_m^n}{\partial x} - \frac{\partial \phi^s}{\partial x} \right), \]
or, dropping the superscript \( m \),
\[ \frac{\partial \phi}{\partial t} = \frac{\partial \phi^s}{\partial t} - \dot{c} \left( \frac{\partial \phi}{\partial x} - \frac{\partial \phi^s}{\partial x} \right). \]

Equation (9) is the proposed generalization of the radiation boundary condition. It amounts to no more than a proposal to use Eq. (1) for the local perturbations to the large scale flow, but the above argument suggests that there is no other comparably simple and correct generalization. This suggestion that Eq. (9) is in some sense correct and unique can be supported by considering a simple system in which it is exact.  
We consider Eqs. (10) and (11) (which one may think of as the shallow water equations in one dimension when \( c = \sqrt{gh} \), \( V \) is a basic advecting velocity \( (V \gg u) \), \( u \) is the small perturbation to the velocity and \( \phi \) is the normalized height of the free surface). This system is admittedly simple, but it describes each of the vertical modes of the two-dimensional system considered by Miller and Thorpe, so it is certainly relevant.
\[ \frac{\partial u}{\partial t} = - c \frac{\partial \phi}{\partial x} - V \frac{\partial u}{\partial x}, \]
\[ \frac{\partial \phi}{\partial t} = - c \frac{\partial u}{\partial x} - V \frac{\partial \phi}{\partial x}. \]

For this system the outward moving mode is given by
\[ \phi_+ = \frac{1}{2} (\phi + u), \]
and the inward moving mode by
\[ \phi_- = \frac{1}{2} (\phi - u). \]

It is easy to show that
\[ \frac{\partial \phi_+}{\partial t} = - (V + c) \frac{\partial \phi_+}{\partial x}, \]
\[ \frac{\partial \phi_-}{\partial t} = - (V - c) \frac{\partial \phi_-}{\partial x}, \]
and the adjectives inward and outward are only justified if \( c > V \). \( \phi_- \) must be given by the large scale environment, denoted by the superscript \( s \),
\[ \phi_- = \frac{1}{2} (\phi^s - u^s). \]
but the outward going part $\phi^*$ is self-determined by the local flow according to Eq. (12). Now we can manipulate these equations
\[
\frac{\partial \phi}{\partial t} = \frac{\partial \phi^*}{\partial t} + \frac{\partial \phi^-}{\partial t}
\]
\[
= -(V + c) \frac{\partial \phi^-}{\partial x} - (V - c) \frac{\partial \phi^*}{\partial x}
\]
\[
= -(V + c) \frac{1}{2} \left( \frac{\partial \phi}{\partial x} + \frac{\partial u}{\partial x} \right) - (V - c) \frac{1}{2} \left( \frac{\partial \phi^*}{\partial x} - \frac{\partial u^*}{\partial x} \right),
\]
and Eq. (11), which is valid for $u^*$ and $\phi^*$ as much as $u$ and $\phi$, can be used to eliminate $\partial u/\partial x$ and $\partial u^*/\partial x$ giving
\[
\frac{\partial \phi}{\partial t} = -(V + c) \frac{\partial \phi}{\partial x} + \frac{\partial \phi^*}{\partial t} + (V + c) \frac{\partial \phi^*}{\partial x}.
\]
This is exactly the same as Eq. (9) with $\dot{c} = V + c$.

REFERENCES


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The analysis of umkehr observations of stratospheric ozone by a ‘Maximum Entropy’ method

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SUMMARY

An application is described of the ‘Maximum Entropy’ (ME) principle to the analysis of umkehr observations of atmospheric ozone. The method is shown to produce results which are comparable with more conventional analyses, and which show a marginally higher correlation with simultaneous ozonesonde measurements than the standard evaluation method. Differences between conventional and ME solutions are attributable to the low intrinsic information content of umkehr observations, and the use of climatological information in addition to observational data in the standard method of analysis. This additional information may also be included in ME solutions, reducing the systematic errors of the simple ME solutions, but possibly degrading that part of the information content of the solution which is due to the observations. Some further meteorological applications of the method are discussed.