A data assimilation experiment and the global circulation during the FGGE special observing periods

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SUMMARY

During the two Special Observing Periods of the First GARP Global Experiment (FGGE) a data assimilation experiment was conducted at the UK Meteorological Office. Throughout the period of the experiment observational data were assimilated dynamically into a numerical forecast model at the four main synoptic hours of each day.

In the first part of the paper, the method of data assimilation is briefly described. Following that, a description is given of the zonally averaged atmospheric circulation, as diagnosed from our analyses. The results of this study are in generally good agreement with those found by previous workers.

1. INTRODUCTION

During the Special Observing Periods (SOPs) of the First GARP Global Experiment (FGGE), the Meteorological Office conducted a data assimilation experiment. The purposes were to evaluate the performance of a method of dynamic data assimilation in a semi-operational environment, and to produce a comprehensive set of global analyses for research purposes.

The method of data assimilation will be briefly described, with more complete documentation available as a Meteorological Office internal report (Birch and Lyne 1980). It is essentially that due to Lorenc (1976) and is equivalent to the dynamical relaxation schemes of Hoke and Anthes (1976) and Davies and Turner (1977).

The method of assimilating data directly into a numerical forecast model was used by Charney et al. (1969) and since by numerous other investigators (see Bengtsson 1975). Model variables are replaced by observed values at or near their time of validity. Refinements in methods of interpolation and data insertion have resulted in techniques which approximate closely the dynamical relaxation methods mentioned above. Miyakoda et al. (1978) have described such a system which, in one of its forms, is very similar to the method described here.

The data were assimilated into the model as it was integrated over so-called assimilation periods, and between these periods the model was integrated normally. At each time step during an assimilation period the observed data (or rather their differences from values predicted by the model) were interpolated to model grid points, and these interpolated differences were then used to correct the model values. The variables acted upon were the basic variables of the model – temperature, horizontal wind, humidity mixing ratio and surface pressure.

The SOPs ran from 5 January to 5 March and 1 May to 30 June 1979, though in each case the assimilation was started five days earlier to allow for initial adjustment. The assimilations were continued beyond the end of the second SOP, until 14 August 1979, to provide analyses for future investigations of the south-west monsoon. The initial conditions were interpolated from the Meteorological Office's operational analysis north of about 19°N, and merged with climatological fields in the tropics and Southern Hemisphere. The latter were zonally averaged south of 15°S. A brief account of the day to day running and performance of the assimilation scheme is given in section 2. In sections 3 and 4 the two stages of the assimilation process are described. In section 5 some results of the experiment are presented in the form of a diagnostic study of zonally averaged quantities calculated from the analyses.

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2. Monitoring the Data and the Assimilation

(a) Database

Observations were obtained from the Meteorological Office's Synoptic Data Bank which receives data in real time over the Global Telecommunications System. Data were assimilated into the model at each of the four main synoptic hours, 0000, 0600, 1200 and 1800 GMT. If $T_s$ is a main synoptic hour, then all relevant observations valid between $T_s - 0300$ and $T_s + 0300$ were extracted and stored before being used in the assimilation process. Thus all observations (subject to some selectivity in surface and land observations in data rich areas) were available for assimilation, but the time displacement of any particular observation from the nearest synoptic hour was ignored. An example of a typical distribution of data available for assimilation is given in Fig. 1. The experiment was conducted approximately 24 h behind real time with data cut-off times varying between 8 and 18 h.

Figure 1. Distributions of observations for 00Z on 17th February 1979 showing: (a) surface observations; (b) conventional upper air soundings;
Figure 1. (c) satellite observations; (d) aircraft and constant level balloon observations.

(b) Intervention

The data and model fields were monitored subjectively each day, and, if required, intervention was performed prior to each assimilation cycle. Although in principle this could be done before each six hourly assimilation, practical constraints restricted the frequency to once every 12 h. Thus intervention on both 0000 and 0600 data was performed prior to the assimilation of 0000 data.

Intervention consisted of the manual quality control of data and the creation of bogus observations in data sparse areas, or where the strength and shape of prominent meteorological features were inadequately represented. This commonly included incorporating some of the intervention for the operational analysis in the Northern Hemisphere, and the so-called PAOBs in the Southern Hemisphere. The latter are artificial observations of surface pressure and 1000–500 mb thickness constructed by WMC Melbourne. Additional
guidance for intervention was obtained by hand analysis of the data. In the limited time available, the effort was concentrated on the Southern Hemisphere.

(c) Model assessment

The performance of the assimilation model was monitored on a day to day basis by the intervention team which usually consisted of two scientists. The assimilation was assessed further by running a three- or sometimes five-day forecast once a week from the model fields at a chosen synoptic time, and comparing it with the corresponding operational analysis and forecast. The latter are produced only for the Northern Hemisphere north of about 19°N and so the Southern Hemisphere fields were verified against the Melbourne analyses when these were available. Verification of the tropical region was attempted by comparison with any available analyses and with data, but was inevitably rather unsatisfactory.

In general the assessments showed that the assimilation produced analyses of comparable quality to those of the operational system in the data rich areas of the Northern Hemisphere. Comparison with a selected set of upper air and surface observations indicated that the root mean square height errors of the assimilation were generally greater than those of the operational model, whereas the wind errors were usually less. Table 1 contains a representative example from each SOP, the verification against observations being performed over an area covering much of North America, the North Atlantic and Western Europe. (In each case the statistics are based on approximately 100 upper air observations.

<table>
<thead>
<tr>
<th>Pressure Level (mb)</th>
<th>Mean</th>
<th>Root Mean Square</th>
<th>Root Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Operational</td>
<td>FGGE</td>
<td>Operational</td>
</tr>
<tr>
<td>100</td>
<td>-6.1</td>
<td>8.4</td>
<td>40.3</td>
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<tr>
<td>200</td>
<td>-2.3</td>
<td>-3.1</td>
<td>31.7</td>
</tr>
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<td>300</td>
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<td>850</td>
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<table>
<thead>
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<th>Pressure Level (mb)</th>
<th>Mean</th>
<th>Root Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Operational</td>
<td>FGGE</td>
</tr>
<tr>
<td>Surface Pressure (mb)</td>
<td>0.15</td>
<td>0.26</td>
</tr>
</tbody>
</table>

**Table 1. Analysis Errors for 1200 GMT 28/2/79**

<table>
<thead>
<tr>
<th>Pressure Level (mb)</th>
<th>Mean</th>
<th>Root Mean Square</th>
<th>Root Mean Square</th>
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<td></td>
<td>Operational</td>
<td>FGGE</td>
<td>Operational</td>
</tr>
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<td>100</td>
<td>-0.4</td>
<td>13.3</td>
<td>39.2</td>
</tr>
<tr>
<td>200</td>
<td>0.8</td>
<td>5.6</td>
<td>28.8</td>
</tr>
<tr>
<td>300</td>
<td>0.3</td>
<td>21.2</td>
<td>24.3</td>
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<tr>
<td>500</td>
<td>-1.6</td>
<td>18.6</td>
<td>18.0</td>
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<tr>
<td>700</td>
<td>0.2</td>
<td>15.2</td>
<td>15.6</td>
</tr>
<tr>
<td>800</td>
<td>-2.0</td>
<td>8.0</td>
<td>15.1</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Pressure Level (mb)</th>
<th>Mean</th>
<th>Root Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Operational</td>
<td>FGGE</td>
</tr>
<tr>
<td>Surface Pressure (mb)</td>
<td>-0.04</td>
<td>0.14</td>
</tr>
</tbody>
</table>
at each level and 250 surface observations). The same relative differences in the error level of the two models were also manifest in the subsequent forecasts. These differences are a reflection of the fact that the FGGE model assimilated wind information directly, whilst the operational model used it only indirectly to modify the gradient of the analysed height field (the initial wind field for the operational model is constructed by solving the non-linear balance equation and the omega equation). Conversely, the operational model used height information directly, whilst the FGGE model assimilated temperature and surface pressure observations.

The comparative inferiority of the height analyses may also indicate an inability of the FGGE model to assimilate mass information adequately, the assimilation being univariate with no attempt, for instance, to introduce geostrophic wind increments corresponding to observations of the mass field. It has been demonstrated by the analysis of Hoke and Anthes (1976) that, at shorter wavelengths, mass information is assimilated more readily onto the gravity-inertia modes of the linearized shallow water equations than on to the meteorological mode. Some evidence of a problem in incorporating mass information was also found in a study of the assimilation of buoy data (Shaw 1981).

This difficulty of assimilating mass information has been the subject of recent research, and the reader is referred to the work of Daley and Puri (1980), Daley (1980) and Talagrand (1981) for detailed investigations of the problem.

3. Data preparation

(a) Interpolation

Before observational data could be assimilated into the model, they had first to be expressed in the same terms as the model’s basic variables and interpolated to model grid points. This was accomplished in two stages, the first being to obtain values on $\sigma$-levels by vertical interpolation (the vertical co-ordinate $\sigma$ is the ratio of pressure to surface pressure). At the same time observations such as mean sea level pressure and satellite-derived thickness were converted into basic model variables.

Although different methods of vertical interpolation were used for different types of observations, the general assumption was of linearity in $\log \sigma$. To derive pressure at the model’s surface from a value reported at a standard level (e.g. mean sea level), a constant lapse rate was assumed, with the surface temperature set equal to the lowest level model temperature (when no reported value was available). The pressure of a model level was likewise defined with reference to the model value of surface pressure if no observation was available.

The linear interpolation of the data in the horizontal (or rather their differences from values predicted by the model) was performed at each time step of the assimilation period. The second stage was therefore the calculation of the necessary weights, and this was achieved through a form of optimal interpolation. The details of optimal interpolation have been extensively documented elsewhere (e.g. Gandin 1963; Rutherford 1972) and will not be repeated here.

(b) Automatic Quality Control

During the calculation of the weights, the observations were checked by finding a value at each observation point by interpolation without using the observation itself. An observation was accepted if the magnitude of the difference between it and the interpolated value was less than $c\sqrt{E^2 + \delta^2}$, where $E^2$ is the expected mean square error of the interpolated value (a by-product of optimal interpolation) and $\delta^2$ the expected mean square error of the observation.

The constant $c$ had the value 6 during SOP 1 and 5 during SOP 2. It had been hoped to reduce its value further during SOP 2 by using revised error levels from statistics collected
during SOP 1. However, it was found that smaller values gave rise to unacceptably large numbers of rejections of good data, and this, together with difficulties encountered in deriving realistic error levels from the statistics, suggested that error levels should be related to prevailing synoptic conditions. This was not done during FGGE and a unique value was specified for each observation and level.

Two sweeps through the data were made when checking to avoid the possible rejection of good data on the first sweep through comparison with bad data subsequently rejected during the same sweep. During the second sweep only data which had been rejected previously were subject to quality control.

(c) Data Selection

For reasons of computational economy, the maximum number of observations used in estimating a value at a model grid point was limited to eight. Observations were selected from a circle of influence of approximately three grid lengths (660 km) radius, this circle being divided into quadrants. The best observation from each quadrant was first selected, and then the next best from any quadrant, best in this sense being those observations which, if taken alone, would have the highest weights.

Although this method was adopted to select a widely distributed sample, it could still result in the selection of highly correlated observations situated close together. For this reason a procedure was employed in which a group of observations of the same type were replaced by their arithmetic mean with an accuracy greater than that of the individual observations. The mean square error of this 'super observation' was taken to be $\sigma^2_0/N$, $\sigma^2_0$ being the value appropriate to the type of observation and $N$ the number of observations meaned; that is, no allowance was made for the horizontal correlation of their errors. This procedure was found to be necessary only for aircraft and surface reports, and such observations were meaned if they lay within a half degree latitude – longitude square.

(d) Error Specification

The method of optimal interpolation requires specification of the expected mean square errors of the observations and of the model. In this specification it was assumed that model errors were uncorrelated with observational errors, and that each had no bias.

The observational errors used during SOP 1 are summarized in Table 2, and were generally larger than those suggested by other workers e.g. Bengtsson (1975). This is because some allowance was made for the increase in error due to the synoptic nature of much of the data, and to the interpolation on to model $\sigma$ surfaces.

For SOP 2 revised values were calculated from statistics collected during SOP 1, and these are summarized in Table 3.

<table>
<thead>
<tr>
<th>Observation</th>
<th>Low level</th>
<th>Medium level</th>
<th>High level</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface pressure (all)</td>
<td>2</td>
<td></td>
<td></td>
<td>(mb)$^2$</td>
</tr>
<tr>
<td>Surface wind (ships only)</td>
<td>18</td>
<td></td>
<td></td>
<td>(m s$^{-1}$)$^2$</td>
</tr>
<tr>
<td>Wind (all 'sondes)</td>
<td>8</td>
<td>increasing to</td>
<td>18</td>
<td>(m s$^{-1}$)$^2$</td>
</tr>
<tr>
<td>Wind (aircraft)</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>(m s$^{-1}$)$^2$</td>
</tr>
<tr>
<td>Wind (satellite)</td>
<td>18</td>
<td>increasing to</td>
<td>50</td>
<td>(m s$^{-1}$)$^2$</td>
</tr>
<tr>
<td>Wind (constant level balloons)</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>(m s$^{-1}$)$^2$</td>
</tr>
<tr>
<td>Temperature (all 'sondes)</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>K$^1$</td>
</tr>
<tr>
<td>Temperature (aircraft)</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>K$^1$</td>
</tr>
<tr>
<td>Temperature (satellite)</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>K$^1$</td>
</tr>
<tr>
<td>Temperature (constant level balloons)</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>K$^1$</td>
</tr>
<tr>
<td>Humidity mixing ratio (all 'sondes)</td>
<td>$5 \times 10^{-2}$</td>
<td>$5 \times 10^{-2}$</td>
<td>—</td>
<td>(g/g)$^2$</td>
</tr>
</tbody>
</table>
A DATA ASSIMILATION EXPERIMENT

### TABLE 3. OBSERVATIONAL ERRORS DURING SOP 2

<table>
<thead>
<tr>
<th>Observation</th>
<th>Mean Square Errors</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low level</td>
<td>Medium level</td>
</tr>
<tr>
<td>Surface pressure (land, OWS and Light Vessels)</td>
<td>3</td>
<td>—</td>
</tr>
<tr>
<td>Surface pressure (all other ships)</td>
<td>4</td>
<td>—</td>
</tr>
<tr>
<td>Surface pressure (drifting buoys)</td>
<td>16</td>
<td>—</td>
</tr>
<tr>
<td>Surface wind (all ships)</td>
<td>20</td>
<td>—</td>
</tr>
<tr>
<td>Wind (all 'sondes)</td>
<td>8</td>
<td>increasing to</td>
</tr>
<tr>
<td>Wind (aircraft)</td>
<td>—</td>
<td>50</td>
</tr>
<tr>
<td>Wind (satellite)</td>
<td>16</td>
<td>increasing to</td>
</tr>
<tr>
<td>Wind (constant level balloons)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Temperature (all 'sondes)</td>
<td>5</td>
<td>1.5</td>
</tr>
<tr>
<td>Temperature (aircraft)</td>
<td>—</td>
<td>8</td>
</tr>
<tr>
<td>Temperature (satellite)</td>
<td>6</td>
<td>decreasing to</td>
</tr>
<tr>
<td>Temperature (constant level balloons)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Humidity mixing ratio (all 'sondes)</td>
<td>6 × 10⁻⁴</td>
<td>10⁻⁷</td>
</tr>
</tbody>
</table>

Mean square differences between the model's assimilated values and the observed values were collected and used to estimate values of error covariance for different separations of observation points. These values were extrapolated to zero separation to provide estimates of the mean square model error under the assumption that the observational errors were uncorrelated. Estimates of the observational errors could then be deduced, the total mean square error being the sum of the model and observational errors. This procedure was very similar to that described by Van Maanen (1981) and is described more fully by Lyne (1981). Satellite temperature data were excluded from the estimation of error covariance because of their known horizontal correlation of observational error. This technique could work well only where there were sufficient data to define the statistics adequately. Error estimates for some of the devices, e.g. the constant level balloons, were therefore made subject to additional criteria such as the relative error levels of neighbouring observations and of the model.

Apart from high level satellite derived temperatures, most error values were larger during SOP 2 than during SOP 1. Values for satellite winds were much larger at high levels, and this was found to be due to major errors in regions bordering the jet streams, with much smaller errors elsewhere. This highlights the difficulties in adopting a single value for each type and level of observation. The humidity errors, which were given unrealistically large values during SOP 1 by mistake, were assigned realistic values for SOP 2.

The mean square errors for the model during SOP 1 were derived from statistics collected during Observing System Simulation Experiments (Bromley, 1978), in which the atmosphere and assimilation model were simulated by high and low resolution models respectively. The errors were assumed to be constant in the zonal direction, with the variation in the vertical and north-south directions identical to that implied by the simulation experiments. The magnitudes were increased to ensure that reliable observations had smaller error levels than the model. The model errors were revised for SOP 2 in the light of the statistics calculated during SOP 1, the major changes being a substantial increase in the vector wind error at high levels in the Southern Hemisphere (e.g. 100 m² s⁻² increased to 200 m² s⁻² mean square error at approximately 250 mb and 60°S).

Only satellite observations of temperature were assumed to be horizontally correlated, and, following Bergman and Bonner (1976), the following function was used

\[ \mu = \exp(-k r_{ij}^2) \]

where \( r_{ij} \) is the distance between the \( i^{th} \) and \( j^{th} \) observations and \( k = 12.5 \times 10^{-6} \) km⁻².

The correlation function chosen for the model error was

\[ \mu = (1 - r_{ij}^2/\eta^2)/(1 + 2r_{ij}^2/\eta^2)^{5/2}. \]

The parameter \( \eta \) is some scaling distance, set to 1540 km, which is a little more than the
maximum possible distance between observations in the region of influence, and which ensured that the correlations were always positive.

This correlation function was first suggested by Jones (1976).

4. The assimilation model

The model into which the data were assimilated was the Meteorological Office's high resolution \( \sigma \) co-ordinate general circulation model. This has 11 unequally spaced levels in the vertical and a horizontal resolution of approximately 220 km on an irregular Kurigara-type grid. Standard parametrizations were incorporated for precipitation, deep convection and boundary layer processes, together with climatological estimates of radiative heating and cooling rates. The model is described in a Meteorological Office internal report (Saker 1975).

Data were assimilated dynamically over three hour periods centred on each main synoptic hour. At each time step during such a period the model was integrated according to the following equations

\[
\psi^{*}_{i+\Delta t} = \psi_{i-\Delta t} + 2 \Delta t \ D(\psi^{'}) \\
\psi_{i}' = \psi_{i}' + \alpha(\psi^{*}_{i+\Delta t} - \psi_{i-\Delta t} - 2\psi_{i}') \\
\psi'^{*}_{i+\Delta t} = \psi'^{*}_{i+\Delta t} + \lambda(\psi'^{*}_{i+\Delta t} - \psi^{'}) \\
\psi_{i}' + \Delta t = \psi_{i}' + \Delta t P(\psi^{*}_{i+\Delta t})
\]

These equations represent the application respectively of: the dynamical equations, using a centred time step; time-meaning which prevents the separation of values at successive time steps; adjustment of the model fields towards values interpolated from observations; and lastly the physical equations representing non-adiabatic processes. The time step \( \Delta t \) was 450 s.

The scaling parameter \( \lambda \) is related to the relaxation co-efficient \( K \) employed by Davies and Turner (1977) and Hoke and Anthes (1976) through the equation

\[
\lambda = 2K\Delta t/(1 + 2K\Delta t)
\]

During the assimilation of data, the parameter \( \lambda \) was increased linearly from zero 1·5 h before the synoptic hour to 0·5 on the hour, and then decreased linearly thereafter to zero at the end of the period. The time meaning coefficient \( \alpha \) in Eq. (2) was likewise varied between 0·01, the value used during normal model integration, and its maximum value of 0·1 at the synoptic hour.

5. Diagnostic calculations

(a) Introduction

In this section we examine, on a zonally averaged basis, the general circulation of the atmosphere, as inferred from the analyses produced in this data assimilation experiment. For conciseness most of the results presented refer to the months of January and June, but where similar calculations have been made for February and for May or July, the results were found to be similar to those reproduced here.

Comparisons have been made with climate statistics calculated by Newell et al. (1972) and Oort and Rasmusson (1970, 1971), which were based on several years of observations. Differences between our results and those of previous studies can be partly attributed to year to year variations in the atmospheric circulation and to differences between the analysis methods used. However the improved observational coverage available means that calculations made for the FGGE year should be more reliable than any previous results for a single year. Recent work by Guynn and Le Marshall (1981) suggests that the more intense circulation in the Southern Hemisphere in 1979, as compared with that analysed in previous years, cannot be entirely accounted for by interannual variability.

The results presented in this section, and those from other similar diagnostic
Figure 2. Meridional cross section of the mean zonal wind $\bar{u}$ for January 1979, with westerly wind positive.

Figure 3. Meridional cross section of the mean zonal wind $\bar{u}$ for June 1979.
calculations, have been reported at greater length in a Meteorological Office internal report (Swinbank 1980).

(b) Zonally meaned wind, temperature and kinetic energy

Figures 2 and 3 show the mean zonal wind, \([\bar{u}]\), where the square brackets denote a zonal mean and the overbar a time mean, for January and June. The cross sections are in good agreement with the results found by Newell et al. (1972) and Oort and Rasmusson (1971) (hereafter referred to as \(N\) or \(OR\) respectively). In our analyses the strength of the Northern Hemisphere maximum varied from 39 m s\(^{-1}\) in February to 19 m s\(^{-1}\) in June and July, compared to the annual range found by \(OR\) of 42 to 20 m s\(^{-1}\). In the Southern Hemisphere the seasonal variation was smaller (31 m s\(^{-1}\) in January to 36 m s\(^{-1}\) in July). In the winter analyses in the Southern Hemisphere there is a marked double structure in the tropospheric wind maxima, with the polar tropospheric jet situated beneath the very strong polar night jet.

Between May and July there was a marked increase in the tropical (and Northern Hemisphere subtropical) easterlies through the upper troposphere, which was coupled with a poleward movement in the position of the Northern Hemisphere subtropical jet. This change is very noticeable in the total angular momentum of the atmosphere (Hide et al. 1980), which was found to decrease quite markedly through this period, reaching a minimum in late July.

The mean meridional wind can be used to calculate a stream-function (Figs 4 and 5) which clearly demonstrates the Hadley and Ferrel circulations. From the zonally averaged

Figure 4. Meridional cross section of the vertically integrated mass flux \(\psi\) for January 1979. The contours of \(\psi\) are streamlines which indicate the zonally averaged meridional and vertical motion (the sense of the circulation is anti-clockwise around maxima).
continuity equation

\[
\frac{1}{a \cos \theta} \frac{\partial [\vec{v}] \cos \theta}{\partial \theta} + \frac{\partial [\bar{\omega}]}{\partial p} = 0
\]

it follows that the zonally averaged meridional and vertical motion can be represented with a stream function \( \psi \) defined by

\[
\psi = (2\pi a \cos \theta / g) \int_0^p [\vec{v}] dp
\]

(as displayed here, the circulation is anti-clockwise around maxima in \( \psi \)). From pressure changes at the earth's surface it can be inferred that the net meridional mass drift across any latitude circle cannot be more than a few mms\(^{-1}\), so \( \psi = 0 \) at the earth's surface to a good approximation. Our results have been modified (by adding in a correction to \([\vec{v}]\) which was independent of pressure level) so that there is no net mass flow across any latitude circle. These diagrams demonstrate that, during the periods examined, the circulation is dominated by a single Hadley Cell centred near the equator, with its ascending branch on the summer side. From the results found by N and Oort and Rasmusson (1970), it can be seen that the mass flux is more symmetric at the equinoxes. In the summer hemisphere a weak second Hadley Cell can be seen, and in the mid-latitudes of each hemisphere there is a Ferrel Cell. In most of the cross sections there is evidence of a very weak direct circulation in the polar regions.
From our analyses the strengths of the Hadley circulation showed a marked temporal variation. For example, the strength of the main Hadley cell increased to around $30 \times 10^{10} \text{kg s}^{-1}$ in mid-January and then decreased, at first quickly and then more gradually to $15 \times 10^{10} \text{kg s}^{-1}$ at the end of February, whereas Oort and Rasmusson (1970) found an increase in strength between January and February ($19$ and $23 \times 10^{10} \text{kg s}^{-1}$ respectively). Similarly there was a rapid increase in the circulation intensity at the beginning of June, from about $-13 \times 10^{10} \text{kg s}^{-1}$ to around $-26 \times 10^{10} \text{kg s}^{-1}$. Because of these large fluctuations, one might expect that the cell strengths, when averaged over a month, would vary significantly from year to year; taking this factor into account the results from our monthly averages are consistent with the references cited above.

In extra-tropical regions the assessment of $[\vec{v}]$ becomes somewhat less accurate (in general the wind speeds tend to be larger than in the tropics, while the net meridional flow is less). It is in some ways more satisfactory to diagnose $[\vec{v}]$ indirectly from the momentum flux, although in this work the assessment of momentum flux and $[\vec{v}]$ are essentially independent. Our calculations of the Ferrel circulation give similar results to those found by N (which were derived indirectly, from the momentum flux) and Oort and Rasmusson (1970), except that in the Southern Hemisphere the Ferrel cell shown here is slightly stronger and further south than that found by N.

The temperature cross-sections for January and June are given in Figs 6 and 7. The agreement with published zonal averages (e.g. Crutcher 1970; Crutcher et al. 1971) is very good except near the tropopause, where the error in interpolation between adjacent analysis levels becomes significant. The most noticeable features are the seasonal changes in mid-latitudes, accompanied by a change in the position of the low level tropical maximum. At

![Figure 6. Cross section of the mean temperature $[T]$ for January 1979.](image-url)
the tropopause over the equator there is a small seasonal variation (in July the minimum is 201 K), which agrees with the results found by Kidson et al. (1969), although the temperatures found in their analysis at 100 mb are slightly lower than our values. The temperature in the Antarctic stratosphere shows a particularly large seasonal variation, the very low winter temperatures being consistent with the strong polar night jet mentioned previously.

The kinetic energy per unit mass, \( \frac{1}{2}([\bar{u}^2] + [\bar{v}^2]) \) can be divided into three components: zonal kinetic energy, \( \frac{1}{2}([\bar{u}]^2 + [\bar{v}]^2) \); standing eddy KE, \( \frac{1}{2}([\bar{u}^*2] + [\bar{v}^*2]) \); and transient eddy KE, \( \frac{1}{2}([\bar{u}^2] + [\bar{v}^2]) \). (In our notation an asterisk denotes the deviation from a zonal mean, and a prime the deviation from a time mean). Figures 8 and 9 show cross-sections of the total kinetic energy for January and June. Both these cross-sections and cross-sections of the three components (not reproduced here) show good agreement with the values found by OR. Figures 10 and 11 give the kinetic energy divided into the components and vertically averaged between 50 mb and 1000 mb.

In each case the zonal kinetic energy shows a maximum at 30° latitude in winter, corresponding to the subtropical jet. In the Southern Hemisphere winter this is dominated by a second maximum at about 50°S, due to the stratospheric polar night jet and the upper tropospheric polar jet. The latter also makes a large contribution to the KE in the Southern Hemisphere summer, unlike the equivalent Northern Hemisphere feature. These results reflect the much greater seasonal variation of the zonal mean wind in the Northern Hemisphere, compared with the Southern. Considering the transient eddy component, one finds that it is stronger in the Northern Hemisphere, where the zonal component is weaker. The transient eddy KE is more important than the standing eddy KE in mid-latitudes, reflecting, as noted in OR, the meandering character of the polar jet stream.
Figure 8. Cross section of the mean kinetic energy for January 1979.

Figure 9. Cross section of the mean kinetic energy for June 1979.

(c) Meridional Transport of Momentum

The atmosphere can lose or gain angular momentum only at the surface of the earth, through torques due to pressure differences across mountain chains and frictional stress (see Newton 1971). Thus the atmosphere circulates in such a way as to transport angular momentum generated by the eastward torques of the tradewind belt to mid-latitudes, where westerly momentum is lost to the solid earth.
Figure 10. Vertically averaged kinetic energy for January 1979, showing the zonal standing eddy and transient eddy components.

Figure 11. Vertically averaged kinetic energy for June 1979.

To a good approximation, the total transport of angular momentum across a latitude circle is given by the vertical integral of the meridional flux of momentum $\langle \tilde{u} \tilde{v} \rangle$:

$$F = \int_0^{p_*} \frac{2\pi a^2 \cos^2 \theta}{g} [\tilde{u} \tilde{v}] dp$$

(5)
Figure 12. Meridional cross section of $\langle \vec{u} \rangle$, the mean meridional transport of westerly momentum, for January 1979.

Figure 13. Meridional cross section of $\langle \vec{u} \rangle$ for June 1979.

Figures 12 and 13 show the fields of $\langle \vec{u} \rangle$ (in this case the period covered by the January data is 5th to 31st January). There are large poleward fluxes at around 30°N and 30°S centred at around 250 mb. Poleward of these features there are regions of equatorward flux, with the stronger fluxes in the winter hemisphere. These equatorward fluxes were not found in the Northern Hemisphere by $N$ (possibly due to inadequate data coverage),
although OR did find similar fluxes. Another noteworthy feature is that in each case there is a region of cross-equatorial flux towards the summer hemisphere, centred at about 150 mb. Most of the flux in each of these cases is concentrated near the top of the troposphere.

The time-averaged product \( \langle \bar{u}\bar{v} \rangle \) can be split as follows:

\[
\langle \bar{u}\bar{v} \rangle = \langle \bar{u} \rangle \langle \bar{v} \rangle + \langle \bar{u}\bar{v}^* \rangle + \langle \bar{u}^*\bar{v} \rangle
\]

The three terms will be referred to as the toroidal, standing eddy and transient eddy components, respectively. Since data from a single year are considered here, one would expect a greater standing eddy component and a smaller transient eddy component than the results given by N or OR. The three components have each been multiplied by the appropriate constants and vertically integrated between 1000 mb and 100 mb, as in Eq. (5), to give the total angular momentum transport. The results are displayed graphically in Figs. 14 and 15. The toroidal flux is poleward, associated with the Hadley circulation, at 15° latitude in the winter hemisphere, with some equatorward flux associated with the Ferrel cell at around 50° latitude (more particularly in the Southern Hemisphere). The remainder of the transport is accomplished by the eddy terms. In general the transport by standing eddies is less important in the Southern Hemisphere than in the Northern, but the exact partitioning between standing and transient eddies is probably not significant over periods as short as a month.

Comparing these results with the cross-sections of \( \langle \bar{u} \rangle \) in Figs. 2 and 3, one finds that the regions of convergence of angular momentum (positive slope on the graphs) agree well with the areas of low level westerly wind (which one would expect to correspond to sinks of westerly momentum), similarly regions of divergence and easterlies correspond.

One can also relate the angular momentum to the meridional circulation. The rate of change of absolute angular momentum can be written as:

\[
\left[ \frac{\partial M}{\partial t} \right] + \frac{1}{a \cos \theta} \frac{\partial}{\partial \theta} \left\{ \langle M\bar{v} \rangle \cos \theta \right\} + \frac{\partial}{\partial \rho} \left[ M\omega \right] = - \left[ \frac{\partial \phi}{\partial \lambda} \right]
\]
where $M = (\bar{u} + \Omega \alpha \cos \theta) a \cos \theta$ is the angular momentum per unit mass. Friction stress is formally included by interpreting it to be a parametrization of the small scale part of the $[\bar{M}\bar{\omega}]$ term. The last term, which represents the zonal gradient of geopotential is zero above topography, but when vertically integrated gives the torque due to pressure differences across mountains. This equation can be rewritten in terms of $u$ instead of $M$:

$$
\left[ \frac{\partial \bar{u}}{\partial t} \right] + \frac{1}{a \cos^2 \theta} \frac{\partial}{\partial \theta} \left[ \bar{u} \bar{v} \cos^2 \theta \right] + \frac{\partial}{\partial \phi} \left[ \bar{u} \bar{w} \right] - f \bar{v} = - \frac{1}{a \cos \theta} \left[ \frac{\partial \phi}{\partial \lambda} \right].
$$

In the upper troposphere the geopotential gradient term is zero, and the rate of change of zonal wind is found to be small compared with the three remaining terms. Thus, in this situation, there should be a balance between the divergence of the meridional and vertical momentum fluxes and the mean Coriolis acceleration. This relationship allows some assessment of the self-consistency of the analyses; comparison of Figs. 12 and 13 with Figs. 4 and 5 demonstrates that areas of convergence of momentum flux tend to correspond with equatorward $[\bar{v}]$ and the areas of divergence with poleward $[\bar{v}]$. If the meridional circulation is calculated indirectly from the meridional momentum flux (neglecting the vertical flux term), the agreement is good except in the southern part of the Southern Hemisphere, where our values of $[\bar{v}]$ are probably less accurate because of the low density of observations.

On the basis of this study of the zonally-averaged circulation we conclude that our analyses are generally self-consistent, as well as being consistent with the results of previous diagnostic studies. The First GARP Global Experiment has given an unprecedented opportunity to carry out studies of the circulation of the atmosphere, especially in relatively
data sparse areas such as the tropics and the Southern Hemisphere. In the future we hope
to extend this work to give a three dimensional picture of the general circulation, using the
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