The relevance of conditional symmetric instability to the prediction of mesoscale frontal rainbands

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SUMMARY

This paper examines the theory of Conditional Symmetric Instability (Bennetts and Hoskins 1979) and considers its relevance to meso-scale frontal rainbands. The theoretical growth rate of the instability is evaluated from synoptic scale data obtained from a numerical forecast model and compared with observations of the horizontal structure of rainfall observed by radar. One case study is presented in detail and a further 44 cases summarized. Rainbands were observed on 80% of the occasions on which they were predicted.

I. INTRODUCTION

Observational studies (e.g. Browning et al. 1973; Hobbs et al. 1980; Herzegh and Hobbs 1980) have shown that frontal precipitation is frequently organized into bands. The bands may be classified by reference to their scale and to their position and orientation relative to the frontal system (Browning and Harrold 1969; Houze et al. 1976a; Matejka et al. 1980). The larger bands, referred to as types 1a, 1b, 2, 3b, and 4a in the classification of Matejka et al., are typically 50–75 km wide and are found embedded within, and orientated parallel to, the various frontal zones. The smaller bands, types 3a, 4b and 5 are 5–20 km wide and, except for type 5 which is convective in origin and found well behind the cold front, are similarly located but have different orientations depending on their type, see for example Browning and Bryant (1975), James and Browning (1979) and Houze et al. (1976b).

This paper is concerned with the larger bands which, in addition to their common scale, are also associated with regions of potential instability located at the mid-levels of the atmosphere (Browning and Mason 1980). It is the organized release of this instability that selectively enhances the precipitation generated by cross-frontal ascent within the cyclone, and gives rise to the observed banded structure.

Bennetts and Hoskins (1979) suggested that such a process could result from the development of conditional symmetric instability (CSI). This instability is manifest by rolls approximately along the thermal wind, i.e. approximately parallel to the frontal surfaces. As the rolls grow, differential advection overturns the mid-levels of the atmosphere in preferred linear regions, generating conditional gravitational instability. The resulting convection leads to the banded nature of the rainfall. The theory does not indicate a wavelength but numerical integrations have suggested a value of 100–200 km.

The aim of this paper is to assess the relevance of the theory of CSI to the occurrence of rainbands within the atmosphere. The theoretical growth rate is evaluated from synoptic scale data obtained from a numerical forecast model and compared with observations of the horizontal structure of the rainfall measured by the United Kingdom radar network. In Section 2 the relevant aspects of the theory of CSI, the acquisition and interpretation of the radar data and the numerical forecast model are reviewed. In Section 3, one case study is presented in detail and in Section 4 the results of a more general study are summarized.

An alternative theory of rainbands has been advanced by Lindzen and Tung (1976). They describe how a mid-level layer of potentially unstable air can both act as a reflecting lid, permitting the propagation of ducted meso-scale gravity waves and, by the subsequent release of the instability by the waves, allow the generation of rainbands. The growth of the waves depends crucially on the detailed structure of the frontal zone, a feature not resolved.
on the synoptic scale. Therefore the relevance of this theory to the prediction of frontal rainbands is not discussed further.

2. Theory and observations

A brief review is given of the three main aspects of the present study: the theory of CSI, the evaluation of the growth rate using results from a numerical forecast model and the radar observations of the horizontal distribution of rainfall.

(a) Theory of CSI

The word 'conditional' in conditional symmetric instability was used by Bennetts and Hoskins (1979) to indicate that although the energy for the perturbation is derived from the kinetic energy of the basic flow, as with symmetric instability, the presence of growing modes is conditional on there being latent heat release during ascent, i.e. the atmosphere must be at or near saturation. Then the growth rate ($\sigma$), for a small amplitude disturbance, is given by

$$\sigma^2 = -q_w \int \frac{g}{\theta_0} \frac{\partial \theta_w}{\partial z},$$

where all quantities refer to the mean state of the large scale flow, $\partial \theta_w/\partial z$ is the lapse rate of the wet bulb potential temperature and $q_w$ is the vorticity on a wet bulb potential temperature surface and is defined

$$q_w = (fg/\theta_0)(\xi \cdot \nabla \theta_w)$$

where $\xi$ is the absolute vorticity vector, $f$ the coriolis parameter, $g$ the acceleration due to gravity and $\theta_0$ a reference temperature (typically 283 K). $q_w$ is a conserved quantity in a saturated atmosphere provided that frictional and diabatic effects are zero but, because it is difficult to evaluate, is a little used parameter in conventional meteorological analysis.

Throughout this paper the growth rate has been evaluated from the above equations. However it is instructive to rewrite Eq. (1) by making use of the thermal wind relationship. This yields

$$\sigma^2 = -f\zeta + \frac{g}{\theta_0} \frac{(\nabla \theta \cdot \nabla \theta_w)}{\partial \theta_w/\partial z},$$

where $\zeta$ is the vertical component of the absolute vorticity vector and $V_2$ the horizontal gradient operator.

The nomenclature CSI was given to the instability developing in the parametric regions characterized by the two terms on the right hand side of Eq. (2) having a similar magnitude, the instability growing, in theory, when $\sigma^2 > 0$. Clearly, however, in any general evaluation, $\sigma^2$ will also be positive when the atmosphere is either inertially unstable

$$f\zeta < 0, \quad f\zeta \gg g(\nabla \theta \cdot \nabla \theta_w)/(\theta_0 \partial \theta_w/\partial z)$$

or convectively unstable

$$\frac{\partial \theta_w}{\partial z} \rightarrow 0$$

although the growth rate of the latter instability is poorly represented because of the use of the hydrostatic approximation (Bennetts and Hoskins 1979). The three instabilities may be identified on the basis of the above conditions and on the synoptic situation. This is illustrated in Section 3.

Finally, as indicated earlier, the theory predicts growth or decay according as $\sigma^2$ is greater or less than zero. In practice however, the most that can be said is that the instability is more likely to develop for large values of $\sigma^2$, the precise relationship depending on the importance of the features that have been neglected, for example turbulent diffusion and loss of water through precipitation. This point is discussed further in Section 4.
(b) The numerical forecast model

Equation 1 was evaluated using the results from the Meteorological Office operational 10-level rectangle model (Burridge and Gadd 1977) which deals with a limited area of the North Atlantic and Europe and has a grid spacing of 100 km at 60°N. In this model, six to twelve hours integration are required before the humidity and rainfall fields are considered to realistically represent the atmosphere (Jonas 1976). Consequently the forecast fields were used, the initial fields being inadequate to evaluate gradients of $\theta_*$ to a sufficient precision to define the growth rate parameters. Winds, temperature, humidity and their horizontal derivatives were evaluated at 700 mb. Vertical derivatives were obtained from the 850 and 500 mb levels.

In addition, for the reasons discussed at the beginning of Section 2a, the growth rate was evaluated only in regions where the relative humidity exceeded 80%. This choice was somewhat arbitrary but the results were not sensitive to the precise value chosen.

(c) Radar data

The United Kingdom precipitation radar network (Browning 1979) provides an estimate of the precipitation rate every 15 min, at a resolution of 25 km$^2$, over an area that covers much of England and Wales. The distribution of rain at any particular instant is almost always complex. The rain may owe its origins to motion at primary fronts, to orography (Hill et al. 1981), to convective instability, to meso-scale precipitation areas (Browning and Mason 1980), or a combination of these, as well as to motions in frontal rainbands. It is not possible to classify all areas unambiguously and some degree of subjectivity is involved in selecting the areas to be treated as ‘rainbands’. In general, those chosen were required to be coherent in space and time, occur in groups consisting of two or more bands, and have a consistent movement and identification for approximately 6 h. The time scale was both a practical limit imposed by the movement of the bands through the radar network, and yet sufficiently long to identify rain areas caused by orographic and convective events.

3. Case study 29/07/79

This case study has been chosen for detailed discussion because it exhibits many of the features that have been found important in the wider study described in the next section. It illustrates the necessity of distinguishing between the various types of instability (Section 2a) and, because of the position and characteristics of the rainfall, allows the bands to be easily identified (Section 2c).

The spatial distribution of $\sigma^2$, which will be referred to as the growth rate parameter, and was evaluated from the 12 h forecast, is shown in Fig. 1 for 00 GMT on 29/07/79. $\sigma^2$ has units of h$^{-2}$. If $\sigma^2 = 0.1$ h$^{-2}$, then $\sigma = 0.32$ h$^{-1}$ implying that the amplitude of the instability doubles approximately every 2 h. The lowest shown contour is $-0.2$ h$^{-2}$. Instability was never found with $\sigma^2$ below $-0.2$ h$^{-2}$ and consequently the negative contour helps delineate regions of weak stability from the remainder of the field which, for clarity, is left blank. Interest centres on two regions, one to the south west of the United Kingdom ($\sim 50^\circ$N, 5°W) and the other in the North Sea ($\sim 54^\circ$N, 2°E).

(a) South west of the United Kingdom

The position of the central contour (value 1.0 h$^{-2}$, 50°N, 5°W) is close both to the tip of a developing cold frontal wave and to the position of maximum thermal gradient shown in Fig. 2. The $-0.2$ h$^{-2}$ contour encloses a region possessing stability to moist ascent and in which the terms on the right hand side of Eq. (2) are of comparable magnitude. The atmosphere is therefore unstable to CSI but stable to both inertial and convective instabilities. In consequence roll perturbations, manifest by the organization they impose on the
precipitation, are expected to develop with their axes approximately parallel to the thickness lines, which in this region, have a similar orientation to the surface cold front (indicated in Fig. 1).

Figure 3 shows the distribution of precipitation recorded by the radar network at 00 GMT on 29/07/79 (the same time as Fig. 1). In the region 49–51° N, 4–6° W, two type 2 warm sector rainbands are evident. There is a third, weaker band (type 3b) centred at 51° N, 6.5° W, but since there is neither confidence in the precise position of the cold front nor evidence to suggest any fundamental difference between the two types of bands, little is added by the distinction. It is possible, at this early stage in the development of the wave, that all three are of the same category.

The region of precipitation over central southern England (~ 51° N, 2° W) is associated with the warm front and located at the boundary of the region of CSI growth. However it is too small for any assessment of bandedness.

(b) North Sea

As shown in Fig. 1, the region of high growth over the North Sea (~ 54° N, 2° E) is oriented parallel to the cold front and, except in the south, is slightly behind it. Superficially it bears a resemblance to the region discussed in the previous section. However, in this case, the high values delineate a region of moist, convectively unstable air which has a weak horizontal thermal gradient. The atmosphere is unstable to both CSI and convective
instability but since convection has the faster growth rate, it predominates, as confirmed by the synoptic observations of thunderstorms in the area.

The example demonstrates the care that is necessary in differentiating between frontal regions unstable to CSI, and regions where the potential instability of an airmass is released through convection. Such cases have been eliminated from the remainder of the study on the basis of the values of the horizontal temperature gradient and the vertical stability.

One case study in which the convective clouds were organized into bands has been described by Bennetts and Ryder (1982), but such cases are not included in this paper.

4. General Study

All 60 cases of frontal precipitation observed by the United Kingdom radar network from the beginning of 1979, when it first became semi-operational, to October 1980, have been inspected for banded structure. Those in which there was insufficient rain or for which only a small part of the precipitation passed through the radar network, were removed. The remaining 44 cases were classified into banded and non-banded precipitation events (G Monk RRL Malvern – private communication). This information is summarized in Table 1 and listed under the calculated value of the growth rate parameter, $\sigma^2$. 18 cases were identified as non-banded and 26 as banded, the proportion of banded cases highlighting the prevalence of this feature in much of the frontal precipitation falling over the United Kingdom.

Inspection of Table 1 suggests that the threshold value for growth is $\sigma^2 = -0.1 \text{ h}^{-2}$. The existence of a threshold was discussed in Section 2a together with various physical effects that could influence it. In general they would be expected to raise the threshold;
however, the horizontal and vertical grids of the forecast model are only just sufficient to resolve frontal zones and this could lead to the under-estimation of $\sigma^2$, especially in regions of strong gradient. If the results are regrouped into a two by two matrix about $\sigma^2 = -0.1 \, \text{h}^{-2}$, the overall success in predicting banded or non-banded precipitation is 73%. A $\chi^2$ test of significance indicates that the probability of this occurring by chance falls below the 5% level of significance.

It is clear from Table 1 that $\sigma^2$ is better in the prediction of banded precipitation than in identifying occasions in which the rainfall was uniform. A more useful grouping of the results is as follows:

(a) $\sigma^2 \geq 0.2 \, \text{h}^{-2}$
   If $\sigma^2 \geq 0.2 \, \text{h}^{-2}$ then the structure of any frontal precipitation will, almost certainly, be banded.

(b) $0.2 \, \text{h}^{-2} > \sigma^2 \geq 0.0 \, \text{h}^{-2}$
   If $0.2 \, \text{h}^{-2} > \sigma^2 \geq 0.0 \, \text{h}^{-2}$ there is a 75% probability that bands will be identifiable in the radar observations of frontal precipitation.

(c) $\sigma^2 < 0.0 \, \text{h}^{-2}$
   In this regime $\sigma^2$ is of little significance as an indicator of bandedness as there is only a 60% probability that the structure of frontal precipitation will be banded or non-banded according as $\sigma^2$ is greater or less than $-0.1 \, \text{h}^{-2}$.
5. Conclusions

The relevance of the theory of CSI to the organization of frontal precipitation has been examined by evaluating the growth rate parameter $-q_w \theta_0 / (g \partial \theta_w / \partial z)$.

CSI is an instability that develops in regions of strong baroclinicity which have weak stability to wet ascent, conditions typically found within frontal zones, and analysis has been confined to such regions. However, being part of a wider theory which includes as special cases convective and inertial instability, high values of the growth rate parameter can also occur whenever $\partial \theta_w / \partial z \to 0$ or the vertical component of the absolute vorticity vector becomes negative. It is therefore necessary to correctly identify the type of the instability and this is achieved by reference to the synoptic situation and the values of the constituent parts of Eq. 2.

Charts of the parameter ($\sigma^2$), if produced routinely, could be of assistance to forecasters. In very short range forecasts (1–6 h), the mean speed and direction of motion of the major regions of precipitation are calculated from observations. Forecast rainfall is then assessed mainly by advection of the existing patterns. However, in frontal zones, bands of precipitation often travel faster than the frontal system with the consequence that the leading band progressively encounters drier air ahead of the front, and decays. If cognizance is not taken of this, the leading edge of the rain is advected too quickly. CSI fields could help in the early identification of such situations.

The results presented in this paper lend credence to the theory of CSI as an explanation of meso-scale frontal rainbands and provide a method of predicting their occurrence from synoptic scale data.

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