On the droplet distribution near the base of cumulus clouds

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SUMMARY

Lateral entrainment in cumulus clouds is usually assumed to occur continuously as a cloud parcel rises. Computations of the droplet distribution using that assumption are compared with those in which entrainment occurs intermittently, and little difference is found. The width of the droplet distribution is shown to increase as the relative humidity of the entrained air is reduced. The inclusion of the effects of molecular accommodation at the surface of droplets also leads to substantial broadening of the droplet distribution.

1. INTRODUCTION

Observations of the droplet distribution in cumulus cloud show that the rate of growth of the mean droplet radius with height above cloud base is less than that for an isolated parcel of air. Moreover, owing to an excess of small droplets, the observed distribution is considerably broader than the computed distribution for an isolated parcel (Warner 1969a, b). Fitzgerald (1974) finds that the computed distribution is insensitive to the detailed chemical composition of the cloud nuclei. However, he includes the effects of the molecular nature of the transfer of heat and mass across a droplet surface and this results in a somewhat broader distribution than that obtained when molecular accommodation is neglected - a result first noted by Warner (1969b).

Mason and Chien (1962) consider the effects of entrainment of clear air on the development of a cumulus droplet distribution. More detailed calculations by Warner (1973) indicate that the continuous entrainment of clear air results in a distribution with a narrow peak (similar to that for an isolated parcel) ahead of a plateau of entrained droplets, which extends to small sizes. However, the concentration of small droplets is about 10 times smaller than the observed concentration.

Baker et al. (1980) propose a mathematical model of entrainment which yields droplet distributions comparable to those observed by Warner (1969a). One of the assumptions of their model is that the entrainment occurs intermittently. They further postulate that at each entrainment event clear air first mixes with a small fraction of the cloud air and then the resultant air is mixed with the bulk of the cloud at that level. The first mixing process is presumed to result in the evaporation of a given fraction of all cloud droplets and to result in the entrained air becoming just saturated. Moreover, the total concentration of droplets is arbitrarily held constant. Thus the mathematical model contains more assumptions than are necessary to test the concept of intermittent mixing. In the present work we consider detailed calculations of the droplet distribution produced by intermittent mixing, as described qualitatively by Baker et al.

On the basis of observations of cloud microstructure Squires (1958) suggests that lateral entrainment of dry air is dominated by entrainment of air from the region above cloud top. Moreover, Warner (1970) finds that dynamical cloud models based on lateral entrainment do not adequately describe the cloud properties. If the air that dilutes a cloud parcel originates from above cloud top then the properties of the ‘entrained’ air at a given level within a cloud cannot be readily determined for a one-dimensional cloud model; that is, the local properties depend upon the history of the entrained air. We therefore consider the effect on the droplet distribution of arbitrarily relating the properties of the entrained air to the local cloud properties.
2. EQUATIONS FOR INTERMITTENT ENTRAINMENT

Baker et al. (1980) suggest that entrainment into a cumulus cloud occurs intermittently. Thus the adiabatic rise of a cloud parcel is interrupted by entrainment events, which recur with a period of $\tau$. In order to determine the development of the droplet distribution during an entrainment event we assume that the entrainment of mass $m_e$ of environmental air into a mass $m_c$ of cloud air occurs in two stages. First the two air masses, which are at the same pressure, exchange heat so that their temperatures are equalized before mixing takes place. The second stage is an instantaneous mixing of the air masses at uniform temperature and pressure.

The first stage is described by equations for the conservation of heat in each air mass in the form

$$m_c c_p c \frac{dT_c}{dT_e} = m_e L e \frac{d q_{we}}{dT_e} + dQ' ,$$

$$m_e c_p e \frac{dT_e}{dT_e} = m_c L e \frac{d q_{we}}{dT_e} - dQ' ,$$

where $T$ is temperature; $q_{we}$ is the mass fraction of liquid water; $c_p$ is the effective specific heat at constant pressure; $L$ is the latent heat of vaporization for water; $dQ'$ is the heat exchanged; the subscripts $c$ and $e$ refer respectively to cloud and environmental conditions. Introducing a pseudo-time variable $s$, Eq. (1) can be written as

$$c_p \frac{dT_c}{ds} = L e \frac{d q_{we}}{ds} + \frac{dQ}{ds} ,$$

$$c_p e \frac{dT_e}{ds} = L e \frac{d q_{we}}{ds} - \frac{dQ}{ds} ,$$

where $\varepsilon = m_e/m_c$ and $dQ = dQ'/m_c$.

The heat exchange term may be specified arbitrarily, and so to ensure numerical stability when integrating Eq. (2) we take

$$dQ/ds = \epsilon c_p \lambda(T_e - T_c) ,$$

where $\lambda = \min(1, \varepsilon)/(4 \Delta s)$; $\Delta s$ is the integration time step. Equation (3) ensures that thermal equilibrium between the air masses is reached within a sufficient number of time steps. In practice a temperature difference of less than $10^{-4}$ K is reached after 100 time steps. We note that the separate evolution of the droplet distribution in each air mass maintains a temperature difference of at least $10^{-5}$ K for several seconds.

The first (heat-exchange) stage of entrainment is therefore described by solving Eqs. (2)–(3), together with equations for the mass fractions of air, water vapour and liquid water, for the droplet distribution function and for the droplet radius in each air mass. These equations are integrated for an interval $\Delta \tau$, i.e. the heat-exchange time. The time $\Delta \tau$ is generally taken to be 0.1 s with an integration time step $\Delta s$ of 0.001 s, but the effects of using larger values of $\Delta \tau$ are discussed below.

The second stage of entrainment involves the instantaneous mixing of the two air masses. It follows from the conservation of mass that the final values of the mass fractions of dry air $q_d$ and water vapour $q_v$ and the droplet concentration $F_i$ of category $i$ are given by

$$q_d = (q_{d0} + \varepsilon q_{v0})/(1 + \varepsilon) ,$$

$$q_v = (q_{v0} + \varepsilon q_{d0})/(1 + \varepsilon) ,$$

$$F_i = (F_{i0} + \varepsilon F_{i0})/(1 + \varepsilon) .$$

Similarly the total density of the final mixture $\rho$ is found to be

$$\rho = \rho_d + (V_c/V_e) \rho_v/(1 + V_d/V_e) ,$$

where $V_d/V_e = \epsilon \rho_c/\rho_e$; $V_e$ and $V_c$ are the volumes occupied by each air mass before mixing takes place. Following the mixing the cloud parcel immediately commences its adiabatic
rise to the next level of entrainment, which takes place after a period \( \tau \). Thus an entrainment event occurs in the time interval \( \Delta \tau \) and any variations in the individual droplet radii take place during the heat-exchange stage.

The entrainment mass ratio \( \varepsilon \) is related to the average rate of entrainment \( \mu \) by the equation

\[
\mu = \varepsilon / \tau. \tag{6}
\]

The entrainment rate \( \mu \) is often chosen to vary with the dynamical properties of a cloud (or plume). However, the present study is not concerned with the prediction of the cloud dynamics. Because \( \mu \) is typically of order 0.001 s\(^{-1}\) (Warner 1973), all the present calculations are based on this value.

Equations (2)–(6) describe an entrainment event which involves a single mixing event. Baker et al. (1980) further propose that, owing to the fast response of the cloud microphysics compared with the cloud dynamics, an entrainment event may be modelled by two mixing events. The first event preconditions the environmental air before it is finally mixed with the bulk of the cloud. The first mixing is between a mass \( \varepsilon \) of environmental air and unit mass of cloud air. A mass \( \varepsilon_2 \) of the resultant air is then mixed with unit mass of cloud air. If the overall entrainment mass ratio \( \varepsilon \) and the mass ratio \( \varepsilon_1 \) are specified then it is readily found that

\[
\varepsilon_2 = \varepsilon (\varepsilon_1 + 1) / (\varepsilon_1 - \varepsilon). \tag{7}
\]

In the model of Baker et al. (1980) it is assumed that the air at the end of the first mixing event is just saturated. Thus \( \varepsilon_1 \) must be determined by iteration of the first mixing. Using the method of false position (Hamming 1973) we find that only about three iterations are required to obtain saturation to within \( 10^{-5} \) per cent. An alternative is simply to specify a priori the initial mass ratio \( \varepsilon_1 \), and this process is also discussed below.

3. Equations for continuous entrainment

Lateral entrainment into a cumulus cloud is usually assumed to occur continuously. The conservation of total water yields the equation

\[
\frac{dq_v}{dt} + \frac{dq_w}{dt} = - \mu (q_v + q_w - q_{ve} - q_{we}), \tag{8}
\]

where \( t \) is time, \( \mu \) is the entrainment rate and the superscript e refers to the environmental air. The mass fraction of dry air \( q_a \) is given by

\[
\frac{dq_a}{dt} = - \frac{dq_v}{dt} - \frac{dq_w}{dt}. \tag{9}
\]

The concentration per unit mass of air \( F_i \) of particles in category \( i \) is given by

\[
\frac{dF_i}{dt} = - \mu (F_i - F_{ie}). \tag{10}
\]

(We note that the distribution always contains some categories in which droplets are not larger than the equilibrium radius on the Köhler curve; these haze droplets may be referred to as ‘nuclei’.) The rate of change of the mass fraction of liquid water is represented by the equation

\[
\frac{dq_w}{dt} = 4\pi r_0 \sum_{i=1}^{n} F_i r_i^2 \frac{dr_i}{dt} - \mu (q_w - q_{we}). \tag{11}
\]
where \( r_i \) is the droplet radius for category \( i \) and \( \rho_0 \) is the density of water (1 g cm\(^{-3}\)).

The common approximation for the rate of change of droplet radius is (Mason and Chien 1962; Warner 1969b; Mason and Jonas 1974; Rodgers 1979)

\[
\rho_0 r_i \frac{dr_i}{dt} = \left\{ S - r_s/r_i + c_1(r_n/r_i)^3 \right\} \frac{L}{\left( \frac{R_e T}{\kappa T} - 1 \right) \left( \frac{L}{\kappa T} + \frac{R_e T}{D_p} \right)}
\]

where \( S = \rho_d/\rho_s - 1 \), \( r_s = 2\sigma / (\rho_0 R_e T) \), \( \rho_s = \rho_s R_e T \), \( c_1 = \nu \rho_n M_w / (\rho_0 M_n) \), \( \rho_s \) is the saturation density, \( \rho_s \) is the saturation pressure, \( \sigma \) is the surface tension between water and air, \( R_e \) is the gas constant for water vapour, \( r_n \) is the radius of the droplet nucleus, \( v \) is the van't Hoff factor, \( M_w \) is the molecular weight of water, \( M_n \) is the molecular weight of the soluble nucleus, \( \rho_n \) is the density of the nucleus, \( \kappa \) is the thermal conductivity of air and \( D \) is the diffusivity of water vapour in air. Equation (12) is derived under the assumption that the gas is a continuum up to the droplet surface. When the molecular nature of the heat and mass transfer across the surface is taken into account, the thermal conductivity \( \kappa \) and the diffusivity \( D \) in (12) must be modified (e.g. Pruppacher and Klett 1978). When molecular effects are included in the present work the accommodation coefficient is set equal to 0.7 and the condensation coefficient is 0.033.

When entrainment is included, the conservation of heat is expressed by

\[
c_p \frac{dT}{dt} = 4\pi \rho_0 L \sum_{i=1}^{n} F_i r_i^2 \frac{dr_i}{dt} - g \omega - \mu c_{pd}(T - T_a),
\]

where \( g \) is the gravitational acceleration and \( \omega \) is the vertical velocity. The updraught velocity is set at a representative value of 1 m s\(^{-1}\) in the present work.

Equations (8)–(13) are consistent with the intermittent-entrainment Eqs. (2) and (4) in the limit \( \varepsilon \to 0 \) with a finite value of \( \mu \). In particular Eq. (13) which includes the effects of the temperature change during the heat-exchange stage of an intermittent-entrainment event, implies that the entrainment of an infinitesimal parcel of environmental air produces a temperature change as the temperature of the entrained air is adjusted to match the cloud temperature. Latent heat changes in the entrained air are neglected because the environmental air is invariably so dry that there is no significant droplet growth during the heat-exchange stage. Integration of Eqs. (8)–(13) is shown below to yield results comparable with the more usual equations (e.g. Mason and Chien 1962), which include an explicit equation for the rate of change of supersaturation.

Equation (10) implies that the number of droplet categories grows continuously. This numerical problem is usually overcome by introducing new categories only at intervals of about 10 s (Warner 1973) or by assuming the entrained air to be devoid of nuclei but neglecting the consequent dilution of the cloud nuclei. In the present work we extend the interval over which new categories are explicitly introduced by using the formal solution of Eq. (10). It is seen that the concentration \( F \) of a cloud category at time \( t \) is given by

\[
F = F_0 \exp\{-\mu(t - t_0)\},
\]

where the concentration is \( F_0 \) at time \( t_0 \). On the other hand, the concentration of a category that is entrained at time \( t_0 \) is given by

\[
F = F_e[1 - \exp\{-\mu(t - t_0)\}],
\]

where \( F_e \) is the concentration of that category in the environment at time \( t_0 \). Thus Eq. (14) is used to predict the dilution of cloud categories that exist at time \( t_0 \), while Eq. (15) gradually introduces new categories that first appear at time \( t_0 \). (We note that not all the entrained nuclei become droplets.) The set of entrained categories is updated at regular intervals \( t_1 \). Although Eq. (15) is strictly valid only as \( t - t_0 \to 0 \), we find that provided that \( \mu t_1 \) is small
the calculated droplet distribution is rather insensitive to \( t_1 \). The results presented below are obtained with an update time \( t_1 \) of 50 s, but increasing \( t_1 \) to 100 s does not significantly affect the results.

### 4. Nucleus spectrum

The volume concentration of cloud condensation nuclei \( N \) is often related to the peak supersaturation \( S \) experienced by a cloud parcel by the equation (Squires and Twomey 1960)

\[
N = aS^k, \quad \text{where } a \text{ and } k \text{ are constants for a given air mass. If } f(r) \text{ is the droplet distribution density then we also have}
\]

\[
N(r_n) = \rho \int_{r_n}^{\infty} f(r) \, dr,
\]

where \( r_n \) is the radius of the smallest nucleated droplet. A droplet is just nucleated when the extremum of the Köhler curve corresponds to \( dr/dt = 0 \), and so we find from Eq. (12) that

\[
r_n = (r_n/3)(4/c_1S)^{1/2}.
\]

Hence Eqs. (16)–(18) can be combined to yield the theoretical droplet distribution density \( f \) where (e.g. Rodgers 1979)

\[
\rho f(r_n) = (3ak/2r_e)(4/(27c_1))^{k/2}(r_e/r_n)^{1+3k/2}.
\]

Thus the concentration \( F_i \) for droplets with nucleus radii between \( r_{i-1} \) and \( r_i \) is given by

\[
\rho F_i = a\{4/(27c_1)\}^{k/2}\{(r_e/r_i)^{3k/2} - (r_e/r_{i-1})^{3k/2}\}.
\]

The distribution (20) is used in the present work and the initial droplet radii at cloud base are taken to be the nucleus radii. The constants \( a \) and \( k \) in Eq. (16) are set respectively at 3170 cm\(^{-3}\) and 0·6; i.e. the cloud droplet concentration is 200 cm\(^{-3}\) when \( S \) is 1%. Thus the distribution is comparable with that used by Warner (1969b). The smallest nucleus radius is chosen such that some droplets are never nucleated, and so it is generally of order 0·020 \( \mu \)m. The largest nucleus radius \( r_1 \) is set at 0·55 \( \mu \)m. Moreover, no nuclei are assumed to exist with radii larger than 0·6 \( \mu \)m, and so Eq. (12) implies that the concentration \( F_1 \) is equal to 0·63 cm\(^{-3}\). No attempt is made to predict the behaviour of the largest droplets reported by Warner (1969b, Fig. 13). We note first that their concentration is of order 0·01 cm\(^{-3}\), while each curve is deduced from a sample volume of order 10\(^2\) cm\(^3\). Thus the true shape of the droplet distribution is uncertain in this region. Moreover the concentration of these large droplets is so small that they do not contribute significantly to the total liquid water content. They therefore do not affect the supersaturation of a given parcel of air, and they may be due simply to the presence of a few giant nuclei. The fundamental problem under consideration is the high concentration of small droplets.

### 5. Environmental air

The calculations of Warner (1973) assume that the environment in which the clouds grow has a lapse rate of about 7·5 K km\(^{-1}\) and a relative humidity of 80%. We therefore take the properties of environmental air that is laterally entrained to be

\[
T_e = 283 - 7·5z \quad \text{and} \quad q_{oe} = 7·32 - 2·40z,
\]

where \( T_e \) is in K, \( q_{oe} \) is in g kg\(^{-1}\) and \( z \) is in km. The distribution of condensation nuclei in the environment is given by Eq. (12), with the same properties as the nuclei at cloud base.
Entrained nuclei are assumed to have grown to their stable equilibrium point of the Köhler curve. Thus the largest haze droplets, corresponding to a nucleus of 0.55 μm, have a radius of about 0.8 μm in the environmental air.

6. RESULTS AND DISCUSSION

All the computed droplet distributions displayed below are obtained by summing the concentrations \( F_i \) of all droplet categories falling in 1-μm intervals. Thus, as occurs for observed distributions, some dispersion of the actual distribution is induced. The 1-μm intervals are centred on either whole or half-micron radii such that this dispersion is minimized. Calculations with 7, 18 and 31 droplet categories show that the range of the droplet spectrum is represented reasonably well by the seven-category solution at 400 m above cloud base for the case of an isolated cloud parcel. Moreover the solutions are found to compare well with the results of Warner (1973) with no entrainment.

![Droplet distribution density for an entraining cloud at 400 m above cloud base; no molecular accommodation effects; seven droplet categories at \( t = 0 \); average entrainment rate \( \mu \) is 0.001 s\(^{-1}\). —— continuous entrainment, category update time \( t_i \) is 50 s; ---- intermittent entrainment, one mixing event per entrainment event, period between entrainment events \( \tau \) is 50 s, mixing interval \( \Delta \tau \) is 0.1 s, entrainment mass ratio \( \epsilon \) is 0.05; ——— intermittent entrainment, two mixing events per entrainment event, \( \tau = 50 \) s, \( \Delta \tau = 0.1 \) s, \( \epsilon = 0.05, \epsilon_t = 1 \).

It is seen from Fig. 1 that the present model of continuous entrainment also gives results comparable with those of Warner (1973). There is a peak in the droplet distribution, corresponding to the essentially adiabatic growth of the initial cloud droplets, and a broad plateau extending from the peak to smaller droplet sizes. The plateau is produced by the nucleation of entrained aerosols. However, although continuous entrainment yields a broad
distribution, the concentration of small droplets is about 10 times lower than that observed by Warner (1969b).

It is clear from Fig. 1 that intermittent entrainment does not produce results significantly different from those obtained by continuous entrainment. Indeed there is little difference between the use of one-mixing-event and two-mixing-event entrainment models. The results in Fig. 1 are obtained with the period between entrainment events $\tau$ set at 50 s and with the heat-exchange time $\Delta \tau$ equal to 0.1 s. Increasing $\tau$ tends to shorten the plateau in the droplet distribution because the smallest drops catch up with the large ones during the longer periods of adiabatic rise. The environment is slightly cooler than the cloud and so all the cloud drops grow during the heat-exchange stage of entrainment. However, the temperature difference is not large enough to nucleate new droplets in the cloud air. Thus the droplet distribution tends to become narrower as $\Delta \tau$ is increased.

Figure 1 does not include a curve for two-mixing-event entrainment where the air is just saturated after the first mixing event. This is because there is no solution when $\tau = 50$ s and $\Delta \tau = 0.1$ s. Zero supersaturation is attained after the first mixing event at 50 s with $\varepsilon_1$ equal to 0.0135, which is less than the overall mixing mass ratio $\varepsilon$ of 0.05 required for $\mu$ to be 0.001 s$^{-1}$. Thus Eq. (7) implies that insufficient environmental air is entrained by the first mixing to satisfy the overall entrainment rate. This result elucidates the problem that zero supersaturation is rather ephemeral in an evolving system where large droplets respond much more slowly than small ones. We therefore replace the zero-supersaturation criterion
by an a priori specification of $\tau_1$, and Fig. 1 shows the resulting distribution when equal masses of clear and cloud air are mixed in the first mixing event, i.e. when $\tau_1 = 1$.

The present calculations suggest that intermittent entrainment cannot of itself yield broad droplet distributions like those observed by Warner (1969b). Thus the results of Baker et al. (1980) would seem to depend primarily on the assumption that entrainment causes uniform evaporation of a given fraction of all droplet categories. We note that no combination of the explicit mixing events described in Section 2 can produce this phenomenon. Further evidence that intermittent entrainment is unimportant to the results of Baker et al. is that they generally use an entrainment period $\tau$ of 10 s. Calculations with such a small value of $\tau$ are barely different from those with continuous entrainment. Indeed Warner (1973) finds it adequate to introduce new droplet categories only every 10 s in his computations on continuous entrainment. The preceding results suggest that lateral entrainment cannot generate a realistically broad droplet distribution, without the introduction of further radical assumptions such as that of Baker et al. (1980).

The total droplet concentration is observed to be essentially constant with height above cloud base (Warner 1969a). Moreover the concentration is found to be predicted well by the formula of Twomey (1959), which computes the peak supersaturation attained in the adiabatic rise of a cloud parcel. Thus the droplet concentration in a cumulus cloud appears to be determined by the cloud-base conditions. This suggests that entrained nuclei do not play a dominant role in the broadening of the droplet distribution which is seen to occur within the first few hundred metres above cloud base. Indeed the computed curves in Fig. 1 show that, although entrainment yields a broad spectrum of droplets the concentration in each entrained category is about 50 times less than the peak concentration of the original cloud-base droplets. At 400 m above cloud base (i.e. 400 s from $t = 0$) the original air makes up about 60% of the total cloud air when the entrainment rate $\mu$ is 0.001 s$^{-1}$, but only 10% of the total number of nucleated droplets are droplets that originated from entrained nuclei. Thus Fig. 2 shows that the peak of the droplet distribution at 400 m is well predicted when entrained nuclei are neglected – i.e. when the environment is assumed to be nucleus-free. We therefore generally consider nucleus-free entrainment for all the following results.

It would appear that the broadening of the droplet distribution must be explained by some effect on the original cloud-base droplets. Given that continuous entrainment is a reasonable representation of the mixing process it follows that either the equations for droplet growth are erroneous or the properties of the entrained air are not those of the lateral environment of a cumulus cloud. We first assume that the classical droplet-growth Eq. (12) is accurate and consider the effect of varying the entrained air. The laterally entrained air, which is generally a few tenths of a degree cooler than the cloud air and has a relative humidity of about 80%, does retard the growth rate of droplets and so produces a little broadening of the distribution. For example, the range of droplet sizes is from 11.2 to 12.5 $\mu$m for an isolated cloud parcel 400 m above cloud base, but is from 10.3 to 11.8 $\mu$m for the nucleus-free entrainment case in Fig. 2. This suggests that further reduction in the relative humidity of the entrained air may lead to a corresponding increase in the width of the droplet distribution. On the other hand, the entrainment of very dry air near cloud base decreases the peak supersaturation in the cloud parcel and so suppresses the nucleation of some small droplets. Since the adiabatic formula of Twomey (1959) appears to be accurate we therefore assume that entrainment can be neglected up to the level of peak supersaturation (i.e. 15 m above cloud base). This is followed by a 5-m region of linear transition such that the specified entrainment rule applies for $z$ greater than 20 m.

The environment through which cumulus clouds grow is readily observed and so the arbitrary specification of the properties of the entrained air requires some justification. There is some evidence (Squires 1958; Pulach 1979) that the entrained air originates from above or in the vicinity of cloud top rather than from the sides of a cloud. Cloud-top air is often quite dry and has a relatively high potential temperature. However, the precise properties of such air after being transferred by turbulence through the cloud to a given
level clearly depend upon the detailed dynamics of the cloud. A simple model of internal processing of entrained air is to assume that the local properties of entrained air depend upon the local cloud properties. We therefore consider the entrainment rule that the temperature of the entrained air equals the local cloud temperature while its specific humidity is a constant fraction of the cloud saturation specific humidity.

Figure 2 shows the droplet distribution 400 m above cloud base when the entrainment rule is that the entrained air has a relative humidity of 50%. The peak radius is seen to be much less than that for lateral entrainment of air having a relative humidity of 80%, and the distribution is significantly broader. The droplet radius extends from 4-6 to 8-3 μm. The retardation in droplet growth arises because the supersaturation $S$ is of order 0-05% for $z > 50$ m, while $S$ steadily decreases from 0-25 to 0-11% for $z$ between 50 and 400 m for lateral entrainment. Thus the relative humidity of the entrained air can greatly broaden the droplet distribution.

This result is explicable by consideration of a simplified version of the droplet growth equation. As shown by Warner (1969a) for example, at typical atmospheric pressure and temperature and when $r_i$ is much larger than $r_e$ and $r_m$, Eq. (12) simplifies to

$$r_e dr_i/dt \approx S,$$

where $r_i$ is in micrometres, $S$ in per cent and $t$ in seconds. This is a classical result which describes the decrease in growth rate of droplets with increasing size in a constant supersaturation. It predicts that a population of droplets of different sizes experiencing a fixed supersaturation will take up a gradually narrowing distribution as their mean size increases with time. By reducing that supersaturation a broader distribution of generally smaller drops is expected in the same time, as shown in Fig. 2. Of course, if growth at the same reduced supersaturation continues, the narrower distribution will be reached at some later time in the absence of entrained nuclei.

All the above results are computed with the classical droplet-growth Eq. (12), which neglects the effects of molecular accommodation at a droplet surface. Fukuta and Walter (1970) show that these effects retard the rate of growth of small droplets and so they have some broadening effect on the evolution of a droplet distribution. It is seen from Fig. 3 that at 400 m above cloud base the range of droplet radii in an isolated cloud parcel is from 9-8 to 11-9 μm, which is somewhat larger than the range of 11-2 to 12-5 μm when molecular accommodation is neglected (not shown in Fig. 3). The retarded growth rate induced by molecular accommodation causes the peak supersaturation to be larger (0-84 instead of 0-53%) and to occur further above cloud base (20 instead of 15 m). Thus more aerosol particles are nucleated (160 cm$^{-3}$ instead of 120 cm$^{-3}$) when molecular accommodation is taken into account, and this itself contributes to the broadening of the droplet distribution.

Continuous lateral entrainment of nucleus-free air further retards the movement of the peak of the droplet distribution and extends the range of radii from 8-6 to 11-2 μm at 400 m above cloud base (see Fig. 3). The distribution is much broader than the corresponding one in Fig. 2, where the radius range is from 10-3 to 11-8 μm. By introducing an arbitrary entrainment rule, which relates the properties of entrained air to the local cloud properties, we are able to generate distributions comparable with those observed by Warner (1969b) near cloud base. Figure 3 shows that if the entrained air has a relative humidity of 50% then the range of droplet radii at 400 m is from 3 to 8-2 μm for nucleus-free entrainment. When entrained nuclei are accounted for, the range extends from 1-5 to 8-3 μm. The distribution is found to be quite sensitive to the value of the specified relative humidity of the entrained air. If the relative humidity is reduced to 40% then the cloud becomes subsaturated for $z$ greater than 100 m. Increasing the relative humidity to 55% yields a radius range of 4-7 to 8-7 μm for nucleus-free entrainment; i.e. the minimum radius increases greatly because it is sensitive to the supersaturation.

Figure 4 compares the computed distribution at 150 m using the arbitrary entrainment rule with droplet distributions observed by Warner (1969b) between 100 and 200 m above cloud base. Although the peak of the computed distribution is somewhat retarded in comparison with the observations, the width of the distributions is similar. It would there-
Figure 3. Droplet distribution density at 400 m above cloud base when molecular accommodation effects are included. ——— isolated cloud parcel, 21 droplet categories; ——— nucleus-free lateral entrainment, $\mu = 0.001 \text{s}^{-1}$, 21 categories; ——— nucleus-free entrainment, $\mu = 0.001 \text{s}^{-1}$, 21 categories, entrained air properties related to cloud properties (50% r.h. for $z > 25 \text{m}$); ——— continuous entrainment, $\mu = 0.001 \text{s}^{-1}$, 10 initial categories, update time $t_s = 50 \text{s}$, entrained air properties related to cloud properties (50% r.h. for $z > 25 \text{m}$).

Figure 4. Droplet distribution density at 150 m above cloud base. ——— continuous entrainment, $\mu = 0.001 \text{s}^{-1}$, 10 initial categories, update time $t_s = 50 \text{s}$, entrained air properties related to cloud properties (50% r.h. for $z > 25 \text{m}$), molecular accommodation effects included; ——— observed distributions from Warner (1969b, Fig. 13).
Therefore appear that the shape of the main part of the droplet distribution near cloud base can be explained in classical terms, provided that molecular accommodation effects are included and that the relative humidity of the entrained air is low enough to hold the mean supersaturation to values of order 0.05%. Such low values of the supersaturation $S$ are not inconsistent with the measurements of Warner (1968), who finds that the median value of $S$ within 300 m of cloud base is less than 0.1% and that 30% of the data have $S$ less than 0.05%.

7. CONCLUSIONS

Detailed calculations of the behaviour of the droplet distribution imply that intermittent entrainment of clear air into a cumulus cloud does not yield results significantly different from those obtained under the usual assumption of continuous entrainment. The broad distributions calculated by Baker et al. (1980) would therefore seem to be caused primarily by the additional assumptions on the shape of the entrained droplet distribution.

Entrained nuclei within 400 m of cloud base account for no more than 10% of the total droplet concentration, and so any observed broadening of the droplet distribution above that which occurs in an isolated cloud parcel must be explained by effects upon the initial droplets nucleated near cloud base. This means that calculations may assume the entrained air to be nucleus-free, which leads to a substantial reduction in computational effort.

If the entrained air in a cumulus cloud originates from cloud top rather than from the sides of the cloud then the properties of the 'entrained' air at a given level depend upon the cloud dynamics and are not directly related to either the lateral or cloud-top environment. A simple model of the entrainment of air that is internally processed by a cloud is to assume that the properties of the entrained air depend only upon the local cloud properties. By arbitrarily selecting the relative humidity of the entrained air to be low enough to maintain the cloud supersaturation at values of order 0.05%, we find that the computed droplet distribution has a wide peak. Further broadening of the droplet distribution is achieved by including the effects of molecular accommodation at the droplet surface. Then the resultant distributions are comparable with the observations of Warner (1969b) near the base of cumulus clouds.

REFERENCES


