\[ \sum_{i=1}^{n_j} x_{ji} = r_j (\hat{\beta} + \hat{\gamma} + \hat{\delta} \cos(\omega + \phi)) + (\hat{\gamma} + \hat{\delta} \cos(\omega + \phi) - 1) \exp \left( \frac{\hat{\delta} \cos(\omega + \phi)}{\hat{\beta}} \right) \times \]

\[ \times \sum_{i=1}^{n_j} \exp \left( - \frac{x_{ji}}{\hat{\beta}} \right) \] = 0

for \( j = 1, 2, \ldots, 12 \).

This set of equations has then to be solved numerically.

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Christian et al. (1980) (hereafter referred to as C) further extended the work of Gaskell et al., (1978) (hereafter referred to as G) in measuring electrical and microphysical parameters in thunderclouds. The measurements reported by G were made at temperatures warmer than 0°C, while those reported by C were primarily colder than 0°C. The efforts to make these measurements, which are very difficult, should be commended and hopefully will encourage further investigations of this kind.

One important type of measurement they made (and the reason for this comment) was that of charge versus size of precipitation particles. This measurement was carried out with an instrument consisting of a combined induction ring and a shadowgraph size spectrometer mounted below the aircraft wing.

Among the many recorded events they reported were a few cases in which highly charged particles were observed. In fact, the charges these particles carried were so large that they exceeded the maximum charge that can be separated by the inductive-polarization mechanism in the presence of electrical fields of lightning magnitude. These observations were repeatedly quoted by them as important evidence suggesting the ineffectiveness of the induction-polarization mechanism as a major charge separation mechanism in clouds. It is not the purpose of this note to enter into the discussion of the effectiveness of one charging mechanism or the other, but rather to point out a possible problem with the measurements.

In the papers by G and C the authors did not discuss the possible effects of the aircraft on the

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measurements. It seems likely that cloud particles rebounding from and being shed from the propeller may have contaminated their measurements and led to erroneous conclusions.

As noted in these papers, the instrument was mounted below the aircraft wing some distance behind and to the side of the propeller blades. Particles which impact on or are shed from the rotating propeller blades are given tangential velocities upon leaving the blades. The velocities and final trajectories depend upon the duration of contact between the particles and the blade, the point of rebound, the rotation rate of the propeller, the speed of the plane, the phase of the impacting particles (liquid or solid), their size and their density.

In order to see the possible trajectories that particles may have after they leave the propeller, we calculate the trajectories of water drops and graupel particles of different radii following ejection from the tip of the propeller.

For simplicity, since the vertical distance that the particles fall due to gravity within the time frame considered here, is small, we can assume that the particles move in two dimensions. If \( V \) is the velocity of the particle at each point along its trajectory, then:

\[
\frac{dV}{dt} = F_{\text{drag}}
\]  

(1)

Let the air velocity relative to the plane be \( V_a \) (along the direction of flight) and the components of the particle's velocity be \( V_x \) and \( V_y \) which are perpendicular and parallel to the direction of flight, respectively. Under the conditions of high Reynolds number and turbulent flow past a spherical particle (Batchelor 1970):

\[
|F_{\text{drag}}| = 0.5 C_D \pi R^2 \rho_a |V|^2
\]  

(2)

where \( R \) is the particle's radius, \( \rho_a \) the density of air and \( C_D \) is the drag coefficient.

Looking at the components of Eq. (1) and noting that \( m = \frac{4}{3} \pi R^3 \rho \), where \( \rho \) is the density of the particle, we get:

\[
dV_x = - \frac{3 C_D \rho_a}{8 \rho R} V_x |V| dt
\]  

(3)

\[
dV_y = \frac{3 C_D \rho_a}{8 \rho R} |V|(V_a - V_x) dt
\]  

(4)

![Figure 1. Calculated trajectories of water drops (solid curves) and graupel particles (dashed curves) of different radii after leaving the propeller tip with an initial velocity of \( V_a = 253 \text{ m s}^{-1} \), and with the blade perpendicular to the plane of the wings. Particles leaving the blade after it passes the vertical position will be moving with the same trajectories but on surfaces which intersect the plane of the wings at different angles, depending on the position of the tip when the particles depart.](image)
We also note that:

$$|V| = \sqrt{V_x^2 + (V_y - V_0)^2}$$

(5)

The trajectories are obtained after numerically integrating the above equations. The results are presented in Fig. 1. These calculations were carried out with actual measurements (Christian and Moore, private communication) of the radius (96.6 cm) and rotation speed (2500 RPM) of the propeller, and of lateral (2-00 m) and longitudinal (2-62 m) distance from the tip of the propeller to the instrument. A constant drag coefficient ($C_D = 0.4$) was used, since the Reynolds numbers exceed 1000 (e.g. Batchelor 1970) for most particles in the region of interest. The calculations are not meant to be exact, but are presented to demonstrate the types of trajectories that might be expected.

The maximum velocity that could be imparted to a particle leaving the extreme tip of the blade is 253 m s⁻¹. Particles rebounding or shedding from other points along the length of the blade will be given smaller initial velocities. Also, many particles may receive only a fraction of the maximum rebounding energy because of grazing contact with the blade. In addition, particles of different densities (water and ice) will also have different trajectories. In order to demonstrate some of the possible trajectories in Fig. 1, the trajectories are shown for water drops and graupel particles (density of 500 kg m⁻³) coming off with maximum velocity (253 m s⁻¹). Figure 1 demonstrates that both small and large particles can be deflected into the space close to the charge/size instrument depending upon the particle’s density. Obviously, particles that bounce from points on the propeller which are closer to its centre will have smaller initial velocities and hence will not move laterally as far as is shown in Fig. 1. This way even larger particles will be erroneously recorded by the charge/size instrument. The location of the instrument in the plane of the wings is shown in Fig. 1.

Collisions of cloud particles with the propeller can generate new particles in several ways. At temperatures warmer than 0°C cloud droplets collected by the propeller can run to the tip and be shed into drizzle-size drops, thereby producing particles larger than the original cloud droplets. Precipitation-sized particles impacting on the blades may either splash or be broken up, thereby creating more small particles. At temperatures below 0°C ice particles too can be given lateral velocities upon impact with the blades. Such collisions can also modify the charge on the particles. Since the aircraft is often highly charged, shedding or rebounding particles will carry away some of the charge, which, under these conditions, may be higher than the maximum charge predicted by the inductive mechanism even in the presence of high fields. The charge carried by these particles could be either positive or negative, depending upon the sign of the charge on the aircraft, the characteristics of the collision and perhaps local temperature and electric field.

Because of the strong dependence of maximum charge on size ($q \sim f(R^2)$) predicted by the inductive mechanism, smaller particles are expected to carry relatively little charge. However, collision and shedding of particles from the propeller can easily produce high charge on small particles. Thus, if the propeller is introducing artefacts into the charge-size measurements, the effects would be most obvious for the smaller particles. It is in primarily this region where the measurements of G and C show the measured charge to be more than the maximum predicted by the inductive mechanism.

It is difficult to know the degree to which the measurements of G and C may be contaminated by artefacts shed from the aircraft. However, because of the close lateral proximity of the charge/size instrument to the propeller of the aircraft, it seems quite possible that particles shed by the propeller could pass through the sample area of the instrument and be mistaken as representing those in the unaffected cloud space. Shedding of either liquid water or fragments of ice particles from the propeller might readily explain the high charges on particles less than 1 mm diameter reported by G in Fig. 8.

Because of the possible contamination in the charge/size measurements reported by G and C, additional charge/size measurements need to be made from a platform where artefacts can be positively ruled out before conclusions should be drawn with regard to observed charge on particles and the effectiveness of the inductive mechanism.

**References**

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Drs Levin and Dye have raised an important question which should have been addressed in the papers describing the field experiments. If hydrometers are shed from or hit by the propeller then we must consider the possibility that they may subsequently pass through the induction ring and that their spurious charges may be measured. The accompanying note by Moore, Bullock and Winn describes field tests which show that for liquid drops any such centrifuging is entirely absent. The measurements of Gaskell et al. (1978) were made at temperatures warmer than 0°C and thus should not have been affected by the propeller.

The flights reported by Christian et al. (1980) were made at temperatures colder than 0°C and attention was drawn to penetrations B and C (Figs. 12 and 14) of 15 August 1977 in which charges higher than those predicted by the inductive mechanism were measured on particles of diameter between 1 and 2 mm. Firstly, we shall estimate the increase in flux of particles incident upon the charge-size sensor due to interactions with the propeller, and secondly, calculate the probability that these particles can then pass cleanly through the charge amplifier without collision so that the signal from the charge amplifier is acceptable.

The de-icing heaters on the propeller were always used and so no aggregate of ice could build up and then be flung sideways, but particles could be accelerated by a single blow of the propeller in the appropriate direction. Any particle impacting upon the face of the propeller blade will not receive the required radial velocity, so we must consider those incident upon the edge of the blade. If we assume this to be 1 cm thick, and knowing that the propeller turns 42 times a second in which time the aircraft moves forward 50 m then we see that about 1% of the particles in the volume swept out by the propeller may have been hit. According to Levin and Dye’s Figure they may be redistributed up to a distance of 5 metres from the aircraft centre line, that is an area 25 times greater than that swept out by the propeller, which reduces the flux increase to less than 0.1%.

Levin and Dye’s calculations also show that after it hits the propeller the first few metres of the trajectory of a 1 or 2 mm particle is almost a straight line. If the particles are to receive a radial velocity in the horizontal direction they must be hit by the propeller blade when it is approximately vertical. Since the sensor was 262 cm behind the propeller arc, and 200 cm outboard of the aircraft centre line, then these particles should approach the sensor at an angle of 37° to its axis. The diameter of the induction ring was equal to its length so at 45° incidence no particles can pass cleanly through and at 37° the fraction has reached 0.17.

Because the fractional increase in flux is so low and few of these particles would pass cleanly through the device we think it is extremely unlikely that the data in the two Figures are significantly affected by the presence of the propeller. However, it is certainly preferable to site the device further outboard so that interference is impossible.

Another potential source of sampling ambiguity has been identified by Dr P. Ryder (Meteorological Office, Personal Communication). Whereas the charge on all particles passing through the induction ring is detected, only a fraction of these particles passing through the light beam is sized. It is thus possible that a single reported coincidence apparently due to a highly charged small particle could arise from the presence of two particles simultaneously in the device. One particle is highly charged but although large it is not sized because it does not pass through the beam; the second particle is uncharged and small and it is correctly sized. To calculate the probability of this occurring, we note that for a rainfall rate of 10–20 mm h⁻¹ the Marshall-Palmer distribution would predict about 800 m⁻³ drops of diameter above 0.5 mm, and the sample volume of the device is about 125 cc or (1/8000) m⁻³. From Poisson statistics if the charge on a large particle is being measured there is a 5% chance of another drop of size larger than 0.5 mm being present in the device. The actual percentage of spurious coincidences will be lower than this because the measured concentrations of drops > 0.5 mm was nearer 150 m⁻³, and so does not appear to be a serious problem.

References

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