Simulation of storm surges using a three-dimensional numerical model: an application to the 1977 Andhra cyclone

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SUMMARY

A three-dimensional numerical model is developed for the simulation of storm surges generated by tropical cyclones off the east coast of India. Experiments are performed using wind-stress forcing data representative of the 1977 Andhra cyclone and the results are compared with earlier simulations using a depth-averaged model. The three-dimensional model incorporates a turbulence energy closure scheme and is fairly sophisticated in comparison with the depth-averaged approach. Even with the apparently more refined treatment of the frictional processes, there is no substantial difference between simulations performed with the two models.

1. INTRODUCTION

The numerical simulation of storm surges in the Bay of Bengal has been considered by Johns and Ali (1980) and Johns et al. (1981) (hereafter referred to as (I)). In each of these investigations, the dynamical processes were modelled using depth-averaged equations for the horizontal motion. The modelling approach was therefore similar to that employed by various contributors during the last few years and is well documented (Nihoul and Ronday 1976). The simplifying assumptions made in reducing the fully three-dimensional equations to the conventionally used depth-averaged form are discussed by Johns (1981). In particular, these relate to the omission of certain non-linear terms of advective origin in the hydrodynamical equations together with a parametrization of the bottom stress in terms of the depth-averaged motion. In shallow water regions, the omission of these non-linear terms is strictly justifiable only if there is negligible vertical structure in a fluid column. In the case of storm surge phenomena, this seems unlikely as the momentum supporting the flow is transferred across the sea-surface from the atmospheric winds and is communicated to greater depths by vertical turbulent mixing. Thus, we would expect a significant vertical current-structure to evolve during this process. The conventional use of an empirically-based quadratic friction law involving the depth-averaged current raises further uncertainties as to the viability of the depth-averaged model. This is because of the occurrence of a disposable friction coefficient that must be assigned a numerical value together with the fact that such a law presupposes that the bottom stress is a function of purely local flow conditions.

The recognition of these potential deficiencies in the depth-averaged approach has led to the development of mathematical techniques based upon the use of the fully three-dimensional equations. For example, Jelesnianski (1970) and Forristall (1974) have investigated the possibility of combining a two-dimensional depth-averaged model (without non-linear advection) with a locally one-dimensional Ekman model. However, a weakness in this approach is the retention of empirical bottom friction as well as the specification of a constant vertical-exchange coefficient. Moreover, the technique of calculating the vertical structure by a local Ekman model may not be valid in shallow water where non-linear momentum advection is more likely to be significant.

A further contribution to three-dimensional modelling has been made by Nihoul (1977) who also gives a method of combining a depth-averaged and an Ekman model in order to deduce the vertical current-structure at each horizontal point. This study is characterized by the prescription of the form of a depth-dependent vertical-exchange coefficient. The non-linear advective terms may be included by an iterative procedure.
There is, therefore, a reduced empirical input with reference to the bottom stress although
the prescribed form of the vertical-exchange coefficient still represents an undesirable
element of fairly arbitrary empiricism.

In more recent three-dimensional modelling, Davies (1981) has given a method of
solution in which the vertical-exchange coefficient may be related to the depth-averaged
current, this latter quantity being itself determined from the fully three-dimensional
model. This represents a notable advance but it must be emphasized that the method
again employs an empirical bottom friction law as part of the sea-floor boundary condi-
tions. Work reported by Heaps and Jones (1981) is also characterized by a choice of
vertical-exchange coefficient dependent upon the bottom stress being given by an empiri-
cal quadratic law.

It will therefore be noted that none of these three-dimensional models is sufficiently
free from depth-averaged empiricism to provide an independent appraisal of the two
modelling procedures.

Naturally, the final arbiter when deciding upon the merits of different models must
be a comparison with a reliable set of observations. Storm surges in the Bay of Bengal
have, for obvious reasons, not been monitored by tide gauges with the same precision as,
for example, those in the North Sea. However, in (1), a simulation of the surge generated
by the 1977 Andhra cyclone was compared with estimates of flooding that occurred along
part of the coast of Andhra Pradesh. In that study, the value chosen for the bottom
friction coefficient and the validity of the depth-averaging procedure are far from being
uncontentious. Accordingly, in the present work we have developed a new fully three-
dimensional model in order to carry out an independent simulation of the phenomenon.
The model is based upon the turbulence energy closure scheme applied by Johns (1978) in
a simulation of the tidal flow in a channel. Consequently, the formulation of the frictional
mechanism in this new model is conceptually quite different from that used in the depth-
averaged approach. Therefore, a meaningful comparison may be made between simula-
tions using two completely independent modelling procedures.

The results of this comparison are at variance with some of our earlier thinking. In
this, we had surmised that the shallow water evolution of the surge response would be
markedly different in the three-dimensional model because of the full representation of the
vertical current-structure. Further, we had expected that the sea-surface surge response
would be critically dependent on the value chosen for the bottom roughness parameter in
the three-dimensional model. Both of these conjectures proved to be incorrect and we
found a remarkable qualitative and quantitative similarity between the two simulations.
This leads us to conclude that a depth-averaged model is as effective as a fairly sophisti-
cated three-dimensional model in storm surge simulation experiments. Therefore, if the
vertical current-structure is not a primary concern, it does not appear worthwhile replac-
ing the depth-averaged procedure by a more complicated technique with its attendant
substantial increase in computational overheads.

2. FORMULATION

The sphericity of the earth is neglected and we use a system of rectangular Cartesian
coordinates in which the origin, O, is within the equilibrium level of the sea-surface. Ox
points east, Oy north, and Oz is directed vertically upwards. The displaced position of
the sea-surface is given by \( z = \zeta(x, y, t) \) and the position of the sea-floor by \( z = -h(x, y) \). A
western coastal boundary is situated at \( x = b(y) \) whilst there are open-sea boundaries at
\( x = b(y), y = 0 \) and \( y = L \).

The Reynolds-averaged components of velocity, \( u, v \) and \( w \), then satisfy

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - fv = -g \frac{\partial \zeta}{\partial x} + \frac{1}{\rho} \frac{\partial \sigma_x}{\partial z} \tag{1}
\]
\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + fu = -g \frac{\partial \zeta}{\partial y} + \frac{1}{\rho} \frac{\partial \tau_x}{\partial z}. \tag{2}
\]

In Eqs. (1) and (2), \( f \) denotes the Coriolis parameter, the pressure is taken as hydrostatic, \( \rho \) is the density of homogeneous sea-water and \( \tau_x \) and \( \tau_y \) denote the components of horizontal Reynolds stress in the \( x \) and \( y \) directions. We have also omitted the effect of the astronomical tide-generating forces and barometric forcing.

As in (1), the boundary condition at the western coastline leads to
\[
u - vb_1(y) = 0 \quad \text{at} \quad x = b_1(y) \tag{3}
\]
whilst we apply modified versions of our radiation boundary conditions along the open-sea boundaries. These are
\[
\tilde{u} - \tilde{v}b_2(y) - (g/h)^{1/2} \zeta = 0 \quad \text{at} \quad x = b_2(y) \tag{4}
\]
\[
\tilde{v} + (g/h)^{1/2} \zeta = 0 \quad \text{at} \quad y = 0 \tag{5}
\]
and
\[
\tilde{v} - (g/h)^{1/2} \zeta = 0 \quad \text{at} \quad y = L. \tag{6}
\]

In Eqs. (4), (5) and (6), \( \tilde{u} \) and \( \tilde{v} \) denote depth-averaged components of velocity defined by
\[
(\tilde{u}, \tilde{v}) = \frac{1}{H} \int_{-h}^{h} (u, v) \, dz, \tag{7}
\]
where \( H \) is the total depth, \( \zeta + h \).

Boundary conditions at the impermeable sea-floor relate to an absence of fluid slippage and lead to
\[
u = v = w = 0 \quad \text{at} \quad z = -h. \tag{8}
\]

At the sea-surface, the internal Reynolds stress must equal the applied surface wind-stress \( (\tau_x', \tau_y') \). We must therefore have
\[
\begin{align*}
\tau_x &= \tau_x' \\
\tau_y &= \tau_y'
\end{align*} \quad \text{at} \quad z = \zeta. \tag{9}
\]

Also, the surface kinematical condition requires that
\[
\frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} - w = 0 \quad \text{at} \quad z = \zeta. \tag{10}
\]

To accompany (1) and (2), the equation of continuity has the vertically integrated form
\[
\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} (H\tilde{u}) + \frac{\partial}{\partial y} (H\tilde{v}) = 0. \tag{11}
\]

The present formulation is therefore a three-dimensional analogue of the depth-averaged coastal zone model developed in (1) with the inclusion of a modified treatment of conditions along \( y = L \) following that given by Johns et al. (1982). However, (1), (2) and (11) do not in themselves constitute a closed set of equations for the determination of \( u, v \) and \( \zeta \). In (1), closure was effectively attained by considering depth-averaged versions of (1) and (2) and relating the bottom stress to the depth-averaged velocity \( (\tilde{u}, \tilde{v}) \) by an empirically based quadratic friction law containing a disposable friction parameter. In the present work, we follow a procedure applied by Johns (1978) to the calculation of tidal flow in a channel. This involves the parametrization of the internal Reynolds stress in terms of vertical velocity gradients and the turbulence energy density, \( E \), and leads to
\[
\begin{align*}
\tau_x &= K\rho \frac{\partial u}{\partial z} \\
\tau_y &= K\rho \frac{\partial v}{\partial z}
\end{align*} \tag{12}
\]
where the exchange coefficient, $K$, is given by $K = c^{1/4}E^{1/2}$. The constant, $c$, is recommended by Launder and Spalding (1972) as 0.8 and the length scale, $l$, of the vertical mixing process is determined from the similarity law

$$l = \frac{-\kappa E^{1/2}z^{-1}}{d(E^{1/2}l^{-1})/dz}, \quad l = \kappa z_0 \quad \text{at} \quad z = -h. \quad (13)$$

In (13), $\kappa$ is von Kármán’s constant (taken as 0.4) and $z_0$ is the roughness parameter for the elements at the sea-floor. The turbulence energy density satisfies a transport equation describing the ways in which turbulence is generated, dissipated and re-distributed. This has the form

$$\frac{\partial E}{\partial t} + u \frac{\partial E}{\partial x} + v \frac{\partial E}{\partial y} + w \frac{\partial E}{\partial z} = K \left\{ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right\} + \frac{\partial}{\partial z} \left( K \frac{\partial E}{\partial z} \right) - \varepsilon. \quad (14)$$

The first term on the right-hand side of (14) represents the effect of turbulence generation by energy extraction from the Reynolds-averaged flow. The second term represents a vertical turbulent re-distribution of turbulence energy which is assumed to follow a gradient transfer law with the same exchange coefficient as that for momentum exchange. The final term, $\varepsilon$, represents a dissipative effect and is parametrized according to $\varepsilon = c^{3/4}E^{3/2}/l$.

Additional boundary conditions to accompany (14) relate to an assumed absence of turbulence energy transfer across the sea-floor and the sea-surface. These lead to

$$\frac{\partial E}{\partial z} = 0 \quad \text{at} \quad z = -h \quad (15)$$

and

$$\frac{\partial E}{\partial z} = 0 \quad \text{at} \quad z = \zeta. \quad (16)$$

### 3. Coordinate Transformation

The transformation of the coordinates described in this section is a combination of those given in (I) and Johns (1978). The horizontal coordinates are transformed so as to facilitate the implementation of (3) and (4) and this involves the introduction of a new variable, $\xi$, defined by

$$\xi = (x - b_1(y))/b(y) \quad (17)$$

where $b(y) = b_2(y) - b_1(y)$.

The vertical coordinate is similarly transformed to facilitate the implementation of boundary conditions at the sea-floor and the sea-surface. This leads to a new variable, $\sigma$, defined by $\sigma = (z + H)/H$. Taking $\zeta$, $y$, $\sigma$ and $t$ as new independent variables, (1) and (2) then lead to

$$\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial \zeta} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial \sigma} - fu = -\frac{g}{b(y)} \frac{\partial \zeta}{\partial y} + \frac{1}{H \rho} \frac{\partial \tau_x}{\partial \sigma} \quad (18)$$

and

$$\frac{\partial \zeta}{\partial t} + U \frac{\partial \zeta}{\partial \zeta} + v \frac{\partial \zeta}{\partial y} + \omega \frac{\partial \zeta}{\partial \sigma} + fu = -\frac{g}{b(y)} \{b_1'(y) + \xi b'(y)\} \frac{\partial \zeta}{\partial \xi} + \frac{1}{H \rho} \frac{\partial \tau_x}{\partial \sigma}. \quad (19)$$

In (18) and (19),

$$U = \{u - (b_1' + \xi b'\nu)/b(y) \quad (20)$$

and

$$\omega = \sigma_t + u \sigma_y + v \sigma_x + w \sigma_z, \quad (21)$$

where primes and subscripts refer to differentiation.

Eq. (11) leads to
\[
\frac{\partial}{\partial t} (b\zeta) + \frac{\partial}{\partial x} (bH\bar{U}) + \frac{\partial}{\partial y} (bH\bar{v}) = 0,
\]

where \( \bar{U} \) denotes the depth-averaged value of \( U \). Following Johns (1978), \( \omega \) is readily found to satisfy the diagnostic equation

\[
\frac{\partial}{\partial \zeta} \{bH(U - \bar{U})\} + \frac{\partial}{\partial y} \{bH(v - \bar{v})\} + \frac{\partial}{\partial \sigma} (bH\omega) = 0.
\]

In like manner, the energy equation (14) may be transformed to yield

\[
\frac{\partial E}{\partial t} + U \frac{\partial E}{\partial \zeta} + v \frac{\partial E}{\partial y} + \omega \frac{\partial E}{\partial \sigma} = \frac{K}{H^2} \left\{ \left( \frac{\partial \bar{u}}{\partial \sigma} \right)^2 + \left( \frac{\partial \bar{v}}{\partial \sigma} \right)^2 \right\} + \frac{1}{H^2} \frac{\partial}{\partial \sigma} \left( K \frac{\partial E}{\partial \sigma} \right) - \varepsilon.
\](24)

The advantage of this coordinate transformation is that the lateral, sea-floor and sea-surface boundary conditions are expressible in a form ideally suited to our subsequent numerical treatment. Specifically, (3) and (4) are satisfied provided that

\[
U = 0 \quad \text{at} \quad \zeta = 0
\]

and

\[
b(y)\bar{U} - (g/h)^{1/2}\zeta = 0 \quad \text{at} \quad \zeta = 1.
\]

The sea-floor conditions (8) are equivalent to

\[
u = v = \omega = 0 \quad \text{at} \quad \sigma = 0,
\]

and the sea-surface conditions (9), when combined with (12), lead to

\[
\begin{align*}
K \frac{\partial \bar{u}}{\partial \sigma} &= H \bar{T}\zeta/\rho \\
K \frac{\partial \bar{v}}{\partial \sigma} &= H \bar{T}\zeta/\rho
\end{align*}
\]

at \( \sigma = 1 \). (25)

The kinematical condition (10) is equivalent to

\[
\omega = 0 \quad \text{at} \quad \sigma = 1,
\]

and the energy conditions (15) and (16) are satisfied provided that

\[
\partial E/\partial \sigma = 0 \quad \text{at} \quad \sigma = 0 \text{ and } \sigma = 1.
\]

Naturally, (5) and (6) remain unaltered by the coordinate transformation.

Finally, it is convenient to define new prognostic variables by

\[
\begin{align*}
\tilde{u} &= bHu \\
\tilde{v} &= bHv,
\end{align*}
\]

and to re-express (18) and (19) in a conservation form having first introduced into these the parametrization (12). We then find that

\[
\frac{\partial \tilde{u}}{\partial t} + \frac{\partial}{\partial \zeta} (U\tilde{u}) + \frac{\partial}{\partial y} (v\tilde{u}) + \frac{\partial}{\partial \sigma} (\omega\tilde{u}) - f\tilde{v} = -gH \frac{\partial \zeta}{\partial \sigma} \frac{\partial \tilde{u}}{\partial \sigma} + \frac{1}{H^2} \frac{\partial}{\partial \sigma} \left( K \frac{\partial \tilde{u}}{\partial \sigma} \right)
\]

and

\[
\frac{\partial \tilde{v}}{\partial t} + \frac{\partial}{\partial \zeta} (U\tilde{v}) + \frac{\partial}{\partial y} (v\tilde{v}) + \frac{\partial}{\partial \sigma} (\omega\tilde{v}) + f\tilde{u} = -gH \left\{ b \frac{\partial \zeta}{\partial y} - (b' + \xi b') \frac{\partial \zeta}{\partial \zeta} \right\} + \frac{1}{H^2} \frac{\partial}{\partial \sigma} \left( K \frac{\partial \tilde{v}}{\partial \sigma} \right).
\]

Likewise, the introduction of a prognostic energy variable, \( \tilde{E} \), defined by \( \tilde{E} = bHE \), leads to a conservation form of (24) in which
\[
\frac{\partial \tilde{E}}{\partial t} + \frac{\partial}{\partial \zeta} (U \tilde{E}) + \frac{\partial}{\partial y} (\nu \tilde{E}) + \frac{\partial}{\partial \sigma} (\omega \tilde{E}) = \frac{K}{H^3 b} \left( \left( \frac{\partial \tilde{u}}{\partial \sigma} \right)^2 + \left( \frac{\partial \tilde{v}}{\partial \sigma} \right)^2 \right) + \\
+ \frac{1}{H^2} \frac{\partial}{\partial \sigma} \left( K \frac{\partial \tilde{E}}{\partial \sigma} \right) - b H \tilde{e}.
\] (29)

In terms of this ‘\( \sigma \)-coordinate’ formulation, (7) yields the depth-averaged current in the form

\[
(\tilde{u}, \tilde{v}) = \int_0^1 (u, v) \, d\sigma.
\] (30)

The length-scale of the vertical mixing process is obtained from (13) as

\[
l = \kappa \left( \left( \frac{E}{E_0} \right)^{1/2} z_0 + H E^{1/2} \int_0^\sigma E^{-1/2} \, d\sigma \right),
\] (31)

where \( E_0 \) denotes the turbulence energy density adjacent to the sea-floor.

It will be noted that the ‘\( \sigma \)-coordinate’ formulation is characterized by the sea-floor and sea-surface boundary conditions being applied at \( \sigma = 0 \) and \( \sigma = 1 \), respectively. Accordingly, a finite-difference discretization of this new vertical coordinate will lead to a readily implemented numerical scheme of solution. However, our experience has shown that a fine grid-spacing is required both near the sea-floor and the sea-surface in order to resolve adequately the high shears that occur in these regions – especially that near \( \sigma = 0 \). It is therefore expedient to introduce a further transformation of the vertical coordinate to achieve this end. An appropriate form of this is given by

\[
\sigma + \sigma_0 = \sigma_0 \exp \{ \psi(\eta) \},
\] (32)

where \( \sigma_0 (\ll 1) \) is a disposable parameter,

\[
\psi(\eta) = \eta - \frac{1}{2 \eta_s} (1 - \sigma_0) \eta^2
\] (33)

and

\[
\eta_s = \frac{2 \ln(1 + 1/\sigma_0)}{1 + \sigma_0}.
\] (34)

From (32), (33) and (34), it is readily seen that finite-difference increments \( \Delta \sigma \) and \( \Delta \eta \) are related by

\[
\Delta \sigma = \sigma_0 \left\{ 1 - \frac{1}{\eta_s} (1 - \sigma_0) \eta \right\} \exp \{ \psi(\eta) \} \Delta \eta.
\] (35)

Consequently, for a fixed value of \( \Delta \eta \), the corresponding value of \( \Delta \sigma \) is reduced near \( \sigma = 0 \) and \( \sigma = 1 \) in comparison with its value in the mid-depths. Then, by writing \( \beta(\eta) = \partial \sigma/\partial \eta \), we may further transform (27) and (28) to obtain

\[
\frac{\partial \tilde{u}}{\partial t} + \frac{\partial}{\partial \zeta} (U \tilde{u}) + \frac{\partial}{\partial y} (\nu \tilde{u}) + \frac{1}{\beta} \frac{\partial}{\partial \eta} (\omega \tilde{u}) - f \tilde{v} = -g H \frac{\partial \xi}{\partial \eta} + \frac{1}{H^2 \beta} \frac{\partial}{\partial \eta} \left( \phi \frac{\partial \tilde{u}}{\partial \eta} \right)
\] (36)

and

\[
\frac{\partial \tilde{v}}{\partial t} + \frac{\partial}{\partial \zeta} (U \tilde{v}) + \frac{\partial}{\partial y} (\nu \tilde{v}) + \frac{1}{\beta} \frac{\partial}{\partial \eta} (\omega \tilde{v}) + f \tilde{u} = -g H \left\{ b \frac{\partial \xi}{\partial y} - (b' + \xi \beta') \frac{\partial \xi}{\partial \zeta} \right\} + \\
+ \frac{1}{H^2 \beta} \frac{\partial}{\partial \eta} \left( \phi \frac{\partial \tilde{v}}{\partial \eta} \right).
\] (37)
where \( \phi = K/\beta \). Eq. (29) leads to

\[
\frac{\partial \tilde{E}}{\partial t} + \frac{\partial}{\partial \xi} (U \tilde{E}) + \frac{\partial}{\partial \eta} (v \tilde{E}) + \frac{1}{\beta} \frac{\partial}{\partial \eta} (\omega \tilde{E}) = -\frac{\phi}{H^2 b \beta} \left\{ \left( \frac{\partial \tilde{u}}{\partial \eta} \right)^2 + \left( \frac{\partial \tilde{v}}{\partial \eta} \right)^2 \right\} + \]

\[
+ \frac{1}{H^2 b \beta} \frac{\partial}{\partial \eta} \left( \phi \frac{\partial \tilde{E}}{\partial \eta} \right) - b He, \quad (38)
\]

and (30) and (31) show that the depth-averaged velocity and the length-scale are determined from

\[
(\tilde{u}, \tilde{v}) = \int_{0}^{\eta} (u, v) \beta(\eta) \, d\eta \quad (39)
\]

and

\[
l = \kappa \left\{ \left( \frac{E}{E_0} \right)^{1/2} z_0 + HE^{1/2} \int_{0}^{\eta} E^{-1/2} \beta(\eta) \, d\eta \right\}. \quad (40)
\]

The applied surface wind-stress conditions (25) accompanying (18) and (19) lead to

\[
\begin{align*}
\frac{\partial \tilde{u}}{\partial \eta} &= \frac{bH^2 \tau_{\xi}^x}{\rho \phi} \quad \text{at} \quad \eta = \eta_s, \\
\frac{\partial \tilde{v}}{\partial \eta} &= \frac{bH^2 \tau_{\eta}^y}{\rho \phi}
\end{align*}
\quad (41)
\]

whilst, in the '\( \eta \)-coordinate' formulation, the other sea-floor and sea-surface conditions have obvious forms.

4. Numerical solution

The finite-difference discretization of the horizontal coordinates has been described in detail in (1) and it is unnecessary to repeat this here. It is, however, appropriate that we describe the essentials of the discretization process as applied to the vertical coordinate, \( \eta \).

The most important features of this relate to the treatment of the diffusion terms.

In so far as the vertical coordinate and the time variable are concerned, we write

\[
\eta = \eta_k = (k - 1) \Delta \eta, \quad k = 1, 2, \ldots, s, \quad \Delta \eta = \eta_0/(s - 1)
\]

\[
t = t_p = p \Delta t, \quad p = 0, 1, \ldots \quad (42)
\]

and, for any variable \( \chi \),

\[
\chi(\xi, \eta, \eta_k, t_p) = \chi^k_p. \quad (43)
\]

We define vertical differencing and averaging operations by

\[
\delta_{\eta} \chi = (\chi_{k+1/2} - \chi_{k-1/2})/\Delta \eta \quad (44)
\]

\[
\bar{\chi}^n = \frac{1}{4} (\chi_{k+1/2}^n + \chi_{k-1/2}^n) \quad (45)
\]

and a shift operator by

\[
E_i(\chi) = \chi^{i+1}. \quad (46)
\]

The following typical discretizations are then used:

\[
\frac{\partial}{\partial \eta} \left( \phi \frac{\partial \tilde{u}}{\partial \eta} \right) = \delta_{\eta} (\phi \delta_{\eta}(E_i \tilde{u})) \quad (47)
\]
and
\[ \xi = \frac{c^{3/4} E^{1/2} E_r (E)}{H^{3/2} b^{3/2} \xi}. \]

The semi-implicit nature of the evaluations in (47) and (48) guarantees unconditional computational stability with reference to the diffusion and dissipative terms in (36), (37) and (38). A further important feature of (47) is its conservation characteristics and the fact that the wind-stress conditions (41) must be applied at \( \eta = (s - \frac{1}{2}) \Delta \eta \). This latter fact is an additional reason why the vertical grid-spacing near the sea-surface should ideally be as fine as possible.

It will be noted that the implementation of the numerical scheme based upon (47) requires the application of three Gaussian elimination procedures as the solution is advanced through each time-step \( \Delta t \). However, in comparison with an explicit treatment, this is a small price to pay bearing in mind the reduction in \( \Delta t \) that would be necessary with our subsequent ultra-fine grid-spacing near \( \sigma = 0 \).

5. Numerical experiments

We have performed numerical experiments using an identical analysis area, bathymetry and horizontal grid-spacing to that used in (I). This analysis area and the idealized cyclone track are shown in Fig. 1. In relation to the bathymetry, it is sufficient to note that this models a transition from shallow water having a depth \( \sim 10 \text{ m} \) near the coastline to effectively deep water at a distance of approximately 300 km from the coast as in Fig. 3 in (I).

As a result of recent experience, we have changed the formula representing the
associated wind field in accordance with the work reported by Johns et al. (1982). The new representation of the azimuthal wind speed, $V$, is given by

$$ V = \begin{cases} V_0(r/R)^{3/2} & \text{for } r \leq R \\ V_0 \exp\{(R - r)/\alpha\} & \text{for } r > R. \end{cases} \quad (49) $$

Here $V_0$ is the maximum wind speed, $R$ the radius of maximum wind, $r$ is the distance from the centre of the cyclone and $\alpha$ is a length scale fixing the areal extent of the far wind field. Using reports from the India Meteorological Department, the available wind data are represented by choosing $V_0 = 70$ m s$^{-1}$, $R = 80$ km and $\alpha = 240$ km. In comparison with the earlier representation in (I), (49) yields a reduced value for the wind speed at distances from the cyclone centre for which $r \gg R$. In particular, for distances from the cyclone centre in excess of 250 km, the implied values of $V$ are in closer accord with our upgraded estimates of the actual wind speeds.

The idealized cyclone moves across the analysis area with the same translation speed as that used in (I). Its initial position is centred about 1000 km south-east of the position of landfall and this corresponds to $t = 0$ when the system is at rest.

In the work reported here, we have taken $s = 8$ having previously found that an increase in the number of computational levels had no significant effect on the computed surge response.

Our principal object is to compare the results of the multi-level model (MLM) with our earlier depth-averaged model (DM), each of these being characterized by a totally different representation of the frictional mechanism. In our first experiment with MLM, we adopted an overall roughness parameter $z_0 = 1$ cm and took $\sigma_0 = 10^{-4}$. This implies a vertical grid-spacing immediately above the sea-floor of about 10 cm near the coastline and 50 cm in the regions of deepest water. The corresponding values of the vertical grid-spacing near the sea-surface are about 1.8 m and 90 m. In the deepest water, the surface stress is therefore applied at a depth of 45 m. This is deeper than we would have wished but, with an 8-level model having the necessary high resolution near the sea-floor, it is unavoidable. We have, in any case, found that the numerical results are essentially unchanged by an increase in the number of computational levels, which implies that the effective level of application of the wind-stress is not critical.

We then computed the time-variation of the sea-surface elevation at the stations Vishakhapatnam, Divi, Kavali and Pondicherry whose geographical locations are described in (I). The responses have been compared with the corresponding sea-surface elevations determined from DM using a uniform bottom friction coefficient $C = 2.6 \times 10^{-3}$. The results, given in Fig. 2, show a quite remarkable similarity. The differences that do exist are hardly significant although, compared with DM, we note a tendency for MLM to yield a higher positive surge at Kavali by approximately 9%. After the peak surge at Kavali, we note that both MLM and DM predict a rapidly falling sea-surface elevation with DM producing a greater peak negative surge than MLM by about 22%. We also note that the computed phase differences between MLM and DM are insignificant and that the peak elevations in the two models therefore occur at the same time.

It is important to consider this result in the light of our original expectation that the shallow water surge response calculated with MLM would be markedly different from that determined with DM. Firstly, why does the representation of the vertical current-structure in MLM have such an insignificant effect on the results of the calculation? The application of DM is strictly admissible only when there is an absence of vertical current-structure in a column of water. However, as pointed out in section 1, we would expect a vertical structure to evolve as momentum is transferred across the sea-surface subsequently leading to motion at all depths. The key to answering the question lies in the fact that an evolving vertical current-structure invalidates the depth-averaged equations used in DM only when the non-linear momentum advection terms are of significant magnitude
Figure 2. Time variation of surface elevation at coastal stations:
solid line—computed from MLM;
broken line—computed from DM.

compared with the pressure gradient terms. This becomes increasingly the case in the shallow water adjacent to the coastline. However, the bulk of the momentum transfer from the atmosphere to the storm-induced currents takes place over the deeper water where the non-linear advective terms are relatively unimportant. The developing surge response in the deeper water then propagates towards the coastal regions in the form of a long amplifying shallow-water gravity wave. The coastal surge response is not locally generated but is produced by distant wind-stress forcing in regions where the existence of an evolving vertical current-structure does not invalidate the use of the depth-averaged equations. Clearly, this argument will not apply in extensive shallow water regions where local generation must be expected to play a predominant role. Such regions are to be found at the head of the Bay of Bengal where the possibility of a breakdown in the use of the depth-averaged equations must be recognized.
At first, we expected that our choice of value of $z_0$ would have an important effect on the computed surge response. The implication of such a result would be that a detailed knowledge of bottom roughness conditions is essential for incorporation into an effective storm surge model. Accordingly, we carried out a sequence of experiments in which $z_0$ was systematically varied between extreme plausible values. The results of this experiment were illuminating and we found that the surge response with $z_0 = 5\text{ cm}$ was virtually indistinguishable from that with $z_0 = 1\text{ cm}$.

Secondly, then, what is the reason for the relative insensitivity of MLM to variations in the bottom roughness? We expected that the surge response would be highly dependent on horizontal variations in $z_0$, these simulating varying sea-bed roughness conditions. That this is not the case must be attributed to the relative weakness of the sea-bed friction in deep water compared with that in the shallow water coastal regions. Thus, the sea-bed roughness conditions in deep water are completely unimportant and, in our case, it appears that a value of $z_0$ lying between 1 cm and 5 cm is appropriate for the coastal regions where bottom friction is likely to be important.

This result, however, does raise the pertinent question as to whether or not a dissipative mechanism is an essential ingredient in the prediction scheme. Das et al. (1974) answer in the affirmative in relation to storm surge simulation at the head of the Bay of Bengal. Quoting from their paper: 'The total neglect of dissipative forces, in any representative form, would result in large surges not observed in nature.'

It is therefore informative to perform an experiment in our region of interest in which there is a complete neglect of the dissipative mechanism. Clearly, in MLM, internal friction cannot be omitted altogether since it is a necessary mechanism in transferring momentum from the atmosphere into the surge response. Furthermore, our formulation breaks down if $z_0 = 0$. We can, however, omit bottom friction altogether from DM by taking $C = 0$. In such an experiment, we find that the sea-surface elevation along part of the coast increases excessively to about 9 m as the cyclone approaches landfall; this leads to a complete breakdown in the numerical scheme during the resurgence phase when $t \approx 85\text{ h}$. It is therefore clear that a dissipative mechanism must be included in order to preserve realism in the modelling procedure. On the other hand, we have shown that the response of MLM is only weakly dependent on the roughness parameter for values of $z_0$ lying between 1 cm and 5 cm with regard to both its amplitude and phase.

A further question raised by this result is whether the use of DM with a sequence of plausible values of $C$ will produce surge elevations and phases in the same range as those obtained from MLM. We have investigated this by using DM with $C = 1.3 \times 10^{-3}$, $2.6 \times 10^{-3}$ and $5.2 \times 10^{-3}$ and give the corresponding surge responses at Kavali and Divi in Fig. 3.

We note that higher values of the friction coefficient lead to lower surge responses. At Kavali, the lowest value of $C$ considered produces a peak surge about 13% higher than that obtained with $C = 5.2 \times 10^{-3}$. The peak negative surge is also increased by a reduction in the value of $C$. It appears, therefore, that the response of DM is only weakly dependent on the value used for $C$ provided that this lies within the realistic limits suggested in the literature. Furthermore, for these values of $C$, the computed surge response is both qualitatively and quantitatively similar to that determined from MLM with regard to both its amplitude and its phase.

Although our primary interest here is with the important surge-induced sea-surface elevation, it is appropriate to comment briefly on the associated current-structure. Using MLM, we are able to determine the magnitude and direction of the horizontal current at each computational level.

With the forcing produced by a cyclone tracking across the analysis area with a speed of approximately 14 km h$^{-1}$, the time-scale of variations in the applied surface wind-stress is much less than that required to establish anything resembling a steady-state Ekman spiral. Consequently, the evolution in time of the vertical current-structure is complex and does not resemble the familiar well-ordered approach to a steady-state
Ekman spiral that exists in the presence of a temporally uniform applied surface windstress. The complete absence of any observational data on the surge-induced currents with which to compare our computed results makes this a purely academic exercise which contributes nothing to the verification of the model. Nevertheless, on using MLM to deduce the corresponding depth-averaged current from (39), and then comparing this with a direct calculation based on DM, we find that both models yield the same time-evolution for the surge-induced currents. In view of our earlier remarks, we would expect this to be the case for deep water where the non-linear advective terms are small and which, in
consequence, do not invalidate the form of the depth-averaged equations used in DM. In the shallow water coastal regions, where the advective terms are larger, the relatively weak effect of local wind-stress forcing in the surge development does not lead to any significant near-surface vertical current-structure. The only structure is confined to a sea-bed boundary layer whose thickness is much less than the total depth of water. In these circumstances, the surge response consists of a freely propagating gravity wave and, as pointed out by Johns (1981), the necessary conditions are fulfilled for the validity of the depth-averaged equations used in DM.

Naturally, an eventual verification of our model is required by using observed current-structure beneath a surge-generating storm. Appropriate data may be available for currents generated by extra-tropical systems in regions such as Long Island Sound or the North Sea. This, however, is beyond the scope of the present work in which our brief is to evaluate methods of storm surge numerical simulation in the Bay of Bengal.

We believe that our results from MLM demonstrate the efficacy of the depth-averaged model in that part of the Bay of Bengal considered here. We also believe that our three-dimensional turbulence energy model has considerable potential in application to shelf dynamics problems in general and, given observational verification against currents, could produce useful results on the wind-induced current-structure in the surface layers of the ocean.

6. CONCLUDING REMARKS

We have developed a three-dimensional numerical model for the simulation of storm surge phenomena. The internal stresses in the water are related to the vertical gradient of the Reynolds-averaged velocity by means of an energy-based turbulence closure scheme. In comparison with more familiar depth-averaged models (with empirical bottom friction), we believe that our conceptually quite different modelling procedure represents a substantial reduction in empirical input. In this regard, we also believe that it compares favourably with three-dimensional models employing algebraically specified vertical exchange coefficients.

In order to test the model, we have applied it to the surge generated by the 1977 Andhra cyclone. This has already been investigated by Johns et al. (1981) using a depth-averaged model. The results of our experimentation are illuminating. In spite of the apparent sophistication of our new modelling technique and the way in which the dissipative mechanism is included, we find little difference between the new predictions and those based upon the earlier depth-averaged model. Further, we find that the results from the three-dimensional model are relatively insensitive to changes in the value of the bottom roughness parameter and that these results are comparable with those obtained from the depth-averaged model for a range of different values of the empirical bottom friction coefficient.

We therefore conclude that a depth-averaged storm surge model yields an extremely effective procedure for numerical simulation purposes. The additional complexity and substantial computational overheads in the three-dimensional model are simply not worthwhile if the vertical current-structure is not of primary concern.

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