A wave prediction system for real-time sea state forecasting

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SUMMARY

The considerable increase in requirements for sea state forecasts in recent years has led to development of a numerical wave forecasting system in the Meteorological Office. This is based on a wave prediction model which combines the advantages of the parametric technique in predicting a growing wind-sea with those of a discrete spectral model in the swell regime. This is done using a discrete model by parametrizing the nonlinear interactions term in the energy balance equation in a way that reproduces the behaviour of the parametric model. The main difficulty with the method is in the separation of wind-sea and swell required to do this. Propagation of wave energy is performed using an accurate form of the Lax–Wendroff integration scheme. A two-term representation of wave growth is used whilst dissipation is modelled by an explicit whitecapping mechanism. Shallow water effects are included by representations of shoaling, refraction and bottom friction. The operational numerical atmospheric model at the Meteorological Office provides the wind input. An extensive program of evaluation has shown that the results provide high quality guidance with 24-hour forecasts of wave height having a r.m.s. error ranging from 0.6 m in the southern North Sea to 1.0 m east of the Shetlands.

1. INTRODUCTION

In recent years the requirements of marine industry for real-time forecasts of sea state have increased substantially. On the main oceans of the world, efficient routine of ships to save fuel requires accurate wave forecasts, whilst the development of offshore mineral reserves has created a requirement for detailed wave information along continental margins. Wave forecasting techniques were first developed in the 1940s using empirical relationships between local wind speed and wave height (Sverdrup and Munk 1947; Bretschneider 1952a, b). These techniques were designed for application at a single point and permitted only a limited allowance for varying winds during the wave-generation period. The techniques were later developed by Wilson (1955, 1965) to allow for variable winds but the lack of a sound theoretical basis limits their usefulness. Such a theoretical basis was laid by Pierson (1952) and Neumann (1952) by treating the water elevation as a random quantity and then describing its statistics in terms of the spectrum of the surface variance. The behaviour of individual components of this spectrum could then be described by linear propagation theory and the wave height obtained by integrating the spectrum. Longuet-Higgins (1952) gave relations between the integrated spectrum and statistics of wave height. Wave growth was described by an empirical relation between fetch or duration and a cut-off frequency in the Neumann spectrum. These techniques were embodied in a comprehensive method for wave forecasting (Pierson et al. 1955). The theory available for applying energy gain and loss to the spectrum was set on a sound foundation by Gelci et al. (1957) and Hasselmann (1960) in the form of the energy balance equation:

$$\frac{\partial E}{\partial t} + c_g \cdot \nabla E = S$$

where $E(f, \theta, x, t)$ is the energy in a spectral component with frequency $f$ travelling in direction $\theta$ at location $x$ and time $t$, $c_g(f, \theta)$ is the group velocity of the spectral component and $S$ is a function which describes all processes by which energy is gained or lost by a spectral component when viewed by an observer travelling at its group velocity. The principal processes of energy gain are those associated with the wind blowing on the water surface. Theories for this interaction were put forward by Miles (1957, 1960) and Philips (1957). Whitecapping represents a major energy loss, and Pierson and Moskowitz (1964) showed that the wave field reaches a limiting state in steady wind conditions where energy gain from the wind is balanced by energy loss in breaking. Models could then reduce the wind input to zero as this state was approached and avoid the difficulties of an
explicit parametrization of breaking. A number of models have been successfully used based on these techniques (e.g. Pierson et al. 1966). During the 1960s Hasselmann (1967) showed that with a broad spectrum such as the Pierson–Moskowitz limiting spectrum, there should be a contribution to the energy balance from fourth-order wave–wave interactions. Some early attempts to include these as an additional term in the function $S$ were made by Barnett (1968) and Ewing (1971). However, the JONSWAP experiment (Hasselmann et al. 1973) showed that in a fetch-limited sea these interactions are the dominant influence in determining the spectral evolution. The early parametrizations of the nonlinear interactions were not able to reproduce adequately the observed stability of spectral shape in the growing sea.

This shape similarity was, however, exploited in parametric models (Hasselmann et al. 1976) in which discrete spectral components were replaced by a small number of parameters describing the spectral shape. This ensured stability of the spectral shape and also reduced the complexity of the energy balance equation. Unfortunately these models are not applicable to swell propagation, where uncoupled spectral propagation is a very good approximation, or to mixed seas. Attempts have therefore been made to combine the discrete and parametric techniques. Günther et al. (1979) described a hybrid model in which swell and wind–sea were integrated separately with interactions between the two occurring according to prescribed, empirical rules. An alternative technique has been adopted in the model described here. It is based on a discrete spectral formulation but with a wave–wave energy transfer term that is parametrized so as to reproduce closely the main features of the parametric models. These features are the shape similarity during growth, and the relationships between peak frequency, total energy and sharpness of the peak which are observed to be approximately satisfied. Assumptions still have to be made about the interaction of swell and wind–sea, the most important being implicit in the way in which the two are separated.

A complication in wave forecasting for the U.K. continental shelf area is the shallow water of the North Sea. Techniques for computation of the effects of shallow water using linear wave theory were established early in wave forecasting (Pierson et al. 1955) but were aimed at forecasts for specified locations for applications such as coastal erosion or mooring location studies. Considerable theoretical work (e.g. Longuet-Higgins 1956, 1957) has advanced our understanding of the mechanisms of refraction and shoaling. However, these ideas have rarely been applied in large-scale wave forecasting. Cavalieri and Rizzoli (1978) have described a characteristic ray model of the Adriatic Sea which incorporates these effects. The present model treats them in a way which ensures that they are included only on the scale of the model gridlength. This is based on the assumption that perturbations caused by sub-grid-scale bottom features will not affect the grid-scale wave field. Bottom friction has also been included on this basis and assumes a uniform roughness.

2. THE NUMERICAL WAVE MODEL

Following Hasselmann (1960) the evolution of the wave spectrum is described by an energy balance equation

$$dE(f, \theta, x, t)/dt = S(f, \theta, x, t).$$  \hspace{1cm} (2)

In the presence of variable water depth, the total derivative cannot be expanded as in Eq. (1) since both the group velocity $c_g$ and the direction of propagation $\theta$ are functions of time when following a parcel of wave energy. The appropriate expansion in this case is

$$dE/dt \equiv \partial E/\partial t + \nabla \cdot (c_g E) + \partial[(c_g \cdot \nabla \theta)E]/\partial \theta$$  \hspace{1cm} (3)

where $c_g \cdot \nabla \theta = d\theta/dt$ following a parcel of wave energy of given frequency and initial direction. The function $S$ in Eq. (2) may be expanded (Hasselmann et al. 1973):
where \( S_{in} \) represents energy input from the atmosphere, \( S_{ds} \) represents energy loss and \( S_{nl} \) the redistribution of energy within the wave spectrum due to conservative nonlinear interactions. Equation (2) is solved in four stages with the results of each stage providing the input to the next. Combining Eqs. (2) to (4) this may be written schematically as

\[
[[[\frac{\partial E_{ij}}{\partial t} = -V \cdot (c_g E_{ij})] - \partial\{(c_g \cdot V\theta)E_{ij}\}/\partial \theta] + S_{in} + S_{ds}] + S_{nl}]
\]

where \( E_{ij} \) has \( m^2 \text{ Hz}^{-1} \text{ rad}^{-1} \) is the energy in a component of frequency \( f \) Hz and direction \( \theta_j \).

(a) Propagation term: \( \partial E_{ij}/\partial t = -V \cdot (c_g E_{ij}) \)

A modified Lax–Wendroff integration scheme due to Gadd (1978a) is used to integrate this term. For one-dimensional propagation at speed \( c \), it takes the form

\[
E_{j+1/2}^{n+1/2} = \frac{1}{2}(E_j^n + E_{j+1}^n) - \Delta t(c_{j+1} E_{j+1}^n - c_j E_j^n)/2 \Delta x
\]

\[
E_j^{n+1} = E_j^n - \Delta t[(1 + a)(c_{j+1/2} E_{j+1/2}^{n+1/2} - c_{j-1/2} E_{j-1/2}^{n+1/2}) - \frac{1}{2}(c_{j+3/2} E_{j+3/2}^{n+1/2} - c_{j-3/2} E_{j-3/2}^{n+1/2})]/\Delta x
\]

where \( a = \frac{1}{4}[1 - (c_j \Delta t/\Delta x)^2] \), \( j \) signifies spatial position, and \( n \) the time level. Near coasts where some of the required points are not available \( a \) is set to zero, while at boundary points, upstream differences are used to prevent reflection of wave energy. The scheme is stable for \( c \Delta t/\Delta x < 1 \) except for a slow instability noted in Gadd (1980) which has not been detected in the results of the wave model. The propagation speed for wave energy, \( c_g \), is calculated using linear theory

\[
c_g = \frac{1}{2}g \tanh(kH)/k^{0.5}\{1 + 2kH/\sinh(2kH)\}
\]

where the wavenumber, \( k \), is given by

\[
(2\pi f)^2 = gk \tanh kH
\]

and \( H \) is the water depth.

Values of \( c_g \) are precomputed for each frequency in the model at intervals of 2 m depth.

Tests of accuracy of the integration scheme have been presented in Golding (1977) and Vincent and Resio (1979). Figure 1 shows the error in propagation of a Gaussian hump of wave energy. The initial shape was defined as

\[
E(x, y) = P_0 \exp[\{(x - x_0)^2 + (y - y_0)^2\}/L^2]
\]

where \( P_0 \) defines the amplitude and \( L \) the spread. During the subsequent integration, the hump was characterized by two measures, its mean position

\[
\bar{x} = \{\sum \sum x E(x, y)\}/E
\]

and its r.m.s. spread

\[
s = \langle\{\sum \sum \{(x - \bar{x})^2 + (y - \bar{y})^2\} E(x, y)\}/E\rangle^{1/2}
\]

Integrations were performed for a number of values of the Courant number \( \bar{C} = c \Delta t/\Delta x \) where \( c \) is the propagation velocity. The mean error in propagation distances up to 50 gridlengths was less than one gridlength for all values of \( \bar{C} \). The r.m.s. spread, shown in Fig. 1, increases for all values of \( \bar{C} \) but does so least for the highest value. In view of this, the propagation scheme is split so that the slow moving, high frequency components use a multiple of the timestep used by the fastest moving component. The timestep used still satisfies the stability criterion \( \bar{C} \leq 1 \) allowing for variations in gridlength \( \Delta x \), due to grid projection, and speed \( c_g \) due to shallow water.
Refraction: \[ \frac{\partial E_{ij}}{\partial t} = -\frac{\partial}{\partial \theta}(e_{g} \cdot \nabla \theta)E_{ij}/\partial \theta \]

The refraction scheme is based on a continuous form of Snell's law:

\[ \delta (k \sin \alpha)/\partial s = 0 \]  (12)

where \( s \) is a ray path, \( k \) is the wavenumber and \( \alpha \) is the angle between \( s \) and the normal to the depth gradient. If \( \alpha \) is rewritten in terms of \( \theta \), the wave propagation direction on the model grid, and \( \nabla H \), the depth gradient, then Eq. (12) may be written

\[ \frac{\partial \theta}{\partial s} = -\left\{ \left( \frac{\partial H}{\partial x} \sin \theta - \frac{\partial H}{\partial y} \cos \theta \right) / \left( \frac{\partial H}{\partial x} \cos \theta + \frac{\partial H}{\partial y} \sin \theta \right) \right\} \left( \frac{1}{k} \cdot \frac{\partial k}{\partial s} \right) \] \]  (13)

\( \partial k/\partial s \) is obtained by differentiating the relationship between wavenumber and frequency at constant frequency

\[ \frac{\partial k}{\partial s} = -\{ k^2 \text{sech}^2 kH/(\tanh kH + kH \text{sech}^2 kH) \} (\partial H/\partial s). \]  (14)

Substituting into Eq. (13) gives

\[ e_{g} \cdot \nabla \theta = -\left( \frac{\partial H}{\partial x} \sin \theta - \frac{\partial H}{\partial y} \cos \theta \right) \left( -k^2 \text{sech}^2 kH \right) / \left( \tanh kH + kH \text{sech}^2 kH \right). \] \]  (15)

The first term in brackets is computed using centred differences for the \( H \) derivatives. The remaining terms are pre-calculated as a function of \( H \) for each frequency used in the model. Finally, the full refraction term is calculated using a form of upstream difference in \( \theta \) expressed in flux form and with the upstream direction determined by the sign of \( e_{g} \cdot \nabla \theta \). The resulting difference for a single component may be written

\[ [\Delta E]_{g} = -\frac{\partial}{\partial \theta} \{ (e_{g} \cdot \nabla \theta) E \} \Delta t = \left\{ \left[ \min \left( e_{g} \cdot \nabla \theta \right) \cdot E \right]_{\theta + \Delta \theta} \right\} + \left\{ \max \left( e_{g} \cdot \nabla \theta \right) \cdot E \right\}_{\theta - \Delta \theta} - \left\{ (e_{g} \cdot \nabla \theta) E \right\}_{\theta} \Delta t \]  (16)
where $\Delta \theta$ is the angular resolution of the discrete spectrum. This allows for convergence or divergence of energy in a particular direction band as well as uniform turning. To preserve stability the limit

$$|\mathbf{c}_n \cdot \nabla \theta| \leq \Delta \theta / \Delta t$$  \hspace{1cm} (17)

is imposed. In order to avoid excessive accumulation of energy in a particular spectral component due to refraction (as in caustics), each component is tested against the Phillips saturation curve (Phillips 1958) with the constant taken from Pierson and Moskowitz (1964) and a $\cos^2 \theta$ directional distribution assumed:

$$E_p(f) = 0.0005 f^{-5} \text{ m}^2 \text{ Hz}^{-1} \text{ rad}^{-1}.$$  \hspace{1cm} (18)

It is not permitted to gain energy in excess of this value through refraction. Such energy is assumed to be lost through breaking.

This formulation for wave refraction ensures that bottom features whose scale is smaller than that of the model grid will not have an influence on the large-scale wave field. The numerical scheme employed does, however, result in a spreading of the wave energy which will qualitatively resemble that due to random sub-grid-scale features.

A test of the scheme in idealized bottom topography is presented in Fig. 2. In Fig. 2(a), a single component wave front is advancing over half the grid towards a parabolic sea mount. Figure 2(b) shows enhancement of wave height over the mount due to the convergence effect of reduced group velocity and a turning of wave energy caused by refraction. Figure 3 shows the bottom topography of the north-west European continental shelf. Figure 4 shows the influence of the shallow water on propagation of a wave field from a hypothetical storm in the Norwegian Sea. In Fig. 4(a) variations are due only to sheltering by land. Figure 4(b) shows additional variation due to shoaling and refraction (bottom friction was not included). A particularly notable feature is the local maximum at the southern end of the Dogger Bank caused by focussing. The Shetland and Faeroe Islands, which are represented only by shallow water, also have a pronounced effect.

(c) Growth and decay: $\partial E_{ij}/\partial t = S_{in} + S_{ds}$

The energy input, $S_{in}$, is represented by linear and exponential terms following Miles (1957, 1960) and Phillips (1957) and may be written

$$S_{in} = \alpha + \beta E_{ij}.$$  \hspace{1cm} (19)

The linear term represents a direct forcing of the water surface due to turbulent fluctuations in the surface wind. Barnett (1968) and Ewing (1971) used complex expressions based on work by Priestley (1965). However, Snyder et al. (1981) found that the whole of the measured growth fitted well to the $\beta$ term. This suggests that the linear term becomes unimportant very soon after the initiation of wave growth and that a very crude formulation may therefore be appropriate. Such a form is used here:

$$\alpha = \begin{cases} \alpha_1 U^2 \cos^2(\theta - \psi) & \text{for } f = f_{\text{max}}, \quad |\theta - \psi| < 90^\circ \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (20)

where $f_{\text{max}}$ is the highest frequency component represented in the model and $U$, $\psi$ are the wind speed and direction at 19.5 m. The form of $\alpha_1$ is chosen so that the initial energy input is independent of the spectral discretization used in the model:

$$\alpha_1 = 6 \times 10^{-8} / (2\pi f_{\text{max}})$$  \hspace{1cm} (21)

where the numerical constant was obtained by tuning (see below) and has the dimension of time.
Figure 2. Behaviour of a swell front crossing a parabolic sea mount (no friction). The swell is a single component \(f = 0.05 \text{s}^{-1}\) travelling down the diagram. Full lines give wave height, dashed lines give sea depth (both in metres) and arrows show the weighted mean propagation direction. (a) Swell front approaches left half of sea mount; (b) swell front after crossing sea mount.
The exponential term uses the same form as that considered by Snyder et al. and represents the interaction of existing waves with a sheared low level air flow:

$$\beta = \begin{cases} \beta_1 \cdot f \left[ \left( U \cos(\theta - \psi) / c \right) - 1 \right] & \text{if } U \cos(\theta - \psi) / c > 1 \\ 0 & \text{otherwise} \end{cases}$$

(22)

where $c$ is the phase speed of waves with frequency $f$. In normal use this is the deep water phase speed $c = g/(2\pi f)$. However, experiments have been performed with the shallow water form. Little difference was observed in the results though the fact that $c$ is decreased in shallow water should lead to enhanced wave growth. The constant, $\beta_1$, is defined as

$$\beta_1 = 6 \times 10^{-2} \times 2\pi \rho$$

(23)

where $\rho$ is the ratio of air density to water density and the numerical constant was obtained by tuning. It is smaller than the value of 0.2 quoted by Snyder et al. because of the additional input term $\alpha$ and because the wind speed is defined at 19.5 m instead of at 5 m.
Figure 4. Modification of swell propagating from a 12m fully developed sea in the Norwegian Sea (steady state). (a) Deep water; (b) bottom topography from Fig. 3 (no friction).
A primary function of the dissipation term is to limit the growth at the level of the fully developed spectrum, which is normally taken as the Pierson-Moskowitz spectrum (Pierson and Moskowitz 1964)

\[ E_{\text{pm}}(f, U) = \{ag^2/(2\pi)^4f^5\} \exp\{-0.74(g/2\pi U)^4\}. \]  

(24)

The only certain way of achieving that is to make the dissipation explicitly balance the input when the limiting spectrum is reached. In Ewing (1971) this condition is applied to each frequency component in a way that ensures a gradual decrease in net energy input to the component as it reaches its limiting value. This does not permit the energy at any frequency to exceed its fully developed value at any time during growth. However, such an overshoot has been documented as a normal occurrence in the JONSWAP results (Hasselmann et al. 1973). An alternative approach is to model the process causing the energy loss, which is believed to be the intermittent whitecapping observed in a wind-blown sea. A formulation of this type is included in the present model based on the paper by Hasselmann (1974) in which he treated whitecapping as a ‘weak-in-the-mean’ process and obtained the form

\[ S_{\text{as}}(f, \theta) = \Delta(E)f^2E(f, \theta) \]  

(25)

where \( \Delta \) is a function of the entire spectrum. It was expected when formulating the model that this function would be either the total energy \( E \) or the wave height \( E^{0.5} \). However, tuning experiments showed that, independent of the values of the coefficients in the growth terms this form of the dissipation grew too quickly as the wave height grew, and resulted in much too slow wave growth as full development was approached. The difficulty was not fully resolved, but by using \( \Delta = E^{0.25} \), a satisfactory growth curve was obtained. Thus

\[ S_{\text{as}}(f, \theta) = 4 \times 10^{-4}f^2E(f, \theta) \left\{ \int \int E(f, \theta) \, d\theta \, df \right\}^{0.25} \]  

(26)

As expected, this dissipation function did not exactly balance the input when the fully developed spectrum was reached. An explicit reference to the Pierson-Moskowitz spectrum is therefore still included. However, this involves only the total energy in the spectrum. Thus

\[ 0 \leq \bar{S}_{\text{in}} \Delta t \leq E_{\text{pm}} - E_{\text{w}} - S_{\text{as}} \Delta t \]  

(27)

where \( E_{\text{pm}} = (1.4g/U)^{-4} \) and \( E_{\text{w}} \) is the total energy in the wind-sea part of the spectrum (defined below). The three terms contributing to growth and decay were tuned together to reproduce the JONSWAP growth curve when represented as a function of duration. Figures 5(a, b) show the duration and fetch-limited growth curves respectively for a wind speed (at 19.5 m) of 21.54 m s\(^{-1}\). The variables plotted are the non-dimensional energy \( \varepsilon = Eg^2U_r^{-4} \), fetch \( \xi = gxU_r^{-2} \), and duration \( \tau = gU_r^{-1} \) where the velocity scale is \( U_r = 0.0397U_{19.5} = 0.0425U_{10} \). The JONSWAP curves were obtained by rescaling Eq. (2.4) of Hasselmann et al. (1976). The duration-limited line was calculated from the fetch-limited one by assuming a propagation speed of 0.85 times the group velocity of the peak frequency. The resulting equations are:

\[
\begin{align*}
\text{fetch-limited growth:} & \quad \varepsilon = 0.887 \times 10^{-4} \xi, \\
\text{duration-limited growth:} & \quad \varepsilon = 1.165 \times 10^{-6} \tau^{-1.493}. 
\end{align*}
\]

(28)

Figure 5(a) shows that the duration-limited curve follows the JONSWAP curve very well
Figure 5. Growth of a wind-sea in a constant wind \( (U_{10} = 21.54 \text{ m s}^{-1}) \). The logarithmic coordinates are dimensionless measures of: energy \( (\varepsilon = E g^2 U_0^4, U_0 = 0.855 \text{ m s}^{-1}) \); duration \( (\tau = \tau g U_0) \); and fetch \( (\xi = x g U_0^2) \). JONSWAP curves are plotted for comparison. (a) Duration limited.

but with a slightly higher scale. The fetch-limited curve in Fig. 5(b) is closer in terms of scale but does not follow the power law so consistently. Both diagrams show the rapid transition to a fully developed state that results from the dissipation formulation. An advantage of explicitly representing the dissipation term is that a separate formulation of swell dissipation is not necessary. When a fully developed sea moves out of its generating area, the high frequency components will be rapidly damped by the frequency weighting in Eq. (26). However, once the wave field has become well dispersed, the total energy as well as the component energies will be small so that little further energy loss will occur.

A further mechanism for energy loss is included in shallow water. In general, modification of waves occurs in shallow water because of a restriction imposed by the sea floor on the orbital motions of water particles under the wave. Refraction and shoaling occur in the absence of friction and are related to deformation of the orbits from circles to ellipses. However, when the bottom is rough, the motion of the water particles near the bottom is reduced by interaction with the roughness. A full treatment of the various interactions which may remove energy is given in Shemdin et al. (1980). It is likely that different mechanisms are dominant in different parts of a forecast region due to variations in the nature of the sea floor, and the roughness scale will also vary. Unfortunately, detailed mapping of the roughness of the sea floor is not available. The present model therefore uses a single mechanism and assumes uniform roughness. A quadratic bottom friction law
is assumed and the form of the energy loss is based on Hasselmann and Collins (1968) using the approximate form of Collins (1972):

\[ S_{ds} = \Phi_1 gkce/(2\pi(2\pi f \cosh kH)^2) \langle u \rangle E(f, \theta) \]  \hfill (29)

where

\[ \langle u \rangle = \left\{ \int \int (gk/2\pi f \cosh kH)^2 E(f, \theta) \, df \, d\theta \right\}^{-0.5}. \]  \hfill (30)

In practice Eq. (29) is written

\[ S_{ds} = \Phi_1 A(f, H) \left\{ \int \int B(f, H)E(f, \theta) \, df \, d\theta \right\}^{-0.5} E(f, \theta) \]  \hfill (31)

where \( A \) and \( B \) are precalculated as a function of depth for each model frequency. The constant \( \Phi_1 (= 0.005) \) was chosen to provide approximate agreement with the results of Bretschneider (1954) for wave generation in shallow water. The effects of this term are shown in Fig. 6 where duration-limited growth in a 20 m s\(^{-1}\) wind is plotted for a series of water depths. The results presented in that figure imply a maximum wave height in the southern North Sea of about 5 metres.

(d) Nonlinear interactions: \( \partial E_{ij}/\partial t = S_{ni} \)

A number of parametrizations of the nonlinear term have been used in wave models
since its importance was suggested (Hasselmann 1967). Most depend on calculating the energy transfers for a restricted family of spectra. In Barnett (1968) and Ewing (1971) the transfers in the actual model spectrum are then approximated by those of the most similar spectrum in the family. This can only be justified if the difference between the actual spectrum and the family member is irrelevant to the energy transfers. This was shown to be untrue in the JONSWAP results (Hasselmann et al. 1973) where the observed stability of the spectral shape was interpreted as being a result of the interactions. This difficulty was avoided in the parametric model of Hasselmann et al. (1976) by not permitting spectra other than those belonging to the family defined in the JONSWAP results, and calculating the nonlinear transfers for that family. The results of this work indicated that a good approximation to the evolution of the wind-sea could be obtained using a prognostic equation for one of the spectral parameters (e.g. the total energy or the peak frequency), and diagnostic relations for the remainder. This result is exploited in the present model. Firstly, the wind-sea spectrum is separated from the full spectrum at each timestep. The remainder, i.e. the swell, is entirely determined by the preceding steps (a), (b) and (c). However, for the wind-sea spectrum, these define only the total energy. The spectral shape appropriate to this energy level is obtained by first using the diagnostic relations to define the remaining shape parameters \( f_p, \gamma, \sigma \) and then evaluating the shape equation over the appropriate range of parameters. The nonlinear transfers are thus defined implicitly as those required to return the spectrum to this shape. The shape equation is defined as

\[
E(f, \theta) = F(f) \exp \left\{ -1.25 \left( \frac{f_p}{f} \right)^4 + \ln \gamma \exp \left( \frac{f - f_p}{\sigma f_p} \right)^2 \right\} G(f, \theta)
\]  

(32)

where

\[
\int_0^\infty G(f, \theta) = 1
\]

(33)
and

$$\int \int E(f, \theta) \, d\theta \, df = \mathcal{E}. \quad (34)$$

In practice, the integrals are replaced by summations over spectral components and so the normalizing constants cannot be precalculated. The functions $F$, $G$ can take various forms. The normal form for $F$ is

$$F(f) \propto f^{-5} \quad (35)$$

based on the saturation range of Phillips (1958). In that form Eq. (32) is the normal JONSWAP spectrum. However, Thornton (1977) has shown that a more appropriate form for the saturation range in water of arbitrary depth is

$$F(f) \propto e^{2f} f^{-3} \quad (36)$$

where $c$ is the phase velocity of the waves. This would lead to a flatter spectrum at high frequencies, although the peak would still have the sharpness caused by the peak enhancement parameter. An alternative form of the shallow water spectrum has been given by Kitaigorodskii et al. (1975).

The angular spreading function $G$ is used as

$$G(f, \theta) \propto \begin{cases} \cos^2(\theta - \psi) & \text{for } |\theta - \psi| \leq 90^\circ \\ 0 & \text{otherwise.} \end{cases} \quad (37)$$

However Mitsuyasu et al. (1975) have shown that the spreading should vary according to the relation of the frequency to the spectral peak:

$$G(f, \theta) \propto \cos^2 \left\{ \frac{1}{2} (\theta - \psi) \right\} \quad (38)$$

where

$$s = \begin{cases} 11.5 f^{-2.5} & f > f_p \\ 11.5 f_p^{-7.5} f^s & f \leq f_p \end{cases} \quad (39)$$

and $\bar{f} = U/c$.

$U$ is the wind speed and $c$ the phase velocity of waves with frequency $f$.

An advantage of the method of calculating the nonlinear transfers described here is that the variations of shape suggested above can very easily be implemented. Its main limitations lie in the method used for separating the wind-sea spectrum and in the diagnostic relations for the shape parameters.

The basis for the separation of wind-sea is

$$\begin{cases} f \geq 0.8 f_p \\ |\theta - \psi| \leq 90^\circ \end{cases} \quad (40)$$

Since the angular definition is in terms of the wind direction it can easily be applied. However, the frequency constraint depends on a peak frequency which is unknown. In a light wind, the spectral peak will often be in the swell regime, and will always be at a component frequency unless empirical curve fitting is employed. An iterative procedure is therefore adopted which starts with the lowest possible value

$$f_p = f_{pm} \quad (41)$$

Using Eq. (40) the total wind-sea energy $\mathcal{E}_w$ is calculated and is then used to define a new $f_p$ by the approximate relation (cf. Eq. (43))

$$f_p = (2.5 \times 10^{-4} / \mathcal{E}_w)^{0.25} \quad (42)$$

The process could be repeated but in practice it is terminated at this stage. The way in
Figure 7. Modification of a fully developed wave spectrum due to a 90° change of wind direction after (a) 2 h; (b) 6 h; (c) 18 h; (d) 30 h.
Axes are direction ($\theta$) and dimensionless frequency ($f^* = fU_{*}/g$, $U_{*} = 0.855 \text{ m s}^{-1}$).
Contour interval is 0.1 of the peak energy given at the head of each diagram.
which the separation of wind-sea is done is an important part of the design of the model and the above procedure is considered to be only a crude first attempt.

Having defined the wind-sea part of the spectrum, its total energy \( E_w \) is recalculated. The peak frequency and peak enhancement of the JONSWAP spectrum are then calculated from \( E_w \) using formulae which approximate to the single-parameter model of Hasselmann et al. (1976) for small values of \( E_w/E_{PM} \) and tend towards the Pierson-Moskowitz values as it approaches unity:

\[
\begin{align*}
    f_p &= \left\{ (10^{-4} + 7 \times 10^{-4} (\gamma - 1) \sigma) / (E_w)^{0.25} \right\} \\
    \gamma &= 1.0 + 2.3 \{ 1 - (E_w/E_{PM})^2 \}.
\end{align*}
\]

(43)

Using these parameters and the peak width \( \sigma (= 0.08) \), the shape spectrum (32) can be evaluated. The normalizations (33), (34) are carried out at this stage so that the redefined spectrum has exactly the same energy as the original, independent of the discretization. The spectral components in the wind-sea part of the spectrum are then replaced by those calculated from Eq. (32).

The overall effect of this process is to replace the wind-sea part of the spectrum by a JONSWAP spectrum with the same energy. This is done instantaneously which is consistent with the short timescale for adjustment of the frequency spectrum observed in JONSWAP. However, recent work by Resio (1981) and Günther et al. (1981) has shown the importance of treating directional transfers as a slower process in the context of shifting wind directions. Figure 7 shows the behaviour of the present model when the wind direction changes by 90° after generating a fully developed spectrum. The growth of the new wind-sea spectrum appears reasonably realistic in the light of the work cited above except for the sharp cut-off at 90°. However, the results depend on the way in which the wind change occurs and there is scope for further improvement in this area. This is illustrated by the difference that appears when the wind change is in space rather than in time. Figure 8 shows the behaviour of the model in a steady state situation with a 90° change of wind direction along the diagonal. A much sharper change of direction occurs in this case and little swell continues to propagate in the original direction. Figure 9 shows a case in which wind blows on the left half of the area and not on the right. The nonlinear interactions operate on the right edge of the windy area to keep the energy travelling with the wind rather than to let it cross the wind and escape as swell in the windless region. The exact degree of directional coupling has yet to be determined theoretically and it is hoped that when this is done the problems described above will be resolved.

3. Wind input

An important part of any wave forecasting system is the specification of winds. In the Meteorological Office operational system, a two-stage process is used in each twelve-hour run. Firstly, considerable effort is put into specifying the actual wind field over the past twelve hours in order to give an accurate diagnosis of the actual wave conditions at the beginning of the forecast period. This diagnosis is continuously updated and, in archived form, provides a valuable source of climatological information. Secondly, forecast winds are obtained from the operational 10-level forecast model (Burridge and Gadd 1977; Gadd 1978a, b, 1980) up to 36 hours ahead. In all cases where atmospheric model winds are used, the 900 mb values are reduced to 19-5 m using empirical relationships based on the results of Findlater et al. (1966). These are expressed in the form

\[
\begin{align*}
    V_0/V_{900} &= aV_{900}^2 + bV_{900} + c \\
    \theta_0 &= \theta_{900} + d
\end{align*}
\]

(44)
where \(a, b, c, d\) are constants which each take one of five values depending on the lapse rate between 950 mb and the surface. The surface temperature, \(T_0\), is obtained from the mean temperatures of the model's lowest two layers \((T_{850}, T_{950})\) and the sea surface temperature \((T_s)\) in the following manner

\[
T_0 = \begin{cases} 
T_{950} + 0.5h_{950} \gamma_d & \text{if } T_0 < T_s \\
0.1(T_{950} + 0.5h_{950} \gamma_{900}) + 0.9T_s & \text{if } T_0 > T_s \\
T_s & \text{otherwise}
\end{cases}
\] (45)

where \(h_{950}\) is the height of the 950 mb surface, \(\gamma_d\) is the dry adiabatic lapse rate, and \(\gamma_{900}\) is the lapse rate between \(T_{850}\) and \(T_{950}\).

In specifying actual winds for the past twelve hours, the forecast winds already available for that period are improved in a number of ways. Firstly, winds are derived from the mass field analysis at the end of the period. Then observations of surface winds are incorporated at 3-hour intervals using a successive correction analysis technique. At each stage of correction, an orthogonal polynomial fit is made to provide a controlled degree of smoothing at successively smaller scales. The intermediate fields, between the analyses, are adjusted by linearly interpolating between the corrections applied at the analysis times. The initial period of the forecast is also adjusted by a linearly decreasing proportion of the correction made at its start. This procedure was changed by the introduction of a new atmospheric model in September 1982. The new model has a detailed boundary layer specification which removes the need for the empirical reduction from
900 mb. Also it incorporates surface observations in its assimilation run which makes a separate analysis procedure unnecessary.

4. OPERATIONAL RESULTS

There are two versions of the model in use. The following specifications refer to the models in use up to August 1982. A coarse grid model (300 km grid length at 60°N) covers the North Atlantic and North Pacific Oceans north of 20°N. It is a deep water model and uses winds from the 300 km gridlength version of the atmospheric model. It provides forecasts for routeing of ships and also provides the boundary conditions for the continental shelf version. This has a gridlength of 50 km and covers the area shown in Fig. 10. It uses the depth specification shown in Fig. 3 and takes its winds from the 100 km gridlength, limited area version of the atmospheric model. The models use the same spectral discretization into 12 directions (30° spacing) and 11 frequencies (0.05, 0.06, 0.072, 0.086, 0.104, 0.124, 0.148, 0.178, 0.214, 0.256, 0.308 Hz).

The specification of boundary conditions from the coarse grid model is particularly important in forecasts for the west coast of Britain. A study of the flooding event at Portland on 13 February 1979 (Golding 1981), which was successfully forecast, showed that the swell was initially generated by an Atlantic storm at 40°W some 36 h before the flooding occurred. Atlantic swell is also the dominant component in the wave-power resource for the U.K. Considerable work has been done to assess the usefulness of the model results in defining the available wave energy (Winter 1980; Mollison 1980).
A substantial programme of routine verification has been undertaken to compare the model results with observations. Initially this was a general comparison with visual observations from ships. However, the accuracy of these observations was found to be poor so a more detailed comparison was started for specific fixed locations where observations are believed to be more reliable. In most cases the wave observations are made from buoys. The r.m.s. error of the reports is still expected to be at least 10% of wave height in view of the findings of the Seasat-Jason Workshop Report (Jet Propulsion Lab. 1980, pp. 2–67) with a minimum of 0.5 m due to the reporting code. No quality control is exercised on the data except to remove gross errors. Table 1 shows a selection of the results for the calendar year 1981 at 5 locations: Atlantic Weather Ship Lima; three North Sea platforms; and the National Data Buoy DB1 in the south-west approaches to the English Channel. Comparisons are made of wind speed and direction, and wave height. No wind errors are given at analysis time because of the incorporation of the observations in the wind field specification. For the forecast times, a slow growth in error occurs at all stations as forecast time advances. The mean error is generally small and slightly negative. The r.m.s. wind speed error is $3 \text{ m s}^{-1}$ after 12 hours at almost all stations and this must be regarded as unsatisfactory although it will be partly due to sub-grid-scale variation. The same is true of the r.m.s. vector error, which indicates substantial errors in prediction of wind direction. Despite these comments on the accuracy of the wind fields, the wave verification gives consistently low errors, supporting the view that much of the wind error is due to small-scale variation. The mean wave height errors are all zero throughout the
TABLE 1. VERIFICATION STATISTICS FOR FIVE LOCATIONS FOR THE CALENDAR YEAR 1981.
For each station, mean, root-mean-square and root-mean-square vector errors in m s$^{-1}$ are
given for wind velocity (w.v.) and mean and r.m.s. errors in metres for sig. wave heights (w.h.)
(model value = 4/\sqrt{E}). The number of observations in each calculation is given in brackets after
the mean. No wind errors are given for the analysis since observations are incorporated in it.

<table>
<thead>
<tr>
<th>Location</th>
<th>Error type</th>
<th>Analysis</th>
<th>$T + 12$</th>
<th>$T + 24$</th>
<th>$T + 36$</th>
</tr>
</thead>
<tbody>
<tr>
<td>O.W.S. Lima</td>
<td>w.v. mean</td>
<td>-1·0 (703)</td>
<td>-1·5 (702)</td>
<td>-1·5 (699)</td>
<td></td>
</tr>
<tr>
<td>57°N 20°W</td>
<td>r.m.s.</td>
<td>3·7</td>
<td>4·3</td>
<td>4·7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r.m.s.v.</td>
<td>6·0</td>
<td>7·3</td>
<td>8·0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.h. mean</td>
<td>+0·1 (701)</td>
<td>0·0 (700)</td>
<td>-0·1 (699)</td>
<td>-0·2 (696)</td>
</tr>
<tr>
<td></td>
<td>r.m.s.</td>
<td>1·5</td>
<td>1·4</td>
<td>1·6</td>
<td>1·7</td>
</tr>
<tr>
<td>Platform</td>
<td>w.v. mean</td>
<td>-0·3 (619)</td>
<td>-0·4 (619)</td>
<td>-0·6 (617)</td>
<td></td>
</tr>
<tr>
<td>60°N 2°E</td>
<td>r.m.s.</td>
<td>3·3</td>
<td>3·8</td>
<td>4·2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r.m.s.v.</td>
<td>5·6</td>
<td>6·4</td>
<td>7·2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.h. mean</td>
<td>-0·1 (531)</td>
<td>-0·2 (530)</td>
<td>-0·2 (529)</td>
<td>-0·2 (527)</td>
</tr>
<tr>
<td></td>
<td>r.m.s.</td>
<td>0·8</td>
<td>0·9</td>
<td>1·0</td>
<td>1·1</td>
</tr>
<tr>
<td>Platform</td>
<td>w.v. mean</td>
<td>-1·1 (519)</td>
<td>-1·1 (518)</td>
<td>-1·2 (519)</td>
<td></td>
</tr>
<tr>
<td>59°N 1°E</td>
<td>r.m.s.</td>
<td>3·2</td>
<td>3·3</td>
<td>3·9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r.m.s.v.</td>
<td>5·2</td>
<td>5·9</td>
<td>6·6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.h. mean</td>
<td>+0·2 (522)</td>
<td>+0·1 (521)</td>
<td>0·0 (520)</td>
<td>0·0 (521)</td>
</tr>
<tr>
<td></td>
<td>r.m.s.</td>
<td>0·8</td>
<td>0·9</td>
<td>1·0</td>
<td>1·1</td>
</tr>
<tr>
<td>Platform</td>
<td>w.v. mean</td>
<td>-0·1 (606)</td>
<td>-0·1 (607)</td>
<td>-0·1 (605)</td>
<td></td>
</tr>
<tr>
<td>53°N 3°E</td>
<td>r.m.s.</td>
<td>2·8</td>
<td>3·1</td>
<td>3·5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r.m.s.v.</td>
<td>4·6</td>
<td>5·1</td>
<td>5·9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.h. mean</td>
<td>-0·1 (601)</td>
<td>-0·1 (601)</td>
<td>0·0 (600)</td>
<td>-0·1 (599)</td>
</tr>
<tr>
<td></td>
<td>r.m.s.</td>
<td>0·6</td>
<td>0·6</td>
<td>0·6</td>
<td>0·8</td>
</tr>
<tr>
<td>Buoy DB1</td>
<td>w.v. mean</td>
<td>-0·1 (632)</td>
<td>-0·3 (631)</td>
<td>-0·5 (632)</td>
<td></td>
</tr>
<tr>
<td>49°N 9°W</td>
<td>r.m.s.</td>
<td>3·0</td>
<td>3·2</td>
<td>3·6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r.m.s.v.</td>
<td>5·7</td>
<td>6·2</td>
<td>6·9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.h. mean</td>
<td>+0·1 (627)</td>
<td>+0·1 (627)</td>
<td>+0·1 (625)</td>
<td>0·0 (625)</td>
</tr>
<tr>
<td></td>
<td>r.m.s.</td>
<td>0·9</td>
<td>0·9</td>
<td>0·9</td>
<td>0·9</td>
</tr>
</tbody>
</table>

forecast period, while the r.m.s. errors vary from 0·6 m in the southern North Sea to 0·9 m
at DB1 and 1·5 m at O.W.S. Lima. The observations from Lima are made visually and the
wave height for comparison was obtained as

$$H = (H_{seu}^2 + H_{swell}^2)^{1/2}.$$  

Since swell observations are very difficult to make visually, the wave verification for that
station cannot be given much weight. At the remaining stations, a very slight increase in
error occurs during the forecast. There is also a greater error for all forecast times at
stations exposed to the Atlantic. This suggests that the wind specification in the coarse
mesh model may be contributing through the waves passed into the boundaries of the fine
mesh model. Schemes for remote sensing of winds by satellite or radar could be of value
in this respect. The variation of error with wind speed and wave height is shown in Table
2 for DB1. It shows that low wind speeds and wave heights are overestimated while high
values are underestimated. A general increase in r.m.s. error with increasing value of the
observation can also be seen. The large percentage error in the 0–3 m wave height range
can be explained by the 0·5 m resolution of the reporting code. This table shows a closer
correlation between wind and wave accuracy than Table 1. A close link has also been
reported by Bouws et al. (1982) for the southern North Sea where most waves are locally
TABLE 2. Verification statistics for the buoy DBI at 49°N 9°W in ranges of observed values.
For each range of wind speed (w.v.), mean, root-mean-square and root-mean-square-vector errors are given, and for each range of significant wave height (w.h.), mean and r.m.s. errors are given. The number of observations in each calculation is given in brackets after the mean. No wind errors are given for the analysis since observations are incorporated in it.

<table>
<thead>
<tr>
<th>Range (m s⁻¹)</th>
<th>Type</th>
<th>Analysis</th>
<th>T + 12</th>
<th>T + 24</th>
<th>T + 36</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0–10</td>
<td>w.v. mean</td>
<td>+0.3 (412)</td>
<td>+0.4 (412)</td>
<td>+0.5 (410)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r.m.s.</td>
<td>2.7</td>
<td>2.8</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r.m.s.v.</td>
<td>5.1</td>
<td>5.6</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.v. mean</td>
<td>-0.4 (175)</td>
<td>-1.0 (174)</td>
<td>-1.7 (177)</td>
<td></td>
</tr>
<tr>
<td>10–15</td>
<td>r.m.s.</td>
<td>3.2</td>
<td>3.3</td>
<td>3.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r.m.s.v.</td>
<td>5.9</td>
<td>6.3</td>
<td>7.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.v. mean</td>
<td>-2.4 (41)</td>
<td>-3.7 (41)</td>
<td>-4.4 (41)</td>
<td></td>
</tr>
<tr>
<td>15–20</td>
<td>r.m.s.</td>
<td>4.3</td>
<td>5.3</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r.m.s.v.</td>
<td>7.6</td>
<td>9.0</td>
<td>10.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.v. mean</td>
<td>-5.4 (4)</td>
<td>-6.4 (4)</td>
<td>-9.8 (4)</td>
<td></td>
</tr>
<tr>
<td>&gt;20</td>
<td>r.m.s.</td>
<td>6.8</td>
<td>6.5</td>
<td>10.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>r.m.s.v.</td>
<td>15.8</td>
<td>16.8</td>
<td>15.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.h. mean</td>
<td>+0.3 (343)</td>
<td>+0.3 (344)</td>
<td>+0.3 (342)</td>
<td></td>
</tr>
<tr>
<td>0–3</td>
<td>r.m.s.</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.h. mean</td>
<td>+0.1 (258)</td>
<td>+0.1 (257)</td>
<td>+0.1 (257)</td>
<td></td>
</tr>
<tr>
<td>3–6</td>
<td>r.m.s.</td>
<td>1.0</td>
<td>1.1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.h. mean</td>
<td>-0.7 (25)</td>
<td>-0.9 (25)</td>
<td>-1.0 (24)</td>
<td></td>
</tr>
<tr>
<td>6–9</td>
<td>r.m.s.</td>
<td>1.9</td>
<td>1.8</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>w.h. mean</td>
<td>-0.4 (1)</td>
<td>-1.9 (1)</td>
<td>-2.5 (1)</td>
<td></td>
</tr>
<tr>
<td>&gt;9</td>
<td>r.m.s.</td>
<td>0.4</td>
<td>1.9</td>
<td>2.5</td>
<td></td>
</tr>
</tbody>
</table>

generated. The work described there compared the results of the operational wind and wave prediction systems at the British Meteorological Office and KNMI (Netherlands). They concluded that the British wind predictions were significantly better than those produced at KNMI and that this was reflected in the wave height predictions.

A more detailed comparison with other models has been undertaken as part of the international wave model intercomparison experiment (SWAMP) for which Figs. 7, 8, 9 were prepared. The results of this experiment will appear in the proceedings of the Symposium on Wave Dynamics and Radio Probing of the Ocean Surface, Miami, 1981.

5. Conclusion

An operational wave forecasting system has been described in which the atmospheric forecast model of the Meteorological Office is coupled to a wave prediction model. Details of the winds and bottom topography of the North-west European continental shelf are included in a fine mesh model which receives information about swell generated in Atlantic storms through boundary conditions supplied by a coarse grid model. Its accuracy in operational use has been demonstrated and a link shown between the quality of wind and wave predictions. Since the atmospheric model has now been replaced a further improvement in forecast wave accuracy can be expected.

The wave model has been described as a flexible response to the problem of mixing discrete and parametric spectral techniques. Some shortcomings with respect to the defini-
tion of the wind-sea, and the directional coupling, have been identified in section 2 but the model behaviour was shown to be reasonable in idealized situations discussed there. The flexibility of the approach will make it straightforward to replace the existing parametrization of the nonlinear interactions when a better one becomes available. In shallow water, the bottom friction term dominates the other effects included in the model and a local specification of bottom roughness would be an improvement. The increased effects of nonlinear interactions in shallow water should also be included. Another feature of importance in coastal regions is the interaction between waves and currents (Peregrine 1976). Recent advances in the modelling of tides and surges (Flather and Davis 1976; Flather 1981) make the prospect of mutual interaction an exciting possibility for the future.

ACKNOWLEDGMENTS

The work described in this paper has benefitted throughout its development from the constructive criticism of practising forecasters on the Offshore Bench at London Weather Centre, and on the Ships' Routeing Bench in the Central Forecast Office at Bracknell. I am grateful to A. Gadd for his advice and guidance and to J. Ewing, K. Hasselmann, J. Weare and B. Worthington for useful discussions. The preparation of this paper was greatly helped by V. Blackman and J. Ephraums who obtained many of the results presented in it.

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