The steady, linear response of the stratosphere
to tropospheric forcing

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SUMMARY

A linearized, steady-state, 15-level numerical model is used to study the stationary planetary wave response of the stratosphere to thermal and orographic forcing in the troposphere. Zonally symmetric basic states representative of northern hemisphere winter and summer conditions are used. Solutions for each zonal wavenumber are considered separately. Eliassen–Palm cross-sections are used as a diagnostic of wave propagation.

First, several simple thermal and orographic forcing distributions are used in the winter basic state. The response in the stratosphere is sensitive to changes in the zonal wind but its structure is insensitive to the forcing mechanism and to the dissipation used in the model. For representative northern hemisphere thermal and orographic forcing, solutions in winter agree well with observations except for wavenumber one, which has too small an amplitude and an incorrect structure.

The solutions for wavenumbers one and two in the troposphere have small differences from those obtained using a similar 5-level model described by us earlier. For higher wavenumbers, the 15-level and 5-level solutions in the troposphere converge, as these waves are trapped in the troposphere.

For the summer basic state, the planetary wave response has large amplitude in the troposphere only. It is necessary to use vorticity forcing in the upper troposphere to give a solution with a phase variation similar to that observed in summer.

1. INTRODUCTION

After the analytical study of the vertical propagation of planetary waves by Charney and Drazin (1961), which helped to explain much of the observed structure of planetary waves in the stratosphere, there has been continued interest in this field. In general, most recent studies of planetary wave structure in the stratosphere have used three-dimensional numerical models of varying complexity, which allow more realistic conditions to be modelled. These numerical model studies can be broadly split into two groups: linearized perturbation models and general circulation models.

The linearized models are used to study small amplitude planetary-scale perturbations to a realistic zonally symmetric basic state. The first of these models, described by Matsuno (1970), gave solutions for zonal wavenumbers one and two which agreed well with observations though the computed amplitude of wavenumber two was too small. This model was extended by Schuebner and Geller (1977) to investigate the sensitivity of the solutions to variations of zonal wind, static stability and dissipation. Both models used observed height distributions in the upper troposphere as their lower boundary conditions. Thus, they could not be used to investigate the effect of different forcing mechanisms or the propagation of planetary waves from the troposphere into the stratosphere.

The structure of stationary planetary waves has also been studied using general circulation models. Zonal harmonic analyses of time-mean solutions of these models are used to obtain the structure of the steady planetary waves. This method has been used by Kasahara et al. (1973) and Manabe and Terpstra (1974) to study the effects of mountains on the atmosphere. They found that the steady planetary waves in the models were closer to those observed for the case including mountains than for the case without mountains, particularly in the upper troposphere and in the stratosphere. Thermal forcing effects were more important in the lower troposphere. However, the inclusion of mountains modifies

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the thermal forcing in the models and it is not possible to separate the effects of orographic and thermal forcing completely.

Hoskins and Karoly (1981, hereafter HK) studied forced stationary waves in the troposphere using a linearized primitive equation numerical model and investigated the effect of different forcing mechanisms on the horizontal structure of the waves. They found that both thermal and orographic forcing were important in determining the stationary wave structure in the troposphere. A limitation of this model was that its highest level was at 100 mb and it was unable to represent the stratosphere.

In this study, the model used by HK is extended by increasing the number of levels so that an adequate representation of the stratosphere is possible. The extended model is used to investigate the effect of the inclusion of a realistic stratospheric structure on the tropospheric solutions, to study the propagation of planetary waves from the troposphere into the stratosphere and to determine the sensitivity of the wave structure to variations of the forcing, basic state and dissipation.

A description of the numerical model and the method used to obtain stationary solutions is given in the next section. The vertical resolution is tested using a simpler height–longitude model and it is shown to be adequate for representing planetary waves in the stratosphere. The dissipation used in the model is described. Eliassen–Palm cross-sections are used as a diagnostic of planetary wave forcing, propagation and dissipation in the model. The wave propagation is compared with that found in an analytical study by Karoly and Hoskins (1982, hereafter KH) where the ideas of ray tracing and wave propagation in a slowly varying medium were used to investigate the vertical propagation of planetary waves.

Zonally symmetric zonal wind fields representative of northern hemisphere winter and summer conditions are used as basic states for the model. Solutions for simple thermal and orographic forcing distributions in the winter basic state are presented in section 3. A representation of the planetary-scale northern hemisphere thermal and orographic forcing is generated. Solutions for this forcing are compared with observed planetary wave structures in the troposphere and stratosphere from the studies of Chapman and Miles (1981), Hirota (1976), Sato (1980) and van Loon et al. (1973). The variation of the solutions with zonal wavenumber is discussed.

In section 4 the sensitivity of the solutions to variations of dissipation and zonal wind in the winter basic state is investigated. The solutions for the summer basic state are presented in section 5. The model solutions and their comparison with observations are discussed in the final section.

2. Model Description

(a) Basic model and method of solution

The numerical model is an extension of the one used by HK to study the response to thermal and orographic forcing in the troposphere. To permit this model to be used to study the response in the stratosphere, the number of model levels in the vertical is increased to 15. A description of the model and the method of solution is given in HK and the following description includes the modifications used here.

The basic model is the hemispheric, spectral, primitive equation, $\sigma$-coordinate model described by Hoskins and Simmons (1975). Using matrix notation, the linearized primitive equations, with forcing $\mathbf{F}$ and linear dissipation $\mathbf{D}$, can be written in the form

$$\mathbf{X}_n = i\mathbf{A}_n \mathbf{X}_n + D_n \mathbf{X}_n + \mathbf{F}_n$$

for each zonal wavenumber $m$. Here $\mathbf{X}_n$ is a vector comprising all the spectral coefficients of vorticity $\xi_n$, divergence $\delta_n$ and temperature $T_n$ at each level and log surface pressure, $(\ln p_*)_n$. $\mathbf{A}_n$ is a real matrix of size $598 \times 598$ for rhomboidal truncation at wave 26 and
15 levels. The steady response to the forcing $F_m$ is thus

$$X_m = l((A_m - iD_m)^{-1}F_m).$$  \hspace{1cm} (2)$$

After determining the matrix $A_m$ for a particular basic state and matrix $D_m$ for particular dissipation, the solution given by Eq. (2) may be calculated using a routine matrix inversion algorithm.

(b) Vertical resolution

The model in HK used five equally spaced sigma levels with the highest level at 100 mb. Modelling studies of the effect of vertical resolution and the height of the top model level on planetary waves by Nakamura (1976) and Kirkwood and Derome (1977) suggest that the vertical resolution must be sufficient to represent the vertical variations of the basic state and that the top level must be as high as possible.

Here, 15 unequally spaced sigma levels are used with increasing height separation between the levels and the highest level at about 0.003 mb or 86 km. There are four levels up to 300 mb, giving approximately the same tropospheric resolution as in the 5-level model. The expression giving the half-level sigma values is

$$\sigma_{j+1/2} = \sin(j\pi/2NL)\left((j+1)/(NL+1)\right)^{4.6629}, \hspace{1cm} j = 0, 1, \ldots, NL,$$  \hspace{1cm} (3)

where $NL$ is the number of levels. For the full levels the expression is

$$\sigma_j = \frac{1}{2}(\sigma_{j-1/2} + \sigma_{j+1/2}), \hspace{1cm} j = 1, \ldots, NL.$$

The values of pressure, log-pressure coordinate $z = -\ln \sigma$ and approximate physical height at each model level are given in Table 1.

The vertical resolution for the model is tested using a simpler quasi-geostrophic height–longitude model with $\beta$-plane geometry and $\sigma$ coordinates in the vertical. Small amplitude perturbations to a mean zonal flow are considered, with both the flow and perturbations independent of latitude. Steady forced solutions are obtained through a matrix inversion procedure. This model is of a type commonly used in dynamical studies of forced planetary waves in the troposphere and stratosphere, such as by Charney and Drazin (1961) and, more recently, by Nakamura (1976) and Kirkwood and Derome (1977) to study the effect of changes in vertical resolution on the modelling of forced stationary wave.

The basic state has the $\beta$ plane centred at 45° and vertical profiles of zonal flow and temperature which are representative of northern hemisphere winter conditions at this

<table>
<thead>
<tr>
<th>Level</th>
<th>Pressure (mb)</th>
<th>Height (km)</th>
<th>Basic dissipation e-folding time (d)</th>
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<tr>
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<td>3.21 x 10^{-3}</td>
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<td>2.5 5</td>
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<tr>
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<td>15</td>
<td>668</td>
<td>1.14</td>
<td>5 \infty</td>
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</tbody>
</table>

TABLE 1. DISTRIBUTION OF MODEL LEVELS
latitude. These profiles are shown in Fig. 1. The only dissipation used in this model is a Newtonian cooling on the temperature perturbations in the stratosphere (Dickinson 1973) and a representation of the boundary layer in the troposphere. The e-folding times for this dissipation are shown in Fig. 1 also. The tropospheric forcing is based on values used by Bates (1977) for orographic and thermal forcing of zonal wavenumbers one and two.

Attention is centred on the differences between results obtained using two different vertical resolutions. The vertical resolution proposed for the primitive equation model can be uniformly improved by increasing the number of levels in expression (3). The two resolutions used are the proposed 15-level resolution with $NL = 15$ and a higher resolution with $NL = 50$.

The amplitude and phase of the zonal wavenumber-1 perturbation streamfunction for this basic state are shown in Fig. 2. The continuous line represents the solution for the 50-level resolution and the crosses show the solution for the 15-level resolution. Both the amplitude and phase agree very well for the two resolutions, with significant errors occurring at the top level only. Elsewhere, the differences between the solutions are less than 20% for the amplitude and 5° for the phase. The 15-level resolution is able to represent adequately all the amplitude minima and rapid phase variations that occur in the solution for the higher resolution. This comparison is typical of the solutions obtained for other zonal wavenumbers and for other basic states. In the worst case (see Karoly (1980) for details), the solutions for zonal wavenumber two in a basic state representative of mean conditions at 60°N have large differences at the highest two model levels, but at lower levels the differences are small.

These results suggest that the 15-level resolution is sufficient to represent accurately the vertical variations of stationary planetary waves in the troposphere and stratosphere, except perhaps at the highest two levels, above about 0.3 mb or 60 km, where errors may become significant.

(c) Dissipation

The form for the dissipation described in HK is also used here. Biharmonic horizontal diffusion of the form $KV^4X_n$ is included so that critical lines are treated in a linear, dissipative manner: $K = 2.338 \times 10^{16} \text{m}^4 \text{s}^{-1}$ for most experiments.

The linear dissipations on the vorticity, divergence and temperature perturbations are specified at each model level, with e-folding times given in Table 1. At the lowest level, increased dissipation represents boundary layer effects. In the troposphere, thermal dissipation represents the observed damping of standing temperature waves by transient eddies and radiation (Lau 1979). In the stratosphere, thermal dissipation is based on the radiative damping of Dickinson (1973). To prevent the reflection of vertically propagating waves from the upper boundary of the model, increased dissipation is used to form a sponge layer at the highest two model levels. This enhanced dissipation in the mesosphere has some justification as a representation of the effects of breaking gravity waves and tides (Holton and Wehrbein 1980). The sensitivity of the solutions to changes in the dissipation is discussed later.

(d) Eliassen–Palm cross-sections for stationary planetary waves

Eliassen–Palm (EP) cross-sections are meridional sections showing the EP flux $F$ by arrows and its divergence by contours. Edmon et al. (1980, hereafter EHM) reviewed the theory and use of EP cross-sections as a diagnostic of disturbances on a mean zonal wind. Readers are referred to that paper for background and earlier references.

An EP cross-section is a useful diagnostic for forced stationary planetary waves because, when the eddy dynamics are wave-like, $F$ may be treated as the rate of transfer of wave activity from one position to another. For planetary waves on a slowly-varying zonal wind, such that WKB theory is valid, $F$ is parallel to the projection of the local group velocity on the vertical–meridional plane. For stationary disturbances the diver-
Figure 1. Vertical profiles of the zonal-mean (a) zonal wind, (b) temperature and (c) e-folding time scale for the dissipation on temperature perturbations used as the basic state in the height-longitude model. The continuous line shows values used in the 50-level model and crosses values for the 15-level model.
Figure 2. Vertical profiles of zonal wavenumber-one streamfunction (a) amplitude and (b) phase for the basic state in Fig. 1 and both thermal and orographic forcing. The continuous line shows the 50-level model solution and crosses the 15-level solution.
gence of $\mathbf{F}$ is due solely to non-conservative effects in the wave dynamics. Examples of the use of EP cross-sections as a wave diagnostic are given in EHM, Dunkerton et al. (1981) and Palmer (1981).

Here the primitive equation expressions in spherical geometry and pressure coordinates for $\mathbf{F}$ and its divergence from Andrews and McIntyre (1978) are used. The graphical representation of the EP cross-section in EHM uses mass-weighted $\mathbf{V} \cdot \mathbf{F}$ contoured in the latitude–pressure plane and arrows drawn proportional to the EP flux so that the arrows appear nondivergent if $\mathbf{V} \cdot \mathbf{F} = 0$. Such cross-sections are not suitable for regions extending over many scale heights, such as the troposphere and stratosphere, because the pressure coordinate compresses height and the EP flux decreases rapidly with height.

To overcome this problem, log pressure $z = -\ln \sigma$ is used as the vertical coordinate and the EP flux is rescaled by dividing its vertical component by the pressure. The flux arrows retain their correct directions in this coordinate system but appear divergent if $\mathbf{V} \cdot \mathbf{F} = 0$. This rescaling is equivalent to dividing by the density the expressions used in Dunkerton et al. (1981) for EP cross-sections using the log-pressure coordinates. The same expression as in the latitude–pressure plane is used to represent $\mathbf{V} \cdot \mathbf{F}$ with $z$ as vertical coordinate, so contours representing $\mathbf{V} \cdot \mathbf{F}$ in the $(\phi, z)$ plane are related to the acceleration of the zonal flow.

This scaled form of the EP cross-section in log-pressure coordinates is used in this study. It has the disadvantages that the flux arrows do not represent a conserved quantity for steady, conservative waves and that the arrows will appear divergent if $\mathbf{V} \cdot \mathbf{F} = 0$. However, it has the advantage that a single cross-section can be used extending over many scale heights where the arrow directions show the direction of wave propagation, their magnitude is representative of eddy flux amplitudes and contours show the wave-induced acceleration of the mean flow.

3. Northern hemisphere winter basic state

Model solutions are now presented for thermal and orographic forcing in a basic state representative of northern hemisphere winter conditions. The zonal wind is obtained from the analytic form used in KH, based on observations in Oort and Rasmussen (1971), Newell et al. (1972) and COSPAR (1972). Easterly winds exist at the equator at all levels. The flow in the troposphere is similar to that used in the 5-level model. The temperature and surface pressure are obtained from the zonal wind using a balancing procedure described in Hoskins and Simmons (1975). The zonal wind and temperature distributions are shown in Fig. 3. Comparison of this flow with that given by the analytic expressions in KH shows that the vertical resolution of the model is able to represent the vertical variations of the basic state. The forcing distributions used in this model are based on those described in HK but, in this study, the results for each zonal wavenumber are presented separately.

(a) Zonal wavenumber one

(i) Low latitude heating. The model response to zonal wavenumber-one thermal forcing at low latitude is considered first. The meridional distribution of the forcing is that of a cosine squared centred at 15° latitude with meridional extent 7°–23°. The forcing is centred at 0° longitude for all the simple distributions. The vertical profile is proportional to $\sin \pi \sigma$ with maximum at 500 mb ($\sigma = 0.7$). Since the model is linear, the magnitude of the forcing is required only to give ideas on the amplitude of the responses. In this case, the forcing is the zonal wavenumber-one component of the low latitude heating used in HK, having vertically-averaged heating rate of 2.5 K d$^{-1}$ and wavenumber-one amplitude of 0.4 K d$^{-1}$. The heating is balanced by upward motion, as in HK.

The amplitude and phase of the height field perturbation are shown in Fig. 4. The solution at the highest two model levels is not shown because of the doubts about the solution there which were discussed earlier. The amplitude of the height field response
increases with height in the troposphere. At 300 mb (z = 1.2) there are three maxima but only the amplitude of the high latitude maximum increases in the stratosphere to a maximum of 10.7 dm at 80° and 22 km (z = 3.3). The amplitude maximum in the upper stratosphere is located just poleward of the maximum zonal wind at that level. At high latitudes the phase of the height field increases westward with height and, in the stratosphere, the phase tilts westward towards the equator.

The amplitude of the height field gives a misleading impression of the magnitude of the low latitude response. The amplitude of the streamfunction perturbation, shown in Fig. 4(c), gives an indication of the size of flow perturbations in the response. It has large amplitude close to the forcing and it is only in the stratosphere that the high latitude maximum has larger amplitude than the local response.

The EP cross-section for this case, shown in Fig. 4(d), has the largest flux directed upward and equatorward in the forcing region and another region of upward flux at high latitude. The only regions of large flux convergence or divergence are close to the forcing. The direction of the EP flux shows the westward phase tilt of the height field. The upward EP flux and the westward phase tilt are in good agreement with observations (Sato 1980), suggesting that the model is able to represent wave propagation in the atmosphere.

In general, the solution in the troposphere is similar to that for this forcing in the 5-level model. The major differences are that the height field amplitude of this solution is 30% smaller at middle latitudes than for the 5-level solution and the phase is 70° east. The high latitude maximum and the westward phase tilt with height are almost the same for the two models.

(ii) Middle latitude heating. A similar heating is used at middle latitudes, centred at 45° with meridional extent from 30° to 60°. The vertical profile is proportional to \( \sigma^4 \sin \pi \sigma \), with maximum at 820 mb (z = 0.2), between the lowest two model levels. This heating is
Figure 4. Zonal wavenumber-one solution for low latitude heating in the northern hemisphere winter basic state. The solution at the highest two model levels is not shown.

(a) Amplitude of the height field perturbation (contour interval 1 dm)
(b) Longitude of the height field ridge (contour interval 60°)
(c) Amplitude of the streamfunction perturbation in non-dimensional units (contour interval 0.2)
(d) Eliassen-Palm cross-section. The contour interval for the flux divergence is $3 \times 10^{15}$ m$^2$. The maximum vertical flux is $14 \times 10^{13}$ m$^2$ and the maximum meridional flux is $5 \times 10^{13}$ m$^2$. 
the wavenumber-one component of the low-level, middle latitude heating used in HK, with amplitude 0.3 K d$^{-1}$. The heating is mainly balanced by the temperature dissipation in the troposphere. The low-level meridional wind is almost exactly out of phase with the heating so thermal advection cannot balance the heating. The thermal balance for wavenumber one in this case is different from the balance described in HK.

The amplitude of the height field perturbation and the EP cross-section are shown in Fig. 5. The shape of height field response, the phase variation and the EP flux in the stratosphere are all very similar to those for the low latitude heating, indicating that the middle and high latitude response is not very dependent on the latitude of the heating. The importance of dissipation at the forcing region and near the critical line is shown by the EP flux convergence there. The directions of the EP flux show low latitude quasi-horizontal and high latitude vertical wave guides, in qualitative agreement with the ray solutions for the middle latitude source in KH. In comparison with the 5-level solution, the 15-level solution is similar but the height field amplitude is smaller and the phase is shifted east.

Results for the response of simple linear models with fixed horizontal structure to thermal forcing (Pedlosky 1979, pp. 363–371) suggest that, for shallow heating, the response will be small and proportional to the depth of the heating. A number of solutions of this model have been found using the middle latitude heating with different vertical profiles. The response increased with the depth of heating but it was not linearly proportional to the depth. Even very shallow heating produced a significant response in the stratosphere. This difference from the results obtained using models with fixed horizontal structure is due to the possibility of horizontal propagation in this model as well as the vertical propagation allowed in the simpler models.

(iii) Simple orographic forcing. Solutions for forcing by a simple mountain are now described. Its meridional distribution is that of a cosine squared centred at 30° with
meridional extent 18°–52°. This is the simple circular mountain of height 2 km used in HK and the wavenumber-one component, with amplitude 0.3 km, is used as the forcing in this case.

As noted in HK, there is a quadrupole pattern of low-level vertical velocity above the forcing. In this case however, the orographic forcing for wavenumber one is balanced by the boundary layer dissipation, which is different from the balance described in HK. The 5-level solution is quite different from the 15-level solution because of the different forcing balances.

The 15-level height field and EP cross-section are very similar to those for the middle latitude heating away from the forcing region and they are not shown. These similarities show that the wave propagation and the remote response are qualitatively independent of the forcing mechanism.

(iv) Thermal and orographic forcing. In order to compare the model solutions with observations of stationary planetary waves in the atmosphere, a representation of the northern hemisphere thermal and orographic forcing is used in the model. The orographic forcing uses the wavenumber-one component of the spectral representation of the earth orography described by Hoskins (1980). This has maximum amplitude of 0.7 km at 35°, representing the Eurasian plateau, and large amplitude over a wide latitude band. The thermal forcing is a combination of the low and middle latitude heating distributions described earlier. Since the response to heating at the equator is small (HK), tropical thermal forcing is represented by mid-tropospheric heating centred at 15°. The amplitude and phase of each zonal wavenumber component of this heating, given in Table 2, are based on a Fourier analysis of the vertically-averaged heating equivalent to the December–February precipitation distribution at 15°N given in Newell et al. (1972). The middle latitude thermal forcing is represented by low-level heating at 45°. The amplitude and phase of each zonal wavenumber component, given in Table 2, are based on a Fourier analysis of the observed winter heating rates at 45° at 1000 mb and 700 mb given by Lau (1979). This combined forcing gives an estimate of the total stationary wavenumber-one forcing in the northern hemisphere troposphere.

Since the model is linear, the total response is just a combination of the response to each forcing but, because the phases are different, it is not a simple addition of the amplitudes. The response to the thermal forcing alone is of an amplitude comparable with that due to orographic forcing, suggesting that thermal forcing makes an important contribution to forced stationary waves in the stratosphere.

The response to the total forcing is shown in Fig. 6. The phase of the height field agrees well with the observations of van Loon et al. (1973), particularly the vertical tilt at high latitude. This is also shown by the EP cross-section, in which the EP flux direction agrees well with that determined by Sato (1980) from observations of wavenumber one in winter. However, the height field amplitude does not agree well with observations (Hirotta 1976). The amplitude is too small, except in the high latitude troposphere, where it is much too large. In the lower stratosphere, the amplitude maximum is too far poleward but, in the upper stratosphere, it is close to the observed latitude. The stratospheric

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**TABLE 2. NORTHERN HEMISPHERE THERMAL FORCING**

<table>
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<th>Zonal wavenumber</th>
<th>Low latitude heating</th>
<th>Middle latitude heating</th>
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<td></td>
<td>Amplitude (K d⁻¹)</td>
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Figure 6. Wavenumber-one solution for thermal and orographic forcing in winter.
(a) Height field amplitude (contour interval 1 dm)
(b) Longitude of the height field ridge (interval 60°)
(c) EP cross-section (contour interval $10 \times 10^{12}$ m$^2$)
maximum is only 15 dm at 22 km \((z = 3.3)\), much smaller and lower than observed. Possible reasons for the poor representation of the amplitude of wavenumber one in this model are discussed later.

(b) Zonal wavenumber two

The forcing of wavenumber two has about the same amplitude as wavenumber one in the simple distributions. In general, the forcing balance is as described in HK, with advection more important and dissipation less important than for wavenumber one.

The remote responses to the simple wavenumber-two forcing distributions are all similar. The height field amplitude has a tropospheric maximum at about 30° and a high latitude maximum extending into the stratosphere to a height of about 30 km \((z = 4.5)\). This maximum is located higher than and equatorward of that for wavenumber one. The EP cross-section has larger EP flux and flux convergence than for wavenumber one. The 5-level solutions generally have larger tropospheric maxima located poleward and west of those in the 15-level solutions.

The wavenumber-two components of the thermal forcing in Table 2 and the earth orography are combined to give a representation of the total northern hemisphere stationary wavenumber-two forcing. The amplitude and phase of the height field response to the combined forcing are shown in Fig. 7. The shape of the response and the position of the stratospheric maximum are the same as for the simple wavenumber-two forcing distributions and agree well with observations (van Loon et al.) but the amplitude, with maximum of 11 dm at 38 km \((z = 5.7)\), is too small. The phase variation is also in good agreement with observations. The EP cross-section, shown in Fig. 7, is similar to that for wavenumber one in the troposphere. In the stratosphere, the EP flux and flux convergence at the critical line are much larger than for wavenumber one.

In general, the wavenumber-two solutions agree with observations much better than those for wavenumber one. The response away from the forcing does not depend on the forcing mechanism, with both orographic and thermal forcing giving similar stratospheric responses. Although both observations and the ray tracing in KH suggest that the stratospheric maximum should be lower and more equatorward with increasing wavenumber, the model solutions only show the second behaviour. This result is probably affected by the poor model solutions for wavenumber one.

(c) Zonal wavenumber three

For the simple forcing distributions, the wavenumber-three component of the forcing is about 80% of the amplitude of wavenumber one. All the responses are similar. The height field amplitude has a maximum at the mid-latitude tropopause, with decreasing amplitude in the stratosphere. The EP cross-section has large upward flux and flux convergence in the troposphere above the forcing but small amplitude in the stratosphere. Effectively, the response for wavenumber three is trapped in the troposphere in agreement with observations (van Loon et al.) and the ray tracing in HK. The 5-level solution is similar to the 15-level solution, with the same positions for the maxima but larger amplitude.

The solution for the combined northern hemisphere thermal and orographic forcing is shown in Fig. 8. The shape of the response and the position of the maxima agree well with observations and the amplitude is much closer to that observed than for smaller wavenumbers. The phase is also in good agreement, differing by less than 10° longitude at the amplitude maximum from that in van Loon et al. The EP cross-section has larger amplitude in the troposphere than it has for smaller wavenumbers but the amplitude in the stratosphere is small.

Solutions for zonal wavenumber four have been obtained but they are not presented here. The forcing balances are the same as for wavenumber three, the response is confined to the troposphere and the maxima are smaller, lower and equatorward of those for
wavenumber three. The 5-level model solutions for wavenumber four agree very well with the 15-level solutions in the troposphere. This suggests that a model with adequate vertical resolution in the troposphere only should be able to give a reasonable representation of forced stationary waves with wavenumbers three and higher. However, for small
wavenumbers there is vertical propagation out of the troposphere in this basic state and the model representation of these waves is likely to have too large an amplitude and a westward phase shift.

Figure 8. Wavenumber-three solution for thermal and orographic forcing in winter.
(a) Height field amplitude (interval 0-4 dm)
(b) Longitude of the height field ridge (interval 20°)
(c) EP cross-section (interval 10 \times 10^3 \text{ m}^3)
4. **Sensitivity of the solutions**

(a) *Sensitivity to the dissipation*

The form of the dissipation used so far was considered to be the best representation for this model of the dissipation on stationary planetary waves in the atmosphere. The sensitivity of the wavenumber-one solutions is tested by varying the dissipation for all the simple forcing distributions.

Variation of the sponge layer dissipation at the highest two levels results in no changes in the lower levels. There is no apparent reflection from the upper boundary or the region of increased dissipation and the dissipative layer is able to absorb any vertically propagating waves.

Removal of the scale-selective biharmonic diffusion causes large changes. The solutions have a large amplitude, resonant structure since the linear damping is not very effective and the waves reflect from the model boundaries. However, when the magnitude of this dissipation is reduced by half or when it is changed to a higher-order form, the response is increased only slightly. This suggests that inclusion of a scale-selective dissipation in the model is important but that the solutions are not sensitive to the magnitude or form of this dissipation.

The solutions are not very sensitive to the dissipation on temperature perturbations in the model. Increasing the thermal dissipation to values suggested by Blake and Lindzen (1973) causes a slightly reduced response at high latitudes and in the stratosphere, a westward phase shift of the high latitude maximum and a slight reduction of the westward phase tilt with height.

The flow dissipation at the lowest level, representing the boundary layer, is important in the forcing balance for wavenumber one. When it is halved, the amplitude is approximately doubled but the structure is not changed. When it is removed completely, the forcing balance and the local response are changed dramatically but the amplitude is not increased greatly. The sensitivity of the response amplitude to realistic changes in the boundary layer dissipation suggests that the observed amplitudes could be modelled better using smaller dissipation or a shallower boundary layer.

It should be noted that these results are all for wavenumber one, for which the model solutions do not compare well with observations, and the sensitivity to the dissipation may be different for higher wavenumbers.

(b) *Sensitivity to the basic state*

All the solutions described so far have used a single basic state representative of northern hemisphere winter conditions. To investigate the sensitivity of the solutions to changes in the zonal flow, solutions for other representative basic states are discussed. All changes to the basic state affect the local response to the forcing, which determines the wave source, and the medium through which the waves propagate. It is important to separate these two effects.

The first variant of the basic state has the same shape as for winter but stronger westerly winds, with tropospheric maximum of 36 m s\(^{-1}\) and stratospheric maximum of 89 m s\(^{-1}\). A typical solution is for wavenumber-one forcing by the simple mountain at 30°. The height field perturbation, shown in Fig. 9, has smaller amplitude than for the original basic state, but the stratospheric maximum, at 71° and 37 km (z = 5.5), is higher and further equatorward, agreeing much better with observations. The wavenumber-one solutions for the other simple forcings are similar, having smaller amplitude but maxima higher in the stratosphere than before. This is consistent with the model results of Scharmer and Geller (1977) who found increased vertical propagation for stronger westerly winds. The ray solutions in KH suggest that there is reduced equatorward propagation and enhanced vertical propagation for stronger westerly wind.

With the stronger westerly wind, the phase of the high latitude maximum in the troposphere is shifted eastward by about 90° but, at middle latitudes, the phase shift is
much less. This shows the great sensitivity of the phase of the high latitude tropospheric response to variations of the zonal wind. An average solution for a number of different zonal flows would have a larger net tropospheric response in middle latitudes than in high latitudes, where the phase varies more.

The second case has zonal wind almost identical to that used for the original basic state except at the three lowest levels, where the winds are stronger. All the wavenumber-one solutions for this basic state are similar, but quite different from the solutions for the original basic state. In the troposphere, the middle latitude response is larger than before and the high latitude response is smaller. The largest upward EP flux is in middle latitudes (see Fig. 10) and at high latitudes there is weak upward or even downward EP flux. The stratospheric maximum is located much higher than before, at about 47 km (z = 6·8). Although this solution seems to agree better with observations, the amplitude is much smaller than for the original basic state and the phase structure is poor, having eastward tilt with height at high latitude.

A possible explanation for the increased height of the stratospheric maximum for this case can be found by considering the wave propagation arguments in KH. The effective wave source for this case is in middle latitudes rather than at high latitude, as shown by the EP flux. As wave activity propagates upward and poleward from a middle latitude source, it reaches a greater height before reflection from the ray caustic near the pole than
from a high latitude source. Thus the vertical propagation will be larger from a middle latitude source.

The low-level wind field is very important as it determines the local response to any forcing and hence the effective wave source. This model is particularly sensitive to variations of the basic state at the lowest levels.

5. Northern hemisphere summer

A zonal-mean zonal wind distribution representative of summer conditions is used to investigate the forced stationary wave response in the northern hemisphere summer. The zonal flow is based on that used for the summer in KH and is shown in Fig. 11. It has a westerly jet maximum of 14 m s$^{-1}$ at 43° and 190 mb (z = 1.6) with easterly wind above 50 mb (z ~ 3) in the stratosphere and equatorward of 25° in the troposphere.

For the low latitude heating, the response is confined to the forcing region as it is in easterly wind, much as for the equatorial forcing in winter. The surface wind is much weaker than in the winter basic state and the response to orographic forcing is small.

The solution for wavenumber-one, middle latitude, low-level heating is shown in Fig. 12. The forcing balance is the same as for winter, being dominated by the boundary layer dissipation. The largest amplitude height field perturbations occur near the tropo-
Figure 11. Zonal-mean zonal wind (contour interval 5 m s\(^{-1}\)) for the northern hemisphere summer basic state.

Figure 12. Wavenumber-one solution for middle latitude, low-level heating in the northern hemisphere summer.

(a) Height field amplitude (interval 0.2 dm)
(b) EP cross-section (interval \(4 \times 10^3\) m\(^2\))
pause and the amplitude is small in the easterly wind. The shape of the height field and
the positions of the maxima and minima agree well with the observations of wavenumber
one in the northern hemisphere summer by van Loon et al. The major difference is that
the observed height field has eastward phase tilt with height whereas the model solution
has westward phase tilt with height in the troposphere. The EP cross-section shows this
phase structure, with upward flux in the middle latitude troposphere. There is large flux
convergence at the forcing and at the critical line in the stratosphere. The observed EP
cross-section for all stationary planetary waves in the northern hemisphere summer in
EHM is different, having upward flux near the surface only and downward flux in the
upper troposphere, with flux divergence at the tropopause in middle latitudes.

A number of forcing distributions have been used in an attempt to find a forcing
mechanism which gives upper tropospheric EP flux divergence and downward EP flux in
this summer basic state. All thermal forcing distributions give EP flux convergence and
upward flux, even when the maximum heating is at the tropopause. The forcing which
gives an EP cross-section similar to that observed is direct forcing of the vorticity pertur-
bation in the upper troposphere.

The vorticity forcing has meridional distribution of a cosine squared centred at 45°
with extent 23°−67°, vertical profile proportional to (1−σ)6 sin2 ησ, with maximum at
280 mb (z = 1.3) and the wavenumber-one component has amplitude
3.5 × 10−3 × 2πΩ d−1. The vorticity balance at the forcing is maintained by zonal and
meridional advection and the solution is shown in Fig. 13. The height field again has
maxima near the tropopause and small amplitude in the stratosphere. The shape of the
height field agrees with that observed, apart from the absence of the low-level maximum
at middle latitudes. There is eastward phase tilt with height in the upper troposphere. The
EP cross-section has flux divergence just above the forcing region and downward flux in
the upper troposphere. There is upward flux and flux convergence in the lower tropo-

![Figure 13](image_url)

Figure 13. Wavenumber-one solution for middle latitude, upper tropospheric vorticity forcing in the northern hemisphere summer.

(a) Height field amplitude (interval 0-2 dm)
(b) EP cross-section (interval 2 × 10^{13} m²)
sphere at middle latitudes. This agrees well with the observed EP cross-section in EHM.

The agreement between this solution and observations supports the speculation in EHM that upper tropospheric vorticity forcing may play a role in determining the structure of stationary planetary waves in the northern hemisphere summer. Further support is given by the observational study of Holopainen and Oort (1981), who found that the enstrophy of stationary disturbances is forced by large-scale transient eddies in the northern hemisphere middle latitudes during summer. Thermal forcing is also important in summer but, in this model, it does not give the observed EP cross-section.

6. DISCUSSION

Stationary planetary wave solutions in the troposphere and stratosphere have been obtained for orographic and thermal forcing in several basic states using a linearized primitive equation model and a matrix inversion technique. The solution for wavenumber one in winter for combined thermal and orographic forcing does not compare well with observations. The major deficiencies are that the solution has too large an amplitude in the high latitude troposphere and that the amplitude maximum in the stratosphere is too low.

The wavenumber-one solutions are very sensitive to variations of the basic state. Small changes to the low-level zonal wind lead to large changes in the local response to forcing and alter the amplitude structure markedly. Even if the amplitude structure is unaffected, the phase of the high latitude tropospheric response is very sensitive to the zonal wind, moving eastward for increased westerly wind. It may be unrealistic to compare the model stationary solutions for wavenumber one in the winter basic to the observed standing wavenumber-one structure, which is a seasonal average, taking into account the high sensitivity of the model solutions for wavenumber one to the basic state. It may be possible to obtain more realistic model solutions for wavenumber one in the troposphere and stratosphere only by using a time-averaged nonlinear model.

The deficiencies of the solution for wavenumber one do not just occur for this model. The wavenumber-one solutions for the similar 5-level model described in HK also have too large an amplitude in the high latitude troposphere when compared with observations. However, examination of 5-day mean height field analyses in the upper troposphere does show large amplitude for wavenumber one at high latitude on occasions of a displaced polar vortex but the phase of wavenumber one is extremely variable between different periods.

Given the deficiencies of the solution for wavenumber one, the low position of the amplitude maximum in the stratosphere is consistent with a high latitude source in the troposphere rather than a middle latitude source, using wave propagation arguments from KH.

The solutions for wavenumbers higher than one compare well with observations apart from having too small an amplitude. They are not as sensitive to variations of the basic state. Improved solutions with larger amplitude could be obtained using smaller boundary layer dissipation or a shallower boundary layer.

Comparisons between the solutions for the 15-level and 5-level models show that the solutions converge for increasing wavenumber. This suggests that a model which represents the troposphere only should give realistic solutions for wavenumbers three and higher but the solutions for the smaller wavenumbers are likely to have too large amplitude and a westward phase shift at high latitude.

In the winter basic state, the stratospheric solutions for both orographic and thermal forcing are very similar. The model results suggest that the response in the stratosphere is determined by propagation from the perturbation in the troposphere and is independent of the forcing mechanism. It is likely that orographic forcing of stationary planetary waves in the stratosphere is more important than thermal forcing but this study suggests that the remote response to thermal forcing is not negligible, even in the stratosphere.
The solutions for thermal forcing in the summer agree well with the observed amplitude structure but have incorrect phase variation. The observed EP cross-section for all stationary waves in the northern hemisphere summer (EHE) is suggestive of wave forcing in the upper troposphere. The forcing distribution in the model which gives an EP cross-section similar to that observed is direct forcing of the vorticity perturbation in the upper troposphere, which suggests that vorticity forcing may be important in the summer.

Forced stationary wave solutions have been obtained for a basic state representative of autumn conditions but there are some difficulties in understanding these solutions. The zonal flow profile, from KH, has the same shape as the winter flow but smaller amplitude, with a stratospheric jet of 31 m s\(^{-1}\). A typical solution is that for zonal wavenumber-two northern hemisphere thermal and orographic forcing shown in Fig. 14. The height field is generally similar in shape to that for the winter case except that the amplitude is less than half that for winter and the position of the amplitude maximum is higher in the stratosphere. The reduced amplitude in autumn is in agreement with observations and indicates reduced forcing response and increased dissipation in the weaker autumn flow. However, the higher position of the amplitude maximum in autumn than in winter is not in agreement with observations or the ray tracing in KH, both of which suggest that the amplitude maximum should be lower in autumn. The other wavenumber-one and -two solutions in the autumn flow also have smaller amplitude than in winter but amplitude maxima higher in the stratosphere.

Since this difference from observations is not dependent on wavenumber or the forcing, it may be due to the basic state. The EP cross-section in Fig. 14(b) shows wave propagation from an effective source at lower latitude than for the winter case in Fig. 7(c) and this is also apparent in the other autumn solutions. This lower latitude for the tropospheric wave source may lead to an increased height for the stratospheric maximum.

![Figure 14. Wavenumber-two solution for northern hemisphere thermal and orographic forcing in the autumn basic state.](image)

(a) Height field amplitude (interval 0.3 dm)
(b) EP cross-section (interval 2 \(\times\) 10\(^{13}\) m\(^2\))
as in the case with changed low-level winter flow described in section 4(b). This explanation is not altogether satisfactory and the reason for the position of the amplitude maximum in the autumn solutions of this model is a subject for further study.

After this paper was submitted, two related papers, Lindzen et al. (1982) and Ronghui and Gambo (1982), were published. These describe separate three-dimensional model studies of the steady linear response of the atmosphere to thermal and orographic forcing in winter. Their results are generally similar to those presented here, though their solutions for zonal wavenumber one compare better with observations. Lindzen et al. also note the sensitivity of their solutions to small changes of the tropospheric basic state.

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