The effects of horizontal diffusion on baroclinic development in a spectral model

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SUMMARY

Linear and non-linear effects on baroclinic development of linear horizontal diffusion of the form $\kappa V^q$, where $q$ is an integer and $\kappa$, in general, a constant, are investigated using a spectral model and a climatological wintertime northern hemisphere basic state.

An eigenvalue approach shows that the greatest linear sensitivity to the diffusion to be in the high zonal wavenumbers ($m > 12$) which, in the absence of diffusion or with the more scale-selective formulations (large $q$), are more unstable than the intermediate wavenumbers ($m = 7$--9). The $V^4$ diffusion with a decay time scale of 6 h on the shortest retained scale, which has traditionally been used in our model, is shown to affect significantly the linear stability of almost all wavenumbers, whereas the more scale-selective formulations have only a very small effect on $m < 12$.

Without dissipation, integrations exhibit grossly physically unrealistic features after about nine days even with very high resolution, indicating the crucial role played by dissipation in non-linear baroclinic development. Non-linear integrations with dissipation show great sensitivity to the formulation used. Typically about three times more energy is converted from potential to kinetic with a $V^9$ than with a $V^4$ diffusion having a similar decay time scale on the smallest retained spatial scale. It appears that as the dissipation time scale on the large spatial scales is increased while retaining sufficiently short time scales near the truncation limit to prevent the accumulation of energy there, so the amount of potential to kinetic energy conversion approaches a limiting value equal to about five times the energy conversion with the $V^4$ diffusion. In the limiting case the amount of energy lost to dissipation is equivalent to about 45% of that converted from potential to kinetic energy, again emphasizing the crucial role played by dissipation.

It is found that if the decay time scale on the short spatial scales is too long then the integrations exhibit large truncation dependency. This provides a criterion for judging the suitability of a particular choice of dissipation, which might have quite general applicability.

1. INTRODUCTION

With a spectral model extended non-linear integrations can be made without any dissipation, although the solutions quite soon become physically unrealistic due to the accumulation of energy near the truncation limit. Thus some form of diffusion is usually incorporated to parametrize the effects of motions on unresolved scales, assuming that these effects can be represented as a dissipation on the retained scales. Unfortunately the physical basis of such parametrizations is not particularly sound, relying mainly on heuristic arguments but this is perhaps unavoidable given the present uncertainty about the behaviour of the atmosphere itself. Although intuitively and also on the basis of turbulent flow theory (Tennekes 1978) one would expect negligible direct effect of small-scale ($\sim 100$ km say) eddies on the larger scale ($>1000$ km say) flow, the analysis of atmospheric data by Burrows (1976) and Chen and Win-Nielsen (1978) suggests, on the contrary, that the unresolved scales do exert a significant dissipative influence on the large-scale flow.

Considerable effort has been expended by various groups in tuning the dissipation parametrizations in their general circulation (GCM) or forecast models (see for example Williamson (1978), Manabe et al. (1970), Jarraud and Cubasch (1979)). The presence of multiple, possibly interdependent energy sources and sinks in such complex models does however make it rather difficult to isolate the effects of the individual parametrizations and it is not clear that the optimum dissipation formulations have been found. Recently, for example, D. G. Andrews, J. D. Mahlman and R. W. Sinclair (to be published in the J.A.S.) have suggested that the fact that some spectral GCMs appear to give significantly worse results at higher horizontal resolutions than equivalent finite difference models, in contrast to the superiority of the former at lower resolutions, may be attributed to inappropriate levels of dissipation.
In this paper we consider a relatively idealized but nevertheless fundamental problem: the effect of dissipation on the development of baroclinic waves. To simplify the problem further we exclude all energy sources and allow only one energy sink, horizontal diffusion, this being the minimum necessary to ensure that the integrations do not rapidly develop grossly physically unrealistic features. The impact of different linear formulations of the diffusion on both linear and non-linear baroclinic development are investigated and criteria developed for judging the suitability of specific choices. The details of such an investigation are inevitably to some extent rather specific to the particular problem, but the strategy used in the study and the physical insight gained by having treated such a simplified case should be of more general applicability, particularly to dynamical as opposed to general circulation or forecast models.

2. OUTLINE OF THE MODEL AND NUMERICAL EXPERIMENTS

All the investigations described in detail here were based on the 5-level, \( \sigma \)-coordinate spectral primitive equation model with triangular truncation described by Hoskins and Simmons (1975). The model is adiabatic apart from the inclusion of a linear horizontal diffusion applied to the relative vorticity, divergence and temperature. Sections 4 and 5 describe the results of non-linear integrations with this model, the diffusion having been set to zero for those presented in section 4. Section 3 presents linear normal mode results which were obtained from the eigenvalues and eigenvectors of a matrix which can be set up using a romboidal truncation version of the model itself (Hoskins and Karoly 1981). Since the diffusion is linear it may straightforwardly be included in the eigenvalue problem.

Numerous calculations with different initial conditions have been carried out but only the set which exhibited most sensitivity to the diffusion is presented; qualitatively similar but less extreme behaviour was found in the other cases. The initial zonal state for this set was based on the northern hemisphere wintertime circulation statistics compiled by Oort and Rasmussen (1971). The integrations were carried out on one seventh of the hemisphere including only zonal wavenumber \( m = 7 \) and its harmonics, with an initial perturbation consisting of the most unstable \( m = 7 \) normal mode, normalized to give a surface pressure perturbation amplitude of 1 mb.

Linear diffusion of the form \( \kappa \nabla^q \), where \( q \) is an integer, is very simply included in a spectral model and furthermore has a known scale selectivity which increases with increasing \( q \). The e-folding diffusive decay time at total wavenumber \( n \) (the order of the associated Legendre function \( P_n^m \) in the spectral expansion) is given by

\[
\tau(n) = \kappa^{-1} [n(n + 1)a^{-2}]^{-q}
\]

where \( a \) is the radius of the earth. In this study we consider only diffusion of the above form with various values of \( \kappa \) and \( q \), but before presenting the results it is useful to discuss how these values are chosen and what criteria can be used for intercomparisons of the results obtained. Rather than quote values of \( \kappa \) which are not directly physically very understandable we will describe the various choices in terms of the time scale \( \tau \) at the smallest spatial scale retained in our standard truncation (which corresponds to \( n = 42 \)), irrespective of the actual truncation used.

The model dissipation should remove energy from the end of the spectrum at a rate sufficient to prevent a spurious accumulation of energy there, while at the same time not affecting the baroclinically active scales. Ideally the rate of energy dissipation near the truncation limit should be typical of that at which, in the atmosphere, energy is transferred to the scales not resolved by the model. Unfortunately our knowledge of such transfers in the atmosphere is very poor. A very tentative calculation similar to that presented by Barros and Wini-Nielsen (1974), in which on the smaller resolved scales a balance is assumed between dissipation and the flux of enstrophy through the spectrum,
has been performed using the data presented by Chen and Wiin-Nielsen (1978). If a $V^4$ or $V^6$ diffusion formulation is assumed, this calculation suggests that $\tau(n = 42)$ is of order 1 h, although the uncertainty is large since the data used only extend down to $n = 25$. It would be of great interest to repeat such calculations on more modern data sets which might be able to give some reliable information about smaller scales. On the smaller scale we do know that the time scale for the sharpening of fronts is of order 6 h (Hoskins 1982). These results suggest a diffusive decay time scale of several hours on the smallest scales resolved by the model and in fact we have traditionally used a 6 h $V^4$ diffusion. For comparison the following are typical formulations in the spectral models used by other groups or authors, although it should be noted that many of these models also include other dissipative processes apart from horizontal diffusion: 24 h $V^2$ (Bourke 1974; Frederiksen 1981), 62 h $V^2$ (Baer and Alyea 1974), 18 h $V^4$ (Daley 1981), 14 or 36 h $V^4$ (Gordon and Stern 1982), 23 h $V^4$ (Sela 1980) and 200 h $V^4$ (Jarraud and Cubasch 1979).

Figure 1 shows the scale selectivity of most of the diffusion formulations discussed here. In the model the most active baroclinic waves are typically dominated by wavenumbers close to $n = 15$ and have e-folding times of order 2 d. From Fig. 1 it is clear that the 6 h $V^4$ formulation is not really scale-selective enough to avoid significant dissipation of these wavenumbers. Although the 4 h $V^6$ formulation avoids significant dissipation on the dominant baroclinic scales, it is still not clear that dissipation will have a negligible effect on the baroclinic development as it is found that wavenumbers up to about $n = 30$ contribute significantly to the structure of the waves. Thus a series of even more scale-selective diffusions has been considered. Empirically it was found that to maintain acceptably smooth solutions $\tau$ must be less than about one day for the highest eight or so wavenumbers, thus dictating a decrease in the diffusive decay time at $n = 42$ for increasing $q$. In order to meet this criterion and at the same time to ensure negligible effects on
baroclinic development (τ greater than 100 d, say, for n < 30) a V^{36} formulation with τ(42) ~ 50 s would be necessary. Such a short dissipative time scale does not accord with our physical intuition and therefore a 'modified V^{6} diffusion in which κ is allowed to be a function of n has been used as a practical alternative. In theory such an arbitrary choice of scale dependence of the diffusion can lead to undesirable non-local effects when applied to localized structures such as fronts, which involve many spectral components. Simple tests have suggested that this problem may not be too severe but it is difficult to investigate its effect within an actual integration.

Because of the highly idealized nature of the experiments reported here the use of comparisons with atmospheric circulation statistics or analyses to make judgements on the various diffusion formulations is inappropriate and other criteria must be sought. The simplest but most subjective criterion that can be used is a visual one—the degree to which synoptic-scale detail (i.e. frontal structure) is reproduced in the fields of the model variables and the level of small-scale 'noise'. A more objective measure of the latter is obtained from the behaviour of the tail of the energy spectrum. The behaviour of the energy spectrum on scales between those dominated by baroclinic energy generation and those dominated by dissipation could be examined for similarity to the atmosphere or theoretical expectations. However, this is also not a very stringent criterion in view of the fact that the spectral slopes calculated from the atmospheric data (e.g. Burrows 1976) and predicted by theories (e.g. Blumen 1981) vary over the large range from −2 to −4. Furthermore, as our integrations were carried out on a fraction of the hemisphere the spectra obtained are very noisy. Fortunately a more objective, and related criterion can be found.

The dissipation at scales beyond the truncation limit may be regarded as infinite and a satisfactory formulation of the horizontal diffusion should provide sufficient dissipation at the high wavenumber end of the spectrum to provide a smooth transition to this infinite dissipation region. This can be tested by repeating the integrations with a higher truncation and rejecting those formulations for which the results alter significantly with the truncation over the period of integration in which we are interested. This can be regarded as a predictability criterion; Lorenz (1969) has discussed, for a simple model, the relationship between predictability and the spectral slope. Clearly this criterion will only be useful if the basic truncation is such that the essential dynamics are well represented and if the model is intrinsically predictable on the time scale of interest, which should be the case for the model described here.

Finally, an attempt has been made to see if, for dissipation formulations which meet the above criterion, there is convergence to a limiting behaviour in terms of the eddy energy levels reached and the total conversion of potential into kinetic energy as the dissipation time scale on the large scale is increased. The various formulations can then be judged in terms of how closely they approximate this limiting behaviour.

In Table 1 we have summarized the salient aspects of the behaviour of the various diffusion formulations, derived from this study; a NO entry indicates unsatisfactory

<table>
<thead>
<tr>
<th>Diffusion formulation</th>
<th>Noise level acceptable?</th>
<th>Results truncation independent?</th>
<th>Short wave linear/growth rates small?</th>
<th>Dissipative effects local?</th>
<th>Energy dissipation level acceptable?</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 h V^4</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td>24 h V^4</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>60 h V^4</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>4 h V^6</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>1 h V^{16}</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Mod. 6 h V^6</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Mod. 3 h V^6</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>
behaviour. The reader may wish to refer to this table to put specific results of later sections in context.

3. **Linear Effects**

In Fig. 2 the linear growth rate of the most unstable normal mode in the absence of dissipation is shown as a function of zonal wavenumber. The growth rate increases monotonically up to $m = 8$ where there is a weak local maximum. For $m > 11$ the growth rate increases monotonically again so that $m = 20$ is significantly more unstable than $m = 8$. If this remained true in the non-linear regime we would expect to obtain spectra dominated by high wavenumbers, which would conflict with atmospheric observations and results from general circulation type experiments, which normally yield spectra in which the intermediate zonal wavenumbers (6–9, say) dominate over the higher wavenumbers. This problem has been the subject of much discussion (Staley and Gall 1977; Gall and Blakeslee 1977; Simmons and Hoskins 1977) from which it appears that the large growth rates at high wavenumbers may be the result of poor vertical resolution. Gall (1976) suggests that in his GCM experiments the high wavenumber disturbances only attain much smaller amplitudes than the intermediate-scale disturbances, in spite of the larger linear growth rates of the former, due to the effect of the scale-selective, non-linear diffusion included in that model. In contrast, it may be seen from Fig. 2 that the inclusion of a linear $6h \nabla^4$ or $4h \nabla^6$ diffusion in our model drastically reduces the linear growth rates of the high wavenumber disturbances, wavenumbers 17 and above exhibiting no instability in the former case. The more scale-selective formulations are less effective in
damping the high zonal wavenumber disturbances and with both 1 h  $V^6$ and the modified $V^6$ diffusions the high wavenumbers are more unstable than the intermediate ones.

In the absence of dissipation there are typically of order 10 unstable modes for each zonal wavenumber. As the dissipation on the large scales is increased so the number of unstable modes decreases and the distribution of their growth rates becomes more peaked. For $m < 12$ the growth rates, phase speeds and structures of the most unstable mode obtained with diffusions other than 6 h $V^4$ agree well with those obtained without dissipation, the agreement increasing with increasing scale selectivity. On the other hand, with the exception of the growth rate of $m = 7$ (but not its phase speed or structure), the properties of the most unstable mode obtained with the 6 h $V^4$ diffusion agree less well. In particular the stabilizing effect of the $V^4$ diffusion is significant on wavenumbers 8–11, which are virtually unaffected by the other formulations. Furthermore it reduces the growth rate for $m = 3$ and 4 significantly, reflecting the dominance of spectral components with high values of $n$ in the structure of the most unstable mode for these wavenumbers. Hoskins and Revell (1981) have also discussed this aspect of the structure of the most unstable mode at low wavenumbers for a different zonal flow.

From Fig. 2 it may be seen that in some cases the inclusion of dissipation actually increases the growth rate of the most unstable mode. In these cases it was found that the growth rate approaches its non-diffusive value from above as the dissipation on the larger scales is decreased. This behaviour is not fully understood but appears to be connected with subtle changes in the structure of the most unstable mode enabling it to use the available baroclinicity more effectively.

In conclusion it seems that, as far as linear behaviour is concerned, the 4 h $V^6$ and 3 h $V^8$ diffusions appear most satisfactory, not affecting zonal wavenumbers less than 12 significantly but strongly damping the highest wavenumbers.

4. NON-LINEAR INTEGRATIONS WITHOUT DISSIPATION

It is of interest to determine for how long integrations performed without dissipation give realistic results and whether this period can be extended by using a higher truncation. To this end integrations have been carried out with truncation at $n = 42, 63$ and 95. The results are virtually identical up to about day 9 exhibiting barotropically damped baroclinic growth (i.e. with energy conversions from zonal available potential energy to eddy energy and hence to zonal kinetic energy) as noted in previous studies both with and without dissipation (Simmons and Hoskins 1978; Simons 1972). Beyond this time the three solutions diverge from one another although the general trends are broadly similar. Between days 12 and 16 there is a rapid growth in the zonal kinetic energy but this is not accompanied by a sustained decay in the eddy energy. In fact the eddy kinetic energy continues growing, although not monotonically, over the whole period of integration. Beyond day 16 the zonal kinetic energy shows large variations but no significant overall trend upwards or downwards. Thus the barotropic decay phenomenon exhibited in the baroclinic wave life cycle integrations of Simmons and Hoskins (1978) is not reproduced here. This result, that in the absence of dissipation no mechanisms acts to limit the growth of eddy energy, within the constraint of the total energy available, conflicts with the findings of Simons (1972). This discrepancy may be attributable to the severe zonal truncation used in his model or to his period of integration not having been long enough to determine whether an absolute rather than a local eddy energy maximum had been reached.

By day 10 all three integrations have developed a white kinetic energy spectrum for $n$ greater than about 15, this part of the spectrum becoming increasingly dominated by the short wavelengths as the integration progresses. In the fields of the model variables the synoptic-scale structure becomes increasingly dominated by small-scale noise.

There is no suggestion in these results that the period over which the solutions are physically realistic can be extended by using higher resolution. It appears that a critical
stage is reached at about day 9 in which there is a rapid development of energy in the high wavenumbers which, if not controlled by dissipation, quickly dominates the solutions. This cascade of energy to high wavenumbers is consistent with that which is expected as waves are radiated towards a critical line (P. D. Killworth and M. E. McIntyre, personal communication), or in the frontogenetic collapse to singularity at a boundary (Hoskins 1982). Further integrations with dissipation only at the lowest model level, to control the frontogenetic collapse, still exhibit the same behaviour, indicating that frontogenesis alone is not responsible for the observed cascade.

5. **Non-linear integrations with diffusion**

(a) $\nabla^4$ diffusion

We first present results obtained using $\kappa \nabla^4$ diffusion formulations starting with that traditionally used in this model, which has $\tau(42) = 6h$, and then considering two others with larger values of $\tau(42)$ (60h and 24h). The latter two imply significantly reduced dissipation rates on the dominant baroclinically active scales, which should be desirable, but at the expense of reduction of the dissipation rates on the small scales to values which are less consistent with the tentative estimates in section 2 of what atmospheric values might be.

Figure 3 illustrates the time development of the 6h and 60h $\nabla^4$ integrations in terms of the globally averaged eddy (KE) and zonal (KZ) kinetic energy per unit area. In the 60h $\nabla^4$ integration about three times as much energy is converted from zonal available potential energy into eddy energy and finally into zonal kinetic energy. In the 6h $\nabla^4$ case KZ is seen to decrease significantly initially, this being entirely due to the dissipation acting on the basic zonal flow. This diffusive loss continues throughout the integration although for most of the time it is masked by the KE $\rightarrow$ KZ conversion. The total diffusive loss of KZ is equivalent to almost 70% of the net conversion into KZ over the period of the integration for the 6h $\nabla^4$ case but is relatively insignificant (less than 10% of

![Figure 3. Time development of the zonal kinetic energy (KZ) and eddy kinetic energy (KE) in the integrations with 6h and 60h $\nabla^4$ diffusions and truncation at $n = 42$.](image-url)
the respective conversions) for all the other formulations considered in this study. The 6 h $V^4$ integration was repeated with no zonal flow diffusion, when it was found that although the increase in KZ over the integration period was 50% larger, the maximum KE reached was only 10% larger. Thus we must conclude that the effect of the 6 h $V^4$ dissipation on the zonal flow, while undesirably large, is not the cause of the large differences between the 6 h and 60 h $V^4$ solutions.

By even the most subjective criterion the 60 h $V^4$ diffusion appears deficient: the fields of the model variables, particularly the low-level vorticity, are felt to be unacceptably rough, the synoptic-scale structure becoming masked by small-scale noise. The 60 h $V^4$ case has typically three or four orders of magnitude more energy near the truncation limit than the 6 h $V^4$ case. On the other hand the solutions obtained with the 24 h $V^4$ diffusion are felt to be acceptably smooth. However, the gross time development in terms of the globally averaged energy quantities (Fig. 4) is significantly different from the two previous cases. The first peak in the eddy kinetic energy is intermediate between the other two cases, as expected, but there is now a second peak of equal significance, the net result being that the conversion from potential to kinetic energy is comparable with that in the 60 h $V^4$ integration.

![Figure 4. As Fig. 3 but for the runs with 24 h $V^4$ diffusion and truncation at $n = 42$ (T42) and $n = 63$ (T63).](image)

The solutions obtained with 6 h and 24 h $V^4$ diffusions are both subjectively acceptable but show significantly different non-linear time development. It is, however, possible to judge between them, rejecting the latter as deficient, on the basis of the truncation dependence criterion discussed in section 2. Both integrations were rerun with the same diffusion coefficient but with truncation at $n = 63$ (referred to as T63), when it was found that in the 6 h $V^4$ case the solutions were virtually unchanged, whereas the 24 h $V^4$ solutions changed markedly for times greater than about 10 d (Fig. 4). In the T63 integration with 24 h $V^4$ dissipation the second eddy energy peak becomes a relatively insignificant feature and the time development is generally similar to that in the 6 h $V^4$ case, although about twice as much energy is converted as in that case.

Thus the large second eddy energy peak noted in the 24 h $V^4$ case with T42 truncation is in some sense a spurious effect due to there being insufficient dissipation at large $n$
to provide a smooth transition to the effectively infinite dissipation beyond \( n = 42 \). It should be noted that the energy in the second peak is still dominantly in \( m = 7 \) rather than its harmonics; in terms of the total wavenumber it is dominated by \( n = 8 \) and 10. This suggests the importance of interactions between scales near the truncation limit and the synoptic scales.

It is concluded that for T42 truncation, if a \( V^4 \) diffusion is used then the diffusive decay time at \( n = 42 \) must be of order \( 6 \) h if the solutions are not to be oversensitive to the truncation. There is the suggestion that significantly higher levels of energy conversion than in the \( 6h \ V^4 \) case can be obtained if the dissipation time scale on the larger spatial scales is increased but that these higher levels cannot satisfactorily be achieved by use of a \( V^4 \) diffusion.

\[ (b) \quad \text{Comparison of } 6h \ V^4 \text{ and } 4h \ V^6 \text{ results} \]

The use of more scale-selective formulations allow the dissipation rate on the dominant baroclinic scales to be reduced while maintaining a diffusive decay time scale of several hours on the smallest retained scales. The solutions obtained with a \( 4h \ V^6 \) diffusion are acceptably smooth and furthermore satisfy our criterion of truncation independence. The time development (KE is shown in Fig. 5) is similar to that in the \( 24h \ V^4 \) case with T63 resolution although slightly more energy is converted with the \( V^6 \) dissipation.

![Figure 5. Time variation of the eddy kinetic energy for the integrations with truncation at \( n = 42 \) and various diffusion formulations.](image)

Thus we have solutions using two different formulations of the diffusion (\( 6h \ V^4 \) and \( 4h \ V^6 \)) which meet all our criteria of acceptability but nevertheless differ from one another by a factor of about 2.6 in the amount of energy converted. Such sensitivity is much greater than one might intuitively expect by consideration of the typical scale of baroclinic waves and the diffusive decay time scales shown in Fig. 2 and thus we next seek an explanation of this pronounced sensitivity. The results from the \( 6h \ V^4 \) and \( 4h \ V^6 \) integrations start to diverge significantly from one another at about day 10 (see Fig. 5), at
which point the horizontal heat flux associated with the original disturbance has led to the formation of regions of enhanced low-level baroclinicity on the northern and southern flanks of the original strongly baroclinic region. A linear stability analysis with zero dissipation shows that the stability properties of the zonal flows from the two integrations at around this time are very similar. However, the inclusion of a 6 h \( V^4 \) diffusion in the stability analysis reduces the growth rates considerably more than does a 4 h \( V^6 \) diffusion, so that in the most striking case the growth rate with the former is about one half of that with the latter, which was itself some 30% less than the corresponding value without dissipation. This behaviour can be explained by the fact that the disturbances growing on the relatively confined regions of enhanced baroclinicity on the northern and southern flanks of the original strongly baroclinic region are of smaller scale than the original wave, but nevertheless energetically very significant. Therefore the effect of the diffusion is generally larger and the different scale dependencies of the two formulations become more apparent.

It appears that in other integrations which show less sensitivity to the dissipation, these secondary regions of enhanced baroclinicity are dynamically more stable because of the high static stability locally and thus disturbances growing there make little contribution to the total energy conversion.

(c) Formulations with greater scale selectivity

It has been shown that the conversion from potential to kinetic energy during the life cycle of a baroclinic wave may increase considerably if a 4 h \( V^6 \) diffusion is used instead of the 6 h \( V^4 \) formulation. There is no reason to suppose, however, that the amount of energy converted in the former case is the maximum that can be obtained while retaining acceptably smooth solutions. In order to try and obtain some idea of what this maximum might be, further integrations with even more scale-selective formulations of the diffusion (3 h \( V^8 \), 1 h \( V^{16} \) and modified 3 h and 6 h \( V^6 \) have been performed.

With the exception of the latter, these choices yield acceptably smooth, truncation independent solutions and all show a further significant increase in the amount of energy converted over that obtained with 4 h \( V^6 \) diffusion. The variation of the time development of eddy kinetic energy with the diffusion used is illustrated in Fig. 5. The peak values of KE reached between days 15 and 17 in the 1 h \( V^{16} \) and the modified \( V^6 \) runs are very similar and about 60% greater than that achieved in the 4 h \( V^6 \) case. Thereafter the 1 h \( V^{16} \), 3 h modified \( V^6 \) and 6 h modified \( V^6 \) runs (the latter integrated with T63 truncation and not illustrated here) show broadly similar behaviour apart from some discrepancies beyond day 24 which it is felt are not significant. The integrations with the modified 6 h \( V^6 \) diffusion exhibit truncation dependence but are included for comparison with the modified 3 h \( V^6 \) results. In both these cases we would expect no direct effect of the dissipation at all for \( n < 30 \), but the differences in the time development illustrated in Fig. 5 emphasize strongly how interactions involving the smallest scales retained in the truncation would lead to even greater differences.

<table>
<thead>
<tr>
<th>Diffusion formulation</th>
<th>Energy dissipation ((10^8 J m^{-2}))</th>
<th>Energy conversion ((10^5 J m^{-2}))</th>
<th>Dissipation Conversion (\times 100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mod. 6 h ( V^6 )</td>
<td>226</td>
<td>510</td>
<td>44</td>
</tr>
<tr>
<td>Mod. 3 h ( V^6 )</td>
<td>218</td>
<td>490</td>
<td>45</td>
</tr>
<tr>
<td>1 h ( V^{16} )</td>
<td>233</td>
<td>491</td>
<td>48</td>
</tr>
<tr>
<td>3 h ( V^8 )</td>
<td>190</td>
<td>364</td>
<td>52</td>
</tr>
<tr>
<td>4 h ( V^6 )</td>
<td>181</td>
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</tr>
<tr>
<td>6 h ( V^4 )</td>
<td>243</td>
<td>104</td>
<td>233</td>
</tr>
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</table>
tion can affect the large scales over a time scale of about 20 days.

In broad terms these results indicate that as the rate of dissipation on the large scales is reduced, while on the small scales a sufficiently high rate is maintained to keep the solutions acceptably smooth, so these solutions appear to approach a limit in which the total amount of potential energy converted into kinetic energy (per unit area) is about $5 \times 10^5$ J m$^{-2}$. This compares with total conversions of about $3 \times 10^5$ J m$^{-2}$ and $1 \times 10^5$ J m$^{-2}$ in the 4 h $V^6$ and 6 h $V^4$ cases respectively. It is clear from Table 2 that the total dissipation remains approximately constant rather than decreasing as the scale selectivity of the dissipation formulation increases. In fact the dissipation just takes place on progressively smaller spatial scales. As the scale selectivity increases so the percentage of the total conversion which the diffusive losses represent falls rapidly at first but appears to approach a limiting value of about 45%. Thus clearly the limit which the solutions discussed here are approaching is not the limit of no dissipation.

6. SUMMARY AND CONCLUSIONS

The major effect of dissipation on the linear growth rate spectrum for the flow considered here was found to be a stabilization of the higher zonal wavenumbers by an amount decreasing as the scale selectivity of the diffusion formulation was increased. For the 6 h $V^4$, 4 h $V^6$ and 3 h $V^8$ diffusions the maximum growth rate was for $m = 8$, but for the more scale-selective formulations and for the case of no dissipation it occurred at much higher wavenumbers. The $V^4$ diffusion was found not to be scale-selective enough to avoid causing a significant reduction in the growth rates of some zonal wavenumbers less than 12, whereas the other formulations were satisfactory in this respect.

The non-linear integrations discussed showed great sensitivity to the dissipation, as measured for example by the total amount of potential energy converted to kinetic energy, which was five times larger with the modified $V^6$ diffusion than in the 6 h $V^4$ integration. In the idealized case considered here this sensitivity could be explained in terms of the effect of the diffusion on the linear growth rates of vigorous secondary instabilities on the northern and southern flanks of the original disturbance, which was itself relatively insensitive to the dissipation. A similar degree of sensitivity was exhibited in other integrations starting from the same zonal flow but with a random initial perturbation in all wavenumbers. Similar mechanisms to that discussed above must operate in this case but the results, which show the greatest sensitivity to be in the development of the longest waves (zonal wavenumbers 1–4), emphasize also the importance of non-linear interactions between the shortest scales retained in the model and the intermediate and long scales. The importance of such scale interactions is also illustrated by the difference between the integrations with the modified 3 h and 6 h $V^6$ diffusions presented in section 5. Manabe et al. (1970) have reported similar behaviour in their GCM, while Burrows (1976) comments on the apparent importance of such interactions in his analysis of atmospheric data. As Burrows comments, such scale interactions are one of the reasons why sub-grid-scale parametrizations so profoundly affect GCM simulations; we have shown here how rapidly such effects may become important – on time scales of order 10 days.

Evidence was presented that as the scale-selectivity of the diffusion was increased, so the solutions obtained approached a limit in which the total dissipation of energy was equivalent to about 45% of the total conversion from potential to kinetic energy. Also we showed that, in the absence of dissipation, integrations beyond about 9 days exhibited grossly physically unrealistic features even with very high resolution. These results indicate the essential role played by dissipation in the non-linear evolution of the flow. Theoretical support for this conclusion is provided by Andrews and McIntyre (1976) from whose equation 7.1 it can be seen that, in the absence of other diabatic effects, net changes in the zonal flow in a situation where eddy activity grows from and ultimately decays back to negligible values can only result from dissipation.
It is difficult to judge how generally applicable the results of a study such as this are. However, we have demonstrated how much greater than relatively simple considerations might suggest, sensitivity to the diffusion can be in a specific idealized situation and we would expect such sensitivity to be exhibited at least in a certain class of more realistic integrations. We have carried out integrations for various initial zonal flows but have presented only those which exhibited the greatest sensitivity to the diffusion, as clearly any choice must be made on the basis of the behaviour in extreme situations. The sensitivity in the other cases was qualitatively similar but less pronounced. We have presented a method for judging whether a particular formulation is unsatisfactory based on whether the results obtained exhibit truncation dependence. This criterion should be of quite general applicability in models which include no other non-adiabatic processes and in which the fundamental dynamics are well represented with the basic truncation. However, it is only possible to use this criterion to reject a particular diffusion formulation; we have tried to judge between various formulations which satisfy the criterion on the basis of how closely the resulting total energy conversions approximate the limiting value which appears to be approached as increasingly scale-selective diffusions are used. However, a compromise has to be made since the most highly scale-selective formulations have drawbacks, notably the fact that the linear growth rates of the shorter zonal wavelengths exceed those of the synoptic-scale wavelengths and the possibility of non-local dissipative effects with the modified $V^6$ diffusion. From the results of the present study we would discount the 1 h $V^{16}$ and modified $V^6$ dissipations on the above grounds. We feel strongly that the $V^4$ formulation is not scale-selective enough to be satisfactory: with a dissipation time scale of 6 h at $n = 42$ an excessive amount of energy is dissipated, while with longer time scales the solutions are truncation dependent. We have no results which would allow us to judge between the 3 h $V^6$ and 4 h $V^6$ diffusions, which appear equally satisfactory, and indeed there is no reason to suppose that a unique 'best' formulation exists.

In section 2 typical diffusion formulations in other spectral models were quoted; the results of this study lead us to question whether some of these models are sufficiently dissipative. Ultimately the choice of diffusion formulation for use in a different model or in the same model but with a different truncation must be based on detailed comparisons similar to those made here for truncation at total wavenumber 42. On the basis of the present results one might suggest, however, as a sensible starting point for such an investigation, a dissipation such that the shortest retained scale is damped on a time scale of order 3 h, with a damping time of order 100 d on $n = 15$. This would imply that a $V^4$ diffusion might be sufficient with T63 truncation, although a $V^6$ formulation would probably still be preferable. To achieve such scale selectivity with a T21 truncation a $V^{20}$ diffusion would be necessary, although other considerations might prevail with such a severe truncation. For example Sadourny and Hoyer (1982) have suggested that in integrations which do not adequately resolve the baroclinically active scales, a formulation which dissipates potential enstrophy but not energy might be appropriate.

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