The sea breeze with mean flow

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\textbf{SUMMARY}

A quasi-geostrophic mean flow can form the basic state upon which a sea breeze is superimposed; a model is described. With a mean flow perpendicular to the coast the speed of the sea breeze front is shown to be a linear function of the onshore component of the mean flow. When a constant mean flow is introduced the general structure of the sea breeze is unchanged. In particular, for a constant energy input the intensity is also unaltered. In contrast, a shear changes the general qualitative structure but leaves the speed of the front and the intensity of the sea breeze unchanged. These features are consistent with the available potential energy of the system and a constant transfer, per unit area, of this into kinetic energy. Model results and other studies are discussed.

1. INTRODUCTION

Numerical models of the sea breeze have concentrated on modelling days with an atmosphere originally at rest (e.g. Pearce 1955; Estoque 1961; Neumann and Mahrer 1971; Pearson 1973; Dalu 1978). The pioneering studies of Pearce have shown the importance of the Coriolis parameter in eventually limiting the inland extent of the sea breeze. Pearson (1973) and Simpson \textit{et al.} (1977) demonstrated that the vertically integrated buoyancy (temperature) difference between land and sea is the parameter that determines the speed of the sea breeze front. Another measure of the intensity (kinetic energy, velocity, temperature gradient, velocity gradient) of the subsequent flow is the available potential energy (Pearson 1975). These numerical experiments have allowed a better understanding of how the sea breeze develops on an otherwise calm day.

In contrast to the very numerous papers published where the sea breeze is the only flow, days when there is some overall synoptic flow have received less attention. Estoque (1962) extended his original model to include a mean flow, whilst Walsh (1974) concentrated on the effect of the land temperature. This parameter is of smaller importance than the integrated effects. Simpson and Britter (1980) discussed their recent laboratory experiments with general density currents in the presence of mean flow in relation to the sea breeze.

The aim of this paper is to use a fixed heat flux model to determine how the presence of a mean flow changes the sea breeze. The emphasis is on a numerical experiment designed to examine the general properties of the flow. It is not intended to simulate any particular occurrence of the sea breeze, in detail. These numerical experiments have two major objectives. The first is to describe how the speed of the sea breeze front depends on the presence of the mean flow. The second is to test whether the intensity of the sea breeze front varies with the direction of the mean flow (Pielke 1981). In both cases our results are compared with other published work.

In general the atmospheric boundary layer flow under an overall mean flow shows some alteration of direction with height. The exact form of the flow will depend on the boundary formulation involved. A balance exists between Coriolis terms, the pressure gradient, and momentum transfers. This balance is disturbed by the growth of the daytime convective layer. The numerical experiments which are reported in this paper have all been performed in the absence of a complete boundary layer formulation. The aim is to study the interaction between the mean flow and the sea breeze, rather than to study boundary layer modifications. The experiments are consistent with the laboratory models of Simpson \textit{et al.} (1977), with the extra effects of rotation, and are an extension of the numerical experiments of Pearce (1955), Pearson (1973), and Richiardone and Pearson (1983).
Rigorous deductions from numerical experiments require that the numerical schemes conserve the dynamics and energetics of the original equations. The techniques which have been applied in this study conserve exactly the total energy, and correctly model transfer between potential and kinetic energies. Small phase errors are introduced to the wave speeds and consequently positions of each dominant feature can be slightly in error. In particular, major features could have an error of approximately one gridpoint.

2. The model equations

(a) General

The model is an extension of that used previously (Pearson 1973) to include an overall mean flow. The coastline is assumed to be straight with a heat flux constant everywhere over the land. The equations that describe the model are

\[
\frac{Du}{Dt} - fv = -\partial P/\partial x \tag{1}
\]

\[
\frac{Du}{Dt} + fu = \partial P/\partial y \tag{2}
\]

\[
\partial P/\partial z = -\sigma \tag{3}
\]

\[
\partial u/\partial x + \partial w/\partial z = 0 \tag{4}
\]

\[
\frac{D\sigma}{Dt} - N^2w = H(x, z, t) \tag{5}
\]

\(D/Dt\) representing the substantive derivative \(\partial/\partial t + u(\partial/\partial x) + w(\partial/\partial z)\). The heat input is constant in the y direction. As a consequence, no variations parallel to the coast are excited. The model allows the mean pressure to vary along the coast although the perturbation flow is constant in this direction:

\[
\frac{\partial}{\partial y} \left( \frac{\partial P}{\partial x} \right) = \frac{\partial}{\partial y} \left( \frac{\partial P}{\partial y} \right) = 0.
\]

The equations of motion are in a Boussinesq form (Dutton and Fichtl 1969) with rotation included. The hydrostatic approximation has been made as this is valid for the sea–land breeze where the length scale of the motion is larger than the height scale (e.g. Pearson 1973). The symbols \(u, v, w\) are velocities in the \(x, y, z\) directions, with \(u\) perpendicular to a straight coast. The coastline is in the \(y\) direction and \(x\) is positive inland. The sign convention for \(f\), the Coriolis parameter, is for the northern hemisphere. The buoyancy equation is valid in general for any deviations in density. For the sea breeze, which normally occurs on a dry summer day, it is convenient to consider changes in density due to changes in temperature. In a dry atmosphere the negative buoyancy \(\sigma\) is related to the perturbation of potential temperature by

\[
\sigma = -\partial\theta_0/\partial z, \quad \text{with} \quad N = [\{g/\partial\theta_0/\partial z\}^{(d\theta_0/\partial z)}]^{1/2}.
\]

For convenience the buoyancy \(b\) \((= -\sigma)\) is sometimes used. \(P\) is the normalized pressure perturbation: the difference between the pressure at any point and the mean hydrostatic value if no flow occurs, divided by density.

\(H(x, z, t)\) is a parametrized representation of the buoyancy flux. The major physical process which is not considered in the paper is the effect of the mixing of momentum due to turbulence. It has been shown (Pearson 1973) that although turbulent momentum fluxes may be significant for the very detailed structure of the sea breeze, their presence or absence does not significantly change the general flow pattern, the speed of the sea breeze front, or the inland extent of the flow.

The model integrations use the vorticity form of the equations of motion. The stream
function ($\psi$) and the vorticity ($\zeta$) are related to the original velocities by

$$
u = \partial \psi / \partial z, \quad w = - \partial \psi / \partial z, \quad \zeta = \partial^2 \psi / \partial z^2.$$

This last equation, describing vorticity, is consistent with the hydrostatic approximation.

(b) Geostrophic mean flow

The total fields are a superposition of the mean flow, occurring in the absence of the sea breeze, and the perturbation due to diurnal heating. In these simulations the mean flow is assumed to be effectively constant during the diurnal cycle. Cases where pre-existing fronts move across the coast are omitted. $\dot{w} = 0$ both at the ground and at the effective top of the sea breeze; and large-scale convergence at some levels and divergence at others are omitted, with $\ddot{w} = 0$ everywhere.

These model experiments assume no convergence. The equations that describe the mean flow are:

$$-f\ddot{v} = - \partial \ddot{P} / \partial x$$

$$f\ddot{u} = - \partial \ddot{P} / \partial y$$

$$\partial \ddot{P} / \partial z = - \ddot{\sigma}$$

$$\ddot{u}(\partial \ddot{\sigma} / \partial x) + \ddot{v}(\partial \ddot{\sigma} / \partial y) = 0.$$  

The last equation is the thermodynamic equation for the mean flow, as for these discussions the mean flow is not allowed to have any convergence, or structure, on the same scale as the sea breeze.

(c) Flow parallel to the coast

Flow parallel to the coast, in the coordinate system of the model, is equivalent to $\ddot{u} = 0$, while $\ddot{v}(x, z)$ is specified. In such a mean flow, with a large shear, scale analysis of the flow leads to the set of equations:

$$Du'/Dt - f\dot{v}' = - \partial P'/\partial x - (\partial \ddot{P}/\partial x - f\ddot{v}) = - \partial P'/\partial x$$

$$Dv'/Dt + f\dot{u}' = 0$$

$$\partial P'/\partial z = - \sigma'$$

$$\partial u'/\partial x + \partial w'/\partial z = 0$$

$$D\sigma'/Dt - N^2 w' - M^2 u' = H(x, z, t).$$

Here $u'$, $v'$, $w'$, $P'$, $\sigma'$ represent deviations of the fields from their mean quasi-geostrophic values. $M^2$ represents mean horizontal variations in the buoyancy field consistent with the flow: $M^2 = - \{1 / \rho_0(x) \} \{ \partial P(x) / \partial x \} = f(\partial \ddot{v} / \partial z)$. This is the term which first becomes important for large values of the shear. Terms in the momentum equation, (11), which are small and hence eliminated by the scale analysis are the horizontal advection, by the perturbation of the variations in $\ddot{v}$, $u'(\partial \ddot{v} / \partial x)$, and the vertical transfer of the shear, $w'(\partial \ddot{v} / \partial z)$. For the sea breeze the $M^2$ term is also small and similarly vanishes in the equations. When the flow is parallel to the coast, the equations of motion for the perturbations are in exactly the same form as the original equations of motion. This indicates that the field parallel to the coastline is purely additive; the equations can be integrated as if no flow occurs, and the mean flow then added at the end of the integrations.

The terms that can be neglected, including $M^2$, are the ones discarded by Neumann
He retained both the pressure gradient and the rotation term affecting the mean. However, in this framework they balance and can be removed.

(d) **Mean flow perpendicular to the coast**

The equations of motion for a mean flow perpendicular to the coast imply that \( \bar{u}(z) \) is specified and \( \bar{v} = 0 \). Unlike the situation where flow perpendicular to the coast has all terms negligible, scale analysis does not eliminate the extra terms arising from the presence of a mean flow. The equations of motion that represent this flow are the set (1), (3), (4), (5) together with

\[
Dv/Dt - f(u - \bar{u}) = 0. \tag{15}
\]

(e) **Constant angle mean flow**

The component velocities of a mean flow parallel to the coast are linked through Eqs. (6) to (9). With the deviation of the mean density field from its undisturbed hydrostatic, at rest, values being limited to a function of \((x, y)\), Eq. (9) becomes

\[
\frac{\bar{u}(z)}{\bar{v}(z)} = -\frac{\partial \bar{\sigma}}{\partial x} (x, y) \frac{\partial \bar{\sigma}}{\partial y} (x, y). \tag{16}
\]

This represents a mean flow at a constant angle to the coast, and can be simulated by a combination of the previous two cases. The \( u \) mean flow is incorporated specifically before the integration and the \( v \) flow reincorporated by adding the perturbation to the mean flow.

3. **The heat input**

This paper does not consider the detailed development of the convective mixed layer over the land. The model is designed to test the importance of the mean flow to the sea breeze front. The heat input is parametrized as a separable function of space and time as

\[
H(x, z, t) = -D A(t) B(x) Z(z, t). \tag{17}
\]

The functions \( A(t), B(x), Z(z, t) \) are normalized so that the various heat fluxes can be simulated by variation of the parameter \( D \). That is

\[
B(x) = \begin{cases} 1 & \text{inland} \\ 0 & \text{out to sea} \end{cases}
\]

\[
\int A(t) \, dt = 1
\]

\[
\int Z(z, t) \, dz = 1
\]

the integrals running from 0 to \( \infty \). In the situation where the heating is continuous throughout the day the most important physical parameter during the flow is the current integrated buoyancy flux (Richiardone and Pearson 1983):

\[
TIB(0, t) = -D \int A(t) \, dt. \tag{19}
\]

The approximate value of the speed of the front in the absence of mean flow is

\[
C_0 = \frac{1}{2} \{ TIB(0, t) \}^{1/2} \tag{20}
\]


One of the aims of this analysis is to examine how the speed of the front is affected by the overall mean flow. This can be more simply examined if \( C_0 \) is constant during the
analysis. The functional form of \( A(t) \) is chosen to be zero after a relatively short time, allowing integrations to proceed after the heating has stopped. This allows an estimate of the speed of the front from the time derivative of its position. The functional form of \( A(t) \) is similar to that of Pearson (1973) except that it has been modified to be continuous both at the beginning and at the end of the heat input, i.e.

\[
A(t) = a \frac{e}{t^2 + (t - t_m)^2} - \frac{1}{e} \quad 0 < t < 2t_m
\]

\[
= 0 \quad t > 2t_m.
\]

(21)

The normalization constant \( a \) was evaluated numerically as the sum, from 0 to \( 2t_m \), of the discrete analog to the un-normalized form of \( A(t) \). The choice of \( e \) and \( t_m \) in experiments with no mean flow does not affect the speed of the front nor any significant features of the flow after the heating has finished \( (t > 2t_m) \) (Pearson 1973).

The variation of the heat flux from the ocean to the land is assumed to be a continuous smooth function:

\[
B(x) = (\arctan(x/x_d) + e)/2e \quad -x_m < x < x_m
\]

\[
= 1 \quad x > x_m.
\]

(22)

The constant \( e = \arctan(x_m/x_d) \) has been chosen so that \( B(x) \) is continuous at both \( x_m \) and \( -x_m \). Variations of \( x_m \) and \( x_d \) do not change either the speed of the front or the significant features of the flow (Pearson 1973).

The function \( Z(z, t) \) was chosen to simulate the development of a well-mixed lower layer of height \( h(t) \). Changes in this height over a single timestep were omitted in calculating the form of \( Z \) which would be used at that particular timestep. Thus

\[
Z(z, h(t)) = \begin{cases} 1/h(t) & 0 < z < h, \\ 0 & z > h. \end{cases}
\]

(23)

This requires an estimate of the height \( h(t) \) of the top of the mixed layer. This is found from the Brunt–Väisälä frequency, \( N^2 \), and the current buoyancy at the ground, \( \sigma(x, 0, t) \), by

\[
\sigma(x, 0, t) = -N^2 h(x, t).
\]

(24)

4. The boundary conditions

The ground, \( z = 0 \), is a solid (real) surface and the normal component of velocity must vanish giving \( \psi = 0 \) at \( z = 0 \). The model has a fixed height, \( H \), and this is assumed to be a rigid lid with zero normal velocity, thus \( \psi = c \) at \( z = H \). The sea breeze can extend far from the coast but the limited computer space requires boundaries at a limited distance. The model is truncated at a distance \( x = \pm L \). The boundary conditions that the modeller applies at these points should simulate an effectively unbounded domain. To accomplish this, the radiation boundary conditions for a dispersive wave (Pearson 1974) were generalized to include an overall mean flow (Pearson 1980). The modifications suggested by Orlanski (1976) were found, by numerical experiment, to be unstable for zero mean flow. The further refinement of Klemp and Lilly (1978) is applicable to a single dominant vertical eigenmode when the wavelength in \( x \) is constant in time (e.g. mountain waves) or when this mode has no dispersion in the \( x \) direction \( (f = 0) \).

Two forms of the boundary condition were used. In an early version of the computer code which was used for a \( u \) mean flow with no vertical shear, the boundary conditions were applied in \( x, z \) space to the dominant eigenmode. The more recent model uses the
conditions on the amplitude of the Fourier series expansion in the vertical. The extension of the original idea to include mean flow is simple, but care must be taken in implementation in Fourier transfer space; details are given in an appendix.

5. Techniques for Analysis of Results

To test how the speed of the sea breeze front is affected by the mean flow, the position of the front is estimated at several periods after the heating has finished, and the forward speed is calculated. This speed is evaluated for all the experiments.

As these numerical experiments use only one form of heated layer without entrainment, the height of the mixed layer, the total integrated flux and the approximate speed of the front in the absence of mean flow \( (C_0) \) are interrelated. Following Pearson (1973) and Manins and Turner (1978) the top of the mixed layer over the land is, using Eq. (20),

\[
h = (2D/N^2)^{1/2} = 2^{3/2}C_0N^{-1}. \tag{25}
\]

The change in buoyancy at the ground well inland is

\[
\sigma_g = (2DN^2)^{1/2} = 2^{3/2}C_0N. \tag{26}
\]

These linear relations between the height of the mixed layer, the buoyancy change at the ground and the speed of the front are consequences of allowing a well-mixed layer without entrainment. They are not true if alternative functional forms of \( Z(z, t) \) are used, although the speed of the front and \( D \) are still related (Pearson 1973; Richiardone and Pearson 1983).

In order to test the hypothesis that the intensity of the sea breeze circulation is a function of the mean flow, some objective measures of the intensity are required. The general name ‘intensity’ could refer to point values, gradients of point values, or to some overall measure of intensity.

In this paper the maximum vertical velocity will be used as an example of a point measure of intensity. The maximum over the horizontal, of the horizontal derivative negative buoyancy gradient, is an example of a gradient measure of intensity, and this also describes the intensity of the sea breeze front. These two measures were chosen as they can be easily evaluated with, and without, mean flow. Estimates of \( u, v \) or \( w \), at a particular gridpoint, would depend on the position of the front.

As an overall measure of the intensity, the total kinetic energy in the perturbation can be used if the size of the total circulation is unaltered. The size of the area through which this truly represents the average intensity is taken to be the distance between the front and the original coastline in a coordinate system which moves with the mean flow.

As a cross check on the energetics of the numerical model the kinetic energy, and its fluxes through the boundary, were calculated by integrating the squared velocity field which was output from the numerical experiments. The potential energy and available potential energy fields were calculated from the buoyancy field predicted by the model. The available potential energy can also be evaluated analytically and for this particular form of heat input, with length \( L \) over the land and ocean, the APE (Pearson 1975; Richiardone and Pearson 1983) is given by

\[
APE = \frac{2}{3}LD^{3/2}N^{-1}. \tag{27}
\]

Although all results are to be presented in actual physical units, the values can be non-dimensionalized. In particular the maximum streamfunction scales with the heat input, and inversely on the Brunt–Väisälä frequency as

\[
\psi_{\text{max}} = \mathcal{O}(2^{-3/2}N^{-1}D). \tag{28}
\]

The magnitude of \( u \) can be found from the maximum stream function if the height of the inflow is known. If it is assumed that the depth of the inflow layer is about half the depth
of the heated layer, a very approximate scale for $u$ is $2^{-2}D^{1/2}$.

In a comparison between these model predictions and possible atmospheric observational experiments, it is convenient to convert the total integrated buoyancy into units of the heat flux which has been transferred to the atmosphere. This is given by

$$Q = \theta_0 \rho c_p D/g.$$  \hspace{1cm} (29)

---

**Figure 1.** Maximum buoyancy gradient (horizontal maximum at first gridpoint in the air (s$^{-1}$ m$^{-1}$)) as a function of time. Here 4th order refers to the 4th-order Arakawa's Jacobian and major differences in staggered Jacobian with 2nd-order averages and 2nd-order linear terms.

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**Figure 2.** Maximum vertical velocity (m s$^{-1}$) at lowest gridpoint. Maximum is that found from all horizontal gridpoints. Details as for Fig. 1.
6. Accuracy of the Numerical Model

As the intensity of the flow is to be evaluated quantitatively by examining the output of the numerical integrations it is important to know how the choice of the numerical methods influences these parameters. The numerical techniques chosen for the majority of the experiments was a second-order scheme. However, it was practicable to derive, and program, a true fourth-order scheme. A given set of experimental parameters was chosen and a comparison between the output of each scheme was used to show the magnitude of the numerical errors.

The intensity of the sea breeze front, as measured by the extreme value of the buoyancy gradient, varies between the methods, and oscillates over the period of integration (Fig. 1). Similarly the maximum vertical velocity (Fig. 2) oscillates and varies between the methods. These results indicate that a very significant change in the buoyancy gradient and vertical velocity would be needed to prove that the intensity varied. Changes within the boundary indicated by the differences in numerical methods would indicate that the intensity did not alter; this is however, not a rigorous demonstration.

While the pointwise values showed oscillatory behaviour over time as well as differences between numerical techniques, the total kinetic energy was the same (to more than three figures) with both fourth-order and second-order schemes. Thus kinetic energy is a good estimate of the total overall intensity.

7. Description of Results

(a) \( \bar{u} \) constant in height

The numerical experiments for \( \bar{u} \) constant in height reported here were performed using second-order finite difference schemes with a Coriolis parameter relevant to mid-latitudes (Table 1). The variable parameters, \( N \), \( \bar{u} \) and \( D \), were altered. \( D \) was changed so as to have different speeds of the front, in the absence of the background flow \( \bar{u} \). \( N \) was changed to allow different boundary layer heights and equivalent temperatures at the ground. \( \bar{u} \) was changed in order to assess how the mean flow alters the circulation (Table 2).

<table>
<thead>
<tr>
<th>Number of gridpoints</th>
<th>( 65(\psi, \bar{z}); 64(\sigma, \bar{v}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>in horizontal</td>
<td>17(\psi, \bar{z}); 16(\sigma, \bar{v})</td>
</tr>
<tr>
<td>in vertical</td>
<td></td>
</tr>
<tr>
<td>( \Delta x ): 5 km</td>
<td>( \varepsilon ): 0.75 h</td>
</tr>
<tr>
<td>( \Delta z ): 250 m (( N = 0.01 ))</td>
<td>( x_0 ): 30 km</td>
</tr>
<tr>
<td>( \Delta t ): 30 s</td>
<td>( x_0 ): 15 km</td>
</tr>
<tr>
<td>( t_0 ): 1.5 h</td>
<td>( f ): ( 1 \times 10^{-4} ) s(^{-1} )</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Table 2. Variable Parameters for Experiments that Have a ( \bar{u} ) Mean Flow Constant in Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N ): 0.007071, 0.01, 0.0142 s(^{-1} )</td>
</tr>
<tr>
<td>( \bar{u} ): 0, ( \pm 0.5 ), ( \pm 1 ), ( \pm 1.5 ), ( \pm 2 ), ( \pm 2.5 ), ( \pm 3 ), ( \pm 5 ) m s(^{-1} )</td>
</tr>
<tr>
<td>( D ): 28.1, 50, 78.1, 112 m(^2) s(^{-2} )</td>
</tr>
</tbody>
</table>

The major positive, rigorous conclusion available from the results of these experiments is that for all the various input parameters the observed speed of the front is a linear function of the mean flow (Fig. 3). Furthermore if the observed speed of the front, in the absence of the mean flow, is subtracted from the values, the linearity is statistically validated (Fig. 4).
Figure 3. The relationship between the observed speed of the front and a constant-\( u \) quasi-geostrophic flow. The very high correlation coefficient indicates the statistical significance of the line.

Figure 4. As for Fig. 3 except the actual values of the speed observed at zero mean flow have been subtracted.
If the velocity at a particular point is examined then the perturbations vary significantly between experiments. This variation is due to the front being in a different position, relative to this point, for each experiment. If \( u \) is observed at a particular point, the arrival of the front is seen as a change in both temperature and perturbation velocities. The time at which the front arrives is dependent on the mean flow and the heat input. Perturbation velocities at the arrival differ (Fig. 5). This variation is more a consequence of the differing times of arrival, as the velocity gradients at the front are not constant over time, even in the absence of a mean flow. The maximum vertical velocity, in the front itself at any given time, is not affected by the mean flow. Its variations are similar to those between numerical methods, or for a given experiment the variations over time at the front. The buoyancy gradient is also unchanged.

As the speed of the front is a linear function of the mean flow, the overall size of the perturbation flow is unaltered by the mean. The velocities, relative to the moving coordinate system, are also unchanged, as are the gradients. These constant factors imply that the total kinetic energy of the perturbation in the model domain can be used as a proxy for the local intensity, as well as measuring the overall integrated intensity of the mean flow. The decrease in kinetic energy at later times reflects the advection of the energy outside the region of the calculation (Fig. 6). The constant values of the kinetic energy inside the domain show that the intensity is constant, both overall and pointwise.

(b) \( u \) not constant in height

The numerical experiments with a shear flow discussed in this paper were all performed with \( u \) zero at the ground and a constant shear in the vertical. The experiments, performed with a completely re-programmed numerical model, used a Coriolis parameter applicable to the Venice area (Table 3). The parameters varied between experiments were \( N, D \) and the values of the shear (Table 4).

The vertical shear does not significantly change the speed of the front as measured at the first gridpoint (Fig. 7). The presence of the shear does however subtly change the

![Figure 5. The perturbation \( u \) velocity which is superimposed on the \( u \) mean flow. At first gridpoint in the vertical, and horizontal gridpoint number 43, 50 km from the coast, \( D = 72 \, \text{m}^2 \, \text{s}^{-2} \). Note that the arrival of the front is associated with a sharp change in velocity. For \( u = -2.5 \, \text{m} \, \text{s}^{-1} \) the front does not reach 50 km from the coast during the integration. Variations in perturbation velocity are associated with the different times of arrival.](image)
Figure 6. The kinetic energy in a section of the model domain as a function of $u$ mean flow and time. Points on axis represent 4th-order scheme as in Figs. 1 and 2. The decrease at large times is due to advection outside the domain.

TABLE 3. FIXED PARAMETERS FOR EXPERIMENTS THAT HAVE A $u$-SHEAR FLOW

<table>
<thead>
<tr>
<th>Number of gridpoints</th>
<th>in horizontal</th>
<th>101($\psi$, $\zeta$); 100($\sigma$, v)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in vertical</td>
<td>21($\psi$, $\zeta$); 20($\sigma$, v)</td>
</tr>
</tbody>
</table>

$\Delta x$: 3 km
$\Delta z$: 150 m ($N = 0.01$)
$\Delta t$: 40 s
$t_m$: 1.5 h

$e$: 0.75 h
$x_w$: 24 km
$x_g$: 12 km
$f$: $0.5 \times 10^{-4}$ s$^{-1}$

TABLE 4. VARIABLE PARAMETERS FOR EXPERIMENTS THAT HAVE A $u$-SHEAR FLOW

$u$-shear: 0.000, ±0.001, ±0.002, ±0.003 s$^{-1}$
$D$: 60, 72, 80 m$^2$ s$^{-2}$
$N$: 0.007071, 0.01, 0.01414 s$^{-1}$
structure of the flow and some observable parameters. The changes can be observed as variations in the shape of the contours of the stream function (Fig. 8). Positive shear has only a small effect, decreasing slightly the vertical extent of the flow inland, and increasing it slightly out to sea. The consequences of negative shear, or upper flow from land to sea, are more visible, increasing the vertical extent of the front, forming a secondary maximum aloft.

The u contours also reflect the changed structure of the perturbation flow (Fig. 9). The major change is that the maximum of the return flow aloft is moved more over the ocean with negative shear, and slightly further inland with positive shear. Near the ground the velocity observed at a given point, although slightly different, has its major features unaltered by the shear (Fig. 10).

The change in the structure is reflected in variations in the maximum value of the streamfunction (Fig. 11). No clear pattern emerges. With large negative shear the variations of $\psi_{\text{max}}$ with $D$ are less than those for more moderate shear. However, negative shear is clearly more significant than positive. If the maximum streamfunction for moderate values of shear are plotted against the appropriate dimensional scale, Eq. (28), the significance of the negative shear is much clearer (Fig. 12).

While the detailed structure of the flow is altered by the shear, it appears that overall there is no transfer, on this time scale, of kinetic energy from the mean shear to the perturbation. The total kinetic energy of the perturbation shows no significant variation with shear (Fig. 13). The transfer of the available potential energy, per unit length, to the kinetic energy of the perturbation is constant through these experiments as it was in those with no mean flow (Richardson and Pearson 1983). For these experiments their constant transfer is reflected in the linear growth, with time, of the kinetic energies. Differing values of the shear do not significantly change the slope, or the intercept of straight lines found from a linear regression (Fig. 14). These slopes are proportional to $D^2$. This represents the
Figure 8. Perturbation streamfunction pattern after four hours. Contours from $-300$ to $2100$ m$^2$ s$^{-1}$ at intervals of $200$ m$^2$ s$^{-1}$. (a) With zero $u$ mean flow. (b) With shear of $+0.002$ s$^{-1}$. (c) With shear of $-0.002$ s$^{-1}$.
Figure 9. As in Fig. 8 except for the contours of $u$ velocity. Contours from $-2.7$ to about $6$ m s$^{-1}$ at intervals of $0.5$ m s$^{-1}$.
Figure 10. The perturbation $u$ as a function of time at horizontal gridpoint number 70, 60 km from the coast, $D = 80 \text{ m}^2 \text{s}^{-1}$. (a) The first vertical gridpoint at height 75 m; (b) the third vertical gridpoint at height 375 m.

Figure 11. The maximum streamfunction for $N = 0.01$ with various values of $D$ and shear.
available energy, Eq. (27), being proportional to $D^{3/2}$ and the front moving with $D^{1/2}$, Eq. (20). As the slopes on the linear regressions are effectively the same for each shear, the shear does not change the overall energy transfers.

8. Discussion

(a) Laboratory experiments

The numerical experiments have similarities as well as some significant differences from the laboratory observations of Simpson et al. (1977) and Simpson and Britter (1980). The main similarity is the aim, which is to simplify the real atmospheric flow in an attempt to understand further the more complex real case. The major differences lie in the condition of the ambient atmosphere, and in the techniques used to form the advancing, cooler air. The laboratory experiments use denser fluid to drive the undercutting cool air. These numerical experiments use the heated land surface to entice the circulation. The heating effect has a very different available potential energy from an equal magnitude cooling one (Pearson 1974). Previous numerical experiments indicated that the actual velocity of the front is independent of the sign of the forcing. However, the velocities, inside the flow, and the kinetic energies differ significantly. Although the way that the front is formed does yield a different internal flow, the forward advance is unaltered by the sign, and this advance is the major emphasis in both this paper and those of Simpson et al. and Simpson and Britter.

The major difference between the interpretations, or predictions of the speed of the
front, is in the way that overall density differences are assessed. This paper uses the difference between the original ambient condition and that existing over land to evaluate the parameter $D$. This is equivalent to taking a point well inland and one well out to sea to evaluate a density difference. Simpson et al. and Simpson and Britter use a point just before the front, and one in the flow just behind the front. If the perturbation in $\nu$ is zero at a height $z$, then the densimetric difference between two points $x$ and $x + L$ is

$$u^* = \left| \int \sigma(x + L, z, t) \, dz - \int \sigma(x, z, t) \, dz \right|,$$

the integrals running from 0 to $z$.  

To compare the two techniques the magnitude of the densimetric integral was calculated numerically between a point before the front, and several points behind the front.  

This integral was not constant, even though $D$ and the speed were constant. For example, for $D = 60$ and zero shear, five hours after the start the front is at gridpoint 68. The densimetric integral was 32.8 if the coast was chosen as the point in the flow, 41.5 for gridpoint 60. For a shear of $+0.002$ the integral was 38.5 using the coast and 36.5 with gridpoint 60.  

The large difference between $D$ and the integral is related to the relatively warm
return flow aloft. Calculation of the integral omits this air; $D$ includes it. The difference between the densimetric integral and $D$ explains the difference between the constants $\left(\frac{1}{3}, \sqrt[3]{\frac{1}{2}}\right)$ in different versions of Eq. (20). The variation in the densimetric integral is about 12%, which probably explains the greater scatter when this is used as a parameter than when $D$ itself is used.

Both laboratory measurements and the numerical experiments reported in this paper have a very short period in which the density difference is initiated. Both also show a constant speed of the front. In the atmosphere, where heating is continuous, the position and the velocity of the front are more complex functions of the parameters. The continual increase in the density gradient acts to accelerate the front, while eventually rotation will attempt to slow it down (Pearce 1955; Pearson 1973). Even with this complication, the parameter $D$ allows a good estimate of the position of the front, although the constant in Eq. (20) is different between impulsive and continuous cases (Richardson and Pearson 1983). This modified form can be further adjusted to agree with observations (Bacci et al. 1982).

(b) The angle of the flow

One major result of the analysis is that any mean flow constant in height has essentially a linearly additive effect. The numerical experiments with a $u$ mean flow revealed this a posteriori, and the analysis for the $v$ mean flow indicated that no numerical experiments would be necessary. Neumann (1977) showed that the overall mean flow is important in determining the angle of the flow at any fixed point. Although at first glance
the results may appear contradictory, they are not. Both analyses retain the same terms in the equations of motion and the results are consistent.

The difference occurs in the way the results are analysed. The angle of the flow at any point is determined by the ratio $u/v$. Although the perturbations $u'$ and $v'$, at a given point relative to a coordinate system moving with the mean flow, are unchanged, the total velocities and their ratio, $(\bar{u} + u')/(\bar{v} + v')$, at any given point will be significantly different. Thus, although superimposed $u, v$ fields do not change the perturbation, the angle is altered.

This difference is compounded by the structure of the flow. Consider the flow at a point a distance $X$ from the coast, under a mean flow $\bar{u}$, at time $t$. A coordinate system moving with the mean flow will have moved a distance $\bar{u}t$. Thus the equivalent point, in a sea breeze without mean flow, would be at $X' = X - \bar{u}t$. The point $X'$ has a very different velocity, at time $t$, from that of $X$, even after the front has arrived, as velocities are not constant everywhere behind the front (see Fig. 5).

Neumann also analyses the rate of change of the angle of the flow at any given point and shows that it varies with the mean. This is also true in these experiments, although the rate of change of the velocity components at two appropriate points are unaltered. What may seem to be a conflict occurs because the rate of change of the angle depends on the total magnitude of each component, and because the appropriate point for comparison is one which is a fixed distance, in a moving coordinate frame, from the origin of the density difference.

(c) The intensity of the circulation

The major result of these experiments is that the intensity of the circulation is unaltered by the direction of the flow. This is consistent with the laboratory observations. The reason for this result is that the available potential energy in the experiments was not changed and transfer between this and the kinetic energy of the perturbation was constant. The presence of the perturbation does not in itself lead to any transfer between the kinetic energy of the mean flow into that of the perturbation, on these time scales. On the contrary, on longer time scales the perturbation can lead to both kinetic energy and available potential energy on time and space scales greater than that of the breeze itself (Pearson 1981; Richardson and Pearson 1983).

As these experiments tend to suggest that the direction of the mean flow, in itself, does not alter the intensity of the sea breeze, any observations or hypotheses to the contrary need to be explained by mechanisms not present in this model, or in laboratory experiments. The major requirement of any comparisons between results is that the overall Brunt–Väisälä frequency, $N$, and the heat flux, $D$, must be consistent as these, through Eq. (28), change the available potential energy in these experiments. In addition to these parameters, the actual distribution of the flux in the vertical can alter the available potential energy. For example, in a well-mixed layer, the increase of entrainment leads to an increase in the energy (Richardson and Pearson 1983). As an alternative to the vertical distribution being different everywhere, and for all times, some local variations could occur. These local variations could include the interaction of a continuous heat input and the mean flow to produce larger energies, or alternatively for the shear to interact with a continuous heat flux to produce similar results.

As well as mechanisms which allow the mean flow boundary layer to alter the available potential energy, it is conceivable that some transfer could occur through turbulent exchanges of kinetic energy. As the role of these has been shown to be less important in the sea breeze itself, such mechanisms are very unlikely to occur.

9. Conclusions

The numerical experiments have shown that the difference between the speeds of the sea breeze front on a day with an overall mean flow and an equivalent day with the
atmosphere at rest is a linear function of the mean flow. The shape and the magnitude of the perturbation are unaltered by a constant mean flow. For a constant shear the structure of the perturbation is slightly altered but the overall intensity is unchanged.

The results are consistent with the available potential energy of the perturbation being constant, and a constant transfer into kinetic energy. No transfer occurred from the mean into the perturbation.

For a mean flow parallel to the coast, the speed magnitude of the perturbation velocity component is unchanged. The actual pointwise values can be reconstructed by adding, \textit{a posteriori}, the mean and the perturbation. This procedure can be extended to a mean flow perpendicular to the coast if a coordinate system moving with the front is used as the reference.

\textbf{APPENDIX}

\textit{Boundary conditions}

The radiation boundary condition for the \textit{u} mean flow in Fourier transform space uses the method of stationary phase and the dispersion relation for the internal inertia-gravity waves (Pearson 1974).

The dispersion relation for an internal inertia-gravity wave of frequency \( \omega_0 \) and wave numbers \( k, m \) in the \( x, z \) directions in the absence of mean flow is

\[
\omega_0^2 = f^2 + N^2 k^2/m^2. \tag{A.1}
\]

In the presence of a flow, \( u \), which is slowly varying the dispersion relation in the relevant physical coordinates is

\[
\omega = u \cdot \mathbf{k} + \omega_0. \tag{A.2}
\]

Differentiating yields the group velocity \( C_g \)

\[
C_g = u + C_{g0}, \tag{A.3}
\]

where \( C_{g0} \) is the group velocity in the absence of mean flow. Here \( u \) has only components in the \( x \) direction and the phase speed is

\[
C_p = u + C_{p0}. \tag{A.4}
\]

By the method of stationary phase those waves arriving at the boundary have a group velocity given by

\[
C_g = LT^{-1}. \tag{A.5}
\]

Here \( L \) is the distance of the boundary from the coast and \( T \) is the elapsed time since the most dominant heat input. The group velocity, relative to the mean flow, is then

\[
C_{g0} = LT^{-1} - u. \tag{A.6}
\]

If the estimate of \( C_{g0} \) obtained from (A.6) is negative on the right-hand end (large \( X \)) or positive on the left, then no waves have arrived at the boundary. In this case the conditions of the mean flow are valid and the boundary condition is not applied.

For this dispersion relation (A.1) the phase velocity of each vertical eigenmode is inversely proportional to the group velocity with

\[
C_{p0}(m) = (N^2/m^2)(1/C_{g0}). \tag{A.7}
\]

For the general case a different phase velocity is required for every vertical eigenmode. To accomplish this a numerical Fourier series expansion is applied at the boundary. Before this series is evaluated for the streamfunction \( \Psi \) a mean and a linear trend were subtracted. Call the magnitude of each wave \( \Psi(m) \). Then \( \Psi(m) \) obeys the equation
\[ \frac{\partial \Psi(m)}{\partial t} + \{u + C_{p0}(m)\} \frac{\partial \Psi(m)}{\partial x} = 0. \quad (A.8) \]

This is valid only if \{u + C_{p0}(m)\} is pointing towards the exterior of the model domain, otherwise \(\Psi(m)\) is that of the mean flow. For numerical stability \(\{u + C_{p0}(m)\} \Delta t / \Delta x < 1\) is also required.

After the amplitude of each mode arriving at the boundary has been calculated, the inverse transform is applied. The linear trend is also reintroduced. In the real space the boundary conditions (in \(x, z\)) applies only to the dominant vertical eigenmode and was designed to reflect the long horizontal wavelengths. There are therefore errors in the short wavelengths and to damp these the computer code allows a facility for introducing a region of constant eddy viscosity near the side (and top) boundaries. The value of the eddy viscosity is increased close to the boundary. This facility was not used except where specially noted.

For very large shears, the reflection coefficient is still small. For any wave frequency \(\omega_1\), wavelength \(k_1\), close to the imposed completely transmitted one, the reflection coefficient in the finite difference scheme can be calculated (Pearson 1974). The formula is

\[ |B/A|^2 = \{(Ck_1 - \omega_1)^2 + \left(\frac{1}{2} C \Delta x^2 k^4\right)\}/\{(Ck_1 + \omega_1)^2 + \left(\frac{1}{2} C \Delta x^2 k^4\right)\}. \quad (A.9) \]

The derivation, and application, of these boundary conditions have assumed no vertical shear in either \(u\) or \(v\). For a \(u\)-shear the mean value of the velocity over the whole grid was used when estimating the phase and group velocities. The scale analysis has shown that for \(v\)-shear the extra terms should be neglected. This is also true for the boundary condition. The true dispersion relation is

\[ \omega_0^2 = f^2 + N^2(k^2/m^2) - 2M^2(k/m). \quad (A.10) \]

In this model the ratio of vertical to horizontal wavenumbers is of the order of \(0.5 \times 10^2\). An estimate of the true phase velocity is thus

\[ C_{p0} = (N^2 - 10^2 M^2)C_{\theta 0}^{-1} \text{ m}^{-2}. \quad (A.11) \]

When \(10^2 M^2\) is less than \(N^2\), the error in the phase velocity is at the most 10%. With these estimates of the error in \(C\), the proportion of the amplitude of a single wave that is reflected, \(|B/A|\), is approximately 0.07.

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