Transfer coefficient eddy flux parametrizations in a simple model of the zonal average atmospheric circulation

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SUMMARY

Baroclinic eddy parametrizations of the type proposed by Green in 1970 are applied in a simple climate model which is the zonal average counterpart of the long wave model used by the authors in a recent paper. Three parametrization schemes, each with different specifications of transfer coefficients, are examined. Diabatic forcing is represented by a sinusoidal function of latitude, and the performance of each scheme is gauged by the realism of the eddy flux and zonal mean fields which it produces in the steady state. In broad agreement with numerical simulations of baroclinic eddy life cycles, the best results are obtained when a certain condition of maximum available potential energy release is relaxed. The main feature of the study is the systematic application of fundamental constraints on the eddy vorticity fluxes: these are used to determine free parameters in the functional specification of the transfer coefficients.

1. INTRODUCTION

Because parametrized models of the three-dimensional atmosphere are at an early stage of development (see White and Green 1982—herein referred to as WG1), models of the zonally averaged atmosphere are important as test-beds for large-scale eddy parametrization schemes. In this paper we use a simple zonal average model to examine transfer coefficient parametrization schemes of the type proposed by Green (1970). Our study is more general than the diagnostic analyses carried out by Green and subsequently by White (1977): it uses a time-dependent formulation (although only the final steady states are described here); it accounts for the thermal wind field as well as the surface flow (on the basis of assumed distributions of diabatic heating); and it involves more detailed specifications of the transfer coefficients. The dynamical model is the zonal average counterpart of the three-dimensional formulation used by WG1.

Three different transfer coefficient schemes are investigated. One is similar to that used by WG1, and by Marshall (1981) and Vallis (1982) in their zonal average models of the ocean and atmosphere. The other two schemes are more detailed and self-consistent and have not been applied before.

The rationale of the transfer coefficient parametrization method has been discussed at length by Green (1970) and White (1977); see also WG1, Marshall (1981) and Vallis (1982). Similar large-scale eddy parametrization techniques have been adopted in time-dependent zonal average models by Sela and Wiin-Nielsen (1971), Wiin-Nielsen and Fuenzalida (1975) and Ohring and Adler (1978), but these three studies failed to exploit key constraints on the poleward eddy vorticity flux. The satisfaction of vorticity constraints is the major feature of the parametrization schemes presented here. (Broadly similar diffuse parametrizations of poleward eddy heat fluxes have, of course, been used in climate models by many workers: for reviews see Willson (1973), Schneider and Dickinson (1974), GARP (1975) and Saltzman (1978).)

In section 2 the structure and operation of the model are outlined. The application of the eddy flux parametrization schemes is considered in general terms in section 3, and a difficulty concerning the heat flux parametrization in our simple dynamical model is discussed. Vallis (1982) has shown that this difficulty does not arise if the primitive
equations—together with spherical geometry—are used; and we are content to adopt a simple compromise solution here. Sections 4, 5 and 6 deal with the application of the three different schemes for specifying the transfer coefficients and contain the main quantitative results of the paper. The most important aspects of the study are summarized in the concluding section 7.

2. Model Formation

(a) The 2-parameter equations

The zonal averages of the quasi-geostrophic vorticity and thermodynamic equations (1) and (2) of WG1 are

\[
\frac{\partial}{\partial t} \left( \frac{\partial^2 \psi}{\partial y^2} \right) = f_0 \frac{\partial \tilde{w}}{\partial z} - \frac{\partial}{\partial y} \left( \nu_{\text{g}2} \tilde{\psi} \right) + R
\]

(1)

\[
\frac{\partial}{\partial t} \left( \frac{\partial \tilde{\psi}}{\partial z} \right) + \frac{N^2}{f_0} \overline{w} = \frac{g}{f_0} \left\{ \overline{S} - \frac{\partial}{\partial y} (\nu_{\text{g}} \phi') \right\}
\]

(2)

Here the notation is standard. In particular: an overbar indicates a zonal average; a dash, the deviation from it; and \( v_{\text{g}} \) the poleward component of the geostrophic flow. Equation (1) relates the rate of change of zonal average vorticity to the corresponding vortex stretching, the convergence of the poleward eddy vorticity flux \( \nu_{\text{g}2} \tilde{\psi} \) and a further forcing function \( R \). (\( R \), which was not included in WG1, represents an effect to be discussed in section 3.) Equation (2) relates the rate of change of zonal average potential temperature \( \overline{\theta} = \theta_0[1 + (f_0/g) \partial \tilde{\psi}/\partial z] \) where \( \theta_0 = \theta_0(z) \) is a mean state profile of potential temperature) to the zonal average vertical advection, diabatic heating \( \overline{S} \) and the convergence of the poleward eddy flux of entropy \( \nu_{\text{g}} \phi' \). (\( \phi' = -\ln \theta \approx \ln \theta_0 + (\theta - \theta_0)/\theta_0 \); \( \nu_{\text{g}} \phi' \) is proportional to the poleward eddy heat flux.) \( N^2 = (g/\theta_0) d\theta_0/dz \).

Two-parameter forms of Eqs. (1) and (2) are derived by applying the technique described in WG1. The functional representations are:

\[
\overline{\psi}(y, z, t) = \psi_0(y, t) + (z/H) \psi_1(y, t)
\]

(3)

\[
\overline{w}(y, z, t) = w_0(y, t) + (z/H) w_1(y, t) + (z/H) \overline{w}_2(y, t)
\]

(4)

\[
R(y, z, t) = (z/H) R_1(y, t)
\]

(5)

\[
\overline{S}(y, z, t) = S_0(y, t)
\]

(6)

\[
- \nu_{\text{g}2} \tilde{\psi} = V_0(y, t) + (z/H) V_1(y, t)
\]

(7)

\[
- \nu_{\text{g}} \phi' = E_0(y, t)
\]

(8)

and the resulting 2-parameter equations are

\[
\frac{\partial}{\partial t} \left( \frac{\partial^2 \psi_0}{\partial y^2} \right) = -(f_0/H) w_B + \partial V_0/\partial y
\]

(9)

\[
\left( \frac{\partial^2}{\partial y^2} - \frac{12f_0^2}{N^2H^2} \right) \frac{\partial \psi_1}{\partial t} = \frac{6f_0}{H} w_B + \frac{\partial V_1}{\partial y} + R_1 - \frac{12g f_0}{N^2 H} \left( S_0 + \frac{\partial E_0}{\partial y} \right)
\]

(10)

The domain is a channel whose southerly and northerly boundaries are at \( y = 0 \) and \( y = L \) (= \( L_y \) of WG1) and whose depth is \( H \) (see Fig. 1). In deriving Eqs. (9) and (10) a rigid horizontal boundary condition has been applied at \( z = H/2 \) and at \( z = -H/2 \) it
has been assumed that $\overline{w} = w_B$, where $w_B$ is related to the streamfunction by the simple Ekman pumping formula

$$w_B = \left( k_2 f_0 \right) \frac{\partial^2 \psi_0 - \frac{1}{3} \psi_1}{\partial y^2}. \tag{11}$$

The functions $\psi_0, V_0$ etc. appearing in Eqs. (9) and (10) represent the height averages of the respective quantities. Thus $\psi_0$ and $\psi_1$ may be considered as the barotropic and baroclinic parts of the zonal average streamfunction. The 2-parameter model, first proposed for the three-dimensional quasi-geostrophic equations by Eady (1952), bears a close relationship to the more familiar 2-level formulation. See White (1978) and WG1 for further comment. The reason for the specification of the vorticity source function $R$ with zero height average will become apparent in section 3.

(b) Spectral representation

It is convenient to introduce the non-dimensional latitude coordinate $Y = y/L$. Appropriate truncated Fourier expansions of the functions appearing in Eqs. (9) and (10) are

$$(\psi_0/L, \psi_1/L, f_0 L w_B/H, L R_1, g H S_0/f_0 L) = \sum_{n=1}^{n_1} \left( \frac{a_n}{n^2 \pi^2}, \frac{b_n}{n^2 \pi^2}, w_{Bn}, R_n, S_n \right) \cos n \pi Y \tag{12}$$

$$(V_0, V_1, g H E_0/f_0 L^2) = \sum_{n=1}^{n_1} \left( \frac{V_{0n}}{n \pi}, \frac{V_{1n}}{n \pi}, \frac{E_n}{n \pi} \right) \sin n \pi Y. \tag{13}$$

These expansions are essentially the same as those used for the zonal average fields in WG1, but some of the amplitude factors are differently defined. Substitution of Eqs. (12) and (13) into Eqs. (9) and (10) yields $2N$ prognostic equations for the spectral amplitudes $a_n, b_n$ ($n = 1, 2, \ldots, n_1$):

$$da_n/dt = w_{Bn} - V_{0n} \tag{14}$$

$$(1 + \mu^2/n^2 \pi^2) \; db_n/dt = 12\mu^2(S_n + E_n) - 6 w_{Bn} - V_{1n} - R_n. \tag{15}$$

Here $\mu^2 = 12 f_0^2 L^2 / N^2 H^2$. The $w_{Bn}$ are related to the $a_n$ and $b_n$ via Eq. (11).

In numerical integration of Eqs. (14) and (15), $S_1 \neq 0$; but in nearly all cases treated in this paper $S_n = 0$ for $n \geq 2$. Thus the diabatic heating is nearly always represented as
simply proportional to $\cos \pi Y$ (corresponding to low latitude heating and high latitude cooling—note that the diabatic heating here includes the contribution of latent heating). More realistic specifications of the diabatic heating in this model have been examined by White (1979). Figure 4(a) of WG1 summarizes schematically the physical interactions which are represented in the present zonal average model.

The only mathematical complications in applying Eqs. (14) and (15) arise because the parametrizations for the eddy flux convergences do not lead exclusively to linear functions of the dependent variables $\psi_0$ and $\psi_1$ (see section 3). The parametrizations are treated using a combination of Fourier expansion and grid-transform techniques, as described by White (1978).

Truncations $n_T = 3$ to $n_T = 9$ are applied in the integrations to be described in sections 4 to 6. For comparison with models framed in spherical geometry it may be noted that truncation at $n = n_T$ is roughly equivalent to truncation at $n = 2n_T$ in terms of Legendre polynomials $P_n(\cos(\text{latitude}))$. Thus $n_T = 9$ is a fairly high resolution for a parametrized climate model—it is, of course, much higher than the resolutions used in WG1 ($n_T = 3$ or 5).

(c) Time integration procedure

The results presented in sections 4 to 6 are reconstituted using Eqs. (12) and (13) from steady-state solutions of Eqs. (14) and (15) obtained at various spectral truncations and with various parametrization schemes. The initial condition in each case is a state of zero relative motion everywhere ($a_n = b_n = 0; n = 1, 2, \ldots n_T$). Time integration is carried out using the same methods as those applied in WG1. A step of 12 hours (or 6 hours at finer spatial resolutions) is used, and an effectively steady state is reached after several months of simulated time. Typical tropospheric values of the various constants are assumed, as in WG1: $L = 10^8 m$, $H = 10^4 m$, $f_0 = 10^{-5} s^{-1}$, $\beta = 1.5 \times 10^{-11} m^{-2} s^{-1}$, $k_F = 2 \times 10^{-2} m s^{-1}$, $g = 10 m s^{-2}$, $\theta_0 = 300 K$ where it appears in undifferentiated form, and $N^2 H/g = (H/\theta_0) d\theta_0/dz = 0.13$.

3. The eddy flux parametrizations

The parametrizations proposed by Green (1970) for the poleward eddy fluxes of entropy $\phi$ and vorticity $\zeta$ are

$$\bar{\nu}_h \phi = -K_{\psi y} \partial \bar{\phi}/\partial y - K_{\psi z} \partial \bar{\psi}/\partial z$$

$$\bar{\nu}_h \zeta = -K_{\psi y} (\beta - \partial^2 \bar{U}/\partial y^2) + (g f_0 / N^2) (\partial \bar{\phi}/\partial y) (\partial K_{\psi y}/\partial z) + f_0 \partial K_{\psi f}/\partial z.$$  

These forms assume continuous vertical structure, and static compressibility is neglected. $\bar{U} = \bar{U}(y, z, t)$ is the zonal average zonal flow. In practice the factor $\partial \bar{\phi}/\partial z$ in Eq. (16) is replaced by $N^2/g$; indeed this has been done in deriving Eq. (17).

The key quantities in the parametrizations (16) and (17) are the transfer coefficients $K_{\psi y}$ and $K_{\psi z}$ (Green 1970; White 1977). Green suggested that the specification of their spatial variation should be guided by the results of zonal flow instability analyses, with an absolute magnitude being determined by arguments based on idealized energetics of the general circulation and a typical amplification-time/decay-time ratio for baroclinic eddies. Two implications of the approach require comment.

First (as noted by White (1977), section 2), if instability analyses of the zonal average flow are used to specify the spatial variations of the transfer coefficients precisely, then the introduction of transfer coefficients is redundant—one might as well use an energy argument to scale up the various fluxes determined in the instability analyses and apply
them directly in the zonal average model. Thus, inherent in Green's parametrization theory is the notion that instability analyses may give useful guidance as to the spatial variations of the transfer coefficients but do not constrain their variations completely. This is consistent with the baroclinic wave life-cycle simulations carried out by Simmons and Hoskins (1978). They found that the eddy fluxes averaged over a wave's life-cycle are poorly approximated in spatial variation by the fluxes occurring during the early stages of growth. (The result applies especially to the eddy momentum fluxes: in the amplifying stage of a wave they are quite different in spatial variation from their life-cycle averages.)

The second important aspect becomes apparent when any method of completing the specification of the transfer coefficients is considered. In view of the identity

\[ \bar{\nu}_k \zeta = -\partial (u_k \nu_k)/\partial y \]  

(18)

\( u_k \) is the eddy component of the zonal geostrophic flow) it follows that

\[ \int_0^1 \nu_k \zeta \, dY = 0 \]  

(19)

for all \( z \) and \( t \) since \( \nu_k \) must vanish on \( Y = 0, 1 \) according to the usual lateral boundary conditions on quasi-geostrophic flow (see WG1, sections 3 and 4). Condition (19), via Eq. (17), enforces a relation between \( K_{yz} \), \( K_{yz} \) and various zonal mean gradients which must hold at every height and time. Analogous constraints apply also in models framed in spherical geometry (White 1977). Any parametrization of \( \nu_k \zeta \) should indeed obey Eq. (19) at every height and time, and failure to satisfy it implies the presence of spurious momentum sources or sinks. Egger (1975) has enforced Eq. (19) in a parametrization scheme which does not involve transfer or diffusion coefficients.

Our approach here is to exploit the constraints on the vorticity flux expressed by Eq. (19) to provide information about the spatial variations of the transfer coefficients. Using non-controversial qualitative arguments about the dynamics of baroclinic waves we specify some gross features of the transfer coefficients' spatial variations, but the specification is completed by applying Eq. (19). In mathematical terms, the transfer coefficients \( K_{yz}(y, z), K_{yz}(y, z) \) are specified at each time \( t \) with just enough assignable constants to enable the vorticity flux constraints to be satisfied. In this way two potential difficulties partially annihilate one another: the constraints (19) on the eddy vorticity flux must be obeyed; and we do not know very much a priori about the detailed variations of the transfer coefficients appropriate to finite amplitude baroclinic waves.

Sela and Wiin-Nielsen (1971), Wiin-Nielsen and Fuenzalida (1975) and Ohring and Adler (1978) have operated zonal average models based on the 2-level quasi-geostrophic equations. They used potential vorticity forms, with the eddy fluxes of potential vorticity \( \bar{Q} \) at pressure levels \( i = 1, 3 \) specified in terms of 'exchange coefficients' \( K_{Qi} \) by the spherical polar equivalents of

\[ \bar{\nu}_k \bar{Q} = -K_{Qi} \bar{Q} / \partial y. \]  

(20)

Equation (20) is equivalent to Green's (1970) parametrization of \( \bar{v}_k \bar{Q} \), given that \( K_0 = K_{Qp} \). With the potential vorticity equations and Eq. (20) it is possible to integrate in time by specifying only \( K_{Qi} \) (\( i = 1, 3 \)), with no reference to heat fluxes. As noted by Wiin-Nielsen and Sela, this is a very useful simplification. Nevertheless, constraint (19) must still be obeyed. Given the potential vorticity flux at each level, Eq. (19) imposes a constraint on the heat flux at the middle level. In so far as the heat flux need not be specified for time integration of the potential vorticity forms, such a constraint is
unimportant. However, it is readily shown that
\[
(\nu'_g Q')_1 + (\nu'_g Q')_3 = (\nu'_g \bar{Q'})_1 + (\nu'_g \bar{Q'})_3.
\]
(21)
Thus the \( K_{Q_1} \) must obey
\[
\int_0^\ell \left\{ K_{Q_1} \frac{\partial \bar{Q}_1}{\partial y} + K_{Q_3} \frac{\partial \bar{Q}_3}{\partial y} \right\} dy = 0.
\]
(22)
For a more detailed analysis see Marshall (1981). Condition (22) was not applied in the three studies noted above, and consequently spurious momentum sources were present. Wiin-Nielsen and Fuenzalida describe how they found it necessary to adjust the average zonal flow in their model from time to time in order to compensate for the spurious momentum source. This procedure is evidently unsatisfactory. The method used in the present paper obviates the need to resort to such devices.

As noted in WG1, the 2-parameter equations can be cast into forms that are reminiscent of the 2-level potential vorticity forms. Given due attention to the corresponding 2-parameter version of Eq. (22), integration with the potential vorticity equations is probably the most elegant way to proceed. We have chosen, however, to use the barotropic/baroclinic forms (9) and (10), in which the heat and vorticity fluxes appear rather than simply the potential vorticity fluxes. The importance of the vorticity flux constraint (19) is highlighted by this choice; also, the situation is typical of non-geostrophic models, in which it is evidently necessary to parametrize the heat and vorticity fluxes separately.

In the 2-parameter equations (9) and (10) the eddy flux divergences to be parametrized are represented by \( V_0, V_1 \) and \( E_0 \). These quantities are to be expressed in terms of \( \psi_0 \) and \( \psi_1 \). From Eqs. (3), (7), (8), (16) and (17) it follows that
\[
\frac{gH}{f_0 L^2} E_0 = \hat{K}_{\psi_1} \frac{\partial \psi_1}{\partial y} + \frac{12f_0L}{\mu^2} \hat{K}_{\psi_2}
\]
(23)
\[
\frac{gH}{f_0 L^2} V_0 + ZV_1 = \hat{K}_{\psi_1} \left( \beta L^2 + \frac{\partial^3 \psi_0}{\partial y^3} + \frac{\partial^3 \psi_1}{\partial y^3} \right) - \frac{\mu^2}{12} \frac{\partial \psi_1}{\partial y} \frac{\partial \hat{K}_{\psi_2}}{\partial Z} - f_0L \frac{\partial \hat{K}_{\psi_2}}{\partial Z}
\]
(24)
where \( L^2 \hat{K}_{\psi_1} = K_{\psi_1}; \ L \hat{K}_{\psi_2} = K_{\psi_2}; \ L \psi_0 = \psi_0; \ L \psi_1 = \psi_1; \) and \( HZ = z \). In each scheme for specifying the transfer coefficients (see sections 4–6) separability is assumed:
\[
\hat{K}_{\psi_1} = K_h(Y)K(Z)
\]
(25)
\[
\hat{K}_{\psi_2} = J_h(Y)J(Z).
\]
(26)
This is convenient mathematically and seems reasonable for an ensemble of baroclinic wave lifetimes. It is in accord with baroclinic instability analyses of \( \beta \) plane flows that are independent of latitude, but flows with lateral shear do not give separable forms for \( K_{\psi_1} \) and \( K_{\psi_2} \). The assumption of separability is borne out to some extent by Wiin-Nielsen and Sela’s (1971) observational study of eddy fluxes of heat and potential vorticity in the troposphere (see their Figs. 10–13).

The height profile functions \( K \) and \( J \) are linear or quadratic functions of \( Z \) (see sections 4–6). The right-hand sides of Eqs. (23) and (24) therefore contain terms in \( Z^n \), where \( n > 1 \). These terms are replaced by linear functions of \( Z \) according to the least-squares procedure used in WG1 in the derivation of the 2-parameter model. The equalities in Eqs. (23) and (24) are thus to be construed to imply replacement of the right-hand side expressions by the least-squares fits in terms of \( E_0 \) (Eq. (23)) and \( V_0 + ZV_1 \) (Eq. (24)).
The latitude profile functions are chosen so that \( K_{h}(\hat{\lambda}) = J_{h}(\hat{\lambda}) = 1 \). For \( K_{h}(Y) \) we choose in schemes 1, 2 and 3 (see sections 4, 5 and 6)

\[
K_{h}(Y) = \sin^{2} \pi Y
\]  
(27)

representing (i) the expected smallness of \( K_{y} \) in extreme latitudes where temperature gradients and baroclinic activity are small, and (ii) its expected maximum in middle latitudes where the opposite conditions hold. The actual functional form of (27) is suggested by baroclinic instability analyses of \( \beta \) plane flows having no lateral shear (see White 1977). The choice of \( J_{h}(Y) \) varies from scheme to scheme.

The vorticity source function \( R \) in Eq. (1) has not so far been discussed. If \( R = 0 \), it is found that unrealistic results are obtained when Eq. (27) is applied in both the heat and the vorticity flux parametrizations and the diabatic forcing \( S \) is represented as proportional to \( \cos \pi Y \) (see section 2(b)). This is because the divergence of the parametrized eddy heat flux is small in extreme latitudes (and vanishes at \( Y = 0, 1 \)). In the steady state, diabatic effects must therefore be balanced in extreme latitudes by mean advection as represented by the term \( N^{2} \bar{w} / \bar{g} \) in Eq. (2): for the 2-parameter model it follows from Eqs. (2), (4), (6) and (8) that the height-average thermodynamic energy balance in the steady state is

\[
\bar{w}^{zz} \left( = \bar{w}_{0} + \frac{1}{2} \bar{w}_{1} \right) = (g / N^{2}) (S_{0} + \partial E_{0} / \partial y).
\]  
(28)

(The overbar \( \bar{w}^{zz} \) indicates averaging over \( x \) and \( z \).) The mean circulation is, however, also related to the total vorticity forcing. From Eqs. (9), (10) and (28) it follows that \( \bar{w}^{zz} \) must, in the steady state, obey

\[
\bar{w}^{zz} = \left( H / 12 f_{0} \right) (6 \partial V_{0} / \partial y + \partial V_{1} / \partial y + R_{1}).
\]  
(29)

Equation (29) shows that \( \bar{w}^{zz} \) must vanish at \( Y = 0, 1 \) if \( K_{h}(Y) \) is specified by Eq. (27), and \( R_{1} = 0 \). In this case the steady-state heat balance (28) cannot be simultaneously maintained in extreme latitudes if \( S_{0} \) is non-zero there. (With the finite spectral representations (12) and (13) the result of these incompatible requirements is that spurious steady states are attained which depend critically on the truncation \( n_{y} \).) Extensive investigations have revealed that broadly similar difficulties occur if \( K_{h}(Y) \) is represented by other functions which vanish at \( Y = 0, 1 \); the fundamental problem is the inability of the parametrized vorticity flux to drive a mean meridional circulation of sufficient strength to balance the diabatic heating in extreme latitudes.

If, however, Eq. (27) is applied in the vorticity flux parametrization, but \( K_{h}(Y) \) is set equal to unity in the heat flux parametrization, then the diabatic heating can be balanced in extreme latitudes by the divergence of the eddy heat flux, and realistic steady states are obtainable. This procedure is adopted in nearly all the integrations whose final steady states are described in sections 4 to 6. Equations (10), (28) and (29) show that it is precisely equivalent to retaining \( K_{h}(Y) = \sin^{2} \pi Y \) and introducing a vorticity source function \( R_{1} \) of the form

\[
R_{1} = - \frac{12 g f_{0}}{N^{2} H} \kappa \frac{\partial}{\partial Y} \left[ \cos^{2} \pi Y \frac{\partial \bar{\Phi}}{\partial Y} \right]
\]  
(30)

where \( \kappa \) is a quantity which is independent of latitude and is differently defined for schemes 1, 2 and 3 of sections 4, 5 and 6.

In a primitive equation parametrized model formulated in spherical geometry, Vallis (1982) found that the height-integrated angular momentum balance in low latitudes was dominated by the surface stress torque and the mean advection of relative angular
momentum. (This result is consistent with the observational results presented in Table 11 of Oort and Rasmussen (1971) and in Palmén and Newton (1969), chapter 1.) Also, in Vallis's model the driving of the Hadley cell by the parametrized vorticity flux was stronger than in our Cartesian model because the true latitude variation of the Coriolis parameter \( f = -2\Omega \sin(\text{latitude}) \) was included in the explicit dynamical equations. These effects combine to make inclusion of the vorticity source function unnecessary when a primitive equation model with spherical geometry is used: Vallis obtained realistic steady-state fields when the latitude variation of the transfer coefficients was the same in both the heat and vorticity flux parametrizations.

In the light of these arguments and results we are content to put \( K_h(Y) = 1 \) in the heat flux parametrization, and to interpret the replacement as the inclusion of a vorticity source function (given by Eq. (30)) which compensates for the absence of mean relative angular momentum advection in our model and for the weakness of the parametrized vorticity flux in extreme latitudes. (Another possible interpretation of \( R_1 \) is that it represents the cumulus friction effect modelled by Schneider and Lindzen (1976); but in view of the data analysis carried out by Thompson and Hartmann (1979) we favour the interpretation given above.) In this way we are able to illustrate the correct application of the vorticity constraints (19) while still using our very simple Cartesian quasi-geostrophic model.

4. PARAMETRIZATION SCHEME 1

Scheme 1 was used to parametrize the zonal mean eddy fluxes in the long wave model investigated by WG1. It incorporates several somewhat arbitrary specifications and is the least self-contained of the three schemes to be investigated here.

(a) Description

In our 2-parameter model the eddy vorticity flux is required as a linear function of height (see Eq. (7)). The height average \( V_0 \) can be obtained from Eq. (24) as:

\[
V_0(y, t) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left\{ \dot{K}_{zy} \left( \beta L^2 + \frac{\partial^3 \Psi_0}{\partial y^3} + Z \frac{\partial^3 \Psi_1}{\partial Z^3} \right) - \frac{\mu^2}{12} \frac{\partial \Psi_1}{\partial Y} \frac{\partial \dot{K}_{zy}}{\partial Z} \right\} dZ.
\]  
(31)

(Following Green (1970) it has been assumed that \( \dot{K}_{zy} \) vanishes at both limits of the integration.) Since \( \dot{K}_{zy} \) is specified as a separable function of \( Y \) and \( Z \) (see Eq. (25)) it must obey

\[
\left( \frac{\mu^2}{12} \right) \int_{-\frac{1}{2}}^{\frac{1}{2}} \left( \frac{\partial \dot{K}_{zy}}{\partial Z} \right) dZ = -\dot{\gamma} \int_{-\frac{1}{2}}^{\frac{1}{2}} \dot{K}_{zy} dZ
\]  
(32)

in which \( \dot{\gamma} = \dot{\gamma}(t) \) is independent of \( Y \) and \( Z \).

In scheme 1 the third term under the integral sign in Eq. (31) is neglected:

\[
V_0(Y, t) = \left( \beta L^2 + \frac{\partial^3 \Psi_0}{\partial Y^3} + \dot{\gamma} \frac{\partial \Psi_1}{\partial Y} \right) \int_{-\frac{1}{2}}^{\frac{1}{2}} \dot{K}_{zy} dZ.
\]  
(33)

The omission of the term in \( \frac{\partial^3 \Psi_1}{\partial Y^3} \) cannot be rigorously justified—note that \( \dot{K}_{zy} \) is not expected to be symmetric about the mid-level \( Z = 0 \) (Green (1970))—but the approximation will not be serious if the \( \beta \) effect is significant. To complete the specification of \( V_0(y, t) \) we adopt the functional form (27) for \( K_h(Y) \) and apply the vorticity flux
constraint (19) to determine \( \dot{\Psi} \) at each time step:

\[
\dot{\Psi} = -\int K_h(Y) \{ \beta L^2 + \delta^3 \Psi_0 / \partial Y^3 \} dY / \int K_h(Y) (\partial \Psi_0 / \partial Y) dY.
\]  

(34)

Apart from the omission of the term in \( \delta^3 \Psi_0 / \partial Y^3 \), the above parametrization of \( V_0(Y, t) \) is equivalent to that applied in the surface zonal flow problems examined by Green (1970) and White (1977).

To specify \( V_1 \) we merely choose a reasonable height variation for the vorticity flux convergence. Bearing in mind that the observed poleward eddy momentum fluxes (Oort and Rasmussen 1971) are at all seasons greatest near tropopause level and are generally of the same sign for all heights at a given latitude, we put

\[
V_1(Y, t) = 2V_0(Y, t).
\]  

(35)

Thus the eddy vorticity flux vanishes at \( Z = -\frac{1}{2} \) and reaches a maximum (absolute) value at \( Z = +\frac{1}{2} \). Note that \( V_1(Y, t) \) automatically obeys integral constraint (19).

Information about \( \dot{K}_{uv} \) is needed in order to specify the eddy heat flux using Eq. (23). In scheme 1 the term in \( \dot{K}_{uv} \) in Eq. (23) is omitted, and its effect crudely allowed for by simply scaling down the term in \( \dot{K}_{uv} \). We apply Eq. (23) in the form

\[
(gH/f_0 L^2) E_0 = \delta \partial \Psi / \partial Y \int_{-\frac{1}{2}}^{1/2} \dot{K}_{uv} dZ \equiv \delta K_M \partial \Psi / \partial Y
\]  

(36)

with \( \delta = \frac{1}{2} \). Thus the eddy heat flux parametrization in scheme 1 is similar to the essentially diffusive representations which have been used in many zonal average climate models (see Schneider and Dickinson 1974 and Saltzman 1978).

For the reasons discussed in section 3 we set \( K_h(Y) = 1 \) in applying Eq. (25) to Eq. (36). This is equivalent to including a vorticity forcing function of the form (30), with \( \kappa = \delta K_M \), whilst retaining \( K_h(Y) = \sin^2 \pi Y \) throughout.

It remains to estimate an absolute magnitude for \( \dot{K}_{uv} \). We take

\[
K_M = 4 \times 10^7 \Delta \phi / L^2 s^{-1}
\]  

(37)

(where \( \Delta \phi \) is the difference in log(potential temperature) between low and high latitudes). This is similar to the unnumbered equation given at the bottom of p. 175 in Green (1970).

(b) Results

Figure 2 shows the steady-state latitude variations of potential temperature deviation \( \delta \theta = (\theta - \theta_0) \), surface zonal flow \( U_S (\equiv \overline{U}(\frac{1}{2})) \) and tropopause-level zonal flow \( U_T (\equiv \overline{U}(1)) \), when \( S_0 = 0.6 \cos \pi Y \) deg day\(^{-1} \). (\( \theta_0 \) is the domain-average potential temperature defined in section 2(c).) All the latitude variations are qualitatively realistic. The difference in potential temperature between low and high latitudes is about 46°C, and the horizontal gradient is largest (as expected) in middle latitudes. The parametrized eddy vorticity flux forces a familiar pattern of surface zonal winds—westerlies in middle latitudes and easterlies elsewhere, with extremal values of 3–4 m s\(^{-1} \). Thermal wind balance necessitates the existence of a mid-latitude westerly jet at tropopause level, the maximum flow speed being about 30 m s\(^{-1} \).

The results shown in Fig. 2 were obtained with spectral truncation \( n_T = 7 \). Qualitatively similar latitude variations are found when other truncations \( n_T (\geq 3) \) are applied. Table 1 shows the extremal values of the surface and tropopause-level zonal winds obtained for \( n_T = 3, 5, 7 \) and 9. Evidently the convergence with spectral truncation is
Figure 2. Steady-state latitude variations obtained using scheme 1 parametrization. Full line: potential temperature deviation $\delta \theta = \theta - \theta_i$; broken line: tropopause-level zonal flow $U_T$; dotted line: surface zonal flow $U_s$.

rapid, and the accuracy of the $n_T = 3$ results is notable. This rapid convergence is a partial justification of the coarse meridional truncation ($n_T = 3$ or 5) which was applied—together with scheme 1 parametrizations of zonal mean eddy fluxes—in WG1’s long wave climate model.

<table>
<thead>
<tr>
<th>Truncation ($n_T$)</th>
<th>Max. $U_s$ (m s$^{-1}$)</th>
<th>Min. $U_s$ (m s$^{-1}$)</th>
<th>Max. $U_T$ (m s$^{-1}$)</th>
<th>$\Delta \theta$ ($^\circ$C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.91</td>
<td>-2.45</td>
<td>30.18</td>
<td>45.94</td>
</tr>
<tr>
<td>5</td>
<td>3.99</td>
<td>-2.61</td>
<td>30.30</td>
<td>45.94</td>
</tr>
<tr>
<td>7</td>
<td>3.95</td>
<td>-2.58</td>
<td>30.24</td>
<td>45.94</td>
</tr>
<tr>
<td>9</td>
<td>3.92</td>
<td>-2.57</td>
<td>30.20</td>
<td>45.94</td>
</tr>
</tbody>
</table>

Scheme 1 parametrization is applied. $\Delta \theta$ is the difference in potential temperature between the lateral boundaries of the channel domain.

5. PARAMETRIZATION SCHEME 2

Scheme 1 includes several approximations and empirical specifications beyond those inherent in the transfer coefficient parametrization method. In particular, the height variation of the eddy vorticity flux is specified externally, and the effect on the eddy heat flux of vertical displacements of fluid parcels is only crudely allowed for (see Eq. (36)). Furthermore, these two analytical simplifications may well be inconsistent.

In schemes 2 and 3 a consistent treatment is applied: both $K_{sy}$ and $K_{ez}$ are treated in detail on the basis of clear assumptions.
(a) Description

The crucial assumption of scheme 2 is that the ratio \((\tilde{K}_{\psi}/K_{\psi})\) at the middle level \(Z = 0\) takes the value which implies maximum available potential energy release. This corresponds to the mean slope of parcel paths being half that of the mean isentropic surfaces. According to Green's (1970) argument, the condition is enforced by requiring

\[
(\tilde{K}_{\psi}/K_{\psi}) = -g(\partial \overline{\theta}/\partial Y)/2N^2\tau
\]

at \(Z = 0\), for all \(Y\) and \(t\).

For \(\tilde{K}_{\psi}\) the adopted form is

\[
\tilde{K}_{\psi}(Y, Z, t) = (1 - 4Z^2)\tilde{K}_{\psi}(Y, 0, t)
\]

which reflects the expected middle-level maximum of \(\tilde{K}_{\psi}\) and its smallness at \(Z = \pm \frac{1}{2}\) (where parcel paths are constrained to be nearly horizontal).

The functional form applied for \(\tilde{K}_{\psi}\) is

\[
\tilde{K}_{\psi}(Y, Z, t) = 12K_h(Y)\{1 - p_1Z - p_2Z^2\}
\]

where \(K_h, p_1, \) and \(p_2\) are functions of time (see below) and \(K_h(Y)\) is given by Eq. (27).

In deriving from Eqs. (23), (24), (38), (39) and (40) forms suitable for the 2-parameter model we make frequent use of the least-squares procedure whereby terms of degree 2 or more in \(Z\) are replaced by the best-fitting linear functions of \(Z\). After some straightforward algebra we arrive at

\[
(gH/f_0L^2)E_0(Y, t) = K_h(Y)(8 - p_2)\frac{\partial \Psi_1}{\partial Y}
\]

for the height-average eddy flux of entropy and

\[
V_0(Y, t) = K_h(Y)\left\{(12 - p_2)\left(\beta L^2 + \frac{\partial^3 \Psi_0}{\partial Y^3}\right) - p_1\frac{\partial^3 \Psi_1}{\partial Y^3} + \mu^2 p_1\frac{\partial \Psi_1}{\partial Y}\right\}
\]

\[
V_1(Y, t) = K_h(Y)\left[-12p_1\left(\beta L^2 + \frac{\partial^3 \Psi_0}{\partial Y^3}\right) + \frac{20 - 3p_2}{2}\frac{\partial^3 \Psi_1}{\partial Y^3} - 2\mu^2 (2 - p_2)\frac{\partial \Psi_1}{\partial Y}\right]
\]

for the barotropic and baroclinic elements of the eddy vorticity flux. At every time step the quantities \(p_1\) and \(p_2\) are determined by applying the integral constraint (19): in this way the detailed height variation of the transfer coefficient \(K_{\psi}\) (see Eq. (40)) is determined. The quantity \(K_h\) is obtained from

\[
K_h(8 - p_2)\left|\frac{\partial \Psi_1}{\partial Y}\right|_{Y = 1} = \frac{gH}{f_0L^2}a(g^2/N^2)^{1/2}(\Delta \phi)^2.
\]

Equation (44) is readily derived from Eq. (41) by using Green's (1970) Eq. (17). (In Eq. (44) \(a\) is a constant—here taken as \(5 \times 10^{-3}\).)

In the heat flux parametrization (41), \(K_h(Y)\) is set equal to unity for the reasons discussed in section 3. This replacement is equivalent to applying a vorticity source function \(R_1\) of the form (30), with \(\kappa = K_h(8 - p_2)\), whilst retaining \(K_h(Y) = \sin^2\phi Y\). Apart from this necessary treatment of the heat flux parametrization, none of the terms appearing in the parametrizations is omitted or arbitrarily specified in scheme 2. This contrasts markedly with the procedures adopted in scheme 1. In view of the representations (39) and (40), scheme 2 is consistent with the contention of Green (1970) that
the transfer coefficients are the quantities through which information about baroclinic wave dynamics—including constraints such as Eq. (19)—should be fed into a parametrized model.

(b) Results

Figure 3(a) shows the steady-state latitude variations of potential temperature deviation \( \delta \theta = (\bar{\theta} - \theta_0) \), surface zonal flow \( U_s (= \bar{U}(-1)) \) and tropopause-level zonal flow \( U_T(= \bar{U}(1)) \) obtained when \( S_0 = 0.6 \cos \pi Y \, ^\circ \text{C day}^{-1} \) and the truncation \( n_T = 5 \). All three variations are qualitatively realistic. Extremal values are slightly less than those obtained with scheme 1 (see Fig. 2). Qualitatively similar results are found with truncation \( n_T = 3 \) or 7, and quantitative differences are very small.

The height variation of \( \bar{K}_Y \) in the steady state is shown in Fig. 3(b). \( K(Z) \) exhibits a low-level maximum and decreases to a small positive value at \( Z = 1 \). This behaviour is generally similar to that found in small amplitude baroclinic waves (Green 1970, Fig. 8(a)).

Figure 3(c) shows the latitude and height variation of the parametrized zonal momentum flux \( \bar{u}_x \bar{v}_y \) (obtained by integrating the vorticity flux \( \bar{v}_x \bar{v}_z \) with respect to latitude—see Eq. (18)). Numerical values are generally somewhat less than those obtained

![Figure 3](image-url)

Figure 3. (a) Steady-state latitude variations of \( \delta \theta = \bar{\theta} - \theta_0 \), \( U_T \) and \( U_\tau \) obtained using scheme 2 parametrization. Formas for Fig. 2. (b) Steady-state height variation of the profile function of the transfer coefficient \( \bar{K}_Y \), scheme 2 parametrization applied (see Eq. (40)). (c) Steady-state latitude and height variation of the parametrized eddy flux of zonal momentum; scheme 2 parametrization applied. (Units: \( \text{m}^2\text{s}^{-1} \)).
from tropospheric observations (see Oort and Rasmusson 1971); the discrepancy can, however, be attributed to geometric factors. Much more significant is the height variation of the parametrized flux. Maximum absolute values occur at the lowest level \( Z = -\frac{1}{2} \) and minimum absolute values at the tropopause level \( Z = \frac{3}{2} \). The observed tropospheric momentum fluxes exhibit the opposite behaviour at all seasons—small values at low levels and extrema near tropopause level (Oort and Rasmusson 1971). (Because we have used an incompressible fluid model, comparison with fluxes observed in the compressible atmosphere should incorporate a weighting \( \rho_0 \propto \exp(-z/H_0) \) where \( H_0 \) is the density scale height—see Green (1970), section 6(a). Taking \( H = 8 \) km gives \( \rho_0(-\frac{1}{2})/\rho_0(\frac{3}{2}) = 3.08 \). This is insufficient to alter the qualitative difference in behaviour of the parametrized and observed fluxes. Thus Fig. 3(c) would still show a general decrease with height even if a weighting factor \( \exp(z/H_0) \) were to be applied.)

The unrealistic height variation of the parametrized momentum flux is perhaps not surprising. Scheme 2 hinges on the assumption that maximum available potential energy release occurs at mid-level (\( Z = 0 \)) in the parametrized eddy motion at all times. This behaviour is typical of the early stages of growth of a baroclinic eddy (Simmons and Hoskins 1976) when, indeed, the momentum flux is greatest at low levels. However, in the later stages of an eddy’s life-time the dynamics are quasi-barotropic, the release of available potential energy is small, and the momentum flux occurs predominantly at upper levels (see Simons 1972 and especially Simmons and Hoskins 1978). Thus it seems undesirable to relate the transfer coefficients \( K_{vy} \) and \( K_{oz} \) in such a way as to imply maximum available potential energy conversion at mid-level at all times.

In spite of the unsatisfactory height variation of the parametrized momentum flux, the form of the surface zonal flow \( U_s \) (as well as that of \( U_T \) and \( \theta - \theta_0 \)) is realistic—as noted earlier. This emphasizes that the surface zonal flow in the steady state reflects the latitude variation of the height integral of the eddy momentum flux but is independent of the height distribution of the flux.

6. PARAMETRIZATION SCHEME 3

In scheme 3 maximum theoretical efficiency of potential energy release is not applied. Instead, the height variation of \( K_{vy} \) and the height average value of \( K_{oz} \) are determined entirely by the vorticity constraints (19).

\[(a) \quad \text{Description}\]

\[\dot{K}_{vy} \quad \text{and} \quad \dot{K}_{oz} \quad \text{are assumed to take the separable forms}\]

\[\dot{K}_{vy} = 12\bar{K}K_h(Y)(1 - pZ) \quad \text{(45)}\]

\[8f_0L\dot{K}_{oz} = 12\bar{K}K_h(Y)q(1 - 4Z^2). \quad \text{(46)}\]

In Eqs. (45) and (46), \( \bar{K}, p \) and \( q \) are functions of time only, determined at each time step as outlined below. \( K_h(Y) = \sin^2\pi Y \); note that \( \dot{K}_{vy} \) and \( \dot{K}_{oz} \) have the same cross-channel variation according to Eqs. (45) and (46).

Applying the forms (45) and (46) in Eqs. (23) and (24), using the customary least-squares procedure, we obtain the following parametric forms suitable for the 2-parameter model:

\[\frac{gH}{f_0L^2}E_0(Y, t) = 12\bar{K}K_h(Y) \left[ \frac{\partial \Psi_1}{\partial Y} + \frac{q}{u^2} \right] \quad \text{(47)}\]
\[ V(\bar{Y}, t) = \bar{K}_k(\bar{Y}) \left\{ 12 \left( \beta L^2 + \frac{\partial^3 \Psi_0}{\partial Y^3} \right) + \mu^2 \frac{\partial^3 \Psi_1}{\partial Y^3} - p \frac{\partial^3 \Psi_1}{\partial Y^3} \right\} \]  
(48)

\[ V(\bar{Y}, t) = 12\bar{K}_k(\bar{Y}) \left\{ \left( q + \frac{\partial^3 \Psi_1}{\partial Y^3} \right) - p \left( \beta L^2 + \frac{\partial^3 \Psi_0}{\partial Y^3} \right) \right\}. \]  
(49)

At each time step \( p \) and \( q \) are determined by applying constraint (19) to Eqs. (48) and (49) and solving the resulting inhomogeneous linear equations. \( \bar{K} \) is calculated at each time step from

\[ 12\bar{K} \left\{ \frac{\partial \Psi_1}{\partial Y} \bigg|_{Y=1} + \frac{q}{\mu^2} \right\} = -\frac{gH}{f_0 L^2} a \frac{g^2}{N^2} \langle \Delta \phi \rangle^2 \]  
(50)

which is readily obtained from Eq. (47) and Green’s (1970) Eq. (17).

It will be observed that Eqs. (45) and (46) contain in all 3 functions of time to be determined at each time step. In scheme 2, \( \bar{K}_w \) was related to \( \bar{K}_v \) by Eq. (38), and Eq. (40) contained three such functions. The extra effective constraint in scheme 2 was accommodated by equipping \( \bar{K}_v \) with an extra free constant \( p_2 \). Although \( p_2 \) was not formally redundant, its presence necessitated extensive application of the least-squares procedure in the derivation of Eqs. (41), (42) and (43). Scheme 3 is apparently better posed, for there is no quadratic term in the form assumed for \( \bar{K}_v \) (Eq. (45)).

In the first integration described in section 6(b) the term in \( \sin^2 \pi \bar{Y} (\partial \Psi_1 / \partial Y) \) in Eq. (47) is replaced by \( \partial \Psi_1 / \partial Y \). This is equivalent to introducing into Eq. (10) a vorticity source function \( R_1 \) of the form (30), with \( \kappa = 12\bar{K} \), whilst retaining \( \sin^2 \pi \bar{Y} (\partial \Psi_1 / \partial Y) \) in Eq. (47).

(b) Results

Figure 4(a) shows the steady-state latitude variations of potential temperature deviation \( \delta \bar{\theta} = (\bar{\theta} - \bar{\theta}_0) \), surface zonal flow \( \bar{U}_s = U((-\bar{\bar{Y}}^2) \) and tropopause-level zonal flow \( \bar{U}_T = U((-\bar{\bar{Y}}^2) \) obtained when \( S_0 = 0.6 \cos \pi Y \) day\(^{-1} \), and the truncation \( n_T = 7 \). All the variations are qualitatively realistic: extremal values are generally similar to those obtained using scheme 2, although the surface zonal flow is noticeably weaker (cf. Fig. 3(a)). Qualitatively similar results are obtained with truncation \( n_T = 3, 5 \) or 9, and quantitative differences are small.

The steady-state height variation of \( \bar{K}_v \) is shown in Fig. 4(b). The linear form \( (1 - pZ) \) (see Eq. (45)) cannot reproduce the low-level maximum of \( \bar{K}_v \) found in small amplitude baroclinic waves (Green 1970), but the decrease with height evident in Fig. 4(b) is nonetheless reminiscent of the gross variation found in such waves.

In contrast to the scheme 2 result (Fig. 3(c)) the eddy momentum flux (Fig. 4(c)) has a qualitatively realistic height variation—there is a marked maximum at tropopause level \( (Z = 1) \). If an \( \exp(z/H_0) \) weighting were to be incorporated to allow for static compressibility (see the discussion in section 5(b)) the increase with height would be even stronger. In either case, the maximum value is less than typical observed tropospheric maxima (Oort and Rasmusson 1971); however, the surface flow problems solved by White (1977) suggest that larger maxima would be obtained in a model formulated in spherical geometry.

The results shown in Fig. 4(c) certainly suggest that scheme 3 is preferable to scheme 2. The scheme 3 results are also the more consistent with the baroclinic wave life-cycle integrations carried out by Simmons and Hoskins (1978), (see section 5(b)).
It seems significant, therefore, that scheme 3 contains no specification of maximum available potential energy release.

Figure 5 shows the steady-state latitude and height variations obtained when the diabatic forcing is represented as

\[ S_0(Y) = 0.6(\cos \pi Y - \cos 3\pi Y) \text{ °C day}^{-1} \quad (51) \]

and the truncation \( n_T = 5 \).

In Fig. 5(d) the functional form (51) is plotted together with the variation \( S_0 = 0.6 \cos \pi Y \text{ °C day}^{-1} \) (which has been applied in all the other cases described in this paper). The form (51) vanishes at the lateral boundaries \( Y = 0, 1 \), has extrema near \( Y = 0.3 \) and \( Y = 0.7 \), and vanishes at \( Y = 0.5 \). In this integration \( K_b(Y) \) was specified as \( \sin^2 \pi Y \) in both the heat flux and vorticity flux parametrizations, and the vorticity forcing function \( R_1 \) (see Eq. (10)) was omitted. According to the discussion given in section 3, the transfer coefficient specification in this integration is entirely consistent with theoretical requirements. The results shown in Figs. 5(a)–(c) are qualitatively similar to those shown in Figs. 4(a)–(c)—the general increase in extremal values can be ascribed to the increased extremal values of the diabatic forcing (see Fig. 5(d)). Clearly, if the diabatic forcing vanishes in extreme latitudes it is unnecessary to introduce the vorticity source function \( R_1 \) in order to enhance the mean meridional heat transfer. The results shown in Fig. 5 provide strong support for the arguments put forward in section 3.

7. Concluding Remarks

In this paper a simple dynamical climate model has been used to test three schemes for specifying the transfer coefficients which occur in Green’s (1970) parametrizations of large-scale eddy fluxes. The diabatic heating has been represented by sinusoidal functions of latitude. Scheme 1 (a version of which was used in WGI’s long wave climate
model) involves fixing the height variation of the eddy momentum flux and using a one-dimensional diffusive representation of the heat flux. Schemes 2 and 3 involve no such pragmatic simplifications: the functional forms of the transfer coefficients are defined by applying vorticity constraints at every time step in the integrations. In scheme 2 a condition of maximum potential energy release is applied at mid-level at all times, but in scheme 3 this condition is not applied. Scheme 2 gives an unrealistic height variation for the eddy momentum flux. Scheme 3, however, gives a much more satisfactory result. The difference in behaviour of schemes 2 and 3 is broadly consistent with numerical simulations of baroclinic wave life-cycles (Simmons and Hoskins 1978) which show that particle paths at middle levels are oriented for maximum potential energy release only during the growth phase of the wave.

Pleasing though this result may be, the most important aspect of the study is not the comparison of the different schemes, but the illustration of the technique whereby constraints on the eddy vorticity flux are used to determine the height variation of the transfer coefficients. Scheme 3 could no doubt be improved upon; but whatever changes might be found desirable the vorticity flux constraints should still be exploited. In some published studies (see section 3) difficulty has been experienced both in specifying the spatial variations of transfer (or exchange) coefficients and in satisfying fundamental
constraints on the eddy vorticity flux. We consider that the best resolution of these difficulties is to set them against one another—to use the vorticity constraints to complete the functional specification of the transfer coefficients once reasonable latitude variations have been chosen. At the very least, this is a dynamically consistent approach. The technique is of course applicable to parametrized climate models whose structure (in terms of resolution, basic dynamical formulation, and representation of physical processes) is far more comprehensive than that of the simple 2-parameter quasi-geostrophic model which has been used in this study. For example, in a model with greater vertical resolution, each further degree of freedom in the vertical provides a further constraint of the form (19), and the height variations of the transfer coefficients may be determined in correspondingly greater detail.

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