depends on the thermal and moisture conditions in the cloud environment within the cloud layer. If \( A(\lambda) \) remains constant despite the changing environment the temperature and moisture fields must be coupled. This can be considered as another interpretation of the quasi-equilibrium. It is deduced from sensitivity tests that temperature and moisture must change in the same direction to keep \( A(\lambda) \) constant. Figure 5 reveals that in general the environmental fields during GATE show such a behaviour. Observations during GATE therefore support the closure assumption for the cumulus parameterization scheme by Arakawa and Schubert.

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A new formula for latent heat of vaporization of water as a function of temperature

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**SUMMARY**

Existing formulae and approximations for the latent heat of vaporization of water, \( L_v \), are reviewed. Using an analytical approximation to the saturated vapour pressure as a function of temperature, a new, temperature-dependent function for \( L_v \) is derived.

1. **INTRODUCTION**

An accurate estimate of the latent heat of vaporization of water, \( L_v \), is often required; for example in many meteorological problems where surface energy budgets involve evaporation or transpiration losses. In many instances, a constant value, of approximately \( 2.5 \times 10^3 \) J kg\(^{-1}\), is utilized (e.g. McIntosh and Thom 1969, p. 63). In other meteorological investigations, the temperature dependence of \( L_v \) is ignored; for example, Raphael (1962) (rivers); Ryan et al. (1974) (evaporation from cooling ponds); LeDrew and Reid (1982) and de Bruin (1982) (surface energy budget of
lakes). In an analysis of the basic properties of water by Heggen (1983), no discussion of $L_v$ as a function of $T$ is given.

However, it is important to assess how much variability exists in the value of $L_v$ for the range of temperatures commonly encountered (say up to about 305 K). Figure 1 shows one typical functional form (used in the U.S. Army Corps of Engineers' water quality model CE-QUAL-R1;

![Graph showing variation of latent heat as a function of temperature.](image)

Figure 1. Variation of latent heat as a function of temperature as used in the model CE-QUAL-R1 (after Environmental Laboratory, 1982)).

Environmental Laboratory, (1982)) which shows a decrease in $L_v$ with increasing temperature, given in this case by

$$ L_v = 2.500 \times 10^6 - 2.386 \times 10^4(T-273) \quad (\text{J kg}^{-1}) $$

(after conversion to SI units, using 1 cal = 4.1868 J (Brutsaert 1982, p 39)). For consistency any such formula must be compatible with accepted formulae for $e_v$. Often such formulae are calculated based on an assumption of the value for $L_v$ using the Clausius–Clapeyron equation (see below). In this paper it will be shown that such an exercise results in unacceptable (and unnecessary) errors in the calculated value of $e_v$. However, recent direct analyses of polynomial and exponential ‘best fit’ representations (see e.g. Lowe 1977; Brutsaert 1982) suggest that several accurate analytical representations for $e_v$ are available and these can be used to solve for a new and accurate analytical representation for $L_v$.

2. The Clausius–Clapeyron Equation and the Saturated Vapour Pressure

The Clausius–Clapeyron equation is derived from thermodynamic considerations and relates the saturated vapour pressure, $e_v$ (a function of temperature $T$), to other thermodynamic variables including the latent heat of vaporization, $L_v$:

$$ (1/e_v)(de_v/dT) = M_vL_v/R*T^2 $$

where $M_v$ is the molecular weight of water vapour (=18.016 g mol$^{-1}$; Brutsaert 1982) and $R^*$ is the universal gas constant = 8.3144 J mol$^{-1}$K$^{-1}$. This equation is often used directly to derive $e_v(T)$ by assuming $L_v$ to be a constant. In this case (taking $L_v = 2.5 \times 10^4$ J kg$^{-1}$) Eq. (2) integrates to give

$$ e_v = 610.78 \exp[5417.1(1/273 - 1/T)] \quad (\text{N m}^{-2}). $$

The accuracy of this may be assessed by comparison with the data base found in the Smithsonian Tables (List 1971) and quoted by Glanz and Orlob (1973). This data base, derived by Goff and Gratch (1946), valid for saturation with respect to a plane surface of pure water, is generally regarded (Lowe 1977) as the standard against which other formulae may be judged. Selected
values are shown in Table 1. Equation (3) is seen to overestimate (by about 2½%) at larger $T$ (within the range chosen for meteorological application).

Alternatively some functional dependence of $L_e$ on temperature may be introduced before solution of Eq. (2) (e.g. Eq. (1)). A further possible representation is given by Dake (1972). This is valid for $273 < T < 308$ and is given, after conversion to SI units, by

$$L_e = (2500 \cdot 82 - 2 \cdot 358(T - 273)) \times 10^3 \text{ (J kg}^{-1}\text{).}$$

(4)

Introducing Eqs. (1) and (4) into Eq. (2) gives, after integration, respectively

$$e_e(T) = 610.78 \exp \{6828.6(1/273 - 1/T) - 5.1701 \ln(T/273)\}$$

(5)

$$e_e(T) = 610.78 \exp \{6808.3(1/273 - 1/T) - 5.1085 \ln(T/273)\}.$$  

(6)

Values for these two equations are given in lines 4 and 5 of Table 1. Equations (5) and (6) are seen to be an improvement over Eq. (3) such that the maximum error (between 273 and 308 K) has been reduced from 2.7% to 0.17% (Eq. (6)); 0.08% (Eq. (5)). For many calculations this accuracy may be sufficient. However, assuming that the Smithsonian data base is 'correct', then it is possible to invert the argument to deduce $L_e$ from $e_e$. Lowe (1977) compares and contrasts several formulae for $e_e$ of either logarithmic or polynomial form. He concludes that in general polynomial forms are computationally more efficient, although the exponential form of Richards (see Wigley 1974) was in some ranges more accurate. In the range considered here, the expressions of Lowe and Richards are found to be in error by no more than 0.013%. In a similar comparison by Sargent (1980), a sixth-order polynomial was derived from the original (unpublished) work by Hooper which was found to be accurate to 1 part in 10000 over a temperature range from $-40$ to $+50 \degree C$. Indeed all these representations can be used over wider ranges. However, a simpler and almost as accurate formula valid only for the range 273 to 323 K was presented by Glanz and Orlob (1973):

$$e_e(T) = 2.1718 \times 10^{-6} \exp \{-4157/(T - 33.91)\} \text{ (N m}^{-2}\text{).}$$

(7)

The accuracy of this is claimed (Glanz and Orlob 1973) to be better than 1 part in 10000 in this temperature range (actually better than 2 parts in 10000). Substituting for $e_e$ from Eq. (7) into the Clausius–Clapeyron equation (Eq. (2)) gives, for $L_e$,

$$L_e = 1.91846 \times 10^6(T/(T - 33.91))^2.$$  

(8)

Table 2 compares values at 10 K intervals computed from Eq. (8) in comparison with other formulae quoted above.

3. APPLICATION TO EVAPORATION RATE CALCULATIONS

Perhaps the most used methods of calculating evaporation are variants of Dalton’s (1802) equation:

$$E = f(U)(e_e - e).$$

(9)

The energy loss associated with this, $\phi_e$, is given by

$$\phi_e = \rho L_e E.$$  

(10)

Hence an error of 3% in $L_e$ (by using a constant value) would result in an error of the same
TABLE 2. Value of latent heat of vaporization

<table>
<thead>
<tr>
<th>$T$(K)</th>
<th>273</th>
<th>283</th>
<th>293</th>
<th>303</th>
<th>Max. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smithsonian values (Brutsaert 1982)</td>
<td>2.501</td>
<td>2.477</td>
<td>2.453</td>
<td>2.430</td>
<td>0%</td>
</tr>
<tr>
<td>Eq. (8)</td>
<td>2.5012</td>
<td>2.4764</td>
<td>2.4535</td>
<td>2.4324</td>
<td>0.02%</td>
</tr>
<tr>
<td>cf. constant values of: 2.5 x 10^3 J kg^-1 (McIntosh and Thom 1969, p. 63)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.9%</td>
</tr>
<tr>
<td>585 cal g^-1 = 2.493 x 10^3 J kg^-1 (Perry and Walker 1977, p. 80; quoting Jacobs 1942)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.1%</td>
</tr>
<tr>
<td>Eq. (1)</td>
<td>2.500</td>
<td>2.4761</td>
<td>2.4523</td>
<td>2.4284</td>
<td>0.07%</td>
</tr>
<tr>
<td>Eq. (4)</td>
<td>2.5008</td>
<td>2.4772</td>
<td>2.4537</td>
<td>2.4301</td>
<td>0.03%</td>
</tr>
</tbody>
</table>

Latent heat in 10^3 J kg^-1.

magnitude in $\phi$, and even by using e.g. Eq. (1) or (2) an error of between 0.1 and 0.3% would occur. Although an error of only 0.3% is unlikely to be significant on a small scale (e.g. lake), on a global scale, such an error will result in an inaccuracy of the order of 15 MJ m^-2 per year.

4. CONCLUSIONS

It appears worthwhile to incorporate a temperature dependency into the value for the latent heat of vaporization of water. Formulae already in existence (e.g. Eqs. (1), (4)) may often be acceptable since they give rise to errors less than 1%. However, a more accurate formula has been developed here, which is swift to calculate and which, together with the corresponding formula for the saturated vapour pressure, gives an accuracy better than 2 parts in 10,000 (0.02%) for both $L_\nu$ and $e_\nu$. The use in meteorological calculations is thus recommended of

$$L_\nu(T) = 1.91846 \times 10^4 (T/(T-33.91))^2 \quad \text{(J kg}^{-1})$$

$$e_\nu(T) = 2.1718 \times 10^{10} \exp(-4157/(T-33.91)) \quad \text{(N m}^{-2}).$$

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Charge separation in a Florida thunderstorm

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**SUMMARY**

A simple explanation of the location and movement of the breakdown region in a Florida storm is given in terms of the separation resulting from dynamical equilibria between aerodynamic and electrostatic forces on charged particles.

1. **INTRODUCTION**

The storm discussed in this analysis occurred at about 1900 GMT on 13 August 1978 in the Titusville-Cocoa Airport, Florida. Observations on this thunderstorm appear to be the most complete of any measurements made to date. They were made by three groups led by Lhermitte, Krehbiel and Lennon working together, and the results have been discussed by Lhermitte and Krehbiel (1979). The main features of the observations are summarized briefly as follows. (1) Lightning activity was observed to be associated with a single updraught column for which Doppler radar velocity measurements were obtained. (2) Radar echoes provided locations of precipitation intensities. (3) Lightning Detection and Ranging (LDAR) measurements gave the locations and times of electrical breakdown bursts during the storm. (4) Field change measurements showed that electric moment changes, associated with six intercloud (IC) flashes, were correlated in space and time with definite breakdown bursts located by LDAR. Results from all four techniques are given by Lhermitte and Krehbiel (1979) and by Krehbiel (1981), and will be used as the basis for the present analysis.

2. **BASIC INFORMATION**

Approximate (smoothed) curves of air velocities $v$, height in the updraught column are shown for specific times in Fig. 1. These velocities were obtained directly from Lhermitte and Krehbiel's contours of maximum air velocity, expressed in an altitude-time plot. It is difficult to attribute errors to the velocities represented by the curves, which were drawn through values obtained from