The response of a nonlinear, time-dependent, baroclinic model of the atmosphere to tropical thermal forcing

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(Received 16 November 1983; revised 28 February 1984, Communicated by Professor B. J. Hoskins)

SUMMARY

A multi-level, sigma coordinate, primitive equation atmospheric model has been utilized to study both the tropical and extratropical response to an isolated region of steady thermal forcing in the tropics. The nonlinear response during the first 28 days of the simulation is described. The response can be generally characterized by two distinct components. The first component is a quasi-stationary disturbance which extends eastward and poleward away from the source region along a 'great circle' path. The structure of this disturbance is essentially barotropic away from the source region. The second component is a growing baroclinic wave propagating zonally at mid-latitudes. Significantly, this disturbance is apparently the result of baroclinic instability induced by the quasi-stationary wavetrain. The discussion is predominantly heuristic in form and relies heavily on graphical presentation and quasi-geostrophic theory to interpret the response and individual components of the thermodynamic energy and momentum equations.

1. INTRODUCTION

An intriguing area of research deals with interactions between mid-latitude and tropical regions of the troposphere. Evidence that this is an area of legitimate concern has been provided by observational studies of Bjerknes (1969), Horel and Wallace (1981), and by the modelling studies of Rowntree (1972, 1976) and Julian and Chervin (1978). As with most atmospheric phenomena, the complexities of these interactions preclude detailed analytical resolution and present formidable obstacles to numerical modelling. These characteristics necessitate a multi-discipline research attack, as convincingly argued for by Hoskins (1983), in which the separate, but complementary, techniques of observational analysis and conceptual, theoretical, and numerical models of varying complexity are focused on a common problem. This study is undertaken in that vein; i.e. conceptual understanding is sought through the application of numerical modelling techniques to a parametrization of a complex physical process. The aim is to assist the achievement of a rational explanation of a convoluted atmospheric phenomenon by means of a simulation involving a much simplified, but qualitatively correct, process. This objective is compatible with the views of Lorenz (1982) as to the role of simplified models in atmospheric research.

More specifically, this study deals with the response of a model atmosphere to steady thermal forcing in the form of a tropical heat source of fixed size, location and magnitude. The amplitude of the heating source varies sinusoidally in the vertical, with the maximum at 500 mb. This heat source was chosen as an approximation to the large-scale diabatic heating occurring in the tropics. Ramage (1968) has shown that typically the equatorial regions of South America, Africa and the 'maritime continent' of Indonesia and the Carolines provide the bulk of the equatorial heating by virtue of the latent heat release associated with thunderstorms. Hantel and Baader (1978) demonstrated that below 100 mb, the amplitude of the zonally averaged heating in the tropics varies roughly as a sinusoid in the vertical with a maximum at about 500 mb. Julian and Chervin (1978) have utilized the National Center for Atmospheric Research (NCAR) 5° global atmospheric model to study its response to a stationary warm sea surface temperature anomaly in the equatorial eastern Pacific Ocean (colloquially known as an El Niño event). They found the response to be a thermally direct circulation, driven principally by the latent heat of condensation; in essential agreement with observational data as interpreted by
Cornejo-Garrido and Stone (1977). Thus, in the interest of simplification, the stated heating distribution was chosen to encompass qualitatively these modes of tropical atmospheric heating.

An essentially equivalent thermal forcing has been analysed by means of linearized, steady-state, baroclinic primitive equation models by Hoskins and Karoly (1981), Hendon and Hartmann (1982) and Simmons (1982). The use of a time-dependent, nonlinear, baroclinic primitive equation model for the study reported herein adds a new facet to the informational bank provided by these authors. These three papers (hereinafter referred to as HK, HH and S, respectively) constitute the primary comparison references for this report. However, the reader is strongly encouraged to survey the foundational research by referring to the bibliographies in these papers and also those in Huang (1978) and Julian and Chervin (1978).

2. DESCRIPTION OF THE MODEL

A version of the global, multi-level, sigma coordinate primitive equation model described by Hoskins and Simmons (1975) was adopted for the numerical experiments described in this study. The reader is referred to that paper for detailed description of the model, and only a brief description follows. Horizontal representation of dependent variables is by a triangularly truncated spherical harmonic expansion through zonal wavenumber 21. Five equally spaced sigma levels (0.1, 0.3, 0.5, 0.7, 0.9) were used. A spectral transform method was used in evaluation of the nonlinear terms, and second-order finite-difference approximations were made for vertical derivatives. Semi-implicit time integration was performed, using a 45-minute timestep.

An initial zonal-mean atmospheric state was provided by specification of the zonal-mean wind field. Corresponding temperature and surface pressure fields were then evaluated as an iterative solution of the nonlinear balance equation described by Hoskins and Simmons. The reference wind field is shown in Fig. 1. It is coincident, at the indicated

![Figure 1. Zonally averaged wind field utilized to initiate the simulation, and a cross-section through the centre of the thermal forcing distribution. Tick marks on the vertical axis correspond to the model levels. Approximate height is shown on the left and the equivalent atmospheric pressure on the right. Contours of zonal wind speed are in bold line with speed indicated in m/s. Contours of the thermal forcing are in fine line, without labelling, with 0.2 K/day contour intervals.](image-url)
Figure 2. Zonally averaged initial temperature field compatible with the zonal wind field of Fig. 1. The contouring interval is 10 K.

pressure levels, with the December–February distribution given by Newell et al. (1972), with the exception of an enhancement of the equatorial easterlies by 5 m/s. The associated zonal-mean temperature field is given in Fig. 2.

Dissipation is parametrized by means of Newtonian cooling in the energy equation, Rayleigh friction in the momentum equation, and a scale-selective biharmonic diffusion which is applied to both the energy and momentum equations. This dissipation is applied only to the deviation from the initial state. Thus, the time-dependent model state is damped toward the initial non-divergent, zonally symmetric state.

Thermal and momentum balances for the experiment were evaluated on constant pressure surfaces. Insofar as possible, each component of these equations was expressed in terms of quantities evaluated explicitly by the sigma coordinate primitive equation model. The remaining terms required to complete the sigma-to-pressure coordinate transformation were evaluated by either spectral or finite-difference techniques, consistent with the form used in the primitive equation model. These expressions for the components of the thermal and momentum balance equations were then mapped from sigma to pressure surfaces by second-order interpolation.

3. EXPERIMENT DESIGN

Thermal forcing is represented by a heat source of constant magnitude and fixed location. The source is bounded in the horizontal by an ellipse of minor axis 30° in latitude and major axis 60° in longitude. The centre of the source is located at 15°N latitude and 180° longitude. The amplitude of the heating distribution is modulated in the vertical by \( \sin \sigma \) and has a cosine-squared dependency in the horizontal. The maximum amplitude is 3.2 K/day at 500 mb. A meridional cross-section through the centre of this distribution is shown in Fig. 1, superimposed on the zonal wind field.

According to the analysis of Holton (1979), this amount of heat input is compatible with the latent heat released by large-scale tropical convective systems. It is also consistent with subtropical forcing distributions investigated by HK, HH, and S.
Both the Rayleigh friction and the Newtonian cooling have e-folding times of 5 days at the lowest model level, and 10 days at each of the higher model levels. This agrees with the amount of linear dissipation used by HK for thermal damping, but differs from their nominal situation by the inclusion of momentum damping at all model levels. The value of the biharmonic diffusion coefficient is $1.169 \times 10^{17}$ m$^4$s$^{-1}$.

4. EXPERIMENT RESULTS

The general circulation model was integrated for a total of 28 days. The strength of the thermal forcing was increased from an initial value of zero to full amplitude at model hour 36. It was maintained as constant thereafter. The resulting geopotential height perturbations for days 14, 21 and 28 are shown in Fig. 3 for the 300 mb level.

Figure 3(a) illustrates a disturbance pattern which is very similar to the steady-state linear solutions for a baroclinic model as shown in Fig. 3c of HK, Fig. 3a of HH, and Fig. 3 of S. This pattern originates with a high centred at approximately 20°N 175°E,

![Diagram](image-url)

Figure 3. Perturbations to geopotential height for the 300 mb surface of the northern hemisphere at one week intervals of the simulation. The contouring interval is 5 m. Solid lines indicate positive values of the perturbations, dashed lines indicate negative values, and the dotted line is the zero contour. (This convention will be used in subsequent figures.) The latitude grid is drawn in only at 15° and 35°N. (a) Day 14. (b) Day 21. (c) Day 28.
slightly north-west of the centre of the forcing region. Directly east of this high is a low at 20°N 140°W. The northward and eastward propagating wavetrain of Fig. 3(a) is also a basic component of the disturbance field illustrated in Figs. 3(b) and (c). Comparison of Figs. 3(a), (b) and (c) suggests that this component is quasi-stationary. In subsequent discussion, it will be shown that the phase of this component is nearly constant. The general character of this wavetrain is consistent with those shown in the figures just referenced. However, those figures illustrate a strong growth in amplitude as the disturbance moves into higher latitudes. The lack of a similar growth in amplitude in this study is a consequence of the way in which momentum and thermal dissipation are treated, as has been noted by HK for their linear steady-state model.

The treatment of dissipation in this study differs from the nominal treatment of HK in two aspects: the inclusion of linear damping in the momentum equations at all levels, and in the use of a biharmonic dissipation coefficient which is 5 times their nominal value. However, they did investigate these levels of dissipation separately. They found that including the linear momentum damping at all levels produced negligible change in the pattern of the wavetrain, but reduced the amplitude of the response by 40% in the region of the source and by a factor of 4 at the highest latitudes. The increased biharmonic diffusion produced for them only small changes near the source, but a decrease in amplitude by a factor of 3 in more remote regions. Both HK and S caution that the amplitude of the distant response may be limited by more realistic dissipation models. Such limitations are examined in more depth by HH. They investigated the effects of different parametrizations of dissipation and thermal feedback. Specifically with regard to geopotential height perturbations, their Figs. 7 (inclusion of sensible heating), 12 (increased Rayleigh friction) and 13 (Ekman pumping) all show an absence of amplitude growth with increasing latitude. Such studies indicate that the forms of dissipation used in this investigation are reasonable and consistent with the state-of-the-art for qualitative purposes. Also, the absence of a large amplitude remote response may not constitute an unrealistic situation. The principal support for the choice of dissipation coefficients used herein is that they provide an approximate minimum level of damping adequate to control spectral blocking (i.e. spurious growth of amplitude in the dynamic variables at scales near the truncation limit, as alluded to by McAvaney et al. 1978 and by Puri and Bourke 1974).

The sequence of height plots in Fig. 3 gives the appearance of the superposition of two phenomena: the quasi-stationary component just discussed, and a developing wavetrain moving zonally at approximately 45°N. Evidence to support this contention is provided in Simmons and Hoskins (1979). In that analysis they examined the response of a baroclinically unstable atmosphere to a localized initial perturbation. They found a subsequent development consisting of a wavetrain whose downstream fringe moved at a speed between 75% and 100% of the maximum speed of the zonal flow and with a cyclone spacing of between 55° and 60° in longitude. Their Fig. 1 shows a surface pressure pattern so compatible with the nonsteady component of Fig. 3 that the conclusion may be drawn that the height disturbance field shown in Fig. 3 is essentially composed of two components: a quasi-stationary wavetrain similar to the linearized, steady-state solution of the baroclinic model, and a zonally propagating wavetrain that owes its existence to baroclinic instability. The latter has matured by day 28, and is moving east with its centres aligned along the latitude at which the jet core is located in the 500–700 mb range (see Fig. 1). This alignment will later be demonstrated to be a consequence of thermal advection below 300 mb.

The generation of this zonally propagating wavetrain is an example of the triggering of baroclinic instability by a distant disturbance, as alluded to by Simmons and Hoskins
In this case a mid-latitude instability has been initiated by a localized thermal forcing whose centre is far upstream and 30° in latitude removed from the outbreak of the instability. The assertion that the instability is baroclinic is supported by the results of HK and S, in which the barotropic analogue of this situation produced only a poleward and eastward component, qualitatively in agreement with the present results. Additional data illustrating the baroclinic nature of the non-steady component will be presented in subsequent discussion. The integration was terminated at day 28 in view of the impending resonance which would have enlarged the scope of the present study to consider the nonlinear interactions in the region of the thermal source.

Additional insight into the two-component nature of the geopotential height perturbation can be gained from consideration of its Fourier decomposition in longitude. Such a decomposition was performed for the 500 mb level at 45°N (approximately the latitudinal path of the centre line of the travelling wavetrain). Some features of the decomposition may be anticipated through the study of Fig. 3.

Examination of the quasi-stationary component in Fig. 3(a) indicates a wavelength of about 90°, whereas the travelling component shown in Fig. 3(c) has a wavelength of about 55°. If these wavelengths are characteristic respectively of a quasi-stationary and a travelling component, there should be a noticeable difference in the time histories associated with wavenumbers below, say, five and those with wavenumbers above five. The longer waves should be comparatively constant in amplitude and phase, whereas the shorter waves should exhibit growth in amplitude and a changing phase. Data are presented in Fig. 4 to illustrate that this is what occurred during the final week of the model simulation.

Illustrated in Fig. 4(a) are the time histories of the Fourier coefficients of wavenumbers one through four of the 500 mb geopotential perturbation at 45°N for days 21 to 28. Analogous data are presented in Fig. 4(b) for Fourier coefficients five through eight. The phases associated with both sets of waves are shown in Fig. 4(c). This triad of figures confirms the anticipated results alluded to above. The travelling wave character of the wave packet represented by the superposition of waves five, six and seven is shown in Fig. 4(d). This three-component wave packet qualitatively captures the character of the developing wavetrain.

Further attention will be focused on day 14 because it clearly depicts the quasi-stationary component of the disturbance field while containing the early stages of the wavetrain arising from the instability. Only northern hemisphere plots will be presented since the disturbance is prevented from propagating into the southern hemisphere by virtue of being bounded to the south by tropical easterlies (see Fig. 1). Presented in Fig. 5 are both the geopotential and temperature perturbations for the 300, 500 and 700 mb pressure levels for day 14. The results shown are in substantial agreement with the analyses of HK, HH and S. In a barotropic model the isotherms and isobars are exactly parallel and the waves exhibit no phase tilt with height. From these properties it may be visually inferred from the data in Fig. 5 that the extratropical response is approximately barotropic. However, the response is strongly baroclinic in the vicinity of the source. This behaviour is consistent with the results of S. Simmons further argues, on the basis of a simplified model, that the response far from the source should be barotropic in character.

The most prominent low-latitude phenomenon is the transition from a lower-level warm low slightly north-west of the source, to an upper-level warm high similarly located. Although thermal and momentum balances will be discussed in more detail later, a few observations are appropriate at this point. As an intuitive observation, the small geopotential perturbation in the region of the source at 500 mb implies a weak disturbance
Figure 4. Amplitude and phase of zonal harmonic height waves for the 500 mb surface at 45°N for days 21–28. (a) Amplitude in metres of wavenumbers one through four. (b) Amplitude in metres of wavenumbers five through eight. (c) Phase for wavenumbers one through eight. (d) Amplitude for the superposition of wavenumbers five through seven as a function of longitude.
Figure 5. Perturbations to geopotential height and temperature for various pressure surfaces of the northern hemisphere for day 14 of the simulation. The contouring interval for geopotential height is 5 m and that for temperature 0.25 K. (a) Geopotential height, 300 mb. (b) Geopotential height, 500 mb. (c) Geopotential height, 700 mb. (d) Temperature, 300 mb. (e) Temperature, 500 mb. (f) Temperature, 700 mb.
of the basic zonal wind field. Hence, the thermal anomaly at this location is primarily a direct effect of the diabatic heating (maximum at this level). In contrast, the 700 mb warm thermal anomaly in the source region is in large part due to the cyclonic flow about the low at 170°E. This flow, in the presence of a steep meridional thermal gradient (see Fig. 2), sustains the thermal anomaly by the northward advection of warm equatorial air. A slightly more complex situation occurs at 300 mb. Here the warm cell at 160°E is primarily due to the anticyclonic flow about the high at 170°E, whereas the stronger cold cell at 145°W is maintained by the channelled advection of polar air due to the combined effects of anticyclonic flow about the 170°E high and the cyclonic flow about

Figure 6. Temperature and wind fields as a function of height and longitude for day 14. The contouring interval is 5 K for temperature; 2 m/s for the zonal wind; and 1 m/s for the meridional wind. (a) Temperature, 15°N. (b) Zonal wind, 15°N. (c) Meridional wind, 15°N. (d) Temperature, 35°N. (e) Zonal wind, 35°N. (f) Meridional wind, 35°N.
the 140°W low. The low latitude wind perturbation implied by these data is dynamically compatible with the Walker Circulation (see Gill 1980 and Julian and Chervin 1978). A different perspective of the response can be gained by considering the vertical structure at a fixed latitude. Longitude–height sections are given in Fig. 6 for 15°N (centre of the thermal source) and 35°N (location of the jet core).

Figures 6(a), (b) and (c) show, respectively, temperature, zonal wind, and meridional wind on day 14 at 15°N. Reference to Fig. 3 will demonstrate that this is a latitude where the disturbance is quasi-stationary. A feature common to these three figures is the pronounced westward phase tilt in the vicinity of the source, indicative of a strong baroclinic response. Note the absence of phase tilt in the vertical, downstream from the source (i.e. the zonal wind perturbations at 55–60°W and at 25–30°W).

Figure 6(a) shows thermal perturbations to be essentially confined to the vicinity of the source. Figure 6(b) illustrates the low-level (below 500 mb) inflow and the upper-level outflow associated with a deep tropical heat source. The phenomenon has been discussed by several authors; specifically by S in his generalization of Gill’s (1980) approximate solutions for the vicinity of the forcing region. A notable feature of Fig. 6(c) is the reversal of the meridional winds from principally strong northerly at low level to strong southerly at high level. As discussed earlier, these meridional wind characteristics play a dominant role in the maintenance of temperature anomalies by virtue of the associated thermal advection.

Figures 6(d), (e) and (f) show, respectively, the temperature, zonal wind and meridional wind on day 14 at 35°N. An important distinguishing difference between figures (d), (e) and (f) and figures (a), (b) and (c) is the presence of westward phase tilts downstream from the source, i.e. the zonal wind perturbation at 55–60°W and at 25–30°W. These phase tilts are characteristic of developing baroclinic waves. The development of these waves was illustrated in Fig. 3 by means of the zonally propagating height wavetrain. More detailed discussions are presented in the following sections dealing with thermal and momentum balances.

5. THERMAL BALANCE

The thermodynamic energy equation will be treated in its pressure coordinate form:

\[ \frac{\partial T}{\partial t} = (-v \cdot \nabla T) + (-\omega \frac{\partial T}{\partial p}) + \frac{RT \omega}{c_p} \rho + \text{(forcing)} + \]

\[ + \text{(Newtonian cooling, } -KT') + \text{(diffusion, } -K' \nabla^4 T') \]

where \( v \) is the horizontal velocity on a pressure surface, \( \omega = dp/dt \), \( R \) is the gas constant, \( c_p \) is the specific heat at constant pressure, \( K \) and \( K' \) are specified constants and \( T' \) is the deviation of the local temperature from the initial temperature field. The terms on the right-hand side of this equation represent respectively: horizontal advection; vertical advection; the adiabatic temperature adjustment due to expansion or compression; diabatic heating (by constant thermal forcing in this study); Newtonian cooling (representative of infrared radiative cooling); and biharmonic diffusion (representative of sub-grid-scale processes). These components of the temperature tendency equation are illustrated in Fig. 7 for the 500 mb level of the northern hemisphere on day 14 of the simulation. Results at this level are qualitatively representative of the response throughout much of the troposphere.

Presented in Fig. 7(a) is the temperature tendency, which is characterized by a wavetrain emanating from the forcing region and propagating eastward. The implied temperature wavetrain is the essential element of the propagating baroclinic disturbance.
Figure 7. Components of the thermodynamic energy equation on the 500 mb surface of the northern hemisphere on day 14. (a) Net tendency, contour interval 0.025 K/day. (b) Thermal forcing, contour interval 1 K/day. (c) Horizontal advection, contour interval 0.5 K/day. (d) Vertical advection, contour interval 1 K/day. (e) Adiabatic adjustment, contour interval 1 K/day. (f) Newtonian cooling, contour interval 0.05 K/day. (g) Biharmonic diffusion, contour interval 0.05 K/day.
illustrated in the geopotential height perturbations shown in Fig. 3. The separate contributions can be approximately quantified by noting that the temperature tendency data given in the individual plots of Fig. 7 can be partitioned into two categories whose amplitudes differ by about an order of magnitude. Depicted in Figs. 7(f) and (g) are the effects of Newtonian cooling and biharmonic diffusion, respectively, which are comparable in magnitude to the net tendency within a quadrant roughly centred about the heat source. Outside this region these mechanisms do not contribute significantly to the unsteady component of the disturbance field. Earlier in this discussion it was asserted (with reference to HK) that these damping mechanisms have a significant effect on the amplitude of the quasi-stationary disturbance, but not on its qualitative character. The important point to be stressed is that these admittedly crude parametrizations of dissipative processes exercise primarily a linear control over the amplitude of the total disturbance field and do not dominate the qualitative nature of either the quasi-stationary or the developing baroclinic components. The implication being that the use of these parametrizations (believed appropriate to the simplifying objectives of this study) has not invalidated the results, insofar as producing a qualitatively correct atmospheric response. Thus, within the context of this study, the thermal tendency can be analysed in terms of the four non-dissipative components whose effects are illustrated in Figs. 7(b), (c), (d) and (e). Individually, these effects are approximately an order of magnitude greater than the resultant tendency. They demonstrate a result compatible with the real atmosphere in that adiabatic adjustment compensated to a large degree for the combined effects of advection and diabatic heating. A description of this compensation is facilitated by considering the 'near source' and the 'remote' regions separately.

For the purposes of the ensuing discussion, let the 'near source' region be defined as that sector containing the heat source and bounded by the meridians passing through 120°E and 130°W (shown by the heavy dashed lines on Fig. 7(a)). This sector of Fig. 7(a) is dominated by an elongated region of positive tendency whose ridge follows 20°N from about 120°E to 180°E and then takes a path to the north-east intersecting 35°N at about 145°W. This tendency feature is clearly the origin of the unsteady thermal wavetrain. However, its contribution to the total temperature disturbance in the near source region is substantially masked by the relatively large magnitude of the quasi-stationary component, as illustrated in Fig. 5(e). The adiabatic adjustment depicted in Fig. 7(e) and the thermal dissipation illustrated in Figs. 7(f) and (g) act to attenuate the ridge of positive tendency. Thus, its origins lie in the diabatic forcing and the advection effects shown in Figs. 7(b), (c) and (d).

The heat source is illustrated in Fig. 7(b). As a prescribed constant, it requires no elaboration other than the trivial observation that it does not directly contribute to any asymmetry of the tendency ridge about the 180° meridian. The north-east deflection of the ridge is therefore a consequence of advection. Illustrated in Fig. 7(c) is the effect of horizontal advection of the temperature tendency. The positive anomaly centred near 25°N 170°W has its axis of elongation essentially parallel to the north-eastward segment of the ridge of Fig. 7(a). The positive anomaly of Fig. 7(c) is produced by the channelled type of flow mentioned earlier. This channelled flow can be inferred from the geopotential perturbation shown in Fig. 5(b). The effect of the anticyclonic flow about the high at 25°N 170°W combines with the cyclonic flow about the low at 40°N 180°E to provide a relatively strong north-eastward perturbation velocity. This flow results in transport from a warm region, as can be seen from the combination of the companion temperature perturbation plot in Fig. 5(e) and the basic temperature latitudinal structure shown in Fig. 2. By direct analogy, the correlative southward flow produces cells of negative tendency at about 30°N 150°E and at 15°N 150°W.
The pronounced effect of this advection upon the thermal tendency is a consequence of the fact that the perturbational velocity is comparable in magnitude to the basic zonal flow at low latitudes. To confirm this, refer to Fig. 6. From Fig. 6(b) it can be seen that at 15°N at 500 mb, the basic zonal velocity is zero. Hence, in this vicinity the perturbational velocity and the basic zonal velocity are commensurate quantities. From Fig. 6(e) it can be seen that the perturbational contribution to the total flow has been reduced to 5% or less at 35°N. In this region (near the jet core) horizontal advection is dominated by the basic zonal flow. This point will be pertinent later in discussing the response in the remote region.

The temperature tendency component illustrated in Fig. 7(c) implies horizontal transport of air into a different temperature regime. This transport entails a vertical displacement of air mass by virtue of buoyancy forces. The consequent vertical advection is illustrated in Fig. 7(d). Visual linear superposition of the tendency contributions given in Figs. 7(b) and (c) produces a resultant in the near source region which is almost identically correlated with the data shown in Fig. 7(d). This is confirmation that the stably stratified basic state has not been disrupted to the extent that temperature inversions exist. Thus, rising motion, whether induced by the increased buoyancy associated with direct heating or that associated with horizontal transport into a colder region, results in local warming via advection. The adiabatic adjustment effect shown in Fig. 7(e) exhibits, by its strong negative correlation with the superimposed effects of Figs. 7(b), (c) and (d), the large extent to which those forms of warming (cooling) are compensated by expansional cooling (compressional heating).

The remote region of Fig. 7(a) shows a temperature tendency wavetrain which can be graphically demonstrated to be primarily a result of horizontal advection. To substantiate this assertion, note that the effects depicted in Figs. 7(d) and (e) are in phase (refer to the zero contour lines). This is an illustration that an air parcel advected into a different pressure regime immediately adjusts to that new pressure. Thus, vertical motion, which is essentially normal to the constant pressure surfaces, has its primary thermal advection effect instantly opposed by a thermal expansion effect. In contrast, it can be seen from Figs. 7(c) and (e) (refer to the zero contour lines) that the effects of horizontal advection and adiabatic adjustment are somewhat out of phase. Thus, there are regions in which the effects of horizontal advection are not compensated by adiabatic adjustment, but rather these effects interfere constructively. This is a consequence of the thermal wave lagging the pressure wave. Hence, there are areas where horizontal advection carries warm air into a cooler region at a higher pressure. This is illustrated in Fig. 8 which covers the region north of 15°N between 0 and 90°W.

In this figure the shaded areas indicate those regions in which the horizontal advection and adiabatic effects interfere constructively. The contour lines are those of the thermal tendency (see Fig. 7(a)). The crests of the tendency cells are in phase with the shaded areas or, alternatively, in phase with the uncompensated effects of horizontal advection.

Figure 9 graphically implies the eastward propagation of the growing baroclinic disturbance. This figure applies to the same sector as shown in Fig. 8. Figure 9(a) contains plots of the coincident thermal perturbation contours (from Fig. 5(e)) and those of the thermal tendency (from Fig. 7(a)). The eastward propagation of the disturbance is indicated by the tendency leading the thermal perturbation by a quarter cycle.

Figure 9(b) provides a similar comparison between the geopotential and thermal disturbances. According to Holton (1979), this situation implies a westward tilt with height of the geopotential disturbance in a hydrostatic atmosphere and, for a quasi-geostrophic perturbation, implies "both that the horizontal temperature advection will increase the available potential energy of the perturbation and that the vertical circulation
Figure 8. Temperature tendency and its horizontal advection component at 500 mb on day 14. The region illustrated lies north of 15°N between 270°E and 0° longitude.

Figure 9. Temperature, temperature tendency and geopotential height on the 500 mb surface on day 14. The illustrated region is the same as for Fig. 8. (a) Temperature and temperature tendency. (b) Temperature and geopotential height.
will convert perturbation available potential energy to perturbation kinetic energy."

Further observations with regard to the vertical structure of the temperature tendency can be made visually with the aid of Figs. 10 and 11. Each of these figures represents a vertical profile at fixed latitude and shows the range from 700 to 300 mb and the full cycle of longitude. Figure 10 is for 15°N (location of the centre of the thermal forcing region). Figure 11 is for 35°N (location of the jet core). Each figure is composed of four parts: the net thermal tendency and its principal components of horizontal advection; vertical advection; and adiabatic cooling effects. The fixed latitudes of these plots correspond to the latitude circles drawn on the preceding polar plots. These figures represent but a different perspective of phenomena already discussed for the 500 mb level. Hence, to avoid prolixity, their content will only be highlighted.
Figure 10(a), by comparison with Fig. 11(a), emphasizes the relative quasi-stationary character of the low latitude response. Figure 10(b) depicts a phenomenon in the interval between 180° and 135°W which is similar in character at each of the three pressure levels, but in each case it arises from distinctly different conditions. At 300 mb the pronounced negative tendency at 160°W is predominantly the result of a comparatively strong southward meridional velocity as shown in Fig. 6(c) and indicated by the height perturbations of Fig. 5(a). Advection is from a region which is colder, both by virtue of the latitudinal gradient of the basic temperature field (Fig. 2) and the temperature perturbation (Fig. 5(d)). At 500 mb this magnitude has been reduced by two effects. First, the meridional velocity is substantially weaker (as shown in Fig. 6(c)) and implied in the height data of Fig. 5(b)). Second, the temperature perturbation in this region (as shown in Fig. 5(e)) acts to reduce the latitudinal gradient of the temperature field. At 700 mb, the meridional wind has reversed direction (Figs. 5(c) and 6(c)). However, the horizontal advection effect has not reversed sign since the flatness of the basic temperature field in this region (Fig. 2) has permitted the temperature perturbation (Fig. 5(f)) to cause a local change in the sign of the thermal gradient.

Figures 10(c) and (d) highlight the westward tilt with height of the thermal disturbance in the vicinity of the source and the absence of this indicator of baroclinicity in the remote region. Figures 10(b), (c) and (d), viewed conjointly, demonstrate the substantial magnitude of the effects which essentially cancel out (not shown is the thermal forcing, which is of comparable magnitude). Figure 10(c) also implies upwelling throughout the troposphere in the source region. This is consistent with the height data of Fig. 5(c) and the meridional wind data of Fig. 6(c) which imply convergence at low levels and divergence in the upper troposphere.

The temperature tendency data contained in Fig. 11 differ in some important aspects from that in Fig. 10. The most prominent difference is the reversal in sign of the horizontal advection effect. This reversal can be explained through the use of Figs. 1, 5 and 6(f). From the meridional wind data of Fig. 6(f) and the geopotential perturbation plots of Fig. 5, the meridional velocity can be seen to be very weak at 35°N, whereas Fig. 1 illustrates that this is the latitude of the jet core. Hence, horizontal advection is almost totally by the zonal westerlies. Referral to the temperature perturbation plots of Fig. 5 shows that in the vicinity of 35°N, 180° longitude, the zonal advection involves transport of warm air from the dominant cell of the temperature disturbance. Significant, but less prominent thermal features can be found in the remote region of Fig. 11. These are most apparent at 45°W in Figs. 11(b), (c) and (d). Note the westward tilt with height of the phenomenon in each of these figures; the in-phase relationship between Figs. 11(c) and (d); and the fact that the horizontal advection effect in Fig. 11(b) leads (in terms of phase) the effects shown in Figs. 11(c) and (d). The implications of these features have already been discussed with regard to the thermal tendency at 500 mb.

6. Momentum Balance

The pressure coordinate form of the momentum equation is

$$
\frac{\partial \mathbf{v}}{\partial t} = - (\mathbf{v} \cdot \nabla)\mathbf{v} + \{- \omega \frac{\partial \mathbf{v}}{\partial \eta} + \{- \nabla \phi \} + \{- f (k \times \mathbf{v})\} +
\{\text{Rayleigh friction, } -K\mathbf{v}'\} + \{\text{biharmonic diffusion, } -K' \nabla' \mathbf{v}'\}$$

where $\phi$ is geopotential height, $f$ is the Coriolis parameter, and $k$ is the unit vertical coordinate vector. The constants $K$ and $K'$ were assigned the same values as in the thermodynamic energy equation, and $\mathbf{v}'$ is the deviation of the horizontal velocity from
Figure 12. Zonal components of the momentum equation on the 500 mb surface of the northern hemisphere on day 14. (a) Net zonal wind tendency, contour interval 0.05 m/s per day. (b) Horizontal advection, contour interval 0.5 m/s per day. (c) Vertical advection, contour interval 0.5 m/s per day. (d) Gradient of geopotential, contour interval 2.5 m/s per day. (e) Coriolis acceleration, contour interval 2.5 m/s per day.
Figure 13. As Fig. 12, except the meridional components of the momentum equation. Contour intervals are the same, except for terms (d) and (e) with intervals of 25 m/s per day.
its initial non-divergent state. The first two terms on the right are horizontal and vertical advection, respectively. The third term is the transformed pressure gradient force. The fourth term is the Coriolis force. The last two terms are parametrizations of dissipation mechanisms which act to suppress the deviation, \( v' \). Consistent with the presentation on thermal balance, components of the momentum equation will be illustrated by polar plots for the northern hemisphere at 500 mb on day 14 of the simulation.

Presented in Figs. 12 and 13 are the zonal and meridional components, respectively, of the momentum equation. Rayleigh friction and biharmonic diffusion are not presented graphically. Their contributions are analogous to those of their counterparts in the thermodynamic equation in that the effects are principally restricted to the near source region, are small in relation to the other components, and do not control the qualitative character of the response.

To a high degree, the response is quasi-geostrophic even in the near source region. This is evidenced by a near cancellation between the pressure gradient and Coriolis forces. These terms are illustrated in Figs. 12(d) and (e) and in 13(d) and (e). Note that the magnitude of these components exceeds that of the net tendency by factors of about 50 for the zonal wind and 500 for the meridional wind. The lack of complete cancellation between these two terms is best seen in Figs. 12(d) and (e), since Fig. 13(d) is dominated by the large latitudinal gradient required to maintain the basic zonal flow. Comparison of Figs. 12(d) and (e) will show that the main areas of non-cancellation are the two cells in Fig. 12(c) at about 20°N, 180° and 150°W. However, this non-geostrophic residual is balanced to first order by the advective components of Figs. 12(b) and (c). An analogous argument holds for the meridional wind, although not so clearly apparent.

The vertical advection plots in each of Figs. 12(c) and 13(c) depict a relatively intense cell at about 20°N 170°W with adjacent weaker cells of opposite sign to the west and to the east. Analogous to the discussion of thermal advection, these cells are the consequence of vertical motions arising primarily from changes in buoyancy associated with direct heating for the dominant cell and horizontal transport of colder air for the two weaker cells. The validity of these assertions may be more clearly demonstrated heuristically from Fig. 6. This figure provides graphical data from which vertical derivatives of the zonal and meridional winds may be estimated. These estimates are adequate to confirm the data given in Figs. 12(c) and 13(c) to the extent that in the near source region, the vertical advective effects are opposite in sign for the two velocity fields, and for each field will change sign with a reversal in the direction of the vertical motion.

A similar confirmation of the zonal advection effects in Figs. 12(b) and 13(b) can be demonstrated. For example, consider the horizontal advection term which is illustrated in Fig. 12(b) for the zonal momentum equation. This term has the form \((u/r \cos \theta) \partial u/\partial \lambda + (v/r) \partial v/\partial \theta\) where \(u\) and \(v\) are zonal and meridional winds, respectively, \(r\) is the earth radius, \(\lambda\) is longitude, and \(\theta\) latitude. From Fig. 6(b) it can be inferred that at 15°N the first term is 'small' at 500 mb either by virtue of the smallness of \(u\) or by that of \(\partial u/\partial \lambda\). Hence, horizontal advection is governed by \(v \partial v/\partial \theta\) at this latitude. The joint use of the 500 mb meridional wind from Fig. 6(c) and the latitudinal structure of the basic zonal wind from Fig. 1 implies the horizontal advection which is depicted in the near source region of Fig. 12(b).

The velocity tendencies in the remote region of Figs. 12(a) and 13(a) are dominated by zonal advection. This can be envisioned in several ways. For the zonal wind, it is observable that the horizontal advection component in Fig. 12(b) leads in phase, while the remaining components are mutually in-phase. As in the thermal balance discussion, this could be used to demonstrate graphically that the resultant tendency is dominated by uncompensated horizontal advection. For the meridional wind, this implication is
confirmed by merely noting the coincidence of the tendency crests with those of the horizontal advection from Fig. 13(b).

An alternative visual aid can be provided by employing an approximate form of the thermal wind equation in the remote region,

\[ \mathbf{v}_g \approx (1/f)(\mathbf{k} \times \nabla \bar{T}) \]

where \( \mathbf{v}_g \) is the geostrophic wind and \( \bar{T} \) is the vertically averaged temperature from the ground to the level at which \( \mathbf{v}_g \) is evaluated. Assuming that \( \mathbf{v}_g \) is small at the ground, then approximately

\[ \frac{\partial \mathbf{v}}{\partial t} \approx (1/f)\{\mathbf{k} \times \nabla (\partial \bar{T}/\partial t)\}. \]

For this simulation there is little qualitative distinction between the temperature tendencies at 500 and 700 mb. Thus, visual differentiation of the temperature tendency of Fig. 7(a) produces a coarse, but qualitatively informative, estimate of the velocity tendencies of Figs. 12(a) and 13(a). It is specifically informative with regard to the dominance of horizontal advection since it was earlier demonstrated that this is the controlling component of the thermal response in the remote regions; hence, in this analogy, also of the momentum response there.

As a closing demonstration of the insight that quasi-geostrophic theory can provide into the dynamics of this simulation, consider the following explanation of a phenomenon which is illustrated in the height data of Fig. 3(c). Note that in the remote region the highs and lows of the geopotential wavetrain are displaced from each other in latitude. This is clarified in Fig. 14, which illustrates the quadrant from 0° to 90°E. We believe this relative displacement is a consequence of an effect described succinctly by Holton (1979, p. 113). His analysis deals with the physical interpretation of an idealized developing baroclinic system utilizing the diagnostic omega equation of hydrostatic, quasi-geostrophic theory. The feature pertinent to this discussion is the demonstrated requirement for vertical motion to maintain the hydrostatic temperature field in the presence of differential vorticity advection. This results in rising motion and a positive vorticity tendency over a surface low, and correspondingly, sinking motion and a negative vorticity tendency over a surface high. Holton further discusses (p. 139) the eastward propagation

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Figure 14. Perturbation geopotential height on the 300 mb surface on day 28. The contour interval is 10 m.
of the disturbance in terms of the secondary circulation associated with the divergence term in the quasi-geostrophic vorticity equation.

7. Concluding Remarks

The study described herein was undertaken to assist in achieving a better understanding of the atmospheric response to tropical thermal forcing. It was designed to illuminate, in a qualitative way, some elemental aspects of the processes which distinguish between linear, steady-state, highly parametrized models and the true atmosphere. The practicable goal was to build modestly upon the foundation of insight provided by previous research efforts which were focused on the analysis of simplified aspects of the phenomenon. This objective has been implemented by utilizing parametrizations and thermal forcing essentially equivalent to those of prior studies, but relaxing the constraints of linearity and steady-state. The intent was to proceed cautiously by relaxing those constraints sufficiently to provide for new insight, but not to relax them to an extent that would obscure the relationship between this response and those of earlier works.

Subsequently, a quasi-linear model response was attained which demonstrated aspects of both a quasi-stationary component and a propagating baroclinic disturbance, while remaining amenable to interpretation in terms of established theories. The simulation was terminated at day 28 due to imminent reinforcement of the response in the thermal source region by the propagating baroclinic disturbance. Such reinforcement would most certainly entail strong nonlinear responses and have necessitated a change in the scope of the study.

The quasi-stationary wavetrain extended poleward and eastward from the thermal forcing region along a 'great circle' path. The character of this wavetrain is consistent with that demonstrated in earlier linear, steady-state model studies, but does not exhibit the amplitude growth at higher latitudes characteristic of some of those studies. A new facet of this study, not present in the steady-state model simulations, was the presence of the zonally propagating wavetrain which is an example of the triggering of baroclinic instability by a distant disturbance.

A detailed analysis of the thermal balance of the response was presented through examination of the individual components of the thermodynamic energy equation. In the 'near source' region the resultant thermal tendency depends strongly on the diabatic heating and both vertical and horizontal advection. In contrast, the thermal tendency in the 'remote' region is dominated by horizontal advection. An additional perspective of the thermal balance was provided by longitude-height cross-sections of the thermal tendency components which reveal the vertical structure of the response.

A similar analysis of the momentum balance was presented in terms of the individual components of the zonal and meridional momentum equations. These results confirm the expected approximate balance between the pressure gradient force and the Coriolis force in both the 'near source' and the 'remote' regions. Analogous to the thermal balance results, diagnosis of the momentum balance revealed the importance of both the vertical and horizontal advection in the 'near source' region and the dominant role of horizontal advection in the 'remote' region.

The presented results constitute a relevant addition to a developing body of research primarily in that they unify several facets of this body and demonstrate them in a manner which readily lends itself to graphical presentation and heuristic discourse. Hopefully, they also provide an impetus to investigate additional aspects of the phenomena. One such aspect is an analysis of the response that occurs when the propagating wavetrain interacts with the quasi-stationary component in the region of the thermal forcing. A
second aspect is especially apropos in view of results obtained by Simmons (1982), that is an analysis of the consequences of permitting a longitudinally varying reference state.

ACKNOWLEDGMENTS

We wish to express appreciation to Dr Brian J. Hoskins for many unreserved discussions which were most rewarding in both practical and theoretical content. We are also grateful to Dr James R. Holton for comments on the manuscript.

REFERENCES


Ramage, C. S. 1968 Role of a tropical “maritime continent” in the atmospheric circulation. ibid., 96, 365–370


1976 Response of the atmosphere to a tropical Atlantic Ocean temperature anomaly. ibid., 102, 607–626

Simmons, A. J. S 1982 The forcing of stationary wave motion by tropical diabatic heating. ibid., 108, 503–534

Simmons, A. J. and Hoskins, B. J. 1979 The downstream and upstream development of unstable baroclinic waves. J. Atmos. Sci., 36, 1239–1254