Vorticity balances in the tropics during the 1982–83 El Niño–Southern Oscillation event

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(Received 4 July 1984; revised 5 December 1984)

SUMMARY

The mean vorticity balances in the tropical atmosphere during December 1982 to February 1983, a season of extraordinary anomalies associated with the warming of the east equatorial Pacific Ocean, are determined using the 6-hourly initialized and objectively analysed observations archived at the European Centre for Medium Range Weather Forecasts. The budgets are evaluated at one upper (150 mb) and one lower (850 mb) tropospheric level, with particular emphasis on the balance at 150 mb, near the level of maximum outflow associated with organized deep convection in the tropics.

The balances at the upper and lower levels are found to differ qualitatively from one another. At 850 mb, the zonal flow is weak and the relative vorticity and its gradient small compared with that of the Coriolis parameter. The vorticity balance is therefore essentially a Sverdrup balance between the stretching and horizontal advection of planetary vorticity, and an assumption of linearity about a basic state at rest is not unreasonable. None of this applies, however, at 150 mb.

At 150 mb, the balance is essentially nonlinear, and nearly inviscid, the primary balance obtaining between the stretching and advection of absolute vorticity by the time-mean horizontal flow. Transients play a small though not insignificant role, but terms involving the vertical advection and twisting of vorticity are small enough to be neglected.

The essentially nonlinear character of the upper tropospheric dynamics is highlighted by demonstrating the inability of a diagnostic barotropic model linearized about the observed steady zonal mean flow to simulate the anomalous flow in the central Pacific correctly. The idea of the expansion of material circuits in regions of strong convective outflow, leading to a rapid spin-down of the local absolute vorticity, and the production of a region of almost zero absolute circulation, is of crucial importance. This is seen to be intimately related to the importance of nonlinearity.

The atmospheric heat source associated with latent heat release over the s.s.t. anomaly in the central Pacific is found to exhibit pronounced low-frequency fluctuations, the reality of which is confirmed by a comparison with essentially independent data provided by satellite of the outgoing longwave radiation. The origin of this variability, and its net impact on the time-mean circulation, is not clear at present.

1. INTRODUCTION

The anomalies associated with the El Niño of 1982–83 were amongst the strongest of this century (Quiroz 1983). During the period December 1982 to February 1983, the magnitude of the sea surface temperature (s.s.t.) anomaly in the central and eastern Pacific was such as to result in a local maximum of the actual s.s.t. there, and a shift of the region of maximum tropical rainfall and cloudiness from Indonesia–New Guinea to the central Pacific. One of the challenges of modern general circulation studies is to be able to infer the atmospheric response to such a pattern of anomalous s.s.t. and the associated anomalous tropical heating. This response is of interest for several reasons. First, it accounts for much of the interannual variability in the tropical atmosphere over the Pacific Ocean. Secondly, wave activity in the tropical forcing region can disperse into middle latitudes in meridionally propagating Rossby waves (Hoskins and Karoly 1981; Simmons 1982), explaining at least part of the interannual variability of the wintertime
flow over the north-eastern Pacific, Canada, and the south-eastern United States (Wallace and Gutzler 1981; Horel and Wallace 1981). Finally, the response serves to clarify the role of the atmosphere in the coupled air–sea interactions that govern the maintenance and amplification of an s.s.t. anomaly during any particular El Niño episode (Philander 1983).

Figure 1(a) shows the mean streamfunction during this period at 150 mb, close to the level of maximum outflow associated with organized deep convection in the tropics.

![Mean streamfunction at 150 mb](image)

Figure 1. Mean streamfunction at 150 mb during (a) Dec. 1982–Feb. 1983, and (b) Dec. 1980–Feb. 1981. The difference (a) – (b) is shown in (c). Contour interval is 5×10^4 m^2 s^−1.

The same field during December 1980 to February 1981, nearly representative of climatology, is shown in Fig. 1(b), and the difference in Fig. 1(c). Note the dramatically weaker anticyclonic circulation gyres north and south of the equator in the western Pacific, and the highly anomalous anticyclonic ‘dipole’ pattern near 130°W in the central Pacific. The strength of this dipole was such as to imply anomalous subtropical westerlies in excess of 20 m s^−1, consistent with an east Asian jet that extended well into the eastern north Pacific during this season. The difference map is also suggestive of Rossby wavetrains (rather more obvious in a polar stereographic projection, not shown) that propagate into middle latitudes in both hemispheres from the central Pacific. The identifiable centres of these wavetrains are 15°N 130°W, 35°N 110°W, 40°N 70°W in the northern and 15°S 130°W, 45°S 115°W, 35°S 80°W and 30°S 60°W in the southern hemisphere.

To understand how such dramatic flow anomalies can result from an anomalous s.s.t. distribution it is necessary to consider the relationship between an s.s.t. anomaly and the atmospheric heating anomaly which results from it. This relation is complicated because the heating arises largely from the release of latent heat, which is controlled by the atmospheric dynamics. The atmosphere’s response to this tropical atmospheric heat source is then strongly influenced by the vorticity dynamics. This is because the heating is compensated by the adiabatic cooling of ascent on quite short time scales, reducing the thermodynamic energy equation to a nearly diagnostic relation for the large-scale vertical velocity. Since the divergence equation is also a nearly diagnostic ‘balance’ equation, the vorticity equation remains as the only major prognostic equation for tropical
motions. A determination of the relative importance of the various terms in this equation is clearly of value towards the understanding of the tropical general circulation.

2. DATA AND THEORETICAL CONSIDERATIONS

The vorticity balances discussed in this paper are derived from observations used as input to the high resolution, primitive equation, multi-level, global operational forecast model at the European Centre for Medium range Weather Forecasts (ECMWF). The data assimilation scheme consists of: (i) a 3-dimensional multi-variate optimum interpolation analysis designed to exploit the functional relationships between different meteorological variables; (ii) a nonlinear normal-mode initialization that filters out spurious gravity-wave noise while taking into account the forcing of large-scale vertical motion by organized deep convective heating; and (iii) the forecast model itself, providing a forecast which is used as a 'first guess' for the subsequent analysis, six hours later. Although the data are assimilated on a global 1.875°×1.875° latitude–longitude grid, a coarser (5°×5°) version of this high resolution data is used in the computations presented here. During the analysis, the data are subjected to various quality-control checks on internal consistency, climatological reasonability, consistency with neighbouring observations, consistency with a preliminary analysis at the observation point excluding the observation itself but including all other data, etc. The global analysis domain is divided into 'boxes', each about 660 km to a side and a third of the atmosphere deep. Systems of equations of order as large as 191 may be solved to arrive at the final analysed observations within each box (Lorenc 1981; Hollingsworth et al. 1982). Given the delicate nature of the vorticity balances and the lack of regular observations over large areas of the tropical oceans, one would indeed be strongly discouraged from attempting an empirical determination of the dynamical balances in the tropics without the aid of such a sophisticated data assimilation system.

Consider an atmosphere which is initially at rest. An underlying pool of anomalously warm water will create horizontal pressure gradients causing air to converge towards the pool at low levels and rise. If the air is saturated or nearly so, the water in it will condense as it rises, releasing latent heat. We would thus have an atmospheric heat source directly over the s.s.t. anomaly. Assuming a limitless supply of moisture, this source would be further strengthened by its forcing of vertical motion (horizontal temperature gradients in the tropics being small) which would cause further latent heat release until the heating rate is finally balanced by the adiabatic cooling of ascent in the steady state. The situation could, however, be quite different if the atmosphere is not initially at rest. If, for example, air is initially descending over the warm pool, the anomalously forced vertical velocity may not be large enough to make the total vertical velocity positive, and therefore no latent heat release, or its subsequent intensification, would occur directly over the s.s.t. anomaly. The generation of a heating anomaly over an s.s.t. anomaly thus depends on the presence of atmospheric convergence and divergence patterns, but to the extent that these are forced by the background s.s.t. distribution, we may expect the heating anomaly to be biased towards regions of higher s.s.t.

Figure 2, adapted from Rasmusson and Wallace (1983), shows the s.s.t. anomaly and the actual s.s.t. during the mature phase of the 1982–83 El Niño–Southern Oscillation event. Also shown is the anomalous outgoing longwave radiation (o.l.r.), basically a measure of the cloud-top temperature, for the same period. Negative o.l.r. anomalies indicate higher than normal cloud tops, i.e. more cumulus clouds, enhanced precipitation, and hence positive heating anomalies. The bias of the heating anomaly towards the southern hemisphere is clearly evident.
Since tropical heating tends to be maximum at middle levels and is largely balanced by the adiabatic cooling of ascent, we have, from conservation of mass, an accompanying low-level convergence and upper-level divergence. Upper-level divergence maps are therefore frequently studied to infer the horizontal distribution of tropical heating. Fig. 3 shows the divergence at 150 mb for the 1982-83 northern winter, and an estimate of the total diabatic heating computed as a residual from the thermodynamic equation

\[ H = c_p \int_{50 \text{mb}}^{p_0} \left[ \frac{\partial \bar{T}}{\partial t} + v_2 \cdot \nabla_2 \bar{T} + \left( \frac{L}{L} \right)^{R/\kappa_p} \left( \nabla \cdot \bar{\theta} + \nabla_3 \cdot v_3 \bar{\theta}' \right) \right] \frac{dp}{g} \]  

(1)

where the subscript refers to 2- or 3-dimensional quantities as appropriate, the overbar to the seasonal mean, and \( p_0 = 1000 \text{ mb} \). The rest of the notation is standard. The data used in this calculation were derived from the 12 GMT initialized operational analyses archived at ECMWF.

Also shown in Fig. 3 is the 150 mb divergence for the ‘normal’ winter of 1980-81. The difference, shown in Fig. 3(d), immediately reveals the anomalous convergence over Indonesia and divergence over the central Pacific during 1982-83, consistent with the o.l.r. anomalies shown in Fig. 2(c). The anomalous divergence over the central Pacific is, again, biased towards the southern hemisphere.

In view of the asymmetrical nature of this anomalous heating, it is somewhat intriguing to find that the anomalous streamfunction at 150 mb, shown in Fig. 1(c), is
very nearly antisymmetric about the equator. It is hard to see how a linear model of the dynamics, particularly one that is linearized about a basic state at rest, could yield such a response except in pathological cases.

The difficulty is highlighted, for example, in the model of Gill (1980), an excellent starting point for the analysis. This is a steady state model linearized about a basic state at rest on an equatorial beta plane. The problem is separable in the horizontal and vertical spatial coordinates for separable forcing, the horizontal structure being determined by the shallow water equations, with a different equivalent depth for each vertical mode. Since the vertical derivative of the heating tends to project rather heavily on the first internal mode, only this mode is considered in the analysis.

Apart from its obvious inability to generate an antisymmetrical response to asymmetrical heating, Gill's model fails to account for the observed anomalies at 150 mb in other interesting ways. For example, the vorticity balance in the model is essentially

$$\beta v = -f \nabla \cdot v - \epsilon \zeta$$

(2)

where $f = \beta y$ and $\epsilon$ is a coefficient of Rayleigh friction. For small $\epsilon$, the vorticity source $f \delta \omega / \delta p$ is balanced by $\beta v$, hence the high at upper levels is always to the west of the heating region in this model. The observed anomalies during the northern winter of 1982–83 do not, however, show this westward shift. One way to explain this lack of westward shift within the framework of this model (and other models linearized about a steady zonal mean flow) would be to make $\epsilon$ sufficiently large that the vorticity source term is balanced by the friction term. This would, however, entail reducing the frictional
damping time scale from the already very short values of around 2.5 days (needed to limit the horizontal scale of the response in the model) to even shorter values. Such short time scales are sometimes claimed to be associated with the vertical transport of vorticity in cumulus towers ('cumulus friction'), although general circulation models that do not include these transports in their parametrization of cumulus convection do not appear to have this phase-shift problem (Blackmon et al. 1983; Sardeshmukh and Held 1984).

Another difficulty with this model is that, because of the equatorial beta plane approximation, the response is equatorially trapped. There is hardly any suggestion of exponential decay in the observations as we move away from the equator in the central Pacific; the anomalous subtropical westerlies in both hemispheres are just as strong as the anomalous equatorial easterlies.

Figure 4 shows the absolute vorticity contours at 150 and 850 mb during the same period. The contribution of the relative vorticity is clearly not negligible at the upper levels (a fact noted by Simmons (1982)); indeed $\beta - \bar{u}_y$ is much smaller than $\beta$ over most of the tropics, particularly in large areas over the major centres of convection. This suggests that the flow cannot be reasonably linearized about a basic state at rest.

![Figure 4](image)

Figure 4. Mean absolute vorticity $\zeta + f$ at (a) 150 and (b) 850 mb during D.J.F. 1982/83. Units are $2 \times 10^{-5}$ s$^{-1}$ and negative contours are dashed.

Linearizing about a zonal flow $[\bar{u}](y, z)$ which is a function both of latitude and height, however, renders the problem non-separable which, taken together with the fact that the vertical structure of the observed anomalies does not quite resemble that of the first internal mode (and thus more modes need to be considered), considerably diminishes the elegant simplicity of the model.

It is worth noting the zonal asymmetry at upper levels in the distribution of the absolute vorticity and its meridional gradient, because it suggests that nonlinearities could be important. Note particularly the region over the tropical Atlantic, downstream of Brazil, where 'perturbations' in the southern hemispheric westerlies are large enough (and not particularly anomalous) to modify the zonal mean absolute vorticity significantly, leading to a cut-off low at 15°S 25°W. The contours of absolute vorticity and stream-function are nearly parallel here, indicating approximate conservation of absolute vorticity following the steady horizontal motion, that is, $d(\zeta + f)/dt = \nabla \cdot (\zeta + f) = 0$. One is indeed tempted to think of the flow over this region as a free nonlinear solution to the barotropic vorticity equation. This is consistent with the fact that there is no appreciable upper-level divergence, and hence no generation of upper-level vorticity through vortex-tube stretching:

$$d(\zeta + f)/dt \approx \nabla \cdot (\zeta + f) \equiv - (\zeta + f) \nabla \cdot v \sim 0.$$  \hspace{1cm} (3)
Introducing nonlinearities into a model like Gill’s is, however, not as straightforward as it may seem. It is not, for example, simply a matter of considering the nonlinear version of the shallow water equations for each vertical mode, because one now has to treat the nonlinear interactions between the different vertical modes as well. In other words, the problem becomes, once again, non-separable in the horizontal and vertical coordinates.

One wonders if it is possible to extend Gill’s model while still retaining some of the simplicity of its conceptual framework. Encouraged by the apparently simple description of the flow over the southern Atlantic as a free nonlinear solution, one might consider the nondiabatic barotropic vorticity equation with sources and sinks as a possible candidate:

$$\frac{\partial \zeta}{\partial t} + \mathbf{v} \cdot \nabla (\zeta + f) = - (\zeta + f) \nabla \cdot \mathbf{v} + \text{friction}$$

(4)

with the divergence in the forcing term on the right-hand side specified in terms of the diabatic heating rate $Q$, given the remarkably simple nature of the large-scale heat balance in the tropics:

$$\nabla \cdot \mathbf{v} = \frac{\partial (Q / N^2)}{\partial z}.$$

Figure 5 shows the seasonal mean velocity potential $\chi$ at 150 mb during the winters of 1982–83 and 1980–81 in a format similar to that of Fig. 1. The streamfunction and velocity potential determine the horizontal wind through the relation

$$\mathbf{v} = \hat{k} \times \nabla \psi + \nabla \chi = \mathbf{v}_\psi + \mathbf{v}_\chi.$$

The divergent part of the wind is thus given by $\mathbf{v}_\chi = \nabla \chi$, and the horizontal divergence is $D = \nabla \cdot \mathbf{v} = \nabla \cdot v_x = \nabla^2 \chi$. Velocity potential maps are sometimes shown in the literature to depict the pattern of large-scale divergence. The words ‘large scale’ should however be interpreted with some caution because, as comparing Fig. 5 with Fig. 3 shows, the $\chi$ field only brings out the very largest (i.e. planetary) scales in the divergence pattern.
A comparison of Fig. 5 with Fig. 1 gives an idea of the relative magnitudes of the rotational and divergent components of the wind. The total horizontal wind vectors are shown in Fig. 6. It is evident that even in the western equatorial Pacific, where the divergent motions are the strongest, \( \mathbf{v}_d \) is less than a third of the rotational component \( \mathbf{v}_r \); in other regions it is much smaller. The notion of the Walker cell as a simple divergent, thermally direct overturning in the vertical plane passing through the equator should clearly be viewed with some suspicion. It is therefore not grossly unrealistic to assume that the horizontal wind is nondivergent away from the heating region, an assumption that is needed to justify the consideration of a nondivergent barotropic model.

One should, of course, examine the complete vorticity balance to determine the extent to which the dynamics may be represented by the barotropic vorticity equation. This is done in section 3. Calculations with a diagnostic barotropic model highlighting the importance of nonlinearities are presented in section 4, which concludes with a discussion of the strongly fluctuating atmospheric heat source over the region of nearly constant s.s.t. anomalies in the central Pacific.

3. THE VORTICITY BALANCE AT 150mb

Ignoring external sources and sinks and subgrid-scale processes, the vorticity equation in pressure coordinates may be written

\[
\frac{\partial \zeta}{\partial t} + \mathbf{v} \cdot \nabla (\zeta + f) + \mathbf{f} \cdot \nabla \times \mathbf{v} \frac{\partial \mathbf{v}}{\partial p} = - (\zeta + f) \nabla \cdot \mathbf{v}
\]

where \( \omega = dp/dt \) is the vertical velocity, and \( \mathbf{f} \cdot \nabla \times \mathbf{v} \frac{\partial \mathbf{v}}{\partial p} \) represents vertical advection and twisting. Averaging over the period 1 December 1982 to 28 February 1983, the vorticity tendency term \( d\zeta/dt \) is totally negligible compared to the other terms, and the vorticity balance reduces to

\[
\overline{\mathbf{v} \cdot \nabla (\zeta + f)} + \mathbf{f} \cdot \nabla \times \omega \frac{\partial \mathbf{v}}{\partial p} = - (\zeta + f) \nabla \cdot \mathbf{v}
\]

where the overbar refers to the time mean. Figure 7 shows the terms in this balance computed using the initialized analyses on a 5°×5° grid at 00, 06, 12 and 18 GMT for the 90 days from 1 December to 28 February. Each map in the figure thus represents an average over 360 maps. The horizontal gradients involved are evaluated using centred finite differences. The plots shown are smoothed with a spectral filter of the form \( S_n = \exp[-K(n(n + 1))^2] \) for \( n \leq 24 \), \( S_{24} = 0.1 \), where \( n \) is the total wavenumber on the sphere. In addition to eliminating spurious small-scale noise in the data, such a filter has other desirable properties, discussed in Sardeshmukh and Hoskins (1984).

The vertical advection and twisting terms are not shown because they are found to be very much smaller than the other two terms in the balance (with maximum value of less than 1.5 × 10^{-11} s^{-2}), which may be approximated now as

\[
\mathbf{v} \cdot \nabla (\zeta + f) \approx - (\zeta + f) \nabla \cdot \mathbf{v}.
\]

Figure 6. Mean winds at 150mb during D.J.F. 1982/83.
There are some difficulties with this balance, especially over the Himalayas and in the southern hemisphere. Also, while the pattern of the mean stretching closely resembles that of the mean advection, one may discern a slight tendency for the stretching to be of smaller magnitude, suggesting that the mean large-scale divergences are being slightly underestimated even when the forcing of vertical motion by diabatic heating is taken into account in the ECMWF initialization procedure (this could conceivably turn out to be a problem with the objective analysis scheme, since there are only very slight differences (not shown) between the fields of mean uninitialized and initialized divergences with this new initialization). One should perhaps stress that since the terms involve combinations of spatial derivatives of the wind, the vorticity balance is very sensitive to observation error and inadequacies in the data analysis and initialization schemes and, indeed, provides a very sensitive test for these schemes. Considering this, the balance seems satisfactory, and in any case is considerably better than the Sverdrup balance of Gill

\[ \beta \bar{v} = -f \nabla \cdot \bar{v} \]  

shown in Fig. 8. Note especially the imbalance around 130°W 15–20°S, where one is still sufficiently close to the equator for vortex tube stretching to be relatively ineffectual but
where the strong meridional flow immediately to the east of the heating anomaly causes a large advection of planetary vorticity $\beta \vec{v}$ which, if unchecked, would force the seasonal mean vorticity in the region in less than a day! Note also the great imbalance over the tropical Atlantic in the region of the free nonlinear solution alluded to above, where vortex tube stretching is again small but $\beta \vec{v}$ is large. One may reasonably conclude that (7) is a better upper tropospheric vorticity balance than (8). Note that in large areas in the deep tropics, the balance (7) reduces to a near-trivial zero-equals-zero statement in the time mean because, as mentioned earlier, both the mean absolute vorticity and its horizontal gradient are small there.

One may rewrite (7) as

$$\vec{v}.\nabla(\zeta + f) + \vec{v}'.\nabla \zeta' = -(\zeta + f)\nabla \vec{v} - \zeta' \nabla \vec{v}'$$  \hspace{1cm} (9)

where the primes denote a deviation from the seasonal mean. Figure 9 shows the two terms involving transients; note the smaller contour interval of $2 \times 10^{-11}$s$^{-2}$. Of the two, $-\zeta' \nabla \vec{v}'$ is quite small and may perhaps be neglected. $\vec{v}'.\nabla \zeta'$ is also generally much smaller than $\vec{v}.\nabla(\zeta + f)$, but not near the equator, where it is comparable. The contribution by the transient divergent winds to this transient advection, shown in Fig. 9(c), is not negligible. One could therefore conclude that transients play a secondary role in the overall balance requirements except near the equator, where the absolute vorticity is small enough in the time mean that the mean vorticity source associated with vortex stretching is quite small. Thus, to model the gross features of the time-mean upper-level flow, it may not be unreasonable to assume

$$\vec{v}.\nabla(\zeta + f) = -(\zeta + f)\nabla \vec{v}. \hspace{1cm} (10)$$

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Figure 9. (a) $\vec{v}'.\nabla \zeta'$; (b) $-\zeta' \nabla \vec{v}'$; (c) $\vec{v}'.\nabla \zeta'$; and (d) $\vec{v}.\nabla(\zeta + f)$ at 150 mb during D.J.F. 1982/83. Units are $2 \times 10^{-11}$s$^{-2}$ and negative contours are dashed. The zero contour is not shown.
Figure 10. (a) Mean horizontal advection linearized about the zonal mean zonal flow and (b) $-((\zeta + f)\nabla \mathbf{v}^*)$ at 150 mb during D.J.F. 1982/83. Contouring as in Fig. 7.

One wonders if further simplification is possible, i.e. if one may linearize this longitudinally varying time-mean flow about the zonal mean wind $[u]$: \[
[u]\frac{\partial \zeta^*}{\partial x} + v^* \frac{\partial (\zeta + f)}{\partial y} = -((\zeta + f)\nabla \mathbf{v}^*)
\] (11)
where the square brackets denote a zonal mean and the asterisks a deviation from it, and the overbars denoting time means have been dropped for convenience. Figure 10 shows this balance. The left-hand side of (11) is a much better approximation to $\mathbf{v} \cdot \nabla (\zeta + f)$ than the $\beta \mathbf{u}$ of the Sverdrup balance, substantiating our assertion in the introduction that one cannot linearize about a basic state at rest in the tropical upper troposphere because $\beta - \tilde{u}_y$ is significantly smaller than $\beta$ over large areas of the tropics. While better than the simple Sverdrup balance, (11) has difficulties, especially over the eastern Pacific and certainly over the tropical Atlantic. One must consider the full $\mathbf{v} \cdot \nabla (\zeta + f)$ to obtain a reasonable vorticity balance in these regions.

In summary, (10) appears to be the best that we can do by way of simplifying the vorticity dynamics at 150 mb. However, the linear balance (11) is promising enough to merit closer attention; this is done in the next section. We conclude this section by pointing out that the Sverdrup balance (8) is quite reasonable at lower levels, because the zonal flow $[u]$ is weak enough that vorticity advection by it is small, and $\beta - \tilde{u}_y$ is reasonably well approximated by $\beta$. The balance at 850 mb is shown in Fig. 11. Note the gross imbalance over the Himalayas and the Rockies; it is probably associated with small errors in the interpolation to pressure surfaces in the data analysis scheme, highlighting

Figure 11. As in Fig. 8 but at 850 mb. Units are $2.5 \times 10^{-13} \text{s}^{-2}$. 
once again the difficulty of obtaining good vorticity balances even with high-quality data sets.

4. A linear model of the vorticity balance at 150 mb

Consider the steady barotropic vorticity equation on the sphere, linearized about the zonal mean wind \([u]\):

\[
\frac{[u]}{a \cos \theta} \frac{\partial}{\partial \lambda} \nabla^2 \psi^* + \frac{1}{a \cos \theta} \frac{\partial \psi^*}{\partial \lambda} \left[ \beta + \frac{\partial [\zeta]}{\partial \theta} \right] = -((\zeta + f) \nabla \cdot v - r \nabla^2 \psi^*)
\]

(12)

where \(\psi^*\) refers to the streamfunction, \(r\) is a coefficient of Rayleigh friction, and the square brackets and asterisks denote a zonal mean and a deviation from it respectively. One could perhaps question the modelling of dissipative processes as Rayleigh friction with a constant coefficient, but if (11) is a reasonable vorticity balance, one would expect friction to play a minor role and therefore its precise modelling not to be of crucial importance. One may regard the above model as one which attempts to diagnose the rotational wind pattern from the pattern of divergent winds for a given zonal mean flow. For given \(((\zeta + f) \nabla \cdot v\) and \(r\), we obtain \(\psi^*\) by Fourier analysing in the zonal direction (using 30 zonal harmonics) and solving the resulting meridional structure problem for each Fourier coefficient by the method of Lindzen and Kuo (1969) (with a 2.5° latitude resolution) with \(\psi^*\) set equal to zero at the poles. The solution of (12) for the zonal mean flow of the 1982–83 northern winter and the mean divergence \(\nabla \cdot v\) at 150 mb for the same period is shown in Fig. 12 for \(r^{-1}\) equal to (a) 2-6 and (b) 14-7 days. The observed perturbation streamfunction for the period is shown in Fig. 12(c). The stationary wave pattern is apparently similar for strong and weak damping; only the amplitudes seem to be affected by a change in \(r\), and mainly in mid-latitudes. It is interesting that the mid-latitude pattern for strong damping compares better with the observations than that for weak damping. More relevant, however, is the inadequacy of the solution in the tropics. In the northern hemisphere, the western Pacific high and the Mexican and Atlantic lows have about the right amplitude, but their longitudinal positions are in error by as much as 20° and, more seriously, their southern hemispheric counterparts are missing. The mid-Pacific streamfunction low in the southern hemisphere associated with the s.s.t. anomaly is also reasonably well simulated, although perhaps a little too far south of its observed position, but, again, its counterpart in the northern hemisphere is missing. The linear solutions clearly do not exhibit the striking antisymmetry of the observed pattern about the equator, particularly east of the date-line. Given the latitudinal asymmetry of the forcing and the presence of a narrow band of easterlies (albeit weak, with values of less than 1 m s\(^{-1}\)) between 3° and 7° south of the equator hindering Rossby-wave propagation through it, it is difficult to understand this antisymmetry in terms of the linear model (12).

One may get an idea of the nonlinear effects by forcing (12) with

\[-\nabla \cdot (\zeta^*v^* - [\zeta^*v^*])\]

instead of \(-((\zeta + f) \nabla \cdot v\) with \(r^{-1} = 14.7\) days. These nonlinear terms, which were neglected in forming (11) from (10), are actually quite large, as can be seen in Fig. 12(d). The circulation implied by them, which we may regard as a nonlinear correction to the simulated flow in Fig. 12(b), is shown in Fig. 12(e). Note particularly the correction over the north equatorial Pacific, which clearly brings out the crucial role of nonlinearity in modelling the northern half of the anomalous anticyclonic ‘dipole’ in the region of the maximum s.s.t. anomalies. Note, too, the correction over the south equatorial Atlantic,
where we anticipated the importance of nonlinearity. A major surprise, however, is the strong nonlinear damping of the response in the northern hemispheric middle and high latitudes. There is a strong tendency for the nonlinear correction to cancel a large portion of the linear response in these latitudes, consistent with the fact that we obtain a much better solution (Fig. 12(a)) to (12) for strong rather than weak damping. An explanation for why nonlinear interactions between the quasi-stationary waves should amount to such a simple damping effect on the linear response is not clear to us at present. We may add, however, that both the tropical and extra-tropical ‘sources’ of nonlinearity in Fig. 12(d) are responsible for the large extra-tropical response in Fig. 12(e).

Once the importance of nonlinearities is recognized, a hierarchy of nonlinear barotropic models consistent with the vorticity balance (7) suggests itself. One may consider models with steady or time-dependent horizontal advection of absolute vorticity. One may also consider models with steady (specified $-(\zeta + f)\nabla v$) or time-dependent (specified $\nabla v$ only) forcing of vorticity. This variety of models is possible because of the nature of the transients in the vorticity dynamics and the way in which nonlinearity involving the mean flow becomes important. Terms involving transients are small in the time-mean vorticity balance, but this need not necessarily imply that transients are unimportant in shaping the large-scale flow. It is intriguing, for example, that the major area of convection in the western Pacific amounts to a negligible vorticity source in the

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**Figure 12.** Solution of (12) forced by the observed linearized stretching (Fig. 10(b)) at 150 mb for $r^{-1}$ equal to (a) 2-6 and (b) 14-7 days. The observed perturbation streamfunction is shown in (c). Neglected nonlinear advection and stretching $-\nabla \cdot (\zeta \mathbf{v} \mathbf{v} - [\zeta \mathbf{v} \mathbf{v}])$ is shown in (d) and the solution of (12) forced by them in (e). Units are $5 \times 10^6$ m$^2$s$^{-1}$ in (a), (b), (c) and (e) and $2 \times 10^{-31}$ s$^{-2}$ in (d).
time mean because of the small absolute vorticity there. But to conclude from this that this strong convection plays no significant role in the dynamics of the large-scale flow at upper levels would be premature, since the convection could itself be partly responsible for the small values of the mean local absolute vorticity and thus for the large zonal asymmetry in $\zeta_s$ which leads to significant nonlinear effects. To see this, consider a circular area of strong steady convection $\nabla \cdot \mathbf{v} = D = \text{constant}$, centred on the equator. Ignoring advection, the vorticity tendency from stretching is then $\partial \zeta_s / \partial t = -\zeta_s D$, hence $\dot{\zeta}_s(t) = \dot{\zeta}_s(0) \exp[-Dt]$. Thus the effect of vortex stretching is to destroy the absolute vorticity over the convecting area on a time scale of $1/D = 1/(3 \times 10^{-8}\text{s}^{-1}) \approx 3$ days. (This idea is also implicit in the work of Paegle and Paegle (1976).) Anticyclonic circulation pairs are generated on the two sides of the equator as observed, and the vorticity source $-\zeta_s D$ is switched off and can subsequently appear quite small in the time mean. The effect is perhaps more easily understood by considering the conservation of absolute circulation, $C_s$, on a material circuit $\Gamma$ on an isentropic surface at the convective outflow level. Such a surface is essentially horizontal at this level in the tropics. If $\Gamma$ is initially above the convective region and of small radius, it will expand as long as it remains in the region of divergence, keeping its small, effectively zero, absolute circulation. Thus, in the absence of advection through the convective region, it will become one of zero absolute circulation. Since $C_s = \int_\Gamma \zeta_s dS$, and $\Gamma$ is arbitrary, this zero absolute circulation argument is entirely consistent with that given above for the tendency to produce zero $\zeta_s$ in such a region. In terms of the relative circulation $C$, $C_s = C + 2\Omega (\sin \phi) S$, where $S$ is the horizontal area bounded by the circuit. Therefore zero absolute circulation implies negative and positive relative circulations in the northern and southern hemispheres respectively. These are the anticyclonic circulations referred to above.

As far as the vorticity equation is concerned, a crucial point to note is the time-dependent nature of the vorticity source even for the case of steady convection, i.e. $-\zeta_s(t) D$, instead of $-fD$ or $-((\zeta + f) D$. No steady-state linear model takes this time-dependent nature of the vorticity source into account, and therefore there is no inherent tendency for the source to switch itself off. Hence a large spurious damping mechanism for vorticity may be required in the linear models when $D$ is large, which could be confused with cumulus friction. A time-dependent, nonlinear barotropic model with a time-dependent vorticity source $-(\zeta + f) \nabla \cdot \mathbf{v}$ where $\nabla \cdot \mathbf{v}$ is the specified observed mean divergence therefore seems an attractive possibility.

In summary, the major large-scale tropical vorticity dynamics is contained in the equation

$$\frac{\partial \zeta}{\partial t} + \mathbf{v} \cdot \nabla (\zeta + f) = -(\zeta + f) D \quad (13)$$

where both the nonlinear advection on the left-hand side, and the time-dependent, zonally varying absolute vorticity on the right-hand side, should be taken into account. (It should be noted that we consider this conclusion to be a general one for the tropics, and not restricted only to the anomalous winter discussed in this paper.) For diagnostic steady-state modelling, the equivalent to (12) would be to solve the steady version of the above equation with only the divergent wind specified. This would imply specifying the divergence on the right-hand side and, for consistency, the divergent wind contributing towards advecting the absolute vorticity (not entirely negligible, see Fig. 9(d)) on the left-hand side. Such a divergent barotropic model is currently being developed.

There is one further complication that needs to be recognized before embarking on this modelling programme. This is that the convection and its associated upper-level divergence are by no means steady in time. Figure 13 shows the variance $D^2$ of the horizontal divergence $D = \nabla \cdot \mathbf{v}$ at 150 mb during December 1982 to February 1983. The
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Figure 13. Variance $\overline{D^2}$ of the divergence $D = \nabla \cdot v$ at 150 mb during D.J.F. 1982/83. Units are $2 \times 10^{-11}$ s$^{-2}$. The contour interval is $2 \times 10^{-11}$ s$^{-2}$. Comparing with the map of the mean divergence $\overline{D}$ in Fig. 3, the strongly fluctuating nature of the divergence over each of the major areas of convection over central Africa, Brazil, Indonesia–New Guinea and the mid-Pacific Ocean is immediately apparent. The maximum value of $\overline{D^2}$ is about $10^{-10}$ s$^{-2}$, implying a standard deviation of $10^{-5}$ s$^{-1}$, about three times the local values of $\overline{D}$. It is perhaps worth noting that although $\overline{D^2}$ has such large values, $-\overline{\theta'^2 D'}$, which rarely exceeds $2 \times 10^{-11}$ s$^{-2}$ (see Fig. 9) is only a minor term in the seasonal mean vorticity balance even in the deep tropics. However, as noted above, this does not prove that transient convection plays no major role in determining the seasonal mean flow.

Figure 14. Time series of the area-averaged divergence at 150 mb during D.J.F. 1982/83 over the major centres of convection in (a) the central Pacific, (b) the western Pacific, (c) Africa and (d) South America. Units are $10^{-6}$ s$^{-1}$. 
Figure 14 shows the time series of divergence over these areas of transient convection, where a running mean over five consecutive 6-hourly analyses (i.e. one day) has been employed to filter out diurnal variations. The very different nature of the transient convection in each of the four cases shown is immediately apparent and, most striking of all, the time series over the mid-Pacific shows pronounced low-frequency fluctuations. The peak-to-peak amplitude of this fluctuation is about $5 \times 10^{-6} \text{s}^{-1}$, comparable to the seasonal mean. There is some evidence of the longer time scale in the western Pacific, but the time series there is dominated by the much shorter scale of about 5 days. The 5-day scale dominates completely the time series over Africa and Brazil, although a careful time-series analysis may perhaps reveal the longer time scale even in these regions. Considering, however, that the total variance is about the same in the four time series shown, there is a strong suggestion that the maximum amplitude of the low-frequency variability in the divergence field occurs over the central Pacific, in the region of the maximum anomalous heating associated with the s.s.t. distribution during this El Niño winter. A time series of the vertical velocity at 500 mb (not shown) exhibits this phenomenon as well, indicating that it is indeed related to fluctuations in deep convective heating.

It should be pointed out that a large area of about 30° square to the west of the Galapagos Islands, and to the east of the anomalous heating region in question, is almost devoid of regular observations. One may therefore wonder if the fluctuations as seen in the objectively analysed data are in some way an artifact of the ECMWF analysis scheme itself. To clear any such doubts, the o.l.r. data from the NOAA-7 AVHRR satellite in the 11-5–12-5 μm band is shown in Fig. 15, along with the time series of the ECMWF divergence at 150 mb. The plots shown are area-averaged 5-day running means of the daily 12 GMT values, the area over which the average is taken being the same as in Fig.

**Figure 15.** 5-day running means of the 150 mb divergence (solid line) and outgoing longwave radiation (dashed line) averaged over the area 5°N–25°S 165°W–135°W in the central Pacific during D.J.F. 1982/83. Units are $10^{-8} \text{s}^{-1}$ for the divergence and W m$^{-2}$ for o.l.r.
14(a). Considering the extreme sensitivity of the result to observation and analysis error, the agreement between the two time series is quite remarkable. The peak-to-peak amplitude of the fluctuation in the o.l.r. is about 40 W m\(^{-2}\), comparable, again, to the magnitude of the mean o.l.r. anomaly during this period (Fig. 2(c)). There can thus be little doubt as to the reality of this strong fluctuation in the intensity of the atmospheric heat source over the region of maximum s.s.t. anomalies during this El Niño winter.

The origin of this variability, and its net impact on the time-mean circulation, is still very much a matter of speculation. We may add, however, that it is also evident in the time series of other fields of dynamical interest as far west as China. An attempt to construct a consistent dynamical picture of such low-frequency changes in the flow is a topic of current research.

5. Concluding Remarks

Using the ECMWF initialized operational analyses as a high quality data set, an attempt has been made in this paper to determine the mean vorticity balances in the tropics during the mature phase of the 1982/83 El Niño–Southern Oscillation event. The balances obtained, although not exact by any means, are sufficiently accurate to encourage the belief that terms involving the vertical advection, twisting and subgrid-scale transports of vorticity are small enough to be neglected in the problem of modelling the large-scale quasi-stationary wave pattern. A diagnostic model designed to carry out consistency checks on the vorticity balance lends further support to this view. The vorticity dynamics can therefore be represented by the divergent barotropic vorticity equation (13), but where full nonlinearity, and in particular the time-dependent nature of the vorticity source \(-\zeta(t)D\) should be taken into account, especially at upper levels. Consideration of such a source properly accounts for the expansion of material circuits, the tendency to zero absolute circulation, and the decay of absolute vorticity, all occurring on the time scale \(1/D\). These processes are intimately linked to the importance of nonlinear advection, and become increasingly more relevant for larger \(D\). No linear steady state model takes this time scale into account, and it is therefore possible that the strong spurious damping that is sometimes needed to obtain reasonable answers from such models really mimics the effect of nonlinear stretching (since it tends to act essentially as a local damper on the vorticity source), and not necessarily 'cumulus friction'.

The vorticity balance (13) is thus essentially nonlinear, and nearly inviscid. No further simplification of this balance, such as linearization about a zonal mean flow, may be made without running into difficulties, as highlighted in section 4. Neglecting the ambient flow altogether is grossly unrealistic at upper levels.

The simplest understanding of the atmospheric response to diabatic heating in the tropics would then be that the vertical motion is determined mainly by the heating distribution, the horizontal wind by the vertical motion through the continuity (giving its divergent part) and vorticity (giving its rotational part) equations, the mass or geopotential by the horizontal wind through the divergence 'balance' equation, and the temperature by the geopotential through hydrostatic balance. The last, most delicate, link in this chain is the relationship between the diabatic heating distribution, associated with organized deep convection, and the large-scale structure of the tropical atmosphere.

To model the atmosphere's response to a given distribution of anomalous s.s.t. correctly, it is then not only necessary to take the nonlinear nature of the vorticity dynamics into account, but also vitally important to model the intensity and the horizontal and vertical structure of the associated atmospheric heating with precision. Errors made at this stage, either through inadequacies in a model's convective scheme or, say, through
an underestimation of the magnitude of the s.s.t. anomaly, can rapidly deteriorate the quality of the simulated response. It is interesting, for example, that the anomalous dipole pattern at 150mb over the central Pacific during this El Niño winter was almost completely lost by day 5 of the forecasts made by the operational forecast model at ECMWF. The failure to capture this most dramatic of circulation anomalies, with even this highly sophisticated model, is understandable both in view of the s.s.t. anomaly being underestimated by the operational s.s.t. analyses used at that time, and of the model's inability at that time to maintain the intensity and the vertical structure of the heating due to convection in its correct position beyond 2 or 3 days in the tropics.

ACKNOWLEDGMENTS

The authors are extremely grateful to the Climate Analysis Center of the National Meteorological Center, USA and Dr E. Rasmusson in particular for the provision of the o.l.r. data from the NOAA-7 AVHRR satellite. The work described here comprises part of the Joint Diagnostics Programme with the Meteorological Office using ECMWF data. The advice and encouragement of members of both institutions has been extremely useful. One of us (P.D.S.) has been supported by a grant from the Gassiot Committee which is administered by the Meteorological Office.

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