Measurements and predictions of flow and turbulence over an isolated hill of moderate slope

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(Received 21 March 1984; revised 29 November 1984)

SUMMARY

Field measurements of the mean flow and turbulence statistics in neutral static stability flow over an isolated, roughly circular hill, Blashaval, are presented. Blashaval rises approximately 100 m above the surrounding terrain and has a base diameter of about 800 m. The flow speed at 8 m over the summit is found to be increased by a factor of 1.7 over the upstream value and the flow direction reverses in the lee of the hill. The turbulence structure observed above the summit shows marked variations with height. The observed mean flow is compared with results from a model which makes a careful application of linear theory to real terrain. The turbulence measurements are compared with results from available theories.

1. INTRODUCTION

Our knowledge of flow over hills has been acquired by a combination of field, wind tunnel and theoretical studies. Each type of study has its particular advantages and limitations but together they have led to a steady advancement of knowledge. The field studies are of central importance as only they can verify the success of the other approaches. A particular difficulty with field studies is the lack of suitable sites. To allow interpretation of field data with existing models and theories the site must be fairly ideal, having simple shape and being free from scattered obstacles and irregularities. It is also important that the approaching boundary layer is near to equilibrium. In relation to these criteria the site chosen for the present study is probably one of the best available in the United Kingdom. Nevertheless the factor limiting the quality of the data obtained is still the influence of small-scale irregularities.

One aim of the present work is to extend and verify methods of predicting the mean flow over the hill. It is evident from previous studies (e.g. Mason and Sykes 1979; Walmsley et al. 1982) that useful predictions of the mean flow over gentle topography can be made with the linear theory due to Jackson and Hunt (1975). This theory is invariably applied outside its strict range of validity and here the results are improved by the use of uniformly-valid solutions and selection of the most appropriate velocity scales. The resulting improvements in predictions are confirmed by comparison with the results of a finite difference model; this is applied to gentle slopes but deals with true boundary layer velocity profiles rather than asymptotic scales. In previous studies (e.g. Mason and Sykes 1979) it seems that part of the success of the Jackson and Hunt theory was due to a cancelling of errors involved in estimating relevant velocity scales. A similar refinement of the Jackson and Hunt model has been made independently by Taylor et al. (1983). Their approach is different in detail but seeks the same type of improvements. The refined model has been applied to a representation of the hill, Blashaval, and the results compared with observations. There is good agreement in most regions of the flow and significant discrepancies are confined to the lee of the hill. Similar large discrepancies in flow predictions in the wake of a two-dimensional hill have been found in the wind tunnel study of Britter et al. (1981). The causes of these errors are discussed and the finite difference model is used to examine the importance of nonlinearity.

The second objective of the present study is to provide detailed information on the changes in turbulence structure in flow over the hill. Interest in these turbulence changes
arises from both their local importance and from their influence on the net effect of hills. These net effects, such as total momentum transfer, are difficult to measure directly and the development of adequate theoretical models offers the most promise of providing information. However, whilst various theoretical models agree on predictions of mean flow, they differ widely on net effects depending on the turbulence closure they employ (Sykes 1980). Refined turbulence measurements are thus needed to verify the models to be applied to this problem.

Previous detailed turbulence measurements have been made both in the field (e.g. Bradley 1980) and in wind tunnels (e.g. Britter et al. 1981). The wind tunnel observations are limited to positions remote from the surface where the turbulence changes are characterized by rapid distortion effects. Bradley’s field study provides data both in this outer region of the flow and in the inner region closer to the surface. In the outer region he also finds changes consistent with rapid distortion. Near the surface his results suggest local equilibrium but are subject to some uncertainty owing to the tree cover around the measurement site. The present study seeks to provide information closer to the surface.

2. THEORETICAL MODELS

(a) Review of models

(i) Model A. Jackson and Hunt’s (1975) linear theory considers an asymptotic limit when the flow can be divided into an outer inviscid flow region and an inner layer within which the turbulent stress divergences are significant. This is the limit \( u_*/U_0 \to 0 \) (where \( u_* \) is the square root of the surface stress and \( U_0 \) is the basic flow speed). The linearized equations describing the perturbations in the outer inviscid flow are

\[
U_0(\partial u/\partial x) = -\partial p/\partial x, \quad U_0(\partial w/\partial x) = -\partial p/\partial z, \quad U_0(\partial v/\partial x) = -\partial p/\partial y \tag{1}
\]

with boundary conditions \( w = 0 \) at \( z = \infty \) and \( w = U_0 \partial h/\partial x \) on \( z = 0 \). \( z \) is the vertical coordinate and \( U_0 \) is in the \( x \) direction. \( (u, v, w) \) are velocity perturbations, \( h \) is the height of the orography and \( p \) is the perturbation dynamic pressure. These equations for potential flow can be solved by Fourier transforms and the solution for pressure is

\[
p = p_0 e^{i(kx + my + nz)} \tag{2}
\]

where

\[
h = h_0 e^{i(kx + my + nz)} \tag{3}
\]

\[
p_0 = -k^2 (k^2 + m^2)^{-1/2} U_0^2 h_0 \tag{4}
\]

and

\[
n = i(k^2 + m^2)^{1/2}. \tag{5}
\]

The height scale of the inner layer is found by considering the balance of nonlinear and stress divergence terms in the equations of motion and is

\[
l = 2L \kappa u_*/U_0 \tag{6}
\]

(where \( L \) is the horizontal scale of the topography and \( \kappa \) the von Kármán constant). On this thin scale there is no vertical variation of the pressure field and in consequence of the asymptotic requirement that \( z_0/l \to 0 \), where \( z_0 \) is the roughness length, the shear of the basic velocity profile on the scale \( l \) is negligible and the advection velocity in the inner
layer is $U_0$. The linearized momentum equations in the inner layer are

$$
\begin{align*}
U_0(\partial u/\partial x) &= -\partial p_{z=0}/\partial x + \frac{\partial}{\partial z} \left\{ 2\kappa(z + z_0)u_* (\partial u/\partial z) \right\} \\
U_0(\partial v/\partial x) &= -\partial p_{z=0}/\partial y + \frac{\partial}{\partial z} \left\{ \kappa(z + z_0)u_* (\partial v/\partial z) \right\}
\end{align*}
$$

(7)

where $p_{z=0}$ is the outer layer solution, Eq. (2), at $z = 0$ and a mixing length approximation has been used to parametrize the Reynolds stresses. The details of the flow at small values of $z$ have been considered by Sykes (1980) who showed that a third layer of scale $\sim z_0$ is necessary to match the inner layer flow to the equilibrium layer close to the surface. He found that although the inner layer equations, (7) above, are not strictly rational, the solution agrees with a more formal approach. The solutions to Eqs. (7) are

$$
\begin{align*}
u &= (p_{z=0}/U_0)[1 - K_0(2iz')^{1/2}] / K_0 [2(iz')^{1/2}] \\
\end{align*}
$$

and

$$
\begin{align*}
u &= (lp_{z=0}/kU_0)[1 - K_0(2iz')^{1/2}] / K_0 [2(iz')^{1/2}] \\
\end{align*}
$$

(8)

where $z' = zkU_0/2ku_*$, $z_0' = z_0kU_0/2ku_*$ and $K_0$ is the modified Bessel function.

In what follows we will refer to the solution given by (8), the Jackson and Hunt solution, as model A.

(ii) Model B. As $z' \to \infty$ the inner layer solution (8) tends to the outer layer solution at $z = 0$. It follows that if $p_{z=0}$ in Eq. (8) is replaced by $p$ then we obtain a uniformly valid solution. $p$ is a function of height and its use allows the single solution to tend to the outer layer solution for large $z$ and the inner layer solution for small $z$. We will refer to this solution as model B.

(iii) Model C. When the theory is applied to real topography on a scale of order one kilometre, the conditions of the asymptotic theory are not well satisfied. For typical values of $z_0$ (say $10^{-2}$m) and scale of the topography (say $10^3$m) there is appreciable shear of the basic (upstream) velocity profile for a hill of finite size and the selection of a single value of $U_0$ is somewhat arbitrary and erroneous. Also, on the scale of the inner layer, $l$, there may be an appreciable effect due to the exponential decay on the outer layer scale $(k^2 + m^2)^{-1/2}$. These features indicate that the theory should be applied with caution and accurate results should not be expected. Here a careful choice of the scales involved is considered. This approach is similar to one developed independently by Taylor et al. (1983) which came to our attention whilst this work was being prepared for publication. Only the main features of the present approach are presented here and the reader is referred to Taylor et al. for a more detailed consideration of this type of refinement.

The first part of the refinement is to use the uniformly-valid solution, model B. This will allow a combined representation of the inner layer solution and outer layer exponential decay with height. At a height of 8 m it indicates that a disturbance of wavelength 72 m has decayed to 50% of its surface value. The influence of short-scale disturbances is thus considerably reduced from that given by model A.

The second part of the refinement involves the selection of separate advection velocities for use in the outer and inner layer equations. This selection is quite arbitrary and the choice, though sensible, will be justified empirically by comparison with a 'correct' linear solution. The correct solution is obtained by solving the full equations with a finite difference model. In the asymptotic limit $u_*/U_0 \to 0$ the two should agree exactly. It
would be possible to seek a purely empirical representation of the finite difference results but this is not our intention. This approach is consistent with the pressure being continuous but not a proper matching of the velocity perturbation. A formal analysis which resolves this problem has been given by Hunt, Leibovich and Richards (to be published) and should be considered in future work.

The velocity scale in the outer layer was taken to be the velocity $U(z)$ at the wave scale height $z_0 = (k^2 + m^2)^{-1/2}$. Thus for a horizontal wavelength of 1000 m the velocity at 159 m above the surface is used. The velocity at greater heights will not influence the flow on this scale and the choice of a velocity scale equal to the velocity at much lower heights would underestimate the magnitude of the resulting disturbance. This velocity scale can also be used in the inner layer, i.e. solution B with $U_0 = U(z_0)$. This is referred to as model C.

(iv) Model D. The selection of a single velocity scale for the inner layer is difficult. In the ‘upper’ part of the inner layer the stress divergence is small and the dynamics are essentially those of the inviscid outer flow. There is a local balance between advection and the outer layer pressure field. Close to the surface the stress divergence is important and a velocity scale dependent on the height scale of the stress divergence should be appropriate. Here we anticipate application of the model at heights above the region of significant stress divergence and we select a velocity scale equal to the basic boundary layer velocity at the height at which the flow prediction is required. With the outer layer velocity scale as in model C, i.e. $U_0 = U(z_0)$ and the inner layer velocity scale $U_t = U(H)$, where $H$ is the height under consideration, the solutions for $u$ and $v$ become:

$$
u = \left[ \frac{K_0}{K_0} \left( \frac{2(iz^*)^{1/2}}{2(iz^*)^{1/2}} \right) \right] \right\}
$$

where $z^* = z_0kU_1/2ku_\ast$, $z_0^* = z_0kU_1/2ku_\ast$, $p_0 = -k^2(k^2 + m^2)^{-1/2}U_0^2h_0$. We will refer to this as model D.

(b) Comparison of the results from linear theory and finite difference model solutions

To evaluate the performance of the various models described above, their predictions of flow speed-up are compared with that obtained in a finite difference model. This model is described by Mason and King (1984) and in the present application is effectively identical to the model of Taylor (1977). The equations considered are the momentum equations:

$$
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} &= - \frac{\partial p}{\partial x} + f + \frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{13}}{\partial z} \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + w \frac{\partial v}{\partial z} &= - \frac{\partial p}{\partial x} - f + \frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{23}}{\partial z} \\
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} &= - \frac{\partial p}{\partial z} + \frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{33}}{\partial z}
\end{align*}
$$

and the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0.$$
The Reynolds stresses, $\tau_{ij}$, are parametrized by an eddy viscosity, which is itself derived from a mixing length, i.e.

$$
\begin{align*}
\tau_{ij} &= \nu(D_{ij}/D_{kl} + D_{kl}/D_{ij}) \\
\nu &= \frac{l_{m}^2}{\kappa} \left( \frac{D_{ij}/D_{kl} + D_{kl}/D_{ij}}{D_{ij}/D_{kl} + D_{kl}/D_{ij}} \right)^{1/2} \\
l_{m}^{-1} &= \kappa^{-1}(z_n + z_0)^{-1} + l_0^{-1} \\
l_0 &= 0.0004 |U_g| f^{-1}
\end{align*}
$$

where $z_n$ is the normal distance to the surface, $\partial P_y/\partial y$ is the background pressure gradient and $U_g$ is the geostrophic velocity, $-\nu^{-1} \partial P_y/\partial y$. The model uses a terrain-following coordinate system with 32 grid points in the horizontal and 40 in the vertical. The points in the vertical are spaced at logarithmically-varying intervals with the resolution at the surface being $0.1$ m. Those in the horizontal are uniformly spaced. $z_0$ has been taken to equal $3 \times 10^{-3}$ m, $U_g = 10$ m s$^{-1}$ and the depth of the domain fixed at 4 km. When applied to level terrain, the model generates a planetary boundary layer velocity profile. The only coupling between the $u$ and $v$ components of motion is through the Coriolis term. This occurs on a much longer time scale than flow across the hill and only the $u$ component is involved in the local dynamics. The undisturbed profile of $u$ is well represented, between the surface and a height of about 300 m, by $U_g(z) = (r_{1/2}/\kappa) \ln((z + z_0)/z_0)$ where $r_{1/2} = 0.0875$ m$^2$ s$^{-2}$ here) is the component of the surface stress in the $x$ direction. The model has been applied to flow over a periodic undulation $h = a \cos(2\pi x/L)$ where $a/L$ has been fixed at $10^{-3}$ to give a 'linear' solution. Values of $L = 10^2$, $10^2$ and $10^m$ are considered and the speed changes at heights of 1.77, 7.96 and 30-20 m above the highest point of the undulations are given in Table 1. The fractional speed-up factor $\Delta S = (\Delta u/U(H))/(L/2\pi a)$ where $\Delta u$ is the speed increase over the undisturbed flow at a height $H$ above the surface. This has a maximum value of about 2.0 and decreases with decreasing L and increasing z. With smaller L the decrease with z occurs on a shorter vertical scale. For comparison, the various model solutions given above have been evaluated with $u_*$ equal to the value of $r_{1/2}$ found in the finite difference solution and $U_g(z)$ used to evaluate $U_0$ and $U_1$ in models C and D. For models A and B $U_0$ was set equal to the geostrophic speed. Table 2 shows ratios of $\Delta S$ obtained with the various analytic models A–D, to the corresponding values obtained from the finite difference solution.

With the Jackson and Hunt (1975) inner layer solution (model A) there is reasonable agreement when $L = 10^2$ m but for smaller values of $L$ there is a considerable overestimate especially at $z = 33.20$ m. With the uniformly-valid solution (model B) there is much better overall agreement. This demonstrates the importance of the exponential decay with height. The results for $z = 33.20$ m and $L = 10$ m are not shown as the disturbance has essentially vanished. The main error with model B is an overestimate of speed-up on shorter scales. This is improved by the use of model C which uses a smaller value of $U_0$ on these scales. However model C also tends to underestimate the speed-up on the

<table>
<thead>
<tr>
<th>$z$(m)</th>
<th>$U_0$(m s$^{-1}$)</th>
<th>$L = 10$ m</th>
<th>$L = 100$ m</th>
<th>$L = 1000$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.72</td>
<td>4.52</td>
<td>0.377</td>
<td>1.55</td>
<td>2.24</td>
</tr>
<tr>
<td>7.96</td>
<td>5.68</td>
<td>0.031</td>
<td>0.72</td>
<td>1.85</td>
</tr>
<tr>
<td>33.20</td>
<td>6.89</td>
<td>0.006</td>
<td>0.15</td>
<td>1.17</td>
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TABLE 2. RATIOS OF FRACTIONAL SPEED-UP FACTOR FROM VARIOUS MODELS TO THAT OBTAINED BY FINITE DIFFERENCE MODEL

Model A (Equation (8))

<table>
<thead>
<tr>
<th>$z$ (m)</th>
<th>$L= 10$ m</th>
<th>$L= 100$ m</th>
<th>$L= 1000$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.72</td>
<td>5.7</td>
<td>1.26</td>
<td>0.89</td>
</tr>
<tr>
<td>7.96</td>
<td>15.7</td>
<td>2.40</td>
<td>0.95</td>
</tr>
<tr>
<td>33.20</td>
<td>9.82</td>
<td>1.24</td>
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Model B

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</thead>
<tbody>
<tr>
<td>1.72</td>
<td>1.94</td>
<td>1.25</td>
<td>0.88</td>
</tr>
<tr>
<td>7.96</td>
<td>1.07</td>
<td>1.44</td>
<td>0.90</td>
</tr>
<tr>
<td>33.20</td>
<td>1.22</td>
<td>1.01</td>
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Model C

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<th>$z$ (m)</th>
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<th>$L= 100$ m</th>
<th>$L= 1000$ m</th>
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</thead>
<tbody>
<tr>
<td>1.72</td>
<td>0.90</td>
<td>0.79</td>
<td>0.72</td>
</tr>
<tr>
<td>7.96</td>
<td>0.17</td>
<td>0.91</td>
<td>0.72</td>
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<tr>
<td>33.20</td>
<td>0.80</td>
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Model D (Equation (9))

<table>
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<th>$L= 100$ m</th>
<th>$L= 1000$ m</th>
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</thead>
<tbody>
<tr>
<td>1.72</td>
<td>0.89</td>
<td>1.07</td>
<td>1.13</td>
</tr>
<tr>
<td>7.96</td>
<td>0.14</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>33.20</td>
<td>0.70</td>
<td>0.96</td>
<td></td>
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</tbody>
</table>

Taylor et al. (1983)

<table>
<thead>
<tr>
<th>$z$ (m)</th>
<th>$L= 10$ m</th>
<th>$L= 100$ m</th>
<th>$L= 1000$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.72</td>
<td>0.97</td>
<td>1.01</td>
<td>0.85</td>
</tr>
<tr>
<td>7.96</td>
<td>0.14</td>
<td>0.98</td>
<td>0.89</td>
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<tr>
<td>33.20</td>
<td>0.66</td>
<td>0.91</td>
<td></td>
</tr>
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</table>

larger scales. The under-prediction with model C at $z = 7.96$ m and $L = 10$ m was examined in detail. The discrepancy is due to the less-than-exponential rate of decay of the disturbance with height in the finite difference model. Although of interest this feature is of little practical importance as in such cases the disturbance is very weak.

The use of the inner layer advection velocity $U_i$ in model D gives a further improvement. The remaining errors are due to application of asymptotic theory outside its range of validity. Also shown in Table 2 are results obtained using the model of Taylor et al. (1983) (private communication) whose approach contains features similar to model D and performs with similar accuracy. In earlier work such as Mason and Sykes (1979) the topography was generated analytically and dominated by large scales, i.e. $\sim 10^3$ m. For these long scales it can be seen that model A works quite well. The reason for the development of refinements to this model is to allow better predictions in application to real terrain containing shorter-scale features.

(c) The flow structure

Whilst considering the theoretical models it is of interest to see what variations in turbulence structure they predict. Aspects of the changes to turbulence in flow over hills have been discussed in previous work (Britten et al. 1981; Bradley 1980). Above the inner layer of depth $l$, (here, from Eq. (6), $l \sim 25$ m), the turbulence is not expected to respond to local changes in velocity gradient. The time scale of flow over the topography, $L/U_0$, is faster than the adjustment time scale of the boundary layer eddies, $z/u^\star$, and the turbulence is subject to 'rapid distortion' (Hunt 1973) as the flow streamlines converge
and diverge. The analytic and finite difference models presented above are unable to describe such turbulence changes and always assume local equilibrium with the shear. Such local equilibrium should be appropriate near the surface. The only theory which attempts to provide a full description of the turbulence by matching the local equilibrium region to the rapid distortion region is that due to Sykes (1980).

First we examine the perturbation velocity profiles over the summit of periodic sinusoidal topography. Figure 1 shows these perturbations for models A, B and the linear finite difference model in the case with \( L = 10^3 \text{m} \) and other parameters as noted above. The more arbitrary models C and D are not really appropriate in this context of examining flow structure. There is good agreement between the finite difference model and model B. Model A is unable to give the correct behaviour for large \( z \) but agrees up to the height of the maximum perturbation at \( z/L = 5 \times 10^{-3} \). Above this height the perturbation decays in accord with the behaviour of the outer layer and below it the perturbation is reduced by the diffusion effects described by the inner layer equations. We thus see that for these parameters the diffusion effects are small above a height of about 5 m above the surface. This is in accord with discussion concerning the use of model D above. The height scale usually associated with the velocity maximum and the importance of diffusive effects is \( l \). Although this is mathematically correct, for practical applications it is important to note that the solutions imply that most rapid variation will occur on a height scale \( \sim l/2\pi \).

Figure 2 shows the tangential stress perturbations over the summit of the periodic sinusoidal topography for a series of models. With models A and B and the finite difference model the results are for \( L = 10^3 \text{m} \). The curve shown for Sykes's (1980) theory is based on Fig. 3 of his paper. In models A and B these stress perturbations are given by \( \Delta \tau = \kappa u \epsilon (z + z_0) \partial u/\partial z \). In the finite difference model, \( z + z_0 \) is replaced by a length scale derived from Eq. (11) above and for \( k \gamma \) greater than \( l_0 \) the scale is bounded by \( l_0 \). In Sykes's work the stress is obtained from a full 'second-order' closure model using a transport equation for dissipation to determine length scales. Near the surface all the models show an increase in the magnitude of the surface stress. All the analytic models show the same value but the finite difference solution is significantly different. At
Figure 2. Vertical profile of shear stress perturbation over a crest of the surface $h = a \cos(2\pi z/L)$. $z_o/L = 3 \times 10^{-5}$, $\theta_o = 2\pi a/L$, and $\eta_o$ is the square root of the undisturbed surface stress. The solid curve is the linear finite difference calculation obtained with $a/L = 10^{-3}$. The short-dashed line is the Jackson and Hunt inner layer solution—model A and the long-dashed line is the uniformly-valid Jackson and Hunt solution—model B. The dotted curve is the solution due to Sykes (1980) and is redrawn from a section of his contour plot, Fig. 3.

$z/L \sim 5 \times 10^{-3}$ both model B and the finite difference model show a change in sign at the maximum of the velocity profile and above $z/L \sim 5 \times 10^{-3}$ they show a large but different stress reduction. This is due to the decay of the velocity perturbation with height and the difference between the two cases is almost entirely due to the different mixing lengths used. For flow over a horizontal scale less than the value of $l_0$ used in the finite difference model there would be better agreement. As expected, model A gives the 'correct' behaviour only below the height of the velocity perturbation maximum.

The results given by Sykes require more detailed discussion. They were obtained by solution of his model in an inner layer with a lower equilibrium boundary condition at $z/l = 10^{-2}/2\pi$ and an upper rapid distortion boundary condition at $z/l = 10^2/2\pi$, taken asymptotically to be at the base of the outer flow region. When comparing with the other models we must note that the exponential decay on a scale $L/2\pi$ is not included. Very close to the surface, as implied by the boundary condition, the solution agrees with less refined models. However, at $z \sim l/2\pi$ there is a marked difference. This takes a similar form to the stress minimum seen at greater heights in the other models but its origin is quite different. It is not due to the decay of the perturbation with height, but to the matching of the rapid distortion and local equilibrium regions. It is entirely a product of the second-order closure model rather than local velocity gradients. A dynamical explanation of this surprising behaviour has not been offered. At heights of $\sim 10l/2\pi$, Sykes's theory only shows changes due to rapid distortion, in agreement with dynamical expectations. Allowance for the decay of the perturbation with height would simply lead to this rapid distortion disturbance also decaying exponentially with height.
FLOWS OVER AN ISOLATED HILL

3. EXPERIMENTAL CONDITIONS

(a) The field site

The site selected, the hill called Blashaval, is located on the eastern side of the island of North Uist (57°37'N 7°12'W). Figures 3 and 4 illustrate the relief and water cover over the surrounding 5 and 1.5 km respectively. With winds from the south-west quadrant, there is a fairly uniform fetch extending over a distance of about 15 km. The main inhomogeneities in the surface are the many lakes. For winds from the north-west the nearby hills may be expected to exert some influence on the flow over Blashaval and from the east the boundary layer has had little opportunity to adjust to the land. The terrain in the immediate vicinity of Blashaval is more homogeneous. The vegetation is a uniform cover of heather and grass growing in peat. The main small-scale irregularities are peat cuttings and bluffs about 2 m high around the lakes. Although these influences

Figure 3. A topographic map illustrating the relief and nature of the terrain within 5 km of Blashaval. The contour interval is 20 m and areas of water (both sea and inland lakes) are shown shaded.

Figure 4. A more detailed topographic map of the area within 1.5 km of Blashaval. The contour interval is approximately 7.5 m.
cannot be ignored the results suggest that the smoothness of the terrain at the individual measurement sites is more important in allowing interpretable results.

Figure 5 shows the positions selected for the flow measurements. These have been superimposed upon the topography field used in the theoretical model, the derivation of which is discussed below. The positions were selected to be in the regions of most evenly shaped terrain. The site R is used as a reference site and was located on a peat bog extending to the west of the hill. The nearest unwanted disturbance to this site was the lake edge to the west. Figure 6 shows a photograph of Blashaval taken from a point about 2 km north-west of the summit and illustrates the smooth nature of the terrain.

(b) The prevailing weather conditions

The measurements described in this work were obtained during the five-week period 6 September to 8 October 1982. The weather was unusually windy with a great deal of heavy rain. The predominant wind directions were from the south and west but almost all wind directions occurred during the period. Few data were obtained with 8 m wind speeds less than 5 m s⁻¹ and substantial amounts were obtained with speeds greater than 20 m s⁻¹. There was extensive cloud cover and it proved possible to obtain data only under conditions of neutral static stability.

4. Instrumentation and Data Processing

(a) Mean flow measurements

Mean wind speed and direction at 8 m were measured continuously at the twelve sites marked on Fig. 5 using Gill propeller anemometers (R. M. Young Company, Model
27106) with 180 mm diameter, 300 mm pitch polypropylene propellers. Each mast was erected perpendicular to the local slope and carried a pair of these anemometers, mounted orthogonally on a 25 mm diameter vertical extension to the top of the mast. The propellers were staggered by 280 mm vertically, to minimize mutual interference. The directional response of the anemometers on this mount was studied in a wind tunnel and found to be similar to that for a single Gill anemometer as given by the manufacturers, with the addition of sheltering effects due to the mast extension.

The hourly average of the D.C. voltage output of each Gill anemometer was determined by voltage-to-frequency conversion and accumulating the frequency count over one-hour periods. Given the speed calibration and measured directional response of the system, it is possible to convert these figures into an hourly vector average, speed and direction. To a first approximation, the Gill anemometer exhibits a cosine response, that is it measures the component of the wind along its axis. This is used to establish a first estimate of wind speed and direction. An iterative procedure is then used to correct for the known non-cosine directional response of the system.

Whilst the above non-cosine correction would be exact for instantaneous velocities, it is only approximate when applied to hourly averages. The magnitude of the error will depend on the variability of wind direction throughout the one-hour averaging period. Estimates of the error, based on the typical variance of the wind direction suggest that errors should not exceed ±5% (Horst 1973). In order to check the possible magnitude of this error, the Gill system was compared with a cup anemometer (Vector Instruments A100) at various sites on and around the hill. Additionally, two cup anemometer/vane anemograph systems were located about 12 m and 20 m from the Gill systems at the summit and reference sites, respectively, for the duration of the experiment. Upstream of the hill and on the summit, the speeds measured by the two systems agreed well, with maximum differences of ±5% in speed and ±10° in direction. However, in the lee of the hill, the cup anemometer consistently gave about 10% higher readings than the Gill
system. This cannot be explained by errors due to non-cosine response of the Gill system and is consistent with previous observations (Busch and Kristensen 1976) that cup anemometers tend to 'overspeed' in highly turbulent flows. The results of this intercomparison suggest that the errors in speed and direction caused by the non-cosine response of the Gill anemometers are no greater than those inherent in the more conventional cup anemometer and vane system. Our overall estimate of the error in winds derived from the Gill system, taking into account contributions from calibration, sensor alignment and non-cosine response, give a maximum of ±5% in speed and ±5° in direction.

(b) Profile measurements

In order to study the structure of the undisturbed boundary layer, vertical profiles of wind and temperature to a height of 16 m were recorded at site R throughout the experiment. Wind speed was measured at heights of 2, 4, 8 and 16 m using cup anemometers which produced one pulse per revolution. The number of pulses counted during a two-minute interval was recorded on magnetic tape. The wind direction at 16 m was measured with a Vector Instruments W200 wind vane. Additionally, the temperature differences from 1 to 4 m and from 1 to 16 m were measured using platinum resistance thermometers in radiation shields and were also recorded every two minutes.

To extend the wind speed profile to greater heights, a TALA kite (Approach Fish Inc.) was flown from this site on a number of occasions. This system measures wind speed by sensing the tension in the kite line and is accurate to about ±5%. The kite could reach a height of 200 m.

(c) Turbulence measurements

Turbulence measurements were made at reference site R and the summit site S at heights of up to 16 m using instruments mounted on masts. Two types of instruments were used. The first of these was the hot-film turbulence probe described by Mason and King (1984). This comprised two x-probe hot-film sensors (Prosser Scientific Instruments F150W) mounted on a lightweight wind vane which kept the sensors pointing within 10° of the instantaneous wind direction. The sensors were run at constant temperature by electronic servo circuits and their outputs were used to determine the wind speed and the horizontal and vertical inclination of the wind with respect to the vane. The orientation of the vane itself was measured with a precision potentiometer, thus the components of the wind along fixed axes could be computed. The vane also carried a fast response platinum resistance thermometer which was used to provide continuous ambient temperature corrections for the hot-film data.

Two of these instruments were used simultaneously for turbulence measurements, one mounted at 3 m and the other at 14 m. They were mounted 300 mm upwind of a 50 mm diameter mast and were levelled to within 2°. At frequent intervals the instruments were intercompared at a height of about 1.5 m to provide a check on instrument function.

The frequency response of the hot-film instrument is in excess of 100 Hz; the length constant imposed by the separation of the sensors is a few centimetres. Owing to variations in sensor contamination with dust, the mean speeds measured by this instrument may have a systematic error as large as 0.5 m s⁻¹, but this is of little consequence when measuring turbulent fluctuations.

The second system employed for turbulence measurements used sets of three Gill anemometers to measure the components of the wind. On top of the 16 m mast an orthogonal set was used, while at 3 m and 5 m the set consisted of two anemometers pointing 30° upwards with an angle of 60° between their axes and a third anemometer
which bisected this angle and pointed down at an angle of 30°. The advantages of this non-orthogonal arrangement are twofold. Firstly, providing that the assembly is pointed roughly into the mean wind, none of the anemometers is likely to stall. Secondly, the length constant of a Gill anemometer increases with the angle the wind makes with its axis so the non-orthogonal arrangement produces a better response than the orthogonal one. The anemometers were fitted with 190 mm diameter polystyrene propellers, for which the manufacturers quote a length constant of 0.8 m for axial flow.

The hot-film probe was compared with the Gill system and showed good agreement in the measurement of fluxes and variances. Spectral analysis of the results revealed that the instruments had a similar response for length scales greater than about 5 m; for scales shorter than this the Gill system showed attenuated response relative to the hot-film instrument. This lack of high frequency response limited the use of the Gill system to heights of above 5 m. However, this system does have significant advantages over the hot-film unit; it is more robust, has better stability of calibration and, in contrast to the hot-film sensors, is unaffected by rain.

The output voltages of the turbulence measurement systems were sampled 68 times per second and digitized with 12-bit resolution. The digital data were passed to the recording caravan as a serial code transmitted over a single pair of wires. In the caravan, the data were decoded and sorted by a PDP11/34 computer and written to magnetic disk. Both the mean flow and turbulence instruments were calibrated before and after the experiment.

Turbulence data were collected in runs of between 0.5 and 2 hours duration but were analysed in segments of about 800 s to exclude motions on scales greater than about 5 km. The readings from each instrument were converted to orthogonal velocity components \( u, v, w \) (and also temperature, \( T \), in the case of the hot-film instrument). Mean values of the velocity components were calculated and the coordinate frame of the analysis was then rotated so that \( \bar{v} = \bar{w} = 0 \), i.e. the \( x \) axis lies along the mean wind vector and the \( y \) axis remains horizontal. A linear trend was then removed from the time series for each component before the fluxes, variances, power spectra, co-spectra and cross-spectra were calculated. Each run yielded between two and eight analysis periods; ensemble averages were formed from the results of each period.

5. Observations

(a) Upstream velocity profile

The velocity profile at the upstream site \( R \) provides an important indication of the equilibrium of the lower part of the boundary layer and a measure of the roughness length \( z_0 \), which is needed in the application of the theoretical models. Although site \( R \) was located in a region of very uniform and level terrain, as discussed below the mast was situated on the edge of a gentle slope with a drop of 0.5 m over a distance of about 10 m. The surrounding terrain was soft peat bog and mast erection was possible only on the firm peat occurring at the top of this slope.

The observations obtained from the profile mast were processed to give 1-hour averages of the wind speed at each level and the wind direction at 16 m. It was observed that provided that the 8 m wind speed was greater than 5 m s\(^{-1}\), the ratios of the wind speeds at various heights were independent of wind speed. For wind speeds less than 5 m s\(^{-1}\) the temperature sensors on the mast often indicated significant gradients, but at greater speeds the temperature gradients were insignificant (differences between levels <0.1 K) confirming that stability effects were negligible.
The hourly average wind speeds, for 8 m speeds greater than 5 m s\(^{-1}\), have been grouped into 30° sectors of wind direction and the average ratio of the wind speed at each height to the 16 m wind calculated. Figure 7 displays the resulting ratios. When the wind direction is 80° the wind is blowing directly from the hill towards site R. With flow from this direction the local slope is in the lee of the mast and the changes in velocity profile seen between 60° and 100° are attributed to the influence of the wake of the hill. The exact extent of the influence of the hill on the flow at site R is difficult to deduce as there were no other reference winds. This difficulty is discussed further below. The other distortion to the velocity profile occurs with a wind direction of about 200°. With flow from this direction the wind fetch is ideal but the local slope is just ahead of the mast. For other wind directions, particularly the sector from 230° through 360° to 50°, the profile is very close to logarithmic with a value of \(z_0\) of 0.01 m. The horizontal lines denote the values of flow speed ratio at the measurement heights corresponding to this value of \(z_0\). As noted below, the eddy correlation stress measurements also support this value of \(z_0\).

(b) **Application of linear theory to real terrain**

To apply the analytic solutions to the field site, the topography must be represented numerically. The United Kingdom Ordnance Survey map on a scale of 1:25000 was enlarged photographically, digitized with a manual curve-following digitizer and then transferred to a regular Cartesian mesh using an objective analysis scheme. The topography of interest was confined to a mesh region of about 80×80 points located within a total domain of 256×256 points. The perimeter of the 80×80 domain was not all at the same elevation. To remove this one-grid-length disturbance in the field, the elevation over the remainder of the region was set equal to the average of that on the perimeter of the 80×80 points and a smoothing filter was applied outwards of 70×70 points. Within the remainder of the region a slight smoothing was also applied as it seemed that the manual digitization was a source of some errors. The topographic field generated is shown in Fig. 5 and should be compared with the original map contours shown in Fig.

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Figure 7. Wind speed profiles at the reference site, as a function of wind direction. O: 8 m, \(\triangle\): 4 m, \(\times\): 2 m. The wind speed at each level has been normalized by that at 16 m and the data have been averaged over 30° sections of wind direction. Standard errors are less than the size of the plotted symbols. The horizontal lines show the expected values at each level for a logarithmic profile with \(z_0 = 0.01\) m.
4. The contour-drawing routine used to create Fig. 5 employs no smoothing and lines are placed purely by linear interpolation within grid squares.

With the topography of interest in the centre of this large domain of 256×256 grid points we are able to assume periodicity of solutions without the risk of interference effects from the implied periodic array of hills. This field of elevation on 256×256 points was then transformed into Fourier components by a two-dimensional fast Fourier transform. The analytic solution given in Eq. (9), model D, was applied to each wavenumber in turn and the required velocity perturbation obtained by an inverse Fourier transform. In section 5(c) the model results will be compared with field data. In order to facilitate the comparison, all model results shown here are for a height of 8 m above the surface. To compare with observations, predictions were extracted at the locations corresponding to the instrument sites. These usually fall between mesh points, and interpolation was used to give exact values. The velocity profile used to derive \( U_l \) and \( U_0 \) was given by:

\[
U(z)/U(8) = \left( C_D^2(8)/k \right) \ln\left( (z + z_0)/z_0 \right)
\]

where \( U(8) \) is the speed at 8 m and \( C_D(8) \) and \( z_0 \) were based on observations at the field site, i.e. \( C_D(8) = u_2^2/U^2(8) = 3.58 \times 10^{-3} \) and \( z_0 = 0.01 \) m. Figures 8(a) and (b) show examples of the \( u \) and \( v \) field perturbation for a single wind direction.

Results obtained with the other models were less satisfactory. Those from model A showed a lot of short-scale features, whilst those from models B and C underestimated the perturbation magnitude.

(c) Mean flow observations at 8 m

Observations of the 1-hour averaged flow at each of the sites marked on Fig. 5 were made with the vector mean recording system described above. Each site was instrumented continuously throughout the experiment and the wind speed ratios and direction changes at each site relative to site R were calculated for each hour. In accord with the profile mast results, these data showed no significant dependence on the wind speed for 8 m speeds at site R, \( U_R \), greater than 5 m s\(^{-1}\). The data for speeds much less than 5 m s\(^{-1}\) were limited and subject to large errors due to the variability of light winds.

For a particular orientation of the local topographic slope relative to the upstream wind direction the results obtained from many of the sites proved very similar and suggest that the hill behaves as a nearly axisymmetric feature. Figure 9 presents the results for a representative selection of sites. The difference in speed, \( \Delta = U/U_R \), and direction, \( \theta - \theta_R \), relative to the reference site are shown in each case. The curves were obtained by drawing through the average values found in each 10° sector. For comparison with the observations, the solid curve shows the results obtained from the application of theoretical model D to the digitized terrain.

In the model results the flow at site R is predicted to be within about 1% of the undisturbed speed and within 1° to the undisturbed direction for all wind directions. In reality the topography probably has a greater influence on the flow at site R. At site A there is observed to be a speed reduction of about 50% in the wake of the hill and it seems most unlikely that this will have decayed to a negligible value at site R. This means that for flow directions close to 80° there is some uncertainty in the reference wind speed and speed changes may be exaggerated by perhaps as much as 25%. The effect on the results for sites on the side of the hill must be slight as sites C, G and H all give similar results. The results from the summit do not suggest any significantly anomalous behaviour with a wind direction of 80°. For the summit, site S, Fig. 9 shows the observations obtained with the two types of anemometer. The propeller anemometer system and the
Figure 8. Contours of velocity perturbations at a height of 8 m above the surface obtained by the application of model D (see text) to the digitized terrain contoured in Fig. 3. Figure 8(a) shows the streamwise component of the perturbations and (b) the transverse component. The contour intervals are 0.061 and 0.025 respectively where a value of unity corresponds to the undisturbed flow speed at 8 m. Negative contours are shown dashed. The basic wind direction and map orientation are indicated. For clarity, contours are presented in a domain 2.5 km square rather than the actual 6.7 km square domain used for the calculation.
cup anemometer and vane system were mounted on separate masts about 12 m apart. The values of $U/U_R$ are seen to agree very well with each other, especially in the range of directions from about $120^\circ$ to $330^\circ$. The behaviour from $30^\circ$ to $120^\circ$ suggests that local terrain effects may be causing a difference between the two mast sites. Such small-scale features are not represented in the digitized terrain and must contribute to the differences between the observed speed ratio and that given by theory.

In general the agreement between the observed curves and the model predictions is quite good but for individual hours of data the comparison is not so favourable. The vertical error bar indicates the typical peak-to-peak scatter of the individual hourly

Figure 9. Measurements of 8 m wind speed and direction at the summit site, S, and three other selected sites. Wind speeds have been scaled by that measured at the reference site, $U_R$, and directions are expressed as differences from that measured at site R. The solid lines are predictions from model D; broken lines are measurements, averaged over $10^\circ$ intervals, of wind direction. Error bars show the typical scatter of the observations. On the plot for wind speed at site S, results are shown from both the Gill anemometer system (. . . . . .) and the cup anemometer system (-----).
observations. This scatter is about ±10% in speed and can be compared with a scatter of ±5% observed between the two instruments separated by 20 m at site R. Roughly 50% of the observations fall within half the extent of the error bar. The size of the scatter does not seem to depend on the separation of the sites from the reference site R but it is smallest for all sites on the side of the hill. This suggests that the scatter arises from variations in the speed-up caused by the hill. The most likely cause of such variations is changes in the upstream wind profile with height. Unfortunately this wind profile was measured on three occasions only, and not with great precision. The measurements were made with the TALA kite system described above and suggested a logarithmic profile extending up to 200 m above the surface. On these occasions the individual observations of surface flow at 8 m were close to the overall average values. Bradley's (1980) observations of speed-up over the summit of Black Mountain show a scatter of about ±20% with 30 min average winds. He attributes this scatter to statistical variations between the summit site and the reference site 800 m away. The present results suggest his scatter may also be due to variations in the boundary layer velocity profile and structure.

On the side of the hill all of the measurement sites gave a similar pattern of flow. This is well illustrated by the data from sites B and C in Fig. 9. When the flow is up the slope or around the side of the hill the flow accords with model expectations. The agreement at site C is very good for these flow directions but data from site B show a change in amplitude of disturbance. It is not clear whether this sort of discrepancy is real or due to the limited accuracy of the site survey, which only provided a position accuracy of about 20 m. Movement of the site position up and down the slope by this amount could account for the discrepancy. An obvious feature of the comparison between

![Graph](image)

Figure 10. Calculated fractional speed changes 8 m above the surface \( h = a \cos^2(x \pi/L) \) for \(|x/L| \leq 0.5 \) and \( h = 0 \) for \(|x/L| > 0.5 \) where \( a = 108 \) m and \( L = 750 \) m. For clarity only the central one half of the domain is shown. The hill position and shape (with a vertical exaggeration of 2:2 to 1) are indicated. A value of unity indicates that the flow speed at 8 m is doubled. The solid and dashed curves are obtained with \( z_0 = 0.01 \) m. The solid curve is a nonlinear finite difference solution whilst the dashed curve is the linear finite difference solution (with \( a/L = 1.44 \times 10^{-4} \)) scaled to the above values. The dotted curve is the nonlinear finite difference solution with \( z_0 = 0.1 \) m.
observations and model results is the poor agreement in the lee of the hill. In the wake of the hill the observations show a reduction of flow speed to nearly zero with wind direction changes of nearly 180° demonstrating that flow separation occurs. In contrast, the model gives a flow reduction of only about 25%. The origin of this discrepancy is not clear and needs to be discussed in detail.

Figure 10 shows various predictions of flow speed over a two-dimensional, cosine-squared-shaped ridge on the same scale as that typical of Blashaval (see legend for details). The peak slope of the hill is 24° and equal to the peak slope occurring in the digital representation of the Blashaval topography. Mason and Sykes (1979), using the Jackson and Hunt theory, found that the speed reduction in the wake of a 2-D ridge was about 10% greater than that in the wake of a circular hill with the same section. The ridge used in the present example would thus be expected to give an overestimate of any separation effects occurring with a circular hill. The dashed curve in Fig. 10 is based on the linear finite difference solution obtained with a peak slope of 0·024°. The resulting perturbation has then been multiplied by 10^2 to give a result applicable to the 24° slope; namely a speed-up of about 2·2 and a flow reduction in the lee of about 0·5. The solid curve shows the nonlinear finite difference solution for these parameters. While there is a slightly greater flow reduction in the lee the effects of nonlinearity are surprisingly small and unable to account for the observed flow. In both these cases the value of z_0 was taken to be the value observed at site R, i.e. 0·01 m. Larger values of z_0 reduce the flow speed near the surface and favour flow separation (Mason and King 1985). The dotted curve shows the nonlinear finite difference solution with z_0 increased to 0·1 m. In this case a flow separation occurs in a small region extending up to 5 m from the surface. However, 0·1 m is a gross overestimate of z_0 and its use in model D would result in other velocity changes being overestimated by about 40%. It thus seems that neither nonlinearity nor any reasonable error in estimating z_0 can account for the observed flow separation. Further work is needed to understand this effect and determine its causes. Mechanisms which should be considered are the triggering of separation by small-scale sharp features and the role of the details of the stress changes. A similar discrepancy is observed in the wind tunnel study of Britter et al. (1981) who suggest that an improved turbulence model is needed.

(d) Turbulence measurements

On the occasions when turbulence data were collected at sites R and S, the wind speed U_R had a value between about 6 and 12 m s\(^{-1}\). Neutral static stability was confirmed by direct measurements of the Monin-Obukhov length L = \(-u'_z (kgwT/T)\)\(^{-1}\) (where g is the acceleration due to gravity and k the von Kármán constant). The measured values of L at 14 m were in the range -100 to -4000 m. The range of wind directions was between 180° and 250° and it is appropriate to group all the data together. The turbulence statistics obtained from both sites are summarized in Fig. 11. Data collection at site S was not always simultaneous with that at site R and to effect a comparison between the sites, U_R has been used to normalize the results.

At the upstream site R, u^2 and v^2 are independent of height but \(\overline{w^2}\) and \(\overline{uw}\) show a slight increase with height. This increase is barely significant and may be due to the local slope at this site. The error bars shown are standard errors indicating the scatter of the individual 800 s estimates (about 20 estimates for most points). The values of \(\overline{uw}\) obtained correspond to a value of z_0 of 0·01 ±0·005 m which is in good agreement with the profile mast measurements. The stress ratios \(\alpha_u/u_*\), \(\alpha_v/u_*\) and \(\alpha_w/u_*\) are 2·4, 1·9 and 1·2 respectively at 14 m and in good agreement with previous work (Smith 1975).

The summit data show a number of significant changes. The data points are based
on nearly equal numbers of observations taken from two sites, nominally at the summit but separated by about 30 m to identify any large local effects due to terrain irregularities. The summit region had irregularities of a height about 0.3 m and length scale about 2 m, which would probably exert some influence on the turbulence structure at 1.5 m. Although the data from both sites agreed, the results at 1.5 m should be treated with caution. To explain the changes in turbulence, we may compare with Sykes's (1980) theory and simple theoretical ideas.

Very close to the surface the wind profile should take the usual logarithmic form and the surface stress should correspond to values calculated from the roughness length and the speed at a particular height. The speed at 8 m is above this equilibrium region and cannot be used to give a reliable estimate of surface stress. The various theories provide a more reliable estimate; Fig. 12 shows the stress changes over the summit of a 2-D cosine-squared hill given by the finite difference model. The parameters are the same as with the $z_0 = 0.01$ m cases in Fig. 10 but the hill diameter has been increased to give a peak slope of 18.7°. Above the summit this gives a similar speed-up to that seen in the observations. The solid curve is the nonlinear finite difference solution and the dashed curve a linear finite difference calculation. The surface stress is increased over its undisturbed value by factors of 3.5 and 2.7 respectively. In both cases these values are only reached at heights below about 1 m. In the present observations the increases in $w^2$ and $uw$ at 1.5 m are by factors of 1.4 and 1.6 respectively, relative to their upstream values, and less than would be expected for local equilibrium with the increase in wind speed. The increase in $u^2$ is 3.3 whilst that in $v^2$ is 5.2 and greater than expected. Owing to the possible uncertainty in data from 1.5 m, we are reluctant to place too much emphasis on these results and suggest that further work, over terrain smooth enough to allow reliable measurements to be made at 0.5 m, is needed.

Theoretical considerations suggest that the region between 3 and 5 m ($z \sim 1/2\pi$) lies between the equilibrium layer near the surface and the rapid distortion region above. The observations show that the shear stress in this region is reduced; at 5 m it is 50% of the upstream value. This reduction is greater than that given by the mixing length theory (see Fig. 12), but less than expectations based on Sykes's solution for sinusoidal terrain.
Sykes's solution suggests a 100% stress reduction, and the neglect of nonlinearity in the stress equation may account for the discrepancy. Of the normal stress components, only $\overline{w^2}$ shows any significant minimum and reduces to about the upstream value. $\overline{u^2}$ and $\overline{v^2}$ only show a return towards their upstream values. The mixing length theories imply a fixed ratio of normal stress to tangential stress and cannot account for this behaviour. Sykes's model also gives values close to such fixed ratios. This is at variance with the observations but may, once again, be due to the neglect of nonlinearity rather than the failure of Sykes's theory.

At 16 m, in accord with both theoretical expectations and Sykes's model, the changes are definitely not in local equilibrium and can be compared, albeit rather loosely, with rapid distortion calculations. Rapid distortion theory (e.g. Townsend 1976, p. 76) allows calculation of normal stress changes for the rapid contraction and shear of initially isotropic homogeneous turbulence. Here the upstream turbulence is not isotropic or homogeneous but the success of such calculation in other studies of boundary layer flow over hills (Britten et al. 1981) encourages its use. For flow over a two-dimensional ridge the main effect is a plane straining of the flow so that contraction ratios in the $u, v, w$ directions are $(S, 0, S^{-1})$ where $S$ is the speed-up. For flow over a circular hill the $v$ perturbation has maximum value of about one half of the $u$ perturbation and the expected contraction ratios are $\{S, 1 + \frac{1}{2} \Delta S, S^{-1}(1 + \frac{1}{2} \Delta S)^{-1}\}$ where $\Delta S = S - 1$. This corresponds to a plane strain of $S(1 + \frac{1}{2} \Delta S)^{1/2}$ in the $(u, w)$ plane and an axisymmetric contraction of $1 + \frac{1}{2} \Delta S$ about the $v$ axis. This is approximate but adequate to indicate expected changes.

With the above distortions the turbulence changes can be calculated according to formulae given by Townsend. The observed speed-up at 14 m is about 1.6 and the stress components ($\overline{u^2}, \overline{v^2}, \overline{w^2}$) are expected to change by factors of 0.73, 1.22, 1.43 relative to upstream values for 2-D flow and by factors of 0.82, 1.05, 1.74 for 3-D flow. The observed changes in $\overline{u^2}$ and $\overline{v^2}$ are slight, while $\overline{w^2}$ is observed to increase by a factor of 1.6. These changes

Figure 12. Calculated shear stress changes over the summit of the surface $h = a \cos^2(x \pi/L)$ for $|x/L| \leq 0.5$ and $h = 0$ for $|x/L| > 0.5$ where $a = 108$ m and $L = 1000$ m. $\theta_0 = \pi a/L$ and the value of undisturbed stress is indicated on the graph. $\epsilon_0/L = 10^{-4}$. The solid curve is a nonlinear finite difference solution whilst the dashed curve is the linear finite difference solution (with $a/L = 1.08 \times 10^{-4}$) scaled to the above values.
are in reasonable agreement with the 3-D rapid distortion estimate and suggest that rapid distortion is dominant in determining turbulence changes at 14 m.

These results can be compared with Bradley's (1980) observations over Black Mountain, which has a typical slope of 20° and a base diameter of about 1000 m. The main difference from Blashavall is almost total tree cover and a value of $z_0$ of about 1.1 m. The speed-up over the summit is about 2.1 and consistent with speed-up increasing as $z_0$ is increased (see Fig. 10). Owing to this high value of $z_0$, a value of $f$ for Black Mountain is ~50 m (cf. ~16 m for Blashavall). Bradley's observations at 9 m should thus relate to the Blashavall observations at about 3 m. In fact at 9 m the Black Mountain results suggest the flow is close to local equilibrium with the surface; all the stresses are increased by a factor of about 4 relative to their upstream values. This is at variance with the present observations at 3 m. However, it is possible that Bradley's results at 9 m are influenced by the surrounding tree cover. At 80 m Bradley finds $u^2$, $v^3$ and $w^2$ to be increased over upstream values by 1.2, 1.4 and 2.7 respectively. The above rapid distortion calculation for a 3-D hill with a speed-up of 2.1 gives increases of 0.99, 1.26 and 2.44, which, apart from a slight general underestimation, seem to be in good agreement with observation. Although Bradley obtains a greater speed-up than that seen at Blashavall, his shear stress, $uv$, changes are less; he observed a minimum of about 0.8 of the upstream value at about 40 m. The qualitative agreement between the present study and Bradley's observations is encouraging but it is evident that an improved theoretical interpretation is needed to resolve quantitative differences.

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**Figure 13.** Turbulence power spectra, obtained using the hot-film instruments. (a) Site R, at 3 m. (b) Site R, at 14 m. (c) Site S, at 3 m. (d) Site S, at 14 m. The ordinate is spectral density, $nS(n)$, scaled by stress, $U_1^2$. The abscissa is wavelength, $\lambda$, calculated from frequency, $n$, using Taylor's hypothesis, $\lambda = U/n$. 
Spectra of the turbulence energy components at sites R and S are shown in Fig. 13. Figures 13(a) and (b) show the spectra at the upstream site at 3 and 14 m respectively and (c) and (d) show spectra at the same heights over the summit. The spectra at the upstream site are in good agreement with those found in previous work over level terrain (Kaimal et al. 1972) and are illustrated to contrast with those on the summit. At 3 m above the summit the spectrum is very similar to that obtained upstream. The only significant change is an increase in $w^2$ on the longest scales. At 14 m the $u$ and $v$ spectra remain similar to those found upstream but that for $w$ shows a marked change. There is an obvious increase in energy and shift of the peak of the spectrum towards longer scales. It is hard to quantify the shift but judgement by eye suggests a factor of about two. These changes in the low wavenumbers are qualitatively those expected from rapid distortion of the turbulence, i.e. length scales are increased (in the $u$ direction) with extra energy appearing in the $w$ component. At higher wavenumbers the flow adjusts too fast for a ‘rapid’ response and the turbulence tends to isotropy and adjustment to the underlying surface.

6. CONCLUSIONS

The linear analytic theory of Jackson and Hunt (1975) has been compared with finite difference calculations of planetary boundary layer flow over ridges with small slopes. In accord with the studies of Walmsley et al. (1982) and Taylor et al. (1983), the use of a uniformly-valid solution and careful choice of velocity scales is shown to improve the range of agreement. The velocity scales found to be most appropriate for application to real terrain are based on the assumption that diffusive effects are negligible. The results show this to be a good assumption for heights greater than $\sim l/2\pi$ from the surface (where $l$ is the Jackson and Hunt inner layer scale given by Eq. (6)).

Comparison between the linear theory and observations shows that for topographic slopes of $\sim 24^\circ$ the theory gives good estimates of the flow speed increases. However, both the linear theory and nonlinear finite difference models (with mixing-length turbulence parametrization) fail to predict the extent of the velocity reduction in the lee of the topography.

The turbulence observations over the summit show that in contrast to previous suggestions (e.g. Hunt and Simpson 1982) the equilibrium region very close to the surface does not simply match with the rapid distortion dynamics region occurring at height $\approx l$. At heights of order $l/2\pi$ it is evident that there is a varied response of the stress components. In particular only the shear stress $uw$ and $w^2$ show a minimum. In previous work (Bradley 1980) only $uw$ has shown a minimum. As noted by Sykes (1980) the turbulence dynamics at $z \sim l/2\pi$ are complex and involve all the terms included in second-order closure models. The models using a simple mixing length closure show a tendency for the stress at $z = l/2\pi$ to reduce in response to the reduced velocity shear, but imply corresponding changes in normal stresses. Sykes's model also gives a large stress reduction but does not (at $z = l/2\pi$) show better agreement with observations. The observations correspond to a large amplitude change and should not be taken as a test of Sykes's linear theory. In agreement with wind tunnel observations (e.g. Britter et al. 1981) and Sykes's theory, simple rapid distortion calculations give a good estimate of normal stress component changes at heights $\approx l$.

In summary it is encouraging that the present study has tended to confirm some of the indications and predictions of previous work. However, the behaviour of stresses at heights $\sim l/2\pi$ and the flow structure in the lee of the hill have not been accounted for, and further work is required in these areas.
ACKNOWLEDGMENTS

We would like to thank the staff of the Boundary Layer Research branch (both at the Meteorological Research Unit, Cardington and the Meteorological Office, Bracknell) for their participation in the field experiment and for working under weather conditions which were often far from pleasant. Our thanks are due to Dr A. J. Lapworth and Ms H. M. Schrecker for applying the Jackson and Hunt model to the field site terrain. Finally we must express our gratitude to the crofters of Blashaval for allowing us to use the field site.

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