A fast method for calculating scale-dependent photochemical acceleration in dynamical models of the stratosphere

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SUMMARY

It is shown that photochemical acceleration of the thermal relaxation rate in the stratosphere may be expressed as a matrix multiplication between a square matrix, which may be calculated for quite general conditions, and a vector representing the deviation of temperature from an equilibrium profile. Such a matrix is presented and its form discussed. Use of this matrix allows for temperature perturbations of any vertical scale and thus provides an accurate, as well as fast, method for calculating photochemical acceleration suitable for use in numerical models of stratospheric dynamics. The inclusion of the ozone 9-6 μm band into the heating rate calculations is shown to reduce the photochemical relaxation rate in the upper stratosphere.

1. INTRODUCTION

In the stratosphere the thermal structure and ozone concentration are strongly interdependent. The heating of the atmosphere due to absorption of solar ultraviolet radiation depends on the amount of ozone present but the chemistry of the region acts so as to make the ozone concentration, and hence the heating, inversely correlated with temperature. Because it increases the rate at which a temperature perturbation will relax back to the equilibrium state this effect, first discussed by Craig and Ohring (1958), has become known as 'photochemical acceleration'.

Expanding the solar heating rate \( Q(T, z) \) (dependence on latitude and season implied, \( z = -\log(\text{pressure}) \)) around an equilibrium temperature \( T_e \):

\[
Q(T, z) = Q(T_e, z) + (\partial Q/\partial T)_{T_e} (T(z) - T_e(z))
\]

which may be rewritten

\[
\delta Q(z) = -\alpha(z) \delta T(z)
\]

where \( \delta Q = Q(T, z) - Q(T_e, z) \), \( \delta T = T - T_e \), and the photochemical relaxation coefficient, \( \alpha \), is defined by

\[
\alpha(z) = - (\partial Q/\partial T)_{T_e}.
\]

Numerical models of stratospheric dynamics often do not include photochemical schemes and use fixed ozone profiles to calculate heating rates. Photochemical acceleration is then represented by an additional term of the form of Eq. (2) in the thermodynamic equation.

In writing Eq. (1) it has been assumed that the change in \( Q \) at a particular height depends only on the temperature (and thus ozone) perturbation at the same height. As discussed by Strobel (1977) and Hartmann (1978), however, a change in ozone at a higher level can affect \( Q \) by modifying the flux of radiation reaching level \( z \). In this paper a simple method of including these opacity effects for perturbations of any vertical scale is described.

2. THEORY

The solar flux at altitude \( z \) is given by

\[
F(z) = F(\infty) \exp\left\{ - \int_z^\infty \left[ \sigma_3 p_3(z') + \sigma_2 p_2(z') \right] dz' \right\}
\]

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where $p_3, p_2$ are the partial pressures and $\sigma_3, \sigma_2$ the absorption coefficients of O$_3$, O$_2$. The solar heating rate in degrees/day is given by

$$Q(z) = \frac{g}{c_p, p(z)} \frac{1}{\partial z} \frac{\partial F(z)}{\partial z} = \frac{g}{c_p, p(z)} \frac{\sigma_3 p_3(z)}{\partial F(z)}.$$

The integration over wavelength and zenith angle has been ignored here for simplicity. The change in heating rate $\delta Q$ in response to a change in ozone concentration $\delta p_3$ is given by

$$\delta Q(z) = \frac{g}{c_p, p(z)} \frac{\partial}{\partial z} \{\delta F(z)\} = -\frac{g}{c_p, p(z)} \int_z^\infty \sigma_3 \delta p_3(z') dz'. \tag{4}$$

The first term on the right-hand side of Eq. (4) represents changes in $Q$ due to local changes in $O_3$. If it is assumed that changes in $O_3$ result only from local changes in temperature then this term can be regarded as the conventional expression for photochemical acceleration. Under these circumstances

$$-\alpha(z) \delta T(z) = Q(z) \frac{\delta p_3(z)}{p_3(z)} = \frac{Q(z)}{p_3(z)} \frac{\delta p_3(z)}{\delta T(z)} \delta T(z)$$

i.e. $\alpha(z) = -Q(z)(\partial \ln p_3/\partial T) = Q(z)C(z)/T(z)^2$ where $C(z) = \partial \ln p_3/\partial T^{-1}$ is the O$_3$ temperature coefficient (see for example, Barnett et al. 1975) and would often be assumed independent of height.

The second term on the right-hand side of Eq. (4) represents the opacity effects mentioned in section 1. Hartmann (1978) found that for a uniform (with height) ozone perturbation the opacity term reduced the calculated value of $\alpha$ by over 30% at 40 km. Strobel (1979) has produced a parametrization for $\alpha$ including opacity effects for large vertical scale perturbations as a function of ozone column density.

3. Matrix method

The equilibrium ozone concentration may be expressed (see for example, Barnett et al. 1975) as

$$p_{3e} = a(z)k(z)J(z)^\beta$$

where $a(z)$ is a temperature-independent coefficient, $k(z)$ represents the temperature dependence of the reaction rates, $J(z)$ is the ratio of the photodissociation coefficient of O$_3$ to that of O$_2$, and $\beta$ is a fraction lying between 0.5 (when the chemistry is dominated by Chapman reactions) and 1.0 (when other O$_3$ destruction reactions dominate). Thus

$$\delta p_3/p_3 = \delta k/k + \beta \delta J/J.$$

Now

$$\delta k/k = -\{C(z)/T(z)^2\} \delta T(z) = -\gamma(z) \delta T(z)$$

(see section 2) and, neglecting scattering,

$$J(z) = J(\infty) \exp \left[ -\int_z^\infty \{\sigma_3 p_3(z') + \sigma_2 p_2(z')\} dz' \right]$$
\[ \delta J(z)/J(z) = -\sigma_3 \int_z^\infty \delta p_3(z')dz'. \]

Thus
\[ \delta p_3(z)/p_3(z) = -\gamma(z)\delta T(z) - \beta \sigma_3 \int_z^\infty \delta p_3(z')dz'. \]  \hspace{1cm} (5)

Inserting this expression into Eq. (4) we obtain
\[ \delta Q(z)/Q(z) = -\gamma(z)\delta T(z) - (1 + \beta) \sigma_3 \int_z^\infty \delta p_3(z')dz'. \]

Equation (5) may be used repeatedly for \( \delta p_3 \). In finite difference form (using a first-order mid-point scheme):
\[ \delta Q_i/Q_i = -\gamma_i \delta T_i - \]
\[ -(1 + \beta) \sigma_3 \sum_{j=1/2}^N \Delta z_j p_{3j} \left\{ -\gamma_j \delta T_j - \beta \sigma_3 \sum_{k=j+1/2}^N \Delta z_k p_{3k}( -\gamma_k \delta T_k - \ldots ) \right\} \]

where \( N \) is the top level, \ldots indicates additional terms until the lower limit of the summation is \( N \), and \( \Sigma_{1/2} \) indicates that the first term in the series should be halved.

Now assume \( \Delta z \), the thickness of an element, is constant and define
\[ x_i = \gamma_i p_{3i} \delta T_i \quad \text{and} \quad t_i = \sigma_3 p_{3i} \Delta z \]
then
\[ p_{3i} \delta Q_i/Q_i = -x_i + (1 + \beta) t_i \{ a \} /1 - \Gamma + \Gamma^2 - \ldots + (-\Gamma)^{N-i} x \]  \hspace{1cm} (6)

where \( x \) is a vector with \( N \) elements \( x_i \), \( \{ a \} \) is a row matrix whose first \( N-i \) elements are 1, next element is \( \frac{1}{2} \) and remaining \( (i-1) \) elements are zero, \( I \) is the \( N \times N \) unit matrix, and \( \Gamma \) is an \( N \times N \) matrix whose \( j \)th row is \( \beta t_j \{ a \} \).

Note that higher powers of \( \Gamma \) imply higher orders of element optical depth \( t \). If \( t \) were greater than unity, amplification, rather than damping, of the temperature perturbation could occur. That the value of \( t \) will, however, be less than unity can be deduced by considering the ozone absorption at the wavelength responsible for most absorption at a given height. For example at 25 km (40 km) where the ozone partial pressure is about \( 1.5 \times 10^{-7} \) atm \( (3 \times 10^{-9} \) atm \) strongest absorption occurs at about 310 nm (295 nm) for which the absorption coefficient is approximately \( 2 \times 10^6 \) atm \(^{-1} \) \( (1.5 \times 10^7 \) atm \(^{-1} \)) giving a value for \( t \) of \( 0.3 \Delta z \) \( (0.4 \Delta z) \) which is less than unity for \( \Delta z \) less than about three pressure scale heights, i.e. 22 km. Equation (6) may be written in matrix form
\[ \delta Q = -A \delta T \]  \hspace{1cm} (7)

which relates the change in solar heating at all heights to the temperature perturbation at all heights. \( A \) is the equivalent of the relaxation parameter \( a \). Its diagonal elements correspond to an expression for \( a(z) \) of the form
\[ Q(z)C(z)/T(z)^2 - \text{opacity effects for ozone in element thickness} \frac{1}{2} \Delta z \text{ above } z. \]

The below-diagonal elements of \( A \) contain sums of powers of element optical depth and account for the opacity effect of ozone perturbations above \( z \). In the case of no scattering
the above-diagonal elements of $A$ are zero but if radiation backscattered from the atmosphere below $z$ contributes to the heating rate and photodissociation coefficient at $z$ then this need not remain the case. Thus, it should be possible to construct $A$ for a given latitude, season and $T_e$ and use it to calculate $\delta Q$ for any other temperature structure. In section 5 the construction of such a matrix will be described and in section 7 a method suggested whereby the matrix can be made more generally applicable by reducing its latitudinal and seasonal dependence.

4. RADIATIVE-PHOTOCHEMICAL EQUILIBRIUM

The radiative equilibrium temperature $T_e(z, \lambda, t)$ is attained by an atmosphere in which heating by the absorption of solar radiation and cooling by longwave radiative transfer are balanced. Although this is a hypothetical state much of the middle atmosphere maintains temperatures close to the local $T_e$. To calculate $T_e$ the expression for radiative balance, $dT/dt = Q(T, p, \lambda, z, t) + R(T, \lambda, z, t) = 0$ (where $R$ represents heating by longwave radiative transfer), must be inverted to find $T$. This is usually done by fixing the surface temperature and iterating between $T$ and $R$ (or $Q + R$ if $Q$ is allowed to be a function of $T$) until $Q + R = 0$. The ozone concentration may or may not be allowed to respond. The use of different schemes for calculating $R$ gives a wide range of values for $T_e$ (see Leovy 1964a; Apruzese et al. 1982; Geller 1983; Fels and Schwarzkopf (as quoted by Mahlman and Umscheid 1984)).

At the summer pole, for example, there is a difference of about 10 km in the height of the stratopause between the results of Apruzese et al. and Geller and a difference of perhaps 25 K in its temperature between Apruzese et al. and Fels and Schwarzkopf. At the equatorial stratopause temperatures range from under 270 K (Fels and Schwarzkopf) to about 285 K (Leovy), while at 60° latitude in the winter hemisphere values span nearly 50 K from about 200 K (Fels and Schwarzkopf) to over 240 K (Leovy). These at first, perhaps, surprisingly large differences can, of course, be explained in terms of the various approximations made in the radiative transfer equation and assumptions about the ozone distribution. If, however, the meridional circulation of the middle atmosphere is to be seen as in some sense balancing the deviation of the temperature from the local equilibrium temperature (Andrews 1985) then it is important to note that the choice of radiation schemes implicitly defines $T_e$ and that the latter is not well established. The sensitivity of the meridional circulation of the stratosphere to the specification of heating rates in the lower stratosphere has been discussed by Haigh (1984).

Radiative–photochemical equilibrium also implies a balance between the concentrations of chemical constituents (in particular ozone) and $T_e$. In the present work a detailed photochemical scheme incorporating the major reactions of about twenty-five species of importance in the stratosphere (Pyle, personal communication) has been used. To calculate the temperature and ozone equilibrium fields the radiation schemes described by Haigh (1984) and Pyle’s photochemical scheme were iterated using the Adams–Bashforth time-stepping scheme until the magnitude of the net radiative heating rate was less than 0.01 K d$^{-1}$ everywhere (except in the polar night where the cooling rate did not exceed 0.03 K d$^{-1}$). Inspection of the fields of minor constituents indicated that the photochemistry too had reached equilibrium. To reach this state 1750 iterations were required. The resultant field of $T_e$ for the December solstice is shown in Fig. 1, of diurnally-averaged ozone number density in Fig. 2(a) and of diurnally-averaged ozone column density in Fig. 2(b). The temperature structure is similar to those of the previous authors but is generally colder than the results of Leovy (1964a) and, apart from in the
Figure 1. Zonal-mean radiative–photochemical equilibrium temperature (K) for solstice conditions.

Figure 2(a). Zonal-mean radiative–photochemical equilibrium ozone number density ($10^{12}$ cm$^{-3}$) for solstice conditions.

Figure 2(b). Zonal-mean radiative–photochemical equilibrium ozone column amounts (atm-cm) for solstice conditions. Dashed line shows results of Leovy (1964a), solid line present work.
polar night, warmer than the results of Fels and Schwarzkopf. The former is consistent with the lower equilibrium ozone column amounts predicted by the photochemical scheme in this work and may also be due to the inclusion of water vapour in the longwave radiation scheme. In Fig. 3 is shown the diurnally-averaged solar heating rate ($Q_e$) for the equator corresponding to the equilibrium fields of temperature and ozone. Also in Fig. 3 are shown $T_e$ and the ozone temperature coefficient, $C$, for the equator, the latter being derived from previous photochemical calculations (see Haigh and Pyle 1982 for details). Note that $C$ varies with height due to the predominance of different ozone destruction reactions at different altitudes.

5. Calculation of the matrix

The discussion of section 3 which enabled Eq. (4) to be expressed in linear matrix form as Eq. (7) is the basis of the new fast method of calculating photochemical acceleration. The matrix could, indeed, be calculated by considering individual terms in Eq. (6) but an alternative method is to use photochemical and radiative schemes and deduce the matrix elements empirically. This also allows for the inclusion of backscattered radiation and the ozone 9-6 μm band (see results of section 6) and is the approach adopted here. By perturbing the temperature profile from $T_e$ and calculating the resulting change in heating rate with the ozone distribution fixed at $p_3$, the radiative relaxation coefficient may be calculated. This procedure may be repeated with the ozone distribution also allowed to adjust to the change in temperature and the difference between the two results will give the photochemical component. If $N$ calculations of $\delta Q$ for $N$ profiles $\delta T$ are available then Eq. (7) may be expressed $\delta Q = -A\delta T$, where $\delta Q$, $\delta T$ are $N \times N$ square matrices. In particular if $\delta T = aI$, where $a$ is a temperature perturbation and $I$ the unit matrix, then $A = -(1/a)\delta Q$.

A sample matrix has been calculated using the radiative and photochemical schemes as in section 4 (without including the ozone 9-6 μm band in the longwave cooling—but

![Figure 3. $T_e$: Radiative-photochemical equilibrium temperature (K); $Q_e$: Solar heating rate (K d$^{-1}$) corresponding to radiative-photochemical equilibrium temperature and ozone distribution; $C$: Ozone temperature coefficient (K). All for solstice conditions at the equator.](image-url)
see section 6). With \( N = 13 \) the altitude range used was 2.25 to 8.25 pressure scale heights (105 to 0.261 mb) with intermediate points at equal spacing in \( \log(\text{pressure}) \) (i.e. \( \Delta z = \frac{1}{3} \)). For a temperature perturbation (from \( T_0 \)) of 0.1 K at each level in turn the resultant \( \Delta Q \) was calculated with and without photochemical effects. In the former case time-stepping continued until a new chemical equilibrium was attained. The resulting matrix is listed in Table 1. As might be expected the diagonal elements have the largest magnitude and positive sign accounting for the ozone response to the local temperature change (as discussed in section 3). The below-diagonal elements, representing the effects of temperature perturbation at higher levels, are negative and of smaller but significant magnitude. Above-diagonal elements, representing the effects of temperature perturbation at lower levels on backscattered solar radiation, are present in the lower stratosphere but are of too small magnitude to appear in this format of the table.

The height resolution chosen is adequate from the considerations of section 3 and may also be justified \textit{a posteriori} from study of the matrix elements which would show smaller diagonal elements and larger elements adjacent to and below the diagonal if \( \Delta z \) were too large. To use this method in an arbitrary dynamical model would require calculation of a matrix of grid spacing compatible with the model as the given matrix could not be interpolated in a trivial fashion.

6. INCLUSION OF THE OZONE 9.6\( \mu \)M BAND

Infrared cooling in the upper stratosphere is mainly due to radiation emitted by carbon dioxide in its 15\( \mu \)m band. A smaller contribution, however, is due to the 9.6\( \mu \)m band of ozone. As the amount of cooling depends on the ozone concentration this will be subject to the same type of photochemical coupling as the solar heating. In this case, however, there is a positive feedback with lower temperatures tending to increase the ozone amounts and thus also to increase the cooling. In the lower stratosphere, however, the ozone causes warming in its 9.6\( \mu \)m band by absorbing radiation emitted from the ground and thus any photochemical acceleration here will be positive.

In order to compare the results of this work with those of other authors (see next section) the calculation of the matrix in section 5 did not include a response to the temperature perturbation by the ozone 9.6\( \mu \)m band cooling. Such a matrix has, however, been calculated and is listed in Table 2. Comparison with Table 1 reveals diagonal elements of smaller magnitude in the upper stratosphere and larger magnitude at altitudes below 10 mb, as predicted above. In the lower stratosphere the importance of the below-diagonal (extinction effect) elements is reduced while that of the above-diagonal elements is increased due to the transfer of radiation from lower levels.

7. RESULTS

Using the matrices with and without the 9.6\( \mu \)m coupling the change in net heating rate due to photochemical acceleration was calculated for a perturbation from the equilibrium profile of (a) a uniform 1 K and (b) a wave with amplitude 1 K and vertical wavelength two grid spacings (i.e. one pressure scale height, \( \approx 7.5 \) km). The results, expressed as an equivalent local photochemical acceleration coefficient, \( \frac{(\Delta \delta T)_{\text{eq}}}{\delta T} \), are shown for solstice conditions at the equator in Fig. 4. Also shown is \( \alpha \) calculated using Strobel's (1979) parametrization; this method, while fast and fairly accurate for perturbations of large vertical scale, is not suitable for waves of smaller scale because it assumes that the perturbation scale is of the same order as the ozone scale height. The results of Strobel's method and the matrix method (with no 9.6\( \mu \)m effect) for a uniform
TABLE 1. Photochemical Acceleration Matrix in Units of days$^{-1}$/100

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Ozone 9-6 μm band feedback not included. Level 5 corresponds to 2-25 pressure scale heights (105 mb) and level 17 to 8-25 scale heights (0.261 mb). Other levels are equally spaced in log(pressure).

TABLE 2. As Table 1 but Including Ozone 9-6 μm Band Feedback

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J. D. HAIGH
CALCULATING PHOTOCHEMICAL ACCELERATION

Figure 4. Equivalent photochemical relaxation coefficient calculated for: ---, uniform perturbation, no 9.6 \( \mu m \) coupling; ..., ditto using the parametrization of Strobel (1979); ----, uniform perturbation, including 9.6 \( \mu m \) coupling; ---, short-wavelength perturbation, no 9.6 \( \mu m \) coupling; ----, short-wavelength perturbation, including 9.6 \( \mu m \) coupling.

perturbation compare well between 3 and 1 mb. Above this region Strobel's method produces higher values, probably reflecting the simplified chemistry and constant temperature coefficient used (cf. Fig. 3). Below this region Strobel's method is not applicable because of large ozone optical depths. The peak value of \( \alpha \) for uniform perturbation is about 0.065 d\(^{-1}\), a significant addition to the Newtonian cooling coefficient, which is about 0.2 d\(^{-1}\) in the upper stratosphere for a uniform perturbation. The effect of including the 9.6 \( \mu m \) coupling is to reduce \( \alpha \) by up to 0.015 d\(^{-1}\) with an especially significant effect near 2 mb.

For shorter wavelength perturbations the extinction effect is reduced, the effective \( \alpha \) becomes larger and the impact of including the 9.6 \( \mu m \) coupling is reduced. In the limit of infinitesimal wavelength, as in Hartmann's (1978) 'no extinction' case, \( \alpha \) would revert to its classical value \( CQ/T^2 \). The matrix, like the numerical model in which it would be used, cannot, of course, resolve waves of wavelength less than 2\( \Delta z \).

8. LINEARITY

Implicit in the discussion above is the assumption of linearity. The next term in the expansion of Eq. (1) is \( \frac{1}{2}(\frac{\partial^2 Q}{\partial T^2})_T \delta T (\delta T)^2 \). Following Leovy (1964b) we take the local approximation \( (\frac{\partial Q}{\partial T})_T = -CQ/T^2 \), then

\[
\frac{1}{2}(\frac{\partial^2 Q}{\partial T^2})_T = -\frac{1}{2} (\frac{2}{T_c} + \frac{CQ}{T^2_c})(\frac{\partial Q}{\partial T})_T
\]

thus \( \delta Q = -(\frac{\partial Q}{\partial T})_T \delta T (1 - (\frac{1}{T_c} + \frac{C}{2T^2_c})\delta T) \). The correction is a function of height but taking \( C = 1000 \text{ K} \) and \( T = 250 \text{ K} \) the fractional change in \( Q \) (or \( \alpha \)) is approximately \( -\delta T/100 \text{ K} \). To check this estimate the schemes described above for calculating \( \alpha \) were
repeated using values for $\delta T$ of $\pm 0.1 \, \text{K}, \pm 1 \, \text{K}, \pm 10 \, \text{K}$ both as uniform perturbations and as grid scale waves. The results showed that the error was, indeed, a function of height but that for the uniform perturbation the fractional error was less than Leovy's estimate, being about $-\delta T/500 \, \text{K}$ while the short wavelength perturbation deviated further from linearity with errors of about $-\delta T/50 \, \text{K}$. Except within the polar night the temperature of the middle atmosphere remains quite close (within about $20 \, \text{K}$) of the equilibrium temperature so serious error should not be incurred. In his expression for the thermal relaxation coefficient Dickinson (1973) incorporates a second-order temperature dependence empirically. A similar technique could be used in this case although any attempt to do this is probably not warranted given the assumptions made above.

9. **Applicability**

The above method would appear to present a fast and accurate method for calculating photochemical acceleration but it has one obvious drawback: because of the strong dependence of $\alpha$ on latitude and season a whole range of matrices would seem to be needed. Figure 5 shows, for two different altitudes, how the dominant (local) component of $\alpha$, scaled to unity at the summer pole, varies with latitude under solstice conditions. Also shown, similarly scaled, are $\alpha T_e^2/Q_e$ and $\alpha/Q_e$. It is interesting that the latter of these has very little latitudinal dependence (except near the winter polar night where $\alpha$ is anyhow small). Recalling that locally $\alpha = C Q_e/T_e^2$ the behaviour of $\alpha/Q_e$ indicates that the latitudinal dependences of $C$ and $T_e^2$ must approximately cancel. This enables us to

![Figure 5](image_url)

Figure 5. Variation with latitude of $\alpha$, $\alpha T_e^2/Q_e$ and $\alpha/Q_e$ at two different pressure levels. All values are scaled to unity at the summer pole.
make a simple approximation. Expanding $Q$ as a function of temperature linearly about a standard profile $T_s$ (not a function of latitude, $\lambda$), we have

$$Q(T, \lambda) = Q(T_s, \lambda) - \alpha(\lambda)(T - T_s) = Q(T_s, \lambda) - Q_s(\alpha(\lambda)/Q_s(\lambda))(T - T_s).$$

But $Q_s = Q(T, \lambda)$ (Schoeberl and Strobel 1978), for example, define $\alpha$ in terms of ambient rather than equilibrium values of $T$ and $Q$ and it has already been noted that $\alpha/Q_s$ is approximately independent of latitude, thus the photochemical acceleration term may be expressed in terms of one matrix $A'(\equiv A/Q_s)$ and the departure from a standard temperature profile $T_s$:

$$\delta Q = -Q_s A'(T - T_s).$$

10. CONCLUSIONS

Accurate calculation of diabatic heat sources and sinks is important for dynamical models of the middle atmosphere. Until the present the techniques used to incorporate photochemical acceleration have been able to account only for either, on one extreme, local temperature and ozone perturbations (Blake and Lindzen 1973) or, on the other, perturbations of large vertical scale (Strobel 1979). The effect of variable scale perturbations on the thermal relaxation rate has been parametrized by Fels (1982) but no parametrizations have been constructed for scale-dependent photochemical acceleration of the thermal relaxation rate.

In this work a method which is both computationally fast and accurate for any (model-resolved) vertical scale has been presented. Photochemical acceleration is calculated as a simple matrix multiplication of a precalculated matrix by a perturbation temperature profile. It has been shown that the matrix can be made effectively independent of latitude and season and therefore quite generally applicable. This technique is, of course, very similar to that of the Curtis matrix method sometimes used to calculate infrared cooling rates (Haigh 1980; Fels and Schwarzkopf 1981) and there seems to be no reason in principle why the two matrices could not be combined to further speed up model calculations.

The inclusion of the coupling between temperature, ozone concentration and radiative transfer by ozone in its 9-6 $\mu$m band has been shown to decrease photochemical acceleration above 35 km. This effect can be incorporated into the matrix method.

ACKNOWLEDGEMENTS

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