Comments on ‘Radiative cooling near the top of a cloudy mixed layer’ by F. T. M. Nieuwstadt and J. A. Businger (October 1984, 110, 1073–1078)

By J. W. DEARDORFF

Department of Atmospheric Sciences, Oregon State University, Corvallis, Oregon 97331

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Nieuwstadt and Businger (1984; NB) in their Fig. 1 show an idealized schematic profile of average potential temperature with a jump, $\Delta \theta$, occurring at $z = h$ over a zero height interval. They claim that no modification to this idealized model is needed to account for the vertical variability of cloud top excursions, $\Delta h$ (see their Fig. 2, or Fig. 1 herein), since the important divergence of net radiation occurs within the cloud. However, no investigators dispute that the important radiative flux divergence in this problem occurs locally within the cloud; e.g. see Deardorff (1976, Fig. 1). The relevant point is whether $\Delta h/\lambda$ is significant, where $\lambda$ is NB’s radiative damping length. (This is within the context of the overbar operator representing a horizontal average, or a running horizontal average over an area whose length scale greatly exceeds the cloud depth. In NB’s explanation of their Fig. 1, a horizontal average was implied.) If $\Delta h/\lambda$ is significant, the fraction $r$ which they discussed will be significant, and some of the upper-cloud radiative flux divergence will promote entrainment in the sense envisioned by Lilly (1968) and called ‘direct’ entrainment by NB.

![Figure 1. Depiction of the instantaneous boundary-layer height, as in NB's Fig. 2, except that across dotted regions of entrainment there are no discontinuities in $\theta$ or $\omega$.](image)

In the observations of Roach et al. (1982) it is not clear to me whether or not $\Delta h$ was important relative to a $\lambda$ value of order 50 to 100 m. Although the large-scale excursions of $h$ ranged over about 50 m, the most relevant smaller-scale excursions were unresolved by the acoustic sounder for horizontal scales less than about 210 m. Under different conditions of smaller $\Delta$, and comparable $\lambda$, $\Delta h$ and $r$ must increase, so that it would seem unwise to presume that $r$ is generally near zero.

The rest of my remarks concern NB’s interesting use, in the appendix, of an overbar average which tracks the instantaneous depth, $h(x,y,t)$, of the layer extending up to cloud top. Their Fig. 2 is meant to help explain this kind of average, but fails to include the complications of entrainment. Figure 1 here modifies their figure to indicate regions where entrainment may be occurring, as in Deardorff and Peterson (1980). Would NB’s Eq. (A4) still be deemed valid if these regions were treated?

Finally, it is not clear that Eq. (2), in which no cloud top radiative flux divergence appears, can be relied upon merely because (A4) does not contain the same. In the latter equation with a typical, finite $\Delta h$, the $\delta h/\delta x$ term is of the same order of magnitude as the $\delta h/\delta t$ term, with both being an order of magnitude or more greater than $\Delta_s \delta h/\delta t$ in (2). The uncertainty exists in my mind because the in-cloud radiative flux divergence is presumably an important indirect contributor to the turbulence intensity near cloud top. Hence, the product averages of $u\theta$ and $\delta h/\delta x$ in (A4), and of $\theta$ and $\delta h/\delta t$, may be rather different in the presence of such radiative flux divergence than in its absence.
REFERENCES

Nieuwstadt, F. T. M. and Businger, J. A. 1984 Radiative cooling near the top of a cloudy mixed layer. Ibid., 110, 1073–1078

Reply by F. T. M. NIEUWSTADT and J. A. BUSINGER

We agree that the instantaneous boundary-layer top cannot everywhere be treated as a material surface. Deardorff and Peterson (1980) give a convincing argument that air passes the interface at localized regions whereas the rest of the interface can be treated as a material surface. Let us consider an extension of our equation (A4) that is in agreement with this picture.

As mentioned above we distinguish two regions in a cloud-top interface and each requires different approach. Along the material surface part we adopt the familiar jump in the temperature and humidity profile. Such jump structure can no longer be valid in a spot of active entrainment. There we introduce a so-called mixing region in which the temperature and humidity vary more or less smoothly. The geometry is clarified in Fig. 1 where \( h \) denotes the instantaneous boundary-layer height following the definition of Deardorff and Peterson. The \( h^+ \) is the height where the air can no longer be distinguished from in-cloud air and \( h^- \) forms the boundary with the above-cloud air. The heights \( h^-, h \) and \( h^+ \) fall almost on top of each other along the material surface part of the interface whereas in the entrainment area \( h^- \) and \( h^+ \) enclose the mixing region. The definition of the instantaneous boundary-layer height is extended into a mixing region by \( h^+ = h + \frac{1}{2} \Delta h \), \( h^- = h - \frac{1}{2} \Delta h \), where \( \Delta h = h^+ - h^- \).

With respect to the influence of radiation we can distinguish three contributions, which are indicated by roman numerals in Fig. 1. In region I radiation leads to direct cooling of the cloud layer. This process may generate turbulence by means of downward convection and as a result of this turbulence above-cloud air is entrained. In region II radiation cools the air due to the presence of the cold cloud surface. In addition some cooling may take place by detrained air (Deardorff et al. 1980) but this effect will not be considered here separately. As a consequence of this cooling the temperature inversion across the boundary-layer top decreases, which promotes entrainment. Nieuwstadt and Businger (1984) have shown that this effect is included in the strength of the inversion so that the contribution of above-cloud radiation does not have to be included explicitly in an entrainment relationship. In region III radiation may cool the air during the mixing process. As a result it contributes to entrainment because less turbulent energy is needed to mix the air further downward. Randall (1984) has coined the phrase ‘defensive cooling’ for this process, in contrast with offensive cooling which occurs in region I. Consequently in region III radiation acts as direct entrainment.

Let us now consider how our entrainment relationship is modified when the new geometry of the cloud-top interface is taken into account. To this end we integrate the energy equation between

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Figure 1. Detailed structure of the instantaneous boundary-layer top.
\( h^- \) and \( h^+ \). We treat both regions of the interface separately. Along the material surface part of the interface we obtain a result equivalent to our former equation (A4). In a mixing region we must integrate over the finite thickness \( \Delta h \) and we follow in that case the method of Deardorff (1979). Combination of both integrals and assumption of \( w = 0 \), which was also adopted by Nieuwstadt and Businger (1984), leads to

\[
- \frac{\partial \theta^- \theta^+}{\partial t \theta} \frac{\partial h}{\partial x} - \left[ \frac{(u \theta)^+ - (u \theta)^-}{\partial x} \frac{\partial \theta}{\partial x} - w \overline{\theta} \right] h^- = -f \left[ \frac{\langle \Delta F \rangle}{\rho c_p} + \frac{\partial}{\partial t} \left( \int_{h^-}^{h^+} \theta \, dz \right) + \frac{\partial}{\partial x} \left( \int_{h^-}^{h^+} (u \theta) \, dz \right) \right], \tag{A4'}
\]

where the overbar denotes a horizontal average along the complete instantaneous boundary-layer height both inside and outside mixing regions and where the angular brackets indicate an average only inside mixing regions. The \( f \) is the fraction of the instantaneous cloud top which is occupied by the mixing regions.

On the left-hand side of this equation we find the same terms as in our original entrainment relationship (A4), whereas on the right-hand side we find some extra terms which are due to the presence of mixing regions. When these mixing regions can be considered as stationary and horizontally periodic both integrals in (A4') vanish. In that case the only contribution by radiation remains and we find that radiation in these regions acts indeed as direct entrainment. An explicit calculation of this radiation term becomes quite complicated due to the complex geometry of the mixing regions. However, a first-order estimate may be given as

\[
\langle \Delta F \rangle / \rho c_p = (\epsilon \sigma T^3 / \rho c_p) (\theta^+ - \theta^-),
\]

where \( \sigma \) is the Stefan–Boltzmann constant and \( \epsilon \) is the absorptance of the mixing-region air for infrared radiation. As the mixing region generally consists of dry and therefore clear air the absorptance will be small, say \( \epsilon \approx 0.1 \). In that case a typical value for \( \Delta F / \rho c_p \) is \( 5 \times 10^{-4} \)K m s\(^{-1}\). With a typical value of \( f \), say 5–10\%, we obtain for the entrainment velocity \( \sim 0.01 \) mm s\(^{-1}\), which is generally negligible. In that case equation (A4') reduces to (A4) and our previous conclusion is confirmed, that direct entrainment by radiation can be neglected.

In a final point of his comment Deardorff argues that a term of the form \( \langle (u \theta)^+ - (u \theta)^- \rangle \frac{\partial \theta}{\partial x} \) may depend directly on the cloud top radiation flux, so that in an idealization of (A4) to \( \Delta h \rightarrow 0 \) a direct contribution of radiation to entrainment may appear. However, this term is primarily of a turbulent origin and will depend on radiation only indirectly through its generation of turbulence. Therefore, there seems no immediate reason to interpret this term in the limit \( \Delta h \rightarrow 0 \) as direct entrainment by radiation because the production of turbulence by radiation in this idealized case has been completely accounted for.

In addition we want to emphasize that the correct limiting process towards the idealized case of \( \Delta h = 0 \) should depart from our Eq. (A4'). In this respect we disagree with the introductory remarks made by Deardorff, in which he refers to a horizontal average. The latter averaging procedure seems physically meaningless because it combines the properties of in-cloud and above-cloud air, which are governed by quite different processes. An averaging procedure which follows the local cloud top preserves the characteristics of above and in-cloud air and this corresponds to the basic assumptions of the idealized model.

**REFERENCES**


