A random walk model of dispersion in turbulent flows and its application to dispersion in a valley

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(Received 18 April 1985; revised 23 September 1985)

SUMMARY

A random walk model of the dispersion of a passive tracer in a general three-dimensional turbulent flow is developed. The model is used to simulate dispersion in the Sirhowy valley, South Wales. The results show encouraging agreement with the experimentally measured concentrations, which are generally much lower than those typical of dispersion over flat ground. The simulations show that, for releases from the summit on the upwind side of the valley, the low ground-level concentrations in the valley are primarily a result of the mean flow and not the high turbulence levels in the valley. The simulations have been unable to explain the differences observed between experiments which were carried out in similar conditions; it is possible that the differences are simply a result of the inherent variability in the dispersion. The simulations have enabled estimates of $\sigma_{T}$ and $\sigma_z$ to be made; these quantities are difficult to obtain from the experimental data because of the lack of sufficient elevated samplers and the large horizontal area covered by the cloud.

1. INTRODUCTION

Random walk models are a successful and flexible tool in the investigation of dispersion in turbulent flows. They are particularly suited to the calculation of dispersion in complex flows where many other techniques (e.g. Taylor's (1921) statistical theory, similarity theory or eddy-diffusivity techniques) are inappropriate or invalid. Their ability to model dispersion over flat ground in a variety of stability conditions is well established (e.g. Reid 1979; Wilson et al. 1981; Ley 1982; Ley and Thomson 1983). To date however, little use has been made of these techniques in more complex situations.

During the summers of 1980, 1981 and 1983 the boundary layer branch of the Meteorological Office conducted a number of experiments in the Sirhowy valley, South Wales, with the aim of studying the mean flow and turbulence properties in the valley. Dispersion experiments were carried out at the same time using passive tracers. The results from the 1980 and 1981 experiments are described in Mason and King (1984) (referred to hereafter as MK) and Whitlock et al. (1985), and some of the dispersion data obtained in 1983 are presented in Callander (1986). The Sirhowy valley is, on the larger scales at least, nearly two dimensional and has slopes of up to 30°.

In this paper a random walk model is developed which can predict the dispersion of a tracer within complex three-dimensional flows. To illustrate the capabilities of the model it is applied to the problem of dispersion in the Sirhowy valley and the results from the model are compared with the experimental dispersion data. The random walk model cannot, of course, predict the structure of the mean flow and turbulence; however, given information on the flow structure, the random walk model predicts the dispersion.

2. THE RANDOM WALK MODEL

The random walk method of modelling the dispersion of a passive tracer consists of simulating numerically the motion of many particles of the tracer, in order to build up a picture of the concentration distribution. The trajectory of each particle is simulated by modelling the evolution of the particle's velocity over a succession of time steps. The velocity evolution is assumed to be a Markov process and it is usually assumed that the velocity at one time step depends linearly on that at the previous time step. Hanna (1979) has presented some experimental evidence in support of the linearity assumption. These
assumptions cannot be exactly true since they imply a discontinuous acceleration whose size tends to infinity as the length of the time step is decreased to zero. In the atmosphere however, the accelerations of the fluid elements are large and are significantly correlated only over very short times (of the order of the Kolmogorov time scale – Monin and Yaglom (1975) p. 370, pp. 548–549).

In a real cloud of tracer, the velocities of the fluid elements within the cloud are correlated. In the model however, each particle is assumed to move independently. Hence the concentrations obtained from the model must be regarded as ensemble-average values or, in stationary conditions, as the time-average values resulting from a steady source.

A drawback of the random walk technique is that it assumes a specific form for the Lagrangian correlation statistics. Consider for the moment diffusion in one dimension only. The shape of the Lagrangian correlation is fixed by the model (in homogeneous turbulence it is exponential) and, although physically reasonable (in particular it has the correct inertial subrange form at small times), is not necessarily accurate. However, Pasquill (1974 pp. 131–132) has shown that the dispersion characteristics depend mainly on the velocity variance and the Lagrangian integral time scale and are only weakly dependent on the shape of the correlogram. Provided the model correlogram is not very different from the true one, the model will give satisfactory results.

Throughout most of this section the conditions are assumed to be stationary; an extension to non-stationary conditions is outlined at the end of the section but no application of this is presented in this paper.

One of the most important properties of a random walk model is that it should give the correct steady state distribution of particles in phase space, i.e. the steady state distribution of particles should be the same as the distribution of the air elements. This is referred to below as the steady state condition. For the one-dimensional case with no mean flow, Thomson (1984) showed that there are two ways of achieving this. The difference between the two methods lies in the equations which are used to model the particle trajectories. The first method uses the straightforward

\[ w_{n+1} = \{1 - \Delta t/\tau(z_{n+1})\} w_n + \mu_{n+1} \]
\[ z_{n+1} = z_n + w_n \Delta t \]

while the second uses

\[ w_{n+1}/\sigma_w(z_{n+1}) = \{1 - \Delta t/\tau(z_{n+1})\} w_n/\sigma_w(z_n) + \mu_{n+1}/\sigma_w(z_{n+1}) \]
\[ z_{n+1} = z_n + w_n \Delta t \]

which is equivalent, to first order in \( \Delta t \), to

\[ w_{n+1} = \{1 - \Delta t/\tau(z_{n+1})\} w_n + (\Delta t/\sigma_w)(\partial \sigma_w/\partial z) w_n^2 + \mu_{n+1} \]
\[ z_{n+1} = z_n + w_n \Delta t \]

\( \mu_{n+1} \) is a random variable with density function \( f_{\mu_{n+1}}(\mu) \), \( z_n \) and \( w_n \) are the particle's position and velocity after the \( n \)th time step, \( \Delta t \) is the length of the time step, \( \tau(z) \) is a time scale for the motion and \( \sigma_w^2(z) \) is the variance of the vertical velocity (if the fluctuations in the density of the air are not negligible, \( \sigma_w^2 \) must be taken to be the density-weighted variance, i.e. the Eulerian quantity \( (\rho + \rho')w^2/\rho \) where \( \rho(z) \) is the ensemble-average density, \( \rho' \) is the density perturbation and the overbar indicates an ensemble average). In each case the distribution of \( \mu \) is determined by imposing the steady state condition introduced above.

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The distribution of $\mu$ is of course different for the two models. If the turbulent velocity distribution is Gaussian, then the distribution of $\mu$ is Gaussian for the second method but, in inhomogeneous conditions, non-Gaussian for the first. Hence the advantage which the second method gains by including a term which is nonlinear in $\omega$ is that, if the turbulent velocity distribution is Gaussian, $\mu$ can be chosen to be Gaussian. Because Gaussian random numbers are easy to generate, this makes the second method much easier to apply. There are insufficient experimental data to determine which method gives a more accurate description of the particle trajectories.

(a) Extension to three dimensions

In extending these methods to three dimensions, we consider a very general model for the particle trajectories which includes both (1) and (3) as special cases, namely

$$
\begin{align*}
\mathbf{u}_{n+1}^i &= \mathbf{u}_n^i - \Delta t T^{ij}(\mathbf{x}_{n+1}) \mathbf{u}_n^{j} + \Delta t \left\{ \beta^{ij}(\mathbf{x}_{n+1}) \mathbf{u}_n^i \mathbf{u}_n^j + \gamma^{ijk}(\mathbf{x}_{n+1}) \mathbf{u}_n^i \mathbf{u}_n^j \right\} + \mu_{n+1}^i \\
\mathbf{x}_{n+1} &= \mathbf{x}_n + \mathbf{u}_n \Delta t
\end{align*}
$$

(4)

where $\mathbf{x}$ and $\mathbf{u}$ are the position and velocity of the particle and $T$ is the time scale tensor, the three-dimensional equivalent of $1/\tau$ in (1), (2) and (3). Superscripts indicate Cartesian components and the Einstein summation convention applies. The joint density function of the components of the random vector $\mathbf{u}_{n+1}$ will be denoted by $f_{\mathbf{x}_{n+1}}(\mathbf{u})$ and will be determined by imposing the steady state condition. For the model to be well-defined the tensors $\beta$ and $\gamma$ need to be specified; however, this will not be done at this point in order to allow all models of the form (4) to be considered. If $\beta = \gamma = 0$ the model is simply a three-dimensional extension of (1), as described in Thomson (1984), while if $\gamma \neq 0$ the model is of the same type as (3). The experimental evidence is insufficient to decide which values of $\beta$ and $\gamma$ will yield the most accurate trajectories, and so their values can be chosen to simplify the implementation of the model. The expressions chosen for $\beta$ and $\gamma$ must be such that, in homogeneous turbulence without mean shear, $\beta = \gamma = 0$; $\gamma$ must be zero on symmetry grounds and $\beta$ should be included in the time scale tensor $T$. Of course $\beta$ could be included in the time scale tensor even in inhomogeneous turbulence; however, it turns out to be beneficial not to do this. This point is considered in detail below.

Before proceeding to derive equations for $\beta$, $\gamma$ and $f_{\mathbf{x}}(\mathbf{u})$ we must introduce some notation. Given a density function $\rho(\mathbf{u})$, the corresponding moment-generating function (m.g.f.) will be denoted by $\hat{\rho}(\mathbf{\theta})$, i.e.

$$
\hat{\rho}(\mathbf{\theta}) = \int d\mathbf{u}_1 d\mathbf{u}_2 d\mathbf{u}_3 \exp(\mathbf{u} \cdot \mathbf{\theta}) \rho(\mathbf{u}).
$$

For clarity the equations for the evolution of the particle trajectory will be expressed as an advection step

$$
\begin{align*}
\mathbf{u}_{n+1}^i &= \mathbf{u}_n^i \\
\mathbf{x}_{n+1} &= \mathbf{x}_n + \mathbf{u}_n \Delta t
\end{align*}
$$

(5)

and two velocity-incrementing steps

$$
\begin{align*}
\mathbf{u}_{n+1}^i &= \mathbf{u}_{n+1}^i + \Delta t \left\{ \beta^{ij}(\mathbf{x}_{n+1}) \mathbf{u}_{n+1}^{j} + \gamma^{ijk}(\mathbf{x}_{n+1}) \mathbf{u}_{n+1}^{j} \mathbf{u}_{n+1}^{k} \right\} \\
\mathbf{x}_{n+1} &= \mathbf{x}_{n+1}
\end{align*}
$$

(6)

and
\[u_{n+1}^{i} = u_{n+1}^{ij} + \mu_{n+1}^{i}\] \[x_{n+1}^{i} = x_{n+1}^{ij}\]

(7)

\(g_{n}(x, u)\) will be used to indicate the phase space density function of the particles of air, and the distribution of particles of tracer before and after each of steps (5), (6) and (7) will be denoted as follows:

\[g_{n} \rightarrow g_{n+1}^{'} \rightarrow g_{n+1} '' \rightarrow g_{n+1}\]

(5) \hspace{1cm} (6) \hspace{1cm} (7)

The effect of each of these steps on the phase space distribution of the tracer can now be calculated. The effect of (5) is, by an argument similar to that used by Thomson (1984) for the one-dimensional case, \(g_{n+1}^{'}(x, u) = g_{n}(x - u\Delta t, u)\). By expanding in powers of \(\Delta t\), multiplying by \(\exp(u\cdot\theta)\) and integrating with respect to \(u\), it can be seen that

\[\hat{g}_{n+1}(x, \theta) = \hat{g}_{n}(x, \theta) - \Delta t(\partial \hat{g}_{n}/\partial \theta \cdot dx) + O(\Delta t^{2}).\]  

(8)

To calculate the effect of (6) the substitution \(u^{n} = u^{ij} + \Delta t((\beta^{ij} - T^{ij})u^{ij} + \gamma_{ijk}u^{ij}u^{rk})\) is used:

\[\hat{g}_{n+1}^{''} = \iint \exp(u\cdot\theta)g_{n+1}^{''}(x, u)du^{n1}du^{n2}du^{n3}\]

\[= \iint \exp((u^{ij} + \Delta t((\beta^{ij} - T^{ij})u^{ij} + \gamma_{ijk}u^{ij}u^{rk}))\theta^{j})g_{n+1}^{'}(x, u^{'})du^{n1}du^{n2}du^{n3}.\]

Expanding in powers of \(\Delta t\) and integrating yields

\[\hat{g}_{n+1}^{''} = \hat{g}_{n+1}^{'} + \Delta t\left\{ \theta^{j}(\beta^{ij} - T^{ij}) \frac{\partial}{\partial \theta^{j}} + \theta^{i}\gamma_{ijk} \frac{\partial^{2}}{\partial \theta^{j} \partial \theta^{k}} \right\}\hat{g}_{n+1}^{'} + O(\Delta t^{2}).\]  

(9)

Finally, by using a standard result from the theory of m.g.f.s, the effect of (7) can be found:

\[\hat{g}_{n+1} = \hat{g}_{n+1}^{''} \hat{f}_{x}(\theta).\]  

(10)

Combining (8), (9) and (10) yields

\[\hat{g}_{n+1} = \left[ \hat{g}_{n} + \Delta t\left\{ \theta^{i}(\beta^{ij} - T^{ij}) \frac{\partial}{\partial \theta^{j}} + \theta^{i}\gamma_{ijk} \frac{\partial^{2}}{\partial \theta^{j} \partial \theta^{k}} - \frac{\partial^{2}}{\partial \theta^{j} \partial \theta^{k}} \right\}\hat{g}_{n} \right] \hat{f}_{x}(\theta) + O(\Delta t^{2}).\]

(11)

Imposing the steady state condition \(\hat{g}_{n} = \hat{g}_{n+1} = \hat{g}_{s}\) yields

\[\hat{f}_{x}(\theta) = 1 + \frac{\Delta t}{\hat{g}_{s}}\left\{ \frac{\partial^{2}}{\partial \theta^{j} \partial x^{i}} - \theta^{j}(\beta^{ij} - T^{ij}) \frac{\partial}{\partial \theta^{j}} - \theta^{i}\gamma_{ijk} \frac{\partial^{2}}{\partial \theta^{j} \partial \theta^{k}} \right\}\hat{g}_{s} + O(\Delta t^{2}).\]

(11)

Given \(\beta\) and \(\gamma\) this equation tells us what the distribution of \(\mu\) must be. As indicated above, the values of \(\beta\) and \(\gamma\) can be chosen to make the implementation of the model as simple as possible, with the restriction that the expressions for \(\beta\) and \(\gamma\) must reduce to zero in homogeneous turbulence.

(b) Restriction to Gaussian turbulence

In many situations, and in particular in the simulations performed in section 3 of
this paper, it is assumed that
\[ g_a(x, u) = \rho(2\pi)^{-3/2}(\det \sigma)^{-1/2} \exp\{-\frac{1}{2}(u^i - U^i)\chi^{ij}(u^i - U^i)\} \]  \[ (12) \]

where \( \chi = \sigma^{-1} \) and \( \rho, U \) and \( \sigma \) are functions of \( x \) (i.e. we assume that the velocity part of the distribution is a multivariate normal distribution, Hogg and Craig (1978) p. 405). \( U \) is the density-weighted ensemble average velocity field and \( \sigma \) is the tensor containing the density-weighted covariances of the turbulent velocity fluctuations. In (12) we have chosen the most convenient normalization for \( g_a \), and so it is not strictly a probability density function. \( \rho \) and \( U \) must of course satisfy the continuity equation \( \nabla \cdot (\rho \mathbf{U}) = 0 \). The m.g.f. corresponding to \( g_a \) is
\[ \tilde{g}_a(x, \theta) = \rho \exp(\theta^i U^i + \frac{1}{2} \sigma^{ij} \theta^i \theta^j). \]  \[ (13) \]

In the following, values of \( \beta \) and \( \gamma \) are found which are suitable for situations where (12) and (13) hold.

Substituting (13) into (11) and expanding in powers of the \( \theta^i \) yields
\[ \overline{\mu}^i = \Delta t \left\{ \frac{1}{\rho} \frac{\partial}{\partial x^j} (\rho \sigma^{ij}) + U^j \frac{\partial U^i}{\partial x^j} + T^{ij} U^j - \beta^{ij} U^i - \gamma^{ijm} (U^i U^m + \sigma^{im}) \right\} + O(\Delta t^2) \]

\[ \theta^i \theta^j \mu^i \mu^j = 2 \Delta t \theta^i \theta^j \left\{ \sigma^{ij} \frac{\partial U^i}{\partial x^j} + U^i \frac{\partial \sigma^{ij}}{\partial x^j} + T^{ij} \sigma^{ij} - \beta^{ij} \sigma^{ij} - \gamma^{ijm} (U^i \sigma^{jm} + U^m \sigma^{ij}) \right\} + O(\Delta t^2) \]

\[ \theta^i \theta^j \theta^k \mu^i \mu^j \mu^k = 6 \Delta t \theta^i \theta^j \theta^k \left( \frac{1}{2} \sigma^{ij} \frac{\partial \sigma^{ij}}{\partial x^k} - \gamma^{ijm} \sigma^{ij} \sigma^{km} \right) + O(\Delta t^2). \]

Moments of \( \mu \) of order greater than three are \( O(\Delta t^2) \). If \( \beta \) and \( \gamma \) are chosen to be
\[ \beta^{ij} = \frac{\partial U^i}{\partial x^j} - \frac{1}{2} \frac{\partial \sigma^{ij}}{\partial x^j} \chi^{lm} U^m \]  \[ (14a) \]
\[ \gamma^{ijk} = \frac{1}{2} \frac{\partial \sigma^{il}}{\partial x^k} - \gamma^{ilm} \sigma^{ij} \sigma^{km} \]  \[ (14b) \]

the distribution of \( \mu \) can be taken to be a multivariate normal distribution, the second moments of which are equal to the value they would have in homogeneous turbulence. This simplifies the implementation of the model and yields the following values for the first two moments of the \( \mu^i \):
\[ \overline{\mu}^i = \Delta t \left\{ \frac{1}{2} \frac{\partial \sigma^{ij}}{\partial x^i} + (\sigma^{ij}/\rho) \frac{\partial \rho}{\partial x^i} + T^{ij} U^j \right\} + O(\Delta t^2) \]  \[ (15a) \]
\[ \overline{\mu^i \mu^j} = \Delta t(T^{ij} \sigma^{ij} + T^{ij} \sigma^{ji}) + O(\Delta t^2). \]  \[ (15b) \]

In the above all terms of order \( \Delta t^2 \) have been neglected. For this to be valid, the time step \( \Delta t \) must be small compared with the time scales of the turbulence (i.e. the reciprocals of the eigenvalues of \( T \)) and must also be small enough to ensure that the particle motions adequately resolve any inhomogeneities in \( \rho, U \) and \( \sigma \).

An alternative to (14a) is \( \beta = 0 \); this still leads to a multivariate normal distribution for \( \mu \) but with different values for \( \mu^i \) and \( \mu^i \mu^j \). Such a model will give results identical to those given by the model described above if \( T \) is decreased by an amount equal to the right-hand side of (14a). Hence we see that the time scale tensors required by the two models are different. In practice, however, there is a large degree of uncertainty in the values of the time scales, and the above distinction is academic. It will be seen below that the model based on (14a) is easier to implement.
(c) A more compact form of the model

In the one-dimensional case, Eq. (2) is easier to apply than its equivalent (3). This is because, if (2) is used, the derivative $\partial \rho / \partial z$ does not need to be calculated. We ask "Is there a similarly simple three-dimensional form which would avoid the need to calculate all the components of $\mathbf{B}$ and $\mathbf{\gamma}$?" The first step in simplifying equations (4) is to express them in terms of $u_n - U(x_n)$, which will be denoted by $s_n$. By noting that $U(x_{n+1}) = U(x_n) + \Delta t u_n \cdot \nabla U + O(\Delta t^2)$, and using (14) and (15), (4) becomes

$$s_{n+1}^{j'} = s_n^{j'} + \Delta t \left\{ \frac{1}{2} \left( \frac{\partial \sigma^j}{\partial x_k} \right) \chi^j \sigma^k_{n} + \frac{1}{2} \left( \frac{\partial \sigma^i}{\partial x^l} \right) \left( \sigma^j / \rho \right) (d\rho / dx^l) \right\}$$  \hfill (16a)

$$s_{n+1}^j = s_{n+1}^{j'} - \Delta t T^j_{i} s_{n+1}^{i} + \eta_{n+1}^j$$  \hfill (16b)

$$x_{n+1} = x_n + \left\{ U(x_n) + s_n \right\} \Delta t$$  \hfill (16c)

where $\eta_{n+1}$ is a random vector from a multivariate normal distribution with mean zero and second moments given by $\eta_j \eta^j = \Delta t (T^{jk} \sigma^k_{i} + T^k \sigma_{i}^{j})$. The equation for the evolution of $s$ has been split into two parts; the first part (Eq. (16a)) deals with all the inhomogeneities in $\tilde{g}_s$, while the second part (Eq. (16b)) increments $s$ as if the turbulence was homogeneous. The first of these steps can be further simplified. Consider

$$D_i^j(x_{n+1}) s_{n+1}^{j'} = D_i^j(x_n) s_n^j + \Delta t E_i^j$$  \hfill (17)

where $D$ is a rank-two tensor and $E$ is a vector. This is equivalent, to first order in $\Delta t$, to

$$s_{n+1}^{j'} = s_n^{j'} + \Delta t (\partial (D^{-1})^j_{i} \partial x^k) D_i^j s_n^k + \Delta t (D^{-1})^j_{i} E_i^j$$

and can be made equivalent to (16a) if $D$ is chosen to satisfy

$$D_i^j \partial (D^{-1})^j_{i} \partial x^k = \chi^j \partial \sigma^i / \partial x^k.$$

However, unless the orientation of the principal axes of $\sigma$ is independent of $x$, it is not easy to find such a $D$. A simplification of (16a) can still be found by using two different procedures of the form (17) in alternate time steps:

**procedure 1:**

$$s_{n+1}^{j'} = s_n^{j'} + \Delta t \left\{ \partial \sigma^j / \partial x^l \right\} + (2 \sigma^j / \rho) \partial \rho / \partial x^l \right\}$$  \hfill (18a)

**procedure 2:**

$$\chi^j (x_{n+1}) s_{n+1}^{j'} = \chi^j (x_n) s_n^j.$$  \hfill (18b)

Using these two equations alternately in conjunction with (16b) and (16c) has the same effect as (16) to first order in $\Delta t$.

(d) Extension to non-stationary conditions

In the application of the model described in this paper the conditions are assumed to be stationary. However, the method can be extended to non-stationary conditions by replacing the steady state condition with the demand that the model correctly describes the evolution of the phase space distribution of all particles of air. Of course the resulting concentration field can no longer be thought of as a time-average field, only as an average over the ensemble of flows. What is meant by 'the ensemble of flows' is determined by how much of the flow field is represented deterministically in $U(x,t)$ and how much is represented statistically in $\sigma(x,t)$.

The choice of $\mathbf{B}$ and $\mathbf{\gamma}$ must be made with care in order to ensure that the model results do not depend on the velocity of the reference frame in which the simulation is performed. However, the values of $\mathbf{B}$ and $\mathbf{\gamma}$ which make the model easiest to implement,
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namely

\[ \beta^i = \frac{\partial U^i}{\partial x^j} - \frac{1}{2}(\frac{\partial \sigma^d}{\partial x^j})\chi^{lm} U^m + \frac{1}{2}(\frac{\partial \sigma^d}{\partial t})\chi^{lj} \]

\[ \gamma^{ijk} = \frac{1}{2}(\frac{\partial \sigma^d}{\partial x^k})\chi^{ij} \]

also ensure that the results are independent of the reference frame. With these values the model for the particle trajectories reduces to (18), (16b) and (16c) as for the non-stationary case.

3. **Comparison of the Random Walk Model with Experimental Data Obtained in the Sirhowy Valley, South Wales**

In this section the random walk model described above is applied to the problem of dispersion in the Sirhowy valley and the results are compared with experimental dispersion data. The experimental data consist of twelve dispersion experiments; the six performed in 1981 will be referred to as runs 1 to 6, while those carried out in 1983 will be called runs A to F. The 1981 experiments are described by Whitlock et al. (1985) and some of the results from the 1983 experiments are presented by Callander (1986). Only four of the runs are compared here with the random walk model, namely runs 6, A, E and F. During these runs the atmospheric stability was close to neutral. The unstable cases (runs B, C and D) are not considered because there is currently little available information on the flow in the valley under such conditions. Also runs 1, 2, 3 and 5 were over too short a range to be of interest here, while in run 4 the results may have been spoilt by rain. The source location for each of the runs considered in this paper is shown in Fig. 1, and the duration of each tracer release is given in Table 1. For runs A, E and F the samplers were placed in a two-dimensional array covering a substantial part of the valley surface, while for run 6, in which a line source was used, a single line of samplers was deployed, as indicated in Fig. 1. The technique used for sampling the tracer yielded, at each sampling point, a value for the average concentration over the duration of the run. All the releases were at 2 m above the ground and, except for a few balloon-supported

![Figure 1. Map of the Sirhowy valley showing contours at intervals of 200 feet (61 m). The contours are shown as solid lines except where passing through pits or quarries, where they are indicated by long dashes. The dotted line marks the line along which samplers were placed in run 6. The release points for all the runs considered in this paper (6, A, E and F) are also shown.](image)
TABLE 1. 8 m SUMMIT WIND SPEED AND DIRECTION, AND THE DURATION OF THE TRACER RELEASE FOR EACH OF THE RUNS CONSIDERED IN THIS PAPER

<table>
<thead>
<tr>
<th>Run</th>
<th>Wind speed (m s⁻¹)</th>
<th>Direction (True)</th>
<th>Release duration (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12.7</td>
<td>265</td>
<td>28</td>
</tr>
<tr>
<td>A</td>
<td>7.8</td>
<td>222</td>
<td>68</td>
</tr>
<tr>
<td>E</td>
<td>4.2</td>
<td>66</td>
<td>58</td>
</tr>
<tr>
<td>F</td>
<td>6.7</td>
<td>247</td>
<td>85</td>
</tr>
</tbody>
</table>

samplers, the height of the samplers was about 1 m.

The valley shape is not symmetrical and rises to a sharp crest on the western side. For the numerical simulations, a simplified valley shape was used as shown in Fig. 2. In terms of the coordinate system \((x^1, x^2, x^3)\) (defined in Fig. 2), the valley shape will be denoted by \(x^3 = h(x^1)\).

The wind speed and direction on Bedwellte summit on the western side of the valley is given in Table 1. For run 6, the summit wind was not available; the value given is an estimate based on the wind measured at the crest and the data given in MK. The 8 m summit wind speed will be denoted by \(U_s\).

To apply the random walk model it is necessary to specify the \(\rho\), \(U\), \(\sigma\) and \(T\) fields. Density variations are neglected and \(U\), \(\sigma\) and \(T\) are assumed to depend on \(x^1\) and \(x^3\) only. The mean velocity field is taken from the model described in MK. This model is a two-dimensional (in the sense that \(\partial / \partial x^2\) is zero) finite-difference Navier-Stokes equation model with a Smagorinsky-type (1963) mixing length parametrization of the three-dimensional and subgrid motions. The mixing length \(l\) is given by \(l = 1/(k(z' + z_0)) + 1/l_0\) where \(l_0\) is a specified mixing length, \(z_0\) is the roughness length, \(z'\) is the length of the shortest normal to the underlying terrain and \(k\) is von Kármán’s constant which was taken to be 0.4. The model assumes the same simplified valley cross-section as that used in this paper.

The principal axes \(X^1, X^2, X^3\) of the tensors \(\sigma(x)\) and \(T(x)\) are chosen so that \(X^2\) is parallel to the \(x^2\) axis and \(X^3\) is perpendicular to the terrain vertically below \(x\). This choice of principal axes was made to enable \(T\) to be modelled correctly in the region near the surface where it is strongly anisotropic. These axes are probably incorrect above the region where the flow is in equilibrium with the surface, but because strong

![Figure 2](image)

Figure 2. (a) Cross-section of the valley along the dotted line in Fig. 1. (b) Simplified valley shape used in the simulations together with the axes of the right-handed coordinate system referred to in the text.
anisotropies occur only near the surface this is not a serious error. The decision not to vary the principal axes with height was made in order to simplify the computation. The form of $\sigma$ adopted means that there is no surface stress; in partial justification we note that, in Ley’s (1982) study of dispersion over flat ground, the effect on dispersion of the correlation between horizontal and vertical velocities is small. The surface stress could have been included in $\sigma$ without difficulty, but it would have added to the complexity of the computation. The eigenvalues of $\sigma$ and $T$ will be denoted by $\sigma_1^2$, $\sigma_2^2$, $\sigma_3^2$, $1/\tau_1$, $1/\tau_2$ and $1/\tau_3$.

To simplify the notation, $x$, $y$ and $z$ will often be used to indicate $x^1$, $x^2$ and $x^3$ respectively.

(a) Dispersion from releases on the summit

During runs 6, A and F the tracer was released from Bedwellte summit into a westerly air stream which carried the tracer across the valley. This scenario was modelled in a random walk simulation which will be referred to as simulation I. The $U$ field used in simulation I was generated by assuming a geostrophic wind of 10 m s$^{-1}$ across the valley, with $z_0 = 3$ cm and $l_0 = 40$ m. The streamlines of the flow in the $(x, z)$ plane are given in Fig. 3. Because the surface wind is backed relative to the geostrophic wind, the $U$ field contains a small velocity component parallel to the valley. The maximum 8 m wind across the summit is 7 m s$^{-1}$ and is backed by 21° from the geostrophic direction. The flow field is in good agreement with the observed wind field in westerly cross-valley flows (MK); in particular the size of the recirculation region is about right.

![Direction of mean flow](image)

Figure 3. The streamlines in the $(x, z)$ plane of the mean flow field used in all simulations except simulation IV. The flow is westerly, i.e. from left to right. The shaded area indicates the recirculation region.

In 1981 an extensive set of turbulence data was obtained (MK). Average values of the $\sigma_i/U_i$ and typical values of the positions of the peaks in the turbulence power spectra, $\lambda_i$, in westerly cross-valley flows under neutral conditions are summarized in Table 2. In Table 3 the values obtained during runs A and F are presented. The values of the $\lambda_i$ were obtained from the peaks of the Eulerian time spectra, $I_i^E$, by using Taylor’s frozen eddy hypothesis in the form $\lambda_i = \overline{u_i^E}$, where $\overline{u}$ is the mean wind speed at the measurement site. (Strictly speaking, the values of $\sigma_i$ given in Tables 2 and 3 are not the eigenvalues of $\sigma$ but the standard deviations of the velocity components in a coordinate system.
aligned with the mean wind (MK). This difference is neglected here. The \( \lambda_i \) in the tables refer to the same coordinate system.) These data show that the turbulence levels over the summit are comparable to those over flat ground while the levels in the valley are considerably enhanced relative to the summit values. Above the valley floor the turbulence levels increase with height up to the height of the summit before falling off at greater heights. Near the ground \( \sigma_1 \) and \( \sigma_2 \) are similar to each other and are between 1.5 and 2 times as large as \( \sigma_3 \), with the turbulence becoming more isotropic at a few hundred metres above the summit. The length scales, \( \lambda_i \), show \( \lambda_1 \) and \( \lambda_2 \) to be much larger than \( \lambda_3 \) as expected, the length scale \( \lambda_3 \) being constrained by the presence of the ground. On the summit the spectra of the horizontal components do not show a peak, implying a length scale of at least 2000 m. The turbulent energy levels, \( \sigma_i \), and time scales, \( \tau_i \), used in simulation I were chosen to agree with the measured turbulence data. The exact values used are given in appendix I.

In westerly flows the flow separates from the sharp crest at the valley edge. The release point for run F was on this crest while that for runs 6 and A was about 100 m upwind of the crest. To ensure that the position of the release point in relation to the recirculation eddy is similar in the simulation and in the experiments, the release point in the simulation was chosen to be 100 m from the edge of the recirculation region.

\( \Delta t \) was set equal to the minimum of \( \tau \) and one second. Decreasing the value of \( \Delta t \) by a factor of three made little difference to the results.

The results from simulation I are compared with the experimental data in Figs. 4 and 5. The agreement is very encouraging considering the large scatter in the data.

### TABLE 2. VALUES OF THE NORMALIZED TURBULENCE LEVELS \( \sigma_i/U_i \), AND THE SPECTRAL PEAK LENGTH SCALES, \( \lambda_i \), MEASURED IN 1981 (FROM MK)

<table>
<thead>
<tr>
<th>Height above ground</th>
<th>Valley 12 m</th>
<th>Summit 12 m</th>
<th>Aircraft measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_i/U_i )</td>
<td>0.20</td>
<td>0.14</td>
<td>330 m</td>
</tr>
<tr>
<td>( \sigma_i/U_i )</td>
<td>0.19</td>
<td>0.13</td>
<td>590 m</td>
</tr>
<tr>
<td>( (\lambda_i^2 + \lambda_s^2)_{1/2}/U_i )</td>
<td>0.12</td>
<td>0.07</td>
<td>0.16</td>
</tr>
<tr>
<td>( \lambda_i )</td>
<td>510 m</td>
<td>2000 m</td>
<td>0.12</td>
</tr>
<tr>
<td>( \lambda_s )</td>
<td>240 m</td>
<td>2000 m</td>
<td>0.10</td>
</tr>
<tr>
<td>( \lambda_3 )</td>
<td>50 m</td>
<td>31 m</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 3. VALUES OF THE NORMALIZED TURBULENCE LEVELS \( \sigma_i/U_i \), AND THE SPECTRAL PEAK LENGTH SCALES, \( \lambda_i \), MEASURED DURING RUNS A AND F (1982)

<table>
<thead>
<tr>
<th>Height above ground</th>
<th>Valley 18 m</th>
<th>Balloon above centre of valley</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_i/U_i ) Run A</td>
<td>0.23</td>
<td>0.25</td>
</tr>
<tr>
<td>Run F</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>( \sigma_i/U_i ) Run A</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>Run F</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>( \sigma_i/U_i ) Run A</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td>Run F</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>( \lambda_1 ) Run F</td>
<td>1000 m</td>
<td></td>
</tr>
<tr>
<td>( \lambda_s ) Run F</td>
<td>660 m</td>
<td></td>
</tr>
<tr>
<td>( \lambda_3 ) Run F</td>
<td>330 m</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4. Graph of $C_{max}(x)$ (the largest ground level value of the time-averaged concentration field at a given cross-valley distance $x$ resulting from a source on the summit) against $x - x_s$ (where $x_s$ is the value of $x$ at the source). $C_{max}$ has been normalized by $Q/U_s$, where $Q$ is the source strength and $U_s$ is the 8 m summit wind speed.

- A: Results obtained during run A
- F: Results obtained during run F
- T-T: Typical flat ground values (Turner 1969)
- I-I: Values from simulation I
- Ia-I0: Values from simulation Ia
- IU-IU: Values from simulation IU
- IIf-IIf: Values from simulation IIf

Where simulation I0 differs from simulation I it is shown, for clarity, as a dotted line.

Typical values for dispersion over flat ground are also plotted. These values are calculated from the Gaussian plume model described by Turner (1969), with the choice of the 8 m summit wind as the appropriate mean wind for use in Turner's formula. The simulation results are significantly better than the flat ground values at reproducing the observed dispersion. If the mean wind at the source, or a wind strength representative of the flow in the valley were used in Turner's formula, then the difference between the flat ground values and the observations would of course be larger. The differences between the observed concentrations and Turner's flat ground values are similar to those observed within a canyon by Start et al. (1975).

The sampler arrays used in the experiments were insufficient to determine the
vertical spread, $\sigma_y$, and along-valley spread, $\sigma_z$, of the plume from a point source, but the values obtained from the simulation are plotted in Fig. 6. Here $\sigma_y$ and $\sigma_z$ are defined by

$$\sigma_y^2(x) = \frac{\int \int c(x)y^2 dy \, dz}{C(x)} - \left( \frac{\int \int c(x)y \, dy \, dz}{C(x)} \right)^2$$

$$\sigma_z^2(x) = \frac{\int \int c(x)(z-h)^2 dy \, dz}{C(x)}$$

where $c(x)$ is the time-averaged concentration field and $C(x) = \int \int c(x) \, dy \, dz$. An alternative definition of $\sigma_y$ based on the model's ground level concentrations only,

$$\sigma_y^2(x) = \frac{\int c(x,y,h)y^2 dy}{C_0(x)} - \left( \frac{\int c(x,y,h) \, dy}{C_0(x)} \right)^2$$
where \( C_0(x) = \int c(x, y, h) \, dy \), gives comparable values although they are generally slightly greater, and up to 50\% greater between 50 and 150 metres from the source.

In the centre of the valley the vertical spread given by the model is much greater than that typical of dispersion over flat ground. This is due mainly to the vertical divergence of the mean flow, which balances the reduction in cross-valley velocity; on the far side of the valley, where the streamlines have converged again, the dispersion is comparable with, although still greater than, the flat ground value. The along-valley dispersion given by the model is much greater than the cross-wind spread typical of dispersion over flat ground; this is to be expected because of the high intensity of turbulence in the valley. The rapid increase of the horizontal spread over the first few hundred metres is due to the presence of tracer that has been recirculated in the region of separated flow and has had sufficient time to move a long distance along the valley. The concentrations in the valley are larger than might be expected from the very large values of \( \sigma_y \) and \( \sigma_z \); this is due to the large reduction in flow velocity in the valley which, because \( \nabla \cdot \mathbf{U} = 0 \), increases \( \sigma_z \) without decreasing concentrations, and to the recirculation eddy which allows a given element of tracer to contribute more than once to the time-average concentration.

Figure 5 shows that the concentrations obtained during run 6, and especially those on the far side of the valley, are lower than the model predictions. This is probably due to the finite length of the line source (1.1 km). Because the summit wind direction is about 12\° off the cross-valley direction, the cloud of tracer will have moved a substantial distance to the south in crossing the valley (the cross-valley component of the mean flow is greatly reduced in the valley, whereas the along-valley flow remains comparable to the summit value – MK). Also the length of the line source is comparable to the along-valley spread, \( \sigma_y \), on the far side of the valley, as given in Fig. 6.

![Figure 6](image.png)

The simulation-I concentrations (Fig. 4) are an order of magnitude lower than the values typical of flat ground. In order to investigate whether this enhanced dilution is due primarily to the mean flow or to the high turbulence levels in the valley, two further simulations were performed. These simulations will be called simulations Iσ and IU.
Simulation Iσ was identical to simulation I except that the turbulence parameters σ₁ and τ₁ were set equal to values typical of flat ground (see appendix I for details). In simulation IU the topography was removed and the mean flow was chosen to be \( U^1 = (u_e/k) \ln(z/z_0) \), \( U^2 = U^3 = 0 \) with \( z_0 = 3 \) cm. The turbulence parameters in this simulation were, as functions of \( x \) and \( z = h \), identical to those in simulation I. The results are shown in Fig. 4. They show clearly that the mean flow is the main cause of the enhanced dilution. It is of interest to note that, between 200 and 400 m from the source, the concentrations in simulation Iσ are smaller than those in simulation I; this is presumably because the lower diffusivity means that fewer particles diffuse into the recirculation region and reach ground level.

To try to explain the difference between the concentrations measured during runs A and F, two further simulations were carried out. In these simulations, which we will call simulations IIA and IIF, an attempt was made to model the experiments more closely. The valley site turbulence measurements indicate above average turbulence during run A and below average turbulence during run F. To take account of the values of \( \sigma_1/U_0 \), \( \sigma_2/U_0 \), and \( \sigma_3/U_0 \) used in simulation I were scaled to give the correct values at the valley site. Also, in simulation III, the release point was moved to the edge of the recirculation region and, because the observed summit flow in run F was almost exactly at right angles to the valley, the along-valley component of the mean flow, \( U^3 \), was set equal to zero. Some justification for this is provided by the experimental data gathered by MK which indicate that, for summit wind directions close to the cross-valley direction, the along-valley mean flow varies little with position and has little interaction with the cross-valley flow. The results of these simulations (shown in Fig. 4) show much less difference than that observed between runs A and F, and the reason for the observed differences remains unclear. It is possible that, despite the release time of about one hour, the differences were simply due to the inherent variability in the dispersion; of course the use of longer release times will not necessarily help in obtaining ensemble average concentrations because of the presence of variations in the flow on meso- and synoptic-scales. Simulation IIIf gives lower concentrations than I and IIA near the source because the tracer enters the highly dispersive recirculation region as soon as it is released; at more than 300 m downwind the concentrations are higher than in I and IIA as a result of the lower turbulence levels and the absence of a mean flow along the valley.

The rapid increase in \( \sigma \) near the source is an important aspect of diffusion in the valley. To discover whether this is due to the component of the mean flow along the valley (which makes the streamlines of the mean flow helical in the region of separation) or to the turbulence (which has a long time to act on those elements of tracer which are recirculated), four further simulations were performed. These simulations were identical to simulation I except in the following respects: in simulation IIIU \( U^2 \) was set equal to zero, in IIIσ the turbulent energy was reduced by a factor of two, in IIIτ the time scales were halved and in IIIv \( U^2 \) was set equal to zero and the turbulent energy was halved. The values of \( \sigma_1 \) obtained from these simulations are given in Fig. 6(b). The comparatively low values of \( \sigma_1 \) in simulation IIIU show clearly that a major part of the increase in \( \sigma_1 \) is due to the mean along-valley flow. Simulations IIIσ and IIIτ produce the surprising result that decreasing the diffusivity of the turbulence increases \( \sigma_1 \). This is because the lower diffusivity allows the particles to remain in the recirculating eddy for longer and allows the along-valley mean flow to carry the particles further. This effect does not occur when \( U^2 = 0 \), as can be seen by comparing simulations IIIU and IIIv.

(b) Dispersion from a release on the valley floor

During run E the tracer was released from the floor of the valley with an easterly
wind direction on the summit. This is modelled in simulation IV. In easterly winds the structure of the flow in the valley is a little different from that found in westerly conditions. This is probably due to the absence of a sharp crest on the eastern side of the valley (MK). The most significant difference is that the recirculation region is smaller. By using a roughness length of 3 mm (and leaving the other parameters as they were in the section on releases from the summit) the model of MK generates a flow field which is quite realistic (Fig. 7). The maximum 8 m summit wind in this flow field is 8.7 m s\(^{-1}\) and is backed 16\(^{\circ}\) relative to the geostrophic wind.

![Figure 7](image)

Figure 7. The streamlines in the \((x, z)\) plane of the mean flow used in simulation IV. The flow is easterly, i.e. from right to left. The shaded area indicates the recirculation region.

Another difference concerns the turbulence levels above the valley floor. Balloon measurements conducted in easterly flows (MK) do not show the increase in turbulent energy with height apparent in the westerly flow data in Table 3; instead they show the energy remaining constant or decreasing slightly up to the height of the summit. The only data available on the turbulence levels near the surface during easterly winds in neutral conditions are those obtained at 18 m above the valley floor during experiment E. The measured values of \(\sigma/\bar{U}_s\) were \(\sigma_1/\bar{U}_s = 0.22\), \(\sigma_2/\bar{U}_s = 0.17\) and \(\sigma_3/\bar{U}_s = 0.14\). These values are similar to those found in westerly flows. The \(\sigma\) and \(\tau\) used in simulation IV were chosen to agree with these observations. The exact values used are given in appendix 1. \(\Delta t\) was chosen as in simulation I.

At the release point (see Fig. 1), the observed mean wind was almost parallel to the valley – 1.3 m s\(^{-1}\) at 347\(^{\circ}\) true – and so the release point in the simulation was placed at the point where the separated flow reattaches. At this point the mean wind in the simulation is 2.0 m s\(^{-1}\) along the valley; when scaled with \(\bar{U}_s\) this value is similar to that observed.

The results from simulation IV are presented in Figs. 8 and 9. The results show encouraging agreement with the experimental data, although the concentrations given by the model are generally too large by a factor of about three. Figure 9 also shows concentrations typical of dispersion over flat ground (from Turner's (1969) Gaussian plume model). The simulation results are substantially better than the flat ground values at reproducing the experimentally measured dispersion. The flat ground values overestimate the largest concentrations at a given distance from the source by a factor
of 25 if $U_s$ is chosen as the appropriate wind speed for use in Turner’s formula and by a factor of 80 if the wind speed at the source is used.

4. CONCLUSION

A random walk model has been developed which can be applied to any situation in which $U(x)$, $\sigma(x)$ and $T(x)$ are known or can be realistically estimated. The model has been used to simulate dispersion in the Sirhowy valley, South Wales – a valley in which the flow is complex with separation occurring. Comparisons of the results with the experimental data are encouraging. The experimental and simulated concentrations have been compared with concentrations typical of dispersion over flat ground (Turner 1969). The concentrations from the simulations show much better agreement with the observed concentrations than the flat ground values do. The simulations show that, for releases from the summit, the low ground-level concentrations in the valley are primarily a result of the mean flow and not of the high turbulence levels in the valley.

The simulations have been unable to explain the differences observed between runs A and F which were carried out in similar conditions. It is possible that, despite the release time of about one hour, the differences were simply a result of the inherent variability in the dispersion.
For releases from the summit, the simulations have enabled estimates of $\sigma_y$ and $\sigma_z$ to be made; these are quantities which are difficult to obtain from the experimental data because of the lack of sufficient elevated samplers and the large horizontal area covered by the cloud of tracer. A surprising result is that decreasing the turbulent energy increases $\sigma_y$ by allowing the tracer to remain longer in the recirculating region and to be carried along the valley by the along-valley component of the mean flow.

ACKNOWLEDGMENT

I would like to thank my colleagues in the Boundary Layer Research branch of the Meteorological Office, especially Dr P. J. Mason, Dr J. C. King, Dr B. A. Callander, Dr G. J. Jenkins and Mr J. B. G. Whitlock, who conducted the experiments and made the data available to me. I am also grateful to Mr P. Hollingdale-Smith and Mr A. Gould of the Chemical Defence Establishment who supplied and carried out the chemical analysis of the diffusion samplers. The mean flow fields used in the simulations were supplied by Drs P. J. Mason and J. C. King.
APPENDIX 1: VALUES OF $\sigma_i$ AND $\tau_i$ USED IN THE SIMULATIONS

The turbulent energy levels used in simulation I are as follows (in S.I. units):

$$\sigma_1/U_s = 0.138 + (1 - h/200) \times \begin{cases} 0.064 + 7.51 \times 10^{-2}z & z < 180 \\ 1.17 \times 10^{-3}(350 - z) & 180 < z < 350 \\ 0 & z > 350 \end{cases} \quad (19a)$$

$$\sigma_2/U_s = 0.134 + (1 - h/200) \times \begin{cases} 0.055 + 4.35 \times 10^{-4}z & z < 180 \\ 7.84 \times 10^{-4}(350 - z) & 180 < z < 350 \\ 0 & z > 350 \end{cases} \quad (19b)$$

$$\sigma_3/U_s = 0.1 - 0.03(h/200)\max[2 - z/200, 0] + \begin{cases} 0.016 + 2.25 \times 10^{-4}z & z < 180 \\ 3.32 \times 10^{-4}(350 - z) & 180 < z < 350 \\ 0 & z > 350 \end{cases} \quad (19c)$$

These expressions have been chosen to agree with the values given in Table 2 and the data from the balloon during run F (Table 3). The valley site measurements suggest that turbulence levels during run F were generally below average, and so the values of $\sigma_i/U_s$ measured during run F were scaled up to give the same valley site values as are given in Table 2, before being used to influence the choice of expressions (19).

The time scales, $\tau_i$, used in simulation I are as follows (in S.I. units):

$$\tau_1 U_s = \min[278 + 0.866z + 5.714h, 1594] \quad (20a)$$

$$\tau_2 U_s = \min[131 + 0.858z + 6.457h, 1594] \quad (20b)$$

$$1/(\tau_3 U_s) = 1/183 + 1/(2.53(z - h)). \quad (20c)$$

These expressions give the appropriate linear variation of $\tau_3$ with height near the surface and, assuming the relation

$$\tau_i = 0.11\lambda_i/\sigma_1, \quad (21)$$

agree with the data in Tables 2 and 3. (21) is a combination of Taylor’s hypothesis, the formula $\tau_i/\tau_i^f = 0.44\bar{u}/\sigma_1$ where $\tau_i^f$ is the Eulerian integral time scale (Pasquill 1974, p. 92), and the assumption that the integral scale is one quarter of the spectral peak (Pasquill 1974, p. 59). $\sigma_1$, rather than $\sigma_i$, is used in (21) because the length scales $\lambda_i$ represent the length of the eddies measured in the direction of the mean wind. For computational reasons the value of $\tau_3$ was set equal to one second whenever (20c) yielded a value less than this. It seems unlikely that the exact value of $\tau_3$ in the lowest two metres or so will strongly influence the dispersion on the scale of the valley — indeed reducing the minimum value of $\tau_3$ to 0.3 s makes little difference to the model results.

In simulation I, $\tau_1$, $\tau_2$ and the $\sigma_i$ were set equal everywhere to the values given in (19) and (20) at $h = z = 200$ m (i.e. at the summit). $\tau_3$ was chosen as in simulation I.

The turbulence levels used in simulation IV are as follows (in S.I. units):

$$\sigma_1/U_s = 0.138 + (1 - h/200) \times \begin{cases} 0.083 & z < 200 \\ 4.15 \times 10^{-4}(400 - z) & 200 < z < 400 \\ 0 & z > 400 \end{cases} \quad (22a)$$
\[ \sigma_2/U_*, \begin{cases} 0.039, & z < 200 \\ 1.95 \times 10^{-4}(400 - z), & 200 < z < 400 \\ 0, & z > 400 \end{cases} \] (22b)

\[ \sigma_3/U_*, \begin{cases} 0.037, & z < 200 \\ 1.85 \times 10^{-4}(400 - z), & 200 < z < 400 \\ 0, & z > 400 \end{cases} + \] (22c)

These expressions agree with the values observed during easterly flows; where no observations are available they have been chosen to fit the westerly values given in Table 2. No data on the length scales \( \lambda_i \) are available in easterly neutral flows; in simulation IV the \( \tau_i \) are chosen as in simulation I.

APPENDIX 2: NOTATION

- \( c(x) \): Time-averaged concentration at \( x \)
- \( f_x(\mu) \): The density function of \( \mu \) for the particles at \( x \) (\( f_x(\mu) \) is the one-dimensional equivalent)
- \( g_n \): The density function of the phase space distribution of the particles of air
- \( g_n \): The density function of the phase space distribution of the particles of tracer before applying (5)
- \( g_{n+1}^0 \): As \( g_n \), but after applying (5)
- \( g_{n+1}^1 \): As \( g_n \), but after applying (6)
- \( h \): The height of the ground above \( x = 0 \)
- \( l_0 \): Basic mixing length used in the mean flow model
- \( s_n \): \( u_n - U(x_n) \)
- \( T(x) \): Tensor containing the local Lagrangian time scales at \( x \)
- \( u_n \): Velocity of a tracer particle after the \( n \)th time step (\( w_n \) is the one-dimensional equivalent)
- \( \bar{u} \): Mean velocity recorded at a measurement point
- \( U(x) \): Mean velocity field
- \( U_5 \): 8m summit wind speed
- \( x \): Position vector (the components are sometimes denoted by \( x, y \) and \( z \))
- \( x_n \): Position of a tracer particle after the \( n \)th time step (\( z_n \) is the one-dimensional equivalent)
- \( X \): The principal axes of the tensors \( \sigma \) and \( T \)
- \( z_0 \): Roughness length used in the mean flow model
- \( \beta(x), \gamma(x) \): Tensors used in the random walk model
- \( \Delta t \): Time step used in the random walk model
- \( \eta_n \): Random vector used in the \( n \)th time step
- \( \theta \): Independent variable for moment-generating functions
- \( \lambda_i \): The length scales of the spectral peaks
- \( \mu_n \): Random vector used in the \( n \)th time step (\( \mu_n \) is the one-dimensional equivalent)
- \( \rho(x) \): Mean density
- \( \rho' \): Departure of the density from \( \rho \)
\(\sigma(x)\) Covariance tensor of the turbulent velocities at \(x\)

\(\sigma_i(x)\) Variance of the turbulent velocity component in the direction \(X^i\)

\(\sigma_v(z)\) Variance of the turbulent velocity in the one-dimensional model

\(\sigma_y^2\) Mean square plume width in the \(y\) direction

\(\sigma_z^2\) Mean square height of the tracer particles above the ground

\(\tau(x)\) The local Lagrangian time scale for motion in the direction \(X^i\)

\(\tau(z)\) The local Lagrangian time scale in the one-dimensional model

\(\chi(x)\) Matrix inverse of \(\sigma(x)\)

\(^{\wedge}\) Moment-generating components

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