An observational study of the structure of stratiform cloud sheets:
Part II. Entrainment

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**SUMMARY**

The structure near the top of stratocumulus clouds is investigated with the aid of aircraft data to determine possible processes influencing entrainment. An expression is derived to predict the buoyancy fluctuations which can be produced by mixing at the interface due to evaporative cooling, and the relative influence of cooling by radiation and by evaporation near cloud top is discussed. It is argued that the effects of evaporative cooling are not properly considered in current assessments of the stability of inversions to entrainment. A method for taking this into account in a more realistic manner is proposed.

Entrainment rates derived directly from aircraft flux measurements are compared with various prediction methods. These include the predictions from four entrainment models. These predictions are found to vary widely for a given set of conditions but are generally smaller than the observationally derived values. Some of the consequences of underestimating entrainment and various other shortcomings of the models which are exposed by the comparisons are also discussed.

1. **INTRODUCTION**

Entrainment is the process whereby fluid is exchanged across a density interface bounding a region of turbulent flow. It is of central importance in governing the structure of clouds and must be properly represented in cloud models if realistic behaviour is to be achieved. In particular, models of horizontally extensive layer cloud are very sensitive to the way in which entrainment is specified.

The characteristics of entrainment processes have been observed in laboratory experiments (see Turner 1973) and show that relatively quiescent fluid is engulfed by turbulent motions penetrating across the mean density interface and is then subsequently mixed into the turbulent region. Smaller-scale motion is rapidly damped by the interfacial density gradient so that a sharp interface is maintained which advances into the quiescent fluid, causing the volume of the turbulent region to increase, or in the case of a horizontally extensive mixed layer, causing it to deepen. The rate at which this deepening proceeds has been related to various bulk flow properties in a variety of situations (e.g. mixed layers driven by shear stress or heating at one boundary) and results obtained in similar geophysical boundary layers are now readily interpreted using these concepts. In these fairly simple situations a combination of laboratory and field experiments, theoretical reasoning and numerical models have made it possible to produce formulations expressing the entrainment rate \( w_e \) in terms of local mean variables (e.g. Deardorff 1983). Although many aspects of such parametrizations remain to be tested, the current state of knowledge concerning corresponding cloud-topped boundary layers lags considerably behind, reflecting their increased complexity. Here, latent heat release and cloud top radiative cooling may both provide additional buoyancy giving rise to significant internal sources of turbulent kinetic energy (TKE). Furthermore, while the main sources of TKE in the clear case are often located at the lower boundary with energy supplied to the entrainment interface by turbulent transport through a mixed layer, entrainment in the cloudy case may also be maintained by sources adjacent to the interface itself. Experiments utilizing such a geometry have been carried out in the laboratory by McEwan and Paltridge (1976), but these did not include an analogue of another important feature, water phase change at the interface, which can cause significant density fluctuations through
evaporative cooling. Some aspects of the thermodynamics and possible significance of cloud top evaporative cooling have been addressed by Deardorff (1980a) and Randall (1980a).

Because of the more complicated circumstances, the difficulty of simulating the conditions in the laboratory and the virtual absence of field observations with the necessary range of measurements and accuracy, entrainment into cloudy boundary layers is poorly understood. Nevertheless, because of the importance of entrainment to the evolution of stratiform cloud, a number of models have been developed in an attempt to predict \( w_e \) in the absence of a well-defined parametrization. Such models are based on extensions of concepts derived from cloud-free mixed layers, but because of the additional complexities mentioned above, rest on a much less secure foundation. It is not at all clear that these concepts can be carried over directly to the cloudy case. Despite this, entrainment models are widely used to close the system of equations in models of the cloudy boundary layer.

This paper attempts to identify quantities important in controlling entrainment using data reported in part I of this paper (Nicholls and Leighton 1986, subsequently referred to as NL), where entrainment rates were derived from aircraft measurements in marine stratiform cloud layers. Comparisons with previous approaches are made wherever possible and the observations are also tested against various current entrainment prediction models.

Notation in this paper follows that in NL but a supplementary list of additional and commonly used symbols appears in appendix B.

2. Parametrizations Based on Interfacial Properties

In cloud-free mixed layers, \( w_e \) has been found to depend upon the properties of the turbulence maintaining entrainment and the density difference, \( \Delta \rho \), across the interface (Turner 1973). Many laboratory investigations have been carried out with turbulence being driven by a variety of processes: mechanical stirring with grids (e.g. Rouse and Dodu 1955), surface shear stress (e.g. Kato and Phillips 1969), convection resulting from surface heating (e.g. Willis and Deardorff 1974), and also by shear across the interface itself (Deardorff and Willis 1982). In these experiments involving a single dominant turbulence production mechanism with a length scale of \( l \) and a velocity scale \( c \), dimensional arguments (Turner 1973) suggest that

\[
\frac{w_e}{c} = f(g/\rho)\Delta \rho l/c^2 = f(R_i)
\]

which has been found to be quite successful. In geophysical applications, \( l \) is usually taken as the mixed-layer depth, \( h \), and \( c \) is associated with \( u_\ast \) (surface-shear-driven layers), \( w_\ast \) (convectively driven layers) or \( \Delta V \) (driven by a velocity shear, \( \Delta V \), across the interface). The most widely used results are those for surface shear and convectively driven mixed layers:

\[
\frac{w_e}{u_\ast} = a_1 \frac{1}{R_i} \quad [R_i = (g/\theta_c)\Delta \theta \sqrt{h/u_\ast^2}]
\]

and

\[
\frac{w_e}{w_\ast} = a_2 \frac{1}{R_i} \quad [R_i = (g/\theta_c)\Delta \theta \sqrt{h/w_\ast^2}].
\]

Quoted values of \( a_1 \) vary between 2.5 (Kato and Phillips 1969) and 5 (Kantha et al. 1977) and \( a_2 = 0.2 \) to 0.25 (Driedonks 1982 and Deardorff 1983). However, these equations remain valid only over a certain range, e.g. Kantha et al. suggest \( 90 < R_i < 400 \) for Eq.
(2), and the experimental data are still insufficiently accurate to confirm the exact nature of the dependence on $Ri$ in either case. It has also been suggested that some of the laboratory measurements may have been sensitive to the design of the apparatus and that certain important quantities were not measured (Deardorff 1983). Atmospheric measurements and numerical simulations do, however, broadly support Eqs. (2) and (3) which may also be derived by considering a simplified turbulent kinetic energy balance, either integrated across the entire mixed layer or just across the entrainment region (Driedonks 1982). A similar approach may also be used to deduce interpolation formulae for use when more than one turbulence-generating mechanism is important (Driedonks 1982 and Deardorff 1983).

In the cloud-topped case, the additional effects of radiative transfer and phase change must be considered. For much of the remainder of this section, attention is restricted to convectively dominated stratocumulus characterized by near adiabatic liquid water values and a sharply defined cloud top beneath a strong inversion (e.g. see NL). In such conditions the primary effect of radiative transfer is to destabilize the cloud layer, generating turbulence through the action of buoyancy, as shown in NL. In particular, a positive maximum is generated in the buoyancy flux typically some tens of metres beneath the interface which results in a $w^2$ maximum lower down in the cloud (which may also be further augmented by latent heat release at cloud base). The air within the inversion is also cooled by radiative transfer, but as shown in NL, this is usually a secondary effect being much smaller (6–18%) than the main cloud top cooling rate. Nevertheless, this does act to change the interfacial density jump, $\Delta \theta_r$, with time. Therefore, if the primary effects of radiative cooling and cloud base latent heat release are considered not to affect the mixing processes occurring around the interface directly, but are felt indirectly through the turbulence velocity scale and the value of $\Delta \theta_r$ (other aspects of this interpretation have been discussed by Nieuwstadt and Businger (1984)), then a relationship like Eq. (1) might be useful in describing entrainment into stratiform cloud. Both Deardorff (1980b) and Brost et al. (1982) have attempted such an approach using various forms of the velocity scale, but with little success. Tests with the current data are mentioned in section 2(b) below. However, an unattractive feature of this argument is that no explicit account is taken of possible density fluctuations produced in situ within the entrainment region at cloud top, the only parameter defining the properties of the interface being $\Delta \theta_r$. This is an essential difference between a density interface with cloud and one without. The presence of a cloudy layer means that both radiative and evaporative cooling can have a significant influence on the density fluctuations which result from mixing within the entrainment region near the interface. This implies that entrainment may exhibit an explicit dependence on factors which, for example, influence the degree of evaporative cooling at cloud top.

(a) The inclusion of evaporative effects

Figures 1(a) and (b) show data obtained on a horizontal measurement run near cloud top on flight 528 (data from part of this run were also discussed in NL, e.g. Fig. 4 in that paper). The data are split into two consecutive segments with linear trends removed from all but the top two traces, the remaining fluctuation data being denoted by primes (e.g. $w'$). The mean cloud thickness on this flight was determined to be 190 m (see NL, Table 1) with the first segment (Fig. 1(a)) generally located about 50 m below the average local cloud top (as defined in NL) and the second at about half that distance (as determined by the radiative flux measurements and the method described in NL). In Fig. 1(a), the cloud is almost continuous at flight level with only small excursions about the mean liquid water content of 0.17 g kg$^{-1}$, no obvious subsaturated regions and
Figure 1(a). Time series of data from the MRF C-130 research aircraft on a horizontal run approximately 50 m beneath cloud top on flight 528.
Figure 1(b). As (a) except now only 20 m beneath cloud top. Note that some vertical scales are different from (a).
relatively small fluctuations in $q_T$, $T_v$ and droplet concentration. (Note that $\theta'_v / T'_v \approx 1$, actually 1.04 on this run.) The most clearly defined features in the $w'$ and $T'_v$ time series are the negatively buoyant downdraughts which indicate an overall net positive buoyancy flux during this segment ($\overline{w \theta_v} \approx 2.2 \text{ cm s}^{-1}\text{K}$). In the second segment (Fig. 1(b)), closer to cloud top, there are significant subsaturated regions and large variations in $q_v$, $q_T$, $T_v$ and droplet concentration (N.B. because of this the scales in Figs. 1(b) and 1(a) are different). A comparison of the top three traces shows that evaporation affects the droplet concentration considerably more than the mean volume radius (except where complete evaporation has taken place), an attribute of the inhomogeneous mixing process studied in the laboratory by Latham and Reed (1977). Similar behaviour was observed by Slingo et al. (1982) also in marine stratocumulus. Figure 1(b) also shows that $q_T'$ and $T'_v$ are now strongly anticorrelated. Since the entrainment instability criterion is satisfied on this flight ($\Delta_2 < 0$: Deardorff 1980a; Randall 1980a) this suggests that mixing between inversion and cloudy air is taking place at this level in a way which seldom results in saturation being maintained. This is confirmed by Fig. 1(b) which shows that positive $T_v$ fluctuations are correlated with subsaturated regions. The correlation between vertical velocity fluctuations and the other quantities is much reduced compared with the preceding segment, and there is a net negative buoyancy flux ($\overline{w \theta_v} \approx -0.2 \text{ cm s}^{-1}\text{K}$). The vertical velocity variance is much reduced and the downdraughts seen in the previous segment are absent. These changes in the vertical velocity are more readily apparent in the expanded plots shown in Fig. 2. These show data from parts of the same two segments, but with a 20 s running mean removed.

These and other similar data indicate that the region where significant mixing between cloud and inversion air is occurring occupies a shallow layer at the interface, only a few tens of metres deep, where vertical velocity fluctuations are suppressed and negatively buoyant downdraughts are not yet established. Air from this layer is subsequently incorporated into downdraughts which are observed to be organized fea-

![Figure 2](image-url)  
Figure 2. Vertical velocity measurements from the same runs as Figs. 1(a) (top) and (b) (bottom) plotted on the same, expanded scales. These data have also had a 20 s running mean removed.
tures at slightly lower levels and from there is mixed down into the rest of the cloud. The presence of this shallow layer has also been documented by Caughey et al. (1982; using data obtained by traversing the interface with tethered instrumentation) who called it the entrainment interface layer (EIL). They concluded that entrainment took place as radiatively cooled parcels detached themselves from the base of this layer, drawing in filaments of warmer air from within it, a very similar interpretation to that given above. Mahrt and Paumier (1982) have also analysed some of the properties of this region from aircraft data and found that the thermodynamic properties of air at this level were consistent with mixing having taken place between various fractions of undisturbed inversion and cloud layer air.

These data suggest that an understanding of the effects of cloud top evaporative cooling on entrainment requires some knowledge of the possible range of density fluctuations which can be generated by mixing within the entrainment interface layer.

To assess this, consider a situation in which less dense air with thermodynamic properties \((\theta_1, q_1, q_{1\ell}, \theta_{e1}, \theta_{e2})\) overlies denser air with properties \((\theta_2, q_2, \text{etc.})\) and is separated by a density interface. If a mass \(m_1\) of state 1 is mixed isobarically with a mass \(m_2\) of state 2 defining a mixing ratio

\[
\epsilon = \frac{m_1}{(m_1 + m_2)}
\]

it is fairly straightforward to show (see appendix A) that the buoyancy excess of the mixture over that of state 2 is given by

\[
\delta\theta_v = \epsilon\Delta\theta_c - \epsilon\Delta q_1(L_v/c_p - \psi\theta_{e2}) + (q_{lm} - q_{1\ell})\{L_v/c_p - (1 + \psi)\theta_{e2}\}
\]

where \(\Delta s = s_1 - s_2\). Note that \(q_{lm}\) is the liquid water content of the mixture which is determined by the other independent variables, and also that only two of \(\theta_{e2}, q_{1\ell}\) and \(q_{1\ell}\) are independent if the pressure is specified and state 2 is saturated. Furthermore, although Eq. (5) was derived by considering only a single mixing event between the two states, it can be shown that any combination of mixing between these states or any resulting mixture can always be represented by a single value of \(\epsilon\).

Two example solutions to Eq. (5) are shown in Fig. 3 where \(\delta\theta_v\) is plotted as a function of \(\epsilon\). State 1 is now associated with inversion air (i.e. \(q_{1\ell} = 0\)) and state 2 with cloudy air. The entrainment instability criterion discussed by Deardorff (1980a) and Randall (1980a), i.e. that \(\delta\theta_v < 0\) if \(\Delta z < 0\), is contained within Eq. (5) although the requirement that the mixture remains saturated has been relaxed at the expense of having to specify \(\epsilon\). This criterion is satisfied in example 1 and \(\delta\theta_v < 0\) if \(\epsilon < 0.05\), the minimum occurring at that value of \(\epsilon\) which results in the mixture remaining just saturated. At larger values of \(\epsilon\) the mixture is no longer saturated and at \(\epsilon = 1\), where there is no mixing, \(\delta\theta_v = \Delta\theta_c\) (7.45 K in this case). The position of the kink in the curve depends upon \(q_{1\ell}\), the liquid water content of the cloudy layer, since the mixture will remain saturated at larger values of \(\epsilon\) if \(q_{1\ell}\) is increased. In example 2, \(q_{1\ell}\) has been increased to 0.8 kg kg\(^{-1}\) and state 1 altered to give \(\Delta z > 0\) while keeping \(\Delta\theta_v\) at its previous value. Now \(\delta\theta_v > 0\) for all \(\epsilon\) but because the liquid water content of the cloud and the humidity of the inversion air have been increased, the mixture remains saturated for \(\epsilon < 0.25\).

However, \(\delta\theta_v\) is less in this second example for all \(\epsilon > 0.074\), illustrating that the effects of evaporative cooling on density fluctuations are not well described by the parameters \(\Delta\theta_v\) and \(\Delta z\) as is often assumed. Even if \(\Delta z < 0\), the range of \(\epsilon\) for which negatively buoyant mixtures are produced may be small, especially in thin cloud where the liquid water content is low. Also shown in Fig. 3 is the corresponding curve for the cloud-free situation \((q_{1\ell} = 0\) for the same value of \(\Delta\theta_v\). The difference between this and the other curves shows the temperature reduction caused by evaporative cooling as a function of
\( \varepsilon \) (e.g. at \( \varepsilon = 0.05 \) in example 2 this reduction is 0.25 K as indicated by the difference AA').

Any indicator of interfacial stability should take into account the density fluctuations which can result from evaporative cooling during mixing. However, as Fig. 3 has demonstrated, the range of possible values is not well represented by just two points on the curve (\( \Delta \theta_v \) and \( \Delta z \)) which correspond to two special circumstances (no mixing and mixing which just maintains saturation, respectively). Observations show that all values of \( \varepsilon \) appear to occur within the EIL and it is difficult to conceive of a mechanism which would favour particular values of \( \varepsilon \). On the basis that all values of \( \varepsilon \) are equally important in determining the interfacial stability, a suitable parameter might be defined by

\[
\Delta_m = 2 \int_0^1 \delta \theta_v d\varepsilon.
\]

The numerical factor makes \( \Delta_m = \Delta \theta_v \) in the cloud-free case. This parameter expresses the stability of the interface in the presence of mixing in a physically meaningful way, through the range of density variations which can be generated by mixing in the presence of evaporation, and includes implicitly the effects of cloud liquid water content, inversion...
humidity and all the other quantities which can affect evaporation at the interface. The difference between \( \Delta m \) and \( \Delta \theta \), becomes larger as more evaporative cooling becomes possible and \( \Delta m \to \Delta \theta \), as the reverse occurs, either as the cloud becomes thinner (\( q_2 \to 0 \)) or as the contrasts between the inversion and cloudy air are reduced (\( \Delta \theta, \Delta q \to 0 \)). In the two examples considered above \( \Delta m = 7.1 \text{ K} \) (example 1) and \( \Delta m = 6.2 \text{ K} \) (example 2) which suggests that the latter interface could be more unstable to entrainment.

The relative sizes of density fluctuations produced within the entrainment interface layer by evaporation and radiation may now be compared. At cloud top, longwave radiative losses are primarily due to radiation from the droplets themselves, so the rate of cooling depends strongly on the liquid water content. Direct radiative cooling within the entrainment interface layer is therefore strongly dependent on \( \varepsilon \), the largest effects occurring at \( \varepsilon = 0 \), i.e. in unmixed cloudy air. This serves to emphasize that the primary effects of radiative cooling are felt within the cloud layer and influence entrainment indirectly rather than by directly affecting the mixing processes at the interface. In order to calculate the temperature decrease experienced within the EIL due to radiative cooling, an estimate of the time spent by a parcel within it must be made. This is an uncertain procedure, but taking the thickness of the EIL as \( \lambda = 30 \text{ m} \) and using \( \sigma_w = 0.26 \text{ m s}^{-1} \) (from the run from which Fig. 1(b) was taken) yields a time scale \( \lambda/\sigma_w = 2 \text{ min} \) which seems intuitively reasonable and is similar to those derived by Caughey et al. (1982) using different considerations. Calculations for the clouds studied in NL showed cooling rates of several degrees per hour in the top few tens of metres, falling by approximately an order of magnitude in the clear air just above cloud top. Temperature reductions due to radiative cooling of a few tenths of a degree may therefore be expected if \( \varepsilon \approx 0 \) (as seen in the downdraughts in Fig. 1(a) for example), falling to much smaller values as the liquid water content falls to zero and eventually to a few hundredths of a degree when \( \varepsilon = 1 \). These may be compared with the corresponding temperature reductions caused by evaporation (for the same values of \( \varepsilon \)) shown in examples 1 and 2 in Fig. 3. The decreases due to evaporation (e.g. AA' in example 2) are larger over the whole range of \( \varepsilon \) except near the two extremes \( \varepsilon = 0 \) and \( \varepsilon = 1 \), even for the fairly thin cloud implied in example 1. The generation of temperature reductions by radiative cooling within the EIL is therefore only dominant when little mixing occurs (\( \varepsilon = 0 \) or \( \varepsilon = 1 \)); evaporative cooling dominates otherwise. However, in both examples illustrated in Fig. 3, negative buoyancy (\( \delta \theta_\ell < 0 \)) can only be produced by radiative cooling if \( \varepsilon \) is small, i.e. downdraughts will be formed preferentially from cloudy air which has undergone little mixing with inversion air. Furthermore, in example 1 where \( \Delta z < 0 \), the maximum negative temperature fluctuation which can be produced by evaporation alone (\( -0.015 \text{ K} \)) is small compared with those which may arise from radiative cooling when \( \varepsilon = 0 \) (a few tenths of a degree). The significance of this is discussed further in the next section. However, in terms of their capacity to affect entrainment, the importance of radiative effects in the absence of mixing has already been implicitly recognized: cooling within cloudy air (\( \varepsilon = 0 \)) promotes turbulence (and hence increases \( w_\ell \)) and cooling in clear air above cloud (\( \varepsilon = 1 \)) causes \( \Delta \theta_\ell \) to vary. It is therefore the density fluctuations produced over the remaining range of \( \varepsilon \) values, and which are due mainly to evaporation, which need explicit consideration.

Finally it should be noted that the relative importance of radiative and evaporative cooling may be altered in the case of very thin clouds, for although radiative cooling may decrease if the cloud is no longer optically thick, evaporative effects will also be reduced. However, in such a situation the differences between a cloudy and a cloud-free interface would be much smaller and less significant.
Comparisons with measurements

If $\Delta_m$ is added to the list of parameters governing $w_e$ expressed in Eq. (1) to account for evaporative cooling at cloud top, the entrainment velocity in a convectively mixed cloud layer might then be described by a relationship of the form

$$w_e/w_* = f(Ri_{w*}, \Delta m/\Delta \theta_v).$$

(7)

$\Delta \theta_v$ is still expected to remain an important parameter since it expresses the static stability of the interface (i.e., with no mixing occurring) and will be the density difference felt by eddies with vertical length scales greater than the depth of the entrainment interface layer which do not directly feel the mixing processes occurring on smaller scales. The geometry or penetration of larger-scale hummocks or protrusions into the inversion around which mixing occurs will therefore depend upon $\Delta \theta_v$. The parameter $\Delta m/\Delta \theta_v$ is therefore a measure of the potential for evaporative cooling due to mixing.

In the cloud-free situation $\Delta_m = \Delta \theta_v$, the sensitivity to evaporative cooling disappears and Eq. (7) reduces to the form of Eq. (1). However, the dependence of $w_e/w_*$ on $Ri_{w*}$ will not necessarily follow Eq. (3) because Eq. (7) refers to conditions where a significant part of the buoyancy flux driving entrainment is generated near, but directed away from, the interface. The geometry of this type of cloud-free situation is different from the case where the lower boundary is heated (e.g., the most active convective elements are directed away from the interface, see NL Fig. 12, rather than towards it) and is analogous to the 'dense smoke' simulations of Deardorff (1980b) or the laboratory experiments of McEwan and Paltridge (1976).

The values of these various parameters for the five radiative-convective cases described in NL are listed in Table 1. The range of $Ri_{w*}$ encompassed by these measurements is very small, and in the absence of other data from similar conditions, little can be deduced about any possible $Ri_{w*}$ dependence, other than noting that there appears to be no correlation between variations in $w_e/w_*$ and $Ri_{w*}$. However, a comparison with the values anticipated in a cloud-free layer (given by Eq. (3) which yields $w_e/w_* \sim 10^{-3}$ at these values of $Ri_{w*}$) shows that the measured entrainment velocities are between 4 and 13 times greater in the cloudy cases at the same value of $Ri_{w*}$. This compares with a factor of 30 found by Deardorff (1980b) from numerical simulations, who also suggested in the same paper that an expression similar to Eq. (3) but utilizing $c = \sigma_u$ (the standard deviation of vertical velocity fluctuations near cloud top) as a velocity scale might be

<table>
<thead>
<tr>
<th>Case</th>
<th>$w_e$ (m/s)</th>
<th>$w_*$ (cm/s)</th>
<th>$\Delta \theta_v$ (K)</th>
<th>$\Delta m$ (K)</th>
<th>$w_e/w_* \times 100$</th>
<th>$Ri_{w*}$</th>
<th>$\Delta m/\Delta \theta_v$</th>
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</thead>
<tbody>
<tr>
<td>511</td>
<td>0.57</td>
<td>0.56</td>
<td>6.9</td>
<td>1.4</td>
<td>0.88</td>
<td>270</td>
<td>0.88</td>
</tr>
<tr>
<td>526</td>
<td>0.57</td>
<td>0.71</td>
<td>5.3</td>
<td>1.8</td>
<td>4.1</td>
<td>1.25</td>
<td>0.78</td>
</tr>
<tr>
<td>528</td>
<td>0.63</td>
<td>0.24</td>
<td>7.4</td>
<td>-0.3</td>
<td>6.7</td>
<td>0.38</td>
<td>0.91</td>
</tr>
<tr>
<td>620</td>
<td>0.78</td>
<td>0.44</td>
<td>7.4</td>
<td>1.9</td>
<td>6.5</td>
<td>0.56</td>
<td>0.88</td>
</tr>
<tr>
<td>624</td>
<td>1.18</td>
<td>0.56</td>
<td>7.4</td>
<td>1.6</td>
<td>6.1</td>
<td>0.47</td>
<td>0.82</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$u_e$ (m/s)</th>
<th>$w_e$ (cm/s)</th>
<th>$\Delta \theta_v$ (K)</th>
<th>$w_e/w_* \times 100$</th>
<th>$Ri_{w*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>564</td>
<td>0.8</td>
<td>1.2</td>
<td>5.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>
more useful in predicting $w_e$. Both Caughey et al. (1982) and Nicholls (1984) found

$$\frac{w_e}{\sigma_w} = CRi_w^{-1}$$  \hspace{1cm} (8)

gave good agreement in particular cases with $C \sim 10$. Here $Ri_w = W_{\alpha}^2/\sigma_w^2$ where $\sigma_w$ is measured near cloud top, but since Fig. 11 in NL shows that $\sigma_w^2 = \overline{w^2} = 0.4w_*^2$ in convective conditions, Eq. (8) is equivalent to

$$\frac{w_e}{w_*} \approx 0.25C Ri_w^{-1}.$$  \hspace{1cm} (9)

If $C \sim 10$, this predicts a tenfold increase in $w_e/w_*$ over the corresponding cloud-free value (Eq. (3)) at the same value of $Ri_w$. If $Ri_w \sim 250$, approximately the value encountered on the five convective cases listed in Table 1, Eq. (9) gives $w_e/w_* \sim 10^{-2}$ with $C \sim 10$. While the values listed in Table 1 show this to be a better overall estimate than Eq. (3), the variations between cases cannot be explained in this way due to the small variation of $Ri_w$.

The experiments of McEwan and Paltridge (1976), conducted over a wider range of $Ri_w$ (mainly $400 < Ri_w < 900$), found that although $w_e$ scaled with $w_*$ any dependence on inversion strength (i.e. $Ri_w$) was very weak. If this weak dependence extends to the $Ri_w$ range of the measurements, then the variation of $w_e/w_*$ may largely reflect changes in $\Delta_m/\Delta \theta_v$. This is tested in Fig. 4, where it does appear that the two quantities are correlated in the correct sense: the largest value of $w_e/w_*$ being observed in case 526 where evaporative cooling effects were also largest, and the smallest on flight 528 in the thinnest cloud where evaporation was limited by the relatively low liquid water content at cloud top. This sensitivity is also consistent with the results of McEwan and Paltridge in the limit of negligible evaporative effects ($\Delta_m/\Delta \theta_v = 1$) who found that entrainment due to convective erosion ($w_e/w_*$) was approximately constant at the small value of $6 \times 10^{-4}$ over the range of $Ri_w$ quoted above.

![Figure 4. $w_e/w_*$ against $\Delta_m/\Delta \theta_v$ for the five cases studied: 511 (●), 526 (▼), 528 (■), 620 (▲) and 624 (+). The result of McEwan and Paltridge (1976) is also shown (+).](image-url)
While the uncertainties in determining these quantities from measurements are considerable (see NL), the results do suggest that entrainment may be strongly affected by evaporative effects. However, this influence must be specified in a physically meaningful way, for example note (Table 1) that there is no correlation between $w_e/w_*$ and $\Delta_2$. It has been suggested (e.g. Randall 1980a; Deardorff 1980a) that compliance with the so-called entrainment instability criterion, $\Delta_2 < 0$, is sufficient for a stratiform cloud layer to become convectively unstable and rapidly break up. The data indicate that this criterion should be reappraised. For example, on flight 528 this criterion was satisfied ($\Delta_2 = -0.3$ K) but because the cloud was fairly thin, the maximum cloud top liquid water content was about $0.3$ g kg$^{-1}$. Equation (5) can be used to show that the maximum negative virtual temperature difference which can be generated in this case by evaporative cooling during mixing alone is about $-0.02$ K (at $\varepsilon \approx 0.07$). Figure 1(a), taken from the same flight, shows virtual temperature reductions an order of magnitude greater in downdraughts, which must therefore have been caused by radiative cooling, as noted in the previous section. In this situation, the condition that $\Delta_2 < 0$ presumably results in the predominantly radiatively cooled downdraughts (for which $\varepsilon \approx 0$) being perhaps a few hundredths of a degree cooler than would have been the case had $\Delta_2$ been zero (cf. example 1 in Fig. 3). With such a small relative effect, it is unlikely that a marked change in the cloud dynamics would ensue. In fact the cloud layer, although thinning slowly in this case, was observed in satellite images to remain continuous for several hours. If, perhaps, $\Delta_2$ had been sufficiently negative that evaporative cooling could have produced parcels with a negative buoyancy comparable with that due to radiative cooling, or alternatively had the liquid water content at cloud top been sufficiently high to maintain slightly negative buoyancy over a wider range of $\varepsilon$, then some effect might have been observed. Nevertheless, the condition $\Delta_2 < 0$ appears to be insufficient on its own to indicate the onset of greatly enhanced entrainment and cloud break-up. A similar conclusion was reached by Hanson (1984) from an analysis of aircraft profile data obtained in the eastern Pacific, who reported finding little correlation between the observed fractional cover of stratocumulus sheets (i.e. the number and sizes of breaks in the cloud) and compliance with the condition. However, additional measurements and numerical simulations of similar conditions are required to test these rather speculative ideas.

A value of $w_c$ was also determined for one other case in NL where the mixed layer was maintained by mechanical mixing (case 564). This resulted in very small liquid water contents near cloud top so direct evaporative cooling effects are likely to be minimal. As the mixed layer depth was also less than that of a corresponding steady state Ekman layer ($\sim 0.2 u_*/f = 1.6$ km in this case; Nicholls (1985)), the entrainment rate should be predicted by Eq. (2). The values listed in Table 1 yield $w_e = 0.8$ to $1.6$ cm s$^{-1}$ (as $a_j$ varies from 2.5 to 5), in good agreement with $w_e = 1.2$ cm s$^{-1}$ estimated from the measurements in NL and in contrast with the significant underestimates using Eq. (3) for the convective cases.

It should be noted that nowhere have the possible effects of shear across the interface on the entrainment rates been considered. Although it would be virtually impossible to detect any changes which could be unambiguously attributed to this cause in the presence of other effects without a large body of measurements, the data revealed no compelling evidence that such effects might be important. For example, no significant increases in the velocity variances were observed close to the interface which might be ascribed to the effects of interfacial shear. Neither did the corresponding stress measurements show any systematic behaviour (except case 564 which was dominated by the surface stress) which might also have been expected had TKE generation by this process been important.
3. COMPARISONS WITH ENTRAINMENT MODELS

As yet there is no known way of predicting \( w_c \) empirically from other flow parameters along the lines suggested in the previous section, so most previous cloud simulations have relied upon models to represent the effects of entrainment. By making general assumptions about the entrainment process, usually by relating the potential energy increase of a turbulently mixed layer due to entrainment to features of the energetics of the mixed layers, expressions for \( w_c \) may be obtained which display apparently reasonable behaviour over a range of conditions. The essential details of the most often used entrainment models are discussed below. Their predictions have, up to now, been virtually impossible to assess since only cloud top and cloud base height data have usually been available to validate such models. As the time variations of these quantities are equally sensitive to processes other than entrainment, this rarely constitutes a stringent test, especially when other initial and boundary conditions are also poorly specified. A choice between models therefore often depends less on the knowledge that entrainment velocities are likely to be correctly predicted and rather more on other considerations, e.g. the range of conditions in which the model continues to give sensible values. Furthermore, these closure models usually consider only buoyant production of turbulence to be important in controlling entrainment, the rationale being either that mechanical production is locally small near the interface or that it is exactly balanced by local dissipation. The models discussed below are all restricted in this way, but because five of the six cases presented in NL were dominated by convection, comparisons between these observations and the models should be valid.

(a) The models

Lilly (1968) postulated two bounds on the entrainment rate, the maximum hypothesis

\[
\int_0^h \overline{w\theta_c} \, dz = 0 \quad \text{but} \quad \overline{w\theta_c} \neq 0 \text{ somewhere} \quad (10)
\]

which assumes that all of the buoyancy generated is available to drive entrainment; and the minimum hypothesis

\[
[w\theta_c]_{\text{min}} = 0 \quad \text{but} \quad \int_0^h \overline{w\theta_c} \, dz > 0 \quad (11)
\]

which gives sufficient entrainment to cause a zero buoyancy flux somewhere. It is well known that the latter condition is too restrictive since observations have shown that mechanisms exist to maintain regions where \( \overline{w\theta_c} < 0 \) in both clear and cloudy mixed layers.

Schubert (1976, SH) used a weighted average of Eqs. (10) and (11) such that

\[
\frac{k}{h} \int_0^h \overline{w\theta_c} \, dz + \frac{1}{4}(1 - k)(\overline{w\theta_c})_{\text{min}} = 0 \quad (12)
\]

with \( 0 \leq k \leq 1 \). Solutions to this equation satisfy

\[
\int_0^h \overline{w\theta_c} \, dz \geq 0 \quad \text{and} \quad (\overline{w\theta_c})_{\text{min}} \leq 0.
\]

If \( k = 0.2 \), Eq. (12) reduces to a generally accepted formulation in the case of a clear mixed layer heated from below. However, as mentioned previously, there is no reason why this value should continue to remain valid in the cloudy case where the location of
the sources of turbulent kinetic energy and therefore the nature of the resulting convective motions change considerably.

Deardorff (1976, DD) objected to the use of Eq. (12) as it represents an interpolation between two unrealistic extremes. Instead, he suggested that the buoyancy flux due to entrainment was a fixed fraction of the total buoyant production integrated throughout the mixed layer,

$$\left(\overline{w\theta_e}\right)_{\text{ent}} = -2k/\{(1-k)h\} \int_0^h \overline{w\theta_e}dz. \quad (13)$$

He also assumed that the dry case value of $k = 0.2$ would remain valid in cloudy situations.

Kraus and Schaller (1978, KS) thought that entrainment could be predicted by considering the ratio of the energy lost through work against buoyancy in regions where $w\theta_e < 0$ to that gained by buoyant production in the remaining parts of the mixed layer. Defining this ratio

$$r = -\frac{\int_0^h (w\theta_e < 0)dz}{\int_0^h (w\theta_e > 0)dz} \quad (14)$$

with $p = -(w\theta_e)_{\text{min}}/(w\theta_e)_{\text{max}}$ and $q = -(w\theta_e)_{\text{ent}}/(w\theta_e)_{e}$, they showed that for a cloud-free mixed layer $r = p^2 = q^2$ with $r = 0.04$ if $p$ (= $k$ in this case) = 0.2. They also showed that there is no fixed relationship between $r$, $p$, $q$ or $k$ for cloudy mixed layers so that closure by defining a constant value of $r$ does not imply fixed values of the others. Thus closure by Eq. (14) is not generally equivalent to closure using Eq. (12).

Both Randall (1980b) and Fravalo et al. (1981) have used closures based on Eq. (14) with $r = 0.04$. The former noted that $r = 0$ and $r = 1$ correspond to Eqs. (11) and (10) respectively and also suggested that the approach should be broadened to include mechanical as well as buoyant production.

Stage and Businger (1981a, SB) performed a more complicated decomposition of the buoyancy flux profile than Eq. (14) along the lines originally suggested by Manins and Turner (1978) for the cloud-free case. In this interpretation energy loss to negative buoyancy is not just restricted to regions where $w\theta_e < 0$ or buoyant production to regions where $w\theta_e > 0$ as in Eq. (14), but both are allowed to occur to some varying extent throughout the entire mixed layer. The net sum of these opposing influences must, of course, still yield the original buoyancy flux profile. Different physical processes, e.g. radiation or entrainment, are deemed either to produce turbulent kinetic energy or to convert it back into potential energy with the net dissipation proportional to the net rate of production. By requiring the resulting system to recover the cloud-free, surface heated solution, an entrainment relationship was derived where $w_e$ is again linked to layer-integrated quantities. Comparisons between this scheme and the others listed above were given by Stage and Businger (1981b).

(b) Comparisons with measurements

Comparisons were made between four of the models listed above and the observations reported in NL within the framework of a mixed-layer model (described in Nicholls (1984)). Initial and boundary conditions appropriate to the time at which the flights were made were specified for the five convective cases using the observations and radiative transfer calculations described in NL. Each of the closure methods was selected in turn to diagnose the entrainment velocity from exactly the same conditions in each case. Since this mixed-layer model allows for water transport by rainfall, the closure methods were extended, where necessary, to reflect the fact that rainfall may also affect
the buoyancy flux profiles (as described in Nicholls (1984)), e.g. in the SB scheme, rainfall is always assumed to contribute a negative production term. A value of \( k = 0.2 \) was used in both the DD and SH models and \( r = 0.04 \) in the KS scheme. In the SB method, the ratio of dissipation to production was set to 0.8 (i.e. \( 1 - k \), Stage and Businger (1981a)) which means that all of the models reduce to the same formulation in the cloud-free case. Results derived from these entrainment models and from observations are listed in Table 2.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Reference</th>
<th>Case</th>
<th>Case</th>
<th>Case</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_c ) (cm s(^{-1}))</td>
<td>DD</td>
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<td>0.24</td>
<td>0.28</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>KS</td>
<td>0.39</td>
<td>0</td>
<td>0.26</td>
<td>0.44</td>
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<tr>
<td></td>
<td>SB</td>
<td>0.25</td>
<td>0.11</td>
<td>0.13</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>SH</td>
<td>0.25</td>
<td>0</td>
<td>0.30</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>NL</td>
<td>0.50</td>
<td>0.71</td>
<td>0.24</td>
<td>0.44</td>
</tr>
<tr>
<td>( I ) (m s(^{-1})K)</td>
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<td>4.1</td>
<td>7.5</td>
<td>19.1</td>
</tr>
<tr>
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<td>KS</td>
<td>2.5</td>
<td>5.1</td>
<td>2.7</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>SB</td>
<td>2.9</td>
<td>4.7</td>
<td>3.6</td>
<td>8.9</td>
</tr>
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<td></td>
<td>SH</td>
<td>2.9</td>
<td>5.1</td>
<td>2.5</td>
<td>8.7</td>
</tr>
<tr>
<td></td>
<td>NL</td>
<td>2.2</td>
<td>2.1</td>
<td>2.9</td>
<td>5.5</td>
</tr>
<tr>
<td>( \overline{w\theta_c} )(_{\text{max}} ) (cm s(^{-1})K)</td>
<td>DD</td>
<td>1.8</td>
<td>2.6</td>
<td>2.8</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
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<td>2.8</td>
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<td>1.8</td>
<td>2.5</td>
<td>3.6</td>
</tr>
<tr>
<td>Cloud top (( wq_T ) (10^{-3}) (m s(^{-1}))</td>
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<td>0.5</td>
<td>0.8</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>KS</td>
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<td>0.0</td>
<td>1.2</td>
<td>1.3</td>
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<tr>
<td></td>
<td>SB</td>
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<td>0.2</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
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<td>1.4</td>
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<td>1.5</td>
<td>1.5</td>
<td>1.1</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Overall, the models tend to underestimate \( w_c \). Consequently \( \overline{w\theta_c} \)\(_{\text{max}} \) and the buoyancy integrals (\( I \)) are rather larger than those where \( w_c \) has been derived from observations (denoted NL in the table). However, these are not as sensitive to changes in \( w_c \) as the cloud top water flux, which is significantly below the observed value in most cases. With the exception of case 526, which is discussed further below, the SB model consistently gives the smallest entrainment velocity, about half the observed values. As the uncertainty levels of the latter are estimated as 30–50% (NL), such a difference is significant. The KS scheme gives a consistently large estimate which agrees quite well with the observations. The other two models perform somewhere between KS and SB. In case 528, \( \Delta_2 < 0 \) (Table 1) and no solution is possible in the DD scheme, a weakness already noted by Stage and Businger (1981b). This latter paper also discussed, more generally, the differences in the results predicted by the SB and DD methods for cases with positive surface buoyancy fluxes and \( \Delta_2 > 0 \). The SB scheme was found to predict smaller values of \( w_c \) for all cloud thicknesses, the difference being greater as the fraction of the mixed layer occupied by cloud decreased and as a function of the parameters defining the cloud top interface (actually \( \Delta_{2/1} \), see Appendix B for definition of \( \Delta_{1} \)) became smaller. This analysis explains the differences between the two schemes in cases
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511, 620 and 624, but not in case 526. For this case, in which the buoyancy flux at the base of the mixed layer, \( \overline{w \theta}_o \), was set to a small negative value \((-5 \text{ W m}^{-2})\), \( w_e \) from the SB scheme was found to be considerably less than that given by DD. Furthermore, both the KS and SH schemes gave \( w_e = 0 \) (negative values are not allowed, a zero value is the minimum possible), and yet the value derived from observations is quite large. With such large differences between the models and between these and the observed value, further tests on the sensitivity of the \( w_e \) estimates to small changes in \( \overline{w \theta}_o \) around zero were carried out for this case.

The results are shown in Fig. 5. Three of the schemes (KS, SB and SH) are strongly sensitive over a small range of negative \( \overline{w \theta}_o \), which is reflected in the range of the model estimates in this case. At \( \overline{w \theta}_o = -0.5 \text{ cm s}^{-1} \text{K} \), which was chosen as the bottom boundary condition, \( w_e = 0 \) for both the KS and SH methods because the integrated energy loss to negative buoyancy needed just to maintain the mixed layer is already equal to or greater than that which the entrainment model assumes may occur (i.e. \( r \geq 0.04 \) in the KS scheme) even if \( w_e = 0 \). Figure 5 also shows that the maximum entrainment hypothesis \( (l = 0) \) does not come close to being violated in any of the cases over the range of \( \overline{w \theta}_o \) under consideration. The different sensitivities are related solely to the particular assumptions built into the various models.

For \( \overline{w \theta}_o \geq 0 \), KS gives much greater entrainment than the other models once again. These values are of similar magnitude to that derived from observations in case 526 (0.71 cm s\(^{-1}\)) which suggests that the big difference between this value and those quoted in Table 2 may reflect this sensitivity to the lower boundary condition. A value of \( \overline{w \theta}_o = 0 \) could certainly not be ruled out from the observations (as used in the initial model runs in Nicholls (1984)) which would then have yielded the following estimates for \( w_e \): 0.63 cm s\(^{-1}\) (KS), 0.28 cm s\(^{-1}\) (DD, SB) and 0.14 cm s\(^{-1}\) (SH). These values are in line with the model performance found in the other four cases with KS producing close to the observed value and SB about half the observed values.

However, this also illustrates one of the undesirable aspects of the KS scheme, which is that \( w_e \) is very sensitive to small areas of negative \( \overline{w \theta}_o \) in the subcloud layer, even though the cloud layer buoyancy fluxes remain little changed. Zero entrainment is predicted even though \( l \) is significantly greater than zero and the cloud layer buoyancy fluxes, especially just beneath cloud top, are strongly positive. This is physically unrealistic and suggests that the cloud and subcloud layers must separate under such conditions as described in Nicholls (1984). In a separated configuration energy losses by negative buoyancy fluxes in the subcloud layer are reduced which enables some of the energy produced within the cloud layer to continue to maintain entrainment at cloud top. Models which ignore the possibility of separation occurring and use the KS closure are likely to be unrealistic in these situations.

4. CONCLUDING REMARKS

Aircraft measurements made at the top of convectively driven stratocumulus indicate the presence of a thin layer estimated to be only a few tens of metres deep where significant mixing between cloudy and inversion air occurs. Here the air is often subsaturated, the vertical velocity fluctuations are relatively small and there is a net negative buoyancy flux. Most noticeably, \( q_T \) and \( \theta \), are negatively correlated. This region is referred to as the entrainment interface layer (EIL). Beneath this at a depth of perhaps 50 m, negatively buoyant downdraughts have become established, the vertical velocity variance increases and \( q_T \) is positively correlated with \( \theta \), suggesting that radiatively cooled downdraughts have incorporated some drier air from the upper layer. In view of this, it is argued that
entainment at cloud top might be sensitive to processes which can generate density fluctuations in situ within the EIL in addition to the quantities known to affect entrainment in cloud-free situations, i.e. the nature and intensity of the turbulence driving the entrainment and the static stability of the interface. Both radiative and evaporative cooling are capable of generating such density fluctuations at a cloudy interface and it is their presence which makes mixing in such conditions different to that which occurs at a cloud-free inversion. An expression relating density differences produced by evaporation to the thermodynamic properties of the cloud and inversion layers shows that they are not well defined by either of the two parameters usually considered, $\Delta \theta_e$ and $\Delta_2$. A new measure of the stability of an inversion to mixing processes is derived which implicitly includes all the factors influencing evaporative cooling.

An evaluation of the relative effectiveness of radiative and evaporative cooling for producing temperature reductions within the EIL implies that either effect may dominate, depending upon the amount of mixing which has taken place between the cloudy and the inversion air. Radiative cooling is more important when little mixing occurs, but these consequences are accounted for indirectly, through the effects on the turbulence within the cloud layer and on the static stability of the interface. Otherwise, evaporative cooling dominates and it is suggested that this effect should be included when assessing the stability of an inversion at cloud top to entrainment. The generation of negatively buoyant parcels is dominated by the radiative cooling of air which has undergone little mixing with the inversion, even if $\Delta_2 < 0$, so that cloud layer downdraughts tend to be formed preferentially from such air. It is suggested that the condition $\Delta_2 < 0$ is insufficient to predict the onset of enhanced entrainment.

Methods of predicting $w_e$ in terms of scales involving interfacial parameters were assessed. As noted by previous investigators, the measured entrainment rates were found to be many (4 to 13) times greater than those expected in corresponding cloud-free convective conditions. Since the measurements were all made at similar values of $Ri_w$, no dependence on this parameter could be determined although the variation of $w_e/w_*$ is consistent with a possible dependence on evaporative cooling as suggested above. However, in the absence of further measurements and laboratory or numerical guidance, any such conclusion must remain tentative. Further work is required.
The measurements were also compared with the predictions of four entrainment models. On the whole, these tended to underestimate \( w_0 \) but the method of Kraus and Schaller (1978) produced estimates which were quite close to those derived from the observations. The Stage and Businger (1981a) scheme systematically gave values approximately one half of those observed. This tendency for the entrainment models to underestimate \( w_0 \) has important consequences for mixed layer modelling since Nicholls (1984) showed that increased entrainment tends to promote cloud layer separation which invalidates the single layer approach. Use of these closure models in such applications will therefore tend to reinforce the assumptions built into their formulation.

These comparisons also exposed the considerable differences between the various models, especially in conditions where the surface buoyancy flux is small and negative. The predicted entrainment rates are very sensitive to negative buoyancy fluxes in the subcloud layer and may fall to zero even though a positive maximum buoyancy flux is still predicted near cloud top and the layer-integrated buoyancy production is still positive. This is physically unrealistic; in these situations, observations (e.g. Nicholls 1984; NL) show that the regions of buoyant production become decoupled from the rest of the layer and entrainment continues. Since extensive areas of stratiform cloud are typically located over relatively cool ocean surfaces, this regime is likely to be of considerable importance and a careful appraisal of the behaviour of entrainment closure models in these circumstances is required.

**APPENDIX A**

In the absence of precipitation the variables \( \theta_{e} = \theta + L_{v}q/c_{p} \) and \( q_{T} = q + q_{l} \) are conserved to sufficient accuracy during isobaric mixing involving evaporation and condensation (e.g. Stage and Businger 1981a). \( L_{v} \) and \( c_{p} \) are here regarded as constant. Subscripts 1 and 2 refer to states 1 and 2 defined in section 2(a) and ‘m’ refers to the resultant mixture. Thus

\[
\theta_{em} = \varepsilon \theta_{e1} + (1 - \varepsilon) \theta_{e2} \tag{A1}
\]

\[
q_{Tm} = \varepsilon q_{T1} + (1 - \varepsilon) q_{T2} \tag{A2}
\]

with \( \varepsilon = m_{1}/(m_{1} + m_{2}) \), the mass mixing ratio. Then

\[
\theta_{em} = \theta_{m}(1 + \psi q_{m} - q_{lm}) = \theta_{em} - L_{v}q_{m}/c_{p} + \psi \theta_{em}q_{m} - \theta_{em}q_{lm} \tag{A3}
\]

where terms of order \( L_{v}q/c_{p} \) have been neglected in comparison with those of order \( \theta \). Replacing \( q_{m} \) in Eq. (A3) yields

\[
\theta_{em} = \theta_{em} - q_{lm}(1 + \psi) \theta_{em}/c_{p} + q_{Tm}(\psi \theta_{em} - L_{v}q_{m}/c_{p}). \tag{A4}
\]

Replacing \( m \) by 2 gives an expression for \( \theta_{e2} \) and the buoyancy excess of the mixed state relative to state 2 is proportional to the difference

\[
\theta_{em} - \theta_{e2} = \theta_{em} - \theta_{e2} - (1 + \psi)(\theta_{em}q_{lm} - \theta_{e2}q_{l2}) + L_{v}(q_{em} - q_{l2})/c_{p} + \psi(\theta_{em}q_{Tm} - \theta_{e2}q_{T2}) - L_{v}(q_{Tm} - q_{Tl2})/c_{p}. \tag{A5}
\]

The term \( \theta_{em}q_{Tm} \) can be expanded using Eqs. (A1) and (A2) with the result that

\[
(\theta_{em}q_{Tm} - \theta_{e2}q_{T2}) = \varepsilon \{((\varepsilon - 1) \theta_{e2} - \varepsilon \theta_{e1})(q_{T2} - q_{T1}) - q_{T2}(\theta_{e2} - \theta_{e1})\}. \tag{A6}
\]
Substitution of Eq. (A6) into Eq. (A5) gives
\[
\theta_{vm} - \theta_{s2} = \varepsilon(\theta_{e1} - \theta_{e2})\left(1 - q_{im} + \psi(q_{T2} - q_{im})\right) + \varepsilon(q_{T2} - q_{T1})(L_v/c_p - \psi\theta_{e2}) + \\
+ \left(L_v/c_p - (1 + \psi)\theta_{e2}\right)(q_{im} - q_{i2}) + \psi \varepsilon(\theta_{s2} - \theta_{c1})(q_{T2} - q_{T1}).
\]
Defining the difference \(\Delta s = s_1 - s_2\) and neglecting terms of order \(q\Delta \theta\) in comparison with those of order \(q\theta\) or \(L_v/c_p\) gives
\[
\theta_{vm} - \theta_{s2} = \varepsilon\Delta \theta - \varepsilon\Delta q_1(L_v/c_p - \psi\theta_{e2}) + (q_{im} - q_{i2})(L_v/c_p - (1 + \psi)\theta_{e2})
\]
which is Eq. (5).

**APPENDIX B**

**Supplementary notation (others as NL)**

\(L_v\) Latent heat of vaporization of water

\(R_i\)  \(= g\Delta \rho l/\rho c^2\) (see Eq. (1))

\(R_i\)  \(= g\Delta \theta \theta /\theta \), \(\sigma_w^2\)

\(T_v\) Virtual temperature \(= T(1 + \psi q - q_i)\) = \(T\theta_v/\theta\)

\(w_*\) Convective scaling velocity, see Eq. (6) in NL

\(\sigma_w\) \(=(w^2)^{1/2}\)

\(\Delta \theta_v\) \(\theta_v\) jump at cloud top, see Eq. (1) in NL

\(\Delta_1\) \(= \Delta \theta + \{\theta(1 + \psi) - L_v/c_p\} \Delta q_1\), symbols as in NL

\(\Delta_2\) See Eq. (2) in NL

**References**


Deardorff, J. W. DD 1976 On the entrainment rate of a stratocumulus-topped mixed layer. *ibid.*, 102, 563–582

1980a Cloud top entrainment instability. J. Atmos. Sci., 37, 131–147

1980b Stratocumulus-capped mixed layers derived from a three-dimensional model. Boundary-Layer Meteorol., 18, 495–527


Hanson, H. P. 1984 Stratocumulus instability reconsidered: a search for physical mechanisms. Tellus, 36A, 355–368


Nicholls, S. 1984 The dynamics of stratocumulus: aircraft observations and comparisons with a mixed layer model. ibid., 110, 783–820


Nieuwstadt, F. T. M. and Businger, J. A. Randall, D. A. 1984 Radiative cooling near the top of a cloudy mixed layer. ibid., 110, 1073–1078

1980a Conditional instability of the first kind upside down. J. Atmos. Sci., 37, 125–130

1980b Entrainment into a stratocumulus layer with distributed radiative cooling. ibid., 37, 148–159

Rouse, H. and Dodu, J. 1955 Turbulent diffusion across a density discontinuity. La Houille Blanche, 10, 530–532


1981b A model for entrainment into a cloud-topped marine boundary layer. Part II: Discussion of model behaviour and comparison with other models. ibid., 38, 2230–2242

Turner, J. S. 1973 Buoyancy effects in fluids. Cambridge University Press