Alleviation of a systematic westerly bias in general circulation and numerical weather prediction models through an orographic gravity wave drag parametrization

By T. N. PALMER*, G. J. SHUTTS and R. SWINBANK

Meteorological Office, Bracknell

(Received 9 September 1985; revised 14 March 1986)

SUMMARY

Systematic westerly biases in the northern hemisphere wintertime flow of the Meteorological Office 15-layer operational model and 11-layer general circulation model are described. Evidence that the failure to parametrize subgrid-scale orographic gravity wave drag may account for such biases is presented. This evidence is taken from aircraft studies, surface pressure drag measurements, and studies of the zonally averaged momentum budget. A parametrization scheme is described in which the surface stress is proportional to the near-surface wind speed and static stability, and to the variance of subgrid-scale orography. The stress is absorbed in the vertical by considering the influence of such gravity wave activity on static stability and vertical wind shear. A Richardson-number-dependent wave breaking formulation is devised, and the vertical stress profile determined by a saturation hypothesis whereby the breaking waves are maintained at marginal stability. It is shown that wave breaking preferentially occurs in the boundary layer and in the lower stratosphere.

Results from a simple zonally symmetric model show how the adjustment to thermal wind balance with a wave drag in the stratosphere, warms polar regions by adiabatic descent, and decelerates the mean westerlies in the troposphere.

The influence of the parametrization scheme on integrations of the 11-layer model is described, and found to be generally beneficial.

In a discussion of the reasons why this problem has only recently emerged, it is suggested that the satisfactory northern hemisphere winter circulations of previous, coarser general circulation models were due to a compensation implied by underestimating both the surface drag, and the horizontal flux of momentum by explicitly resolved large-scale eddies.

1. INTRODUCTION

Minimum scales of motion resolvable by numerical weather prediction (NWP) models have steadily decreased over the years as a result of developing computer technology. As a consequence, their predictive skill has improved. However, as wintertime integrations on state-of-the-art NWP models extend beyond the first few days, serious systematic errors in the northern hemisphere flow can develop. Typically these errors appear as extratropical cyclones which develop over the oceans, fail to fill adequately over land, and track across either the American or Asian continent without loss of identity. When averaged over several days the predominant effect at the surface is the appearance of excessively low pressure in high latitudes, and very strong surface westerly winds around mid-latitudes. Figure 1(a) shows a typical example of this, being a p.m.s.l. (pressure at mean sea level) field averaged over days 26–30 of a wintertime integration of a version of the 15-layer Meteorological Office operational model prior to the inclusion of a gravity wave drag parametrization. (Details of this, and other integrations of the Meteorological Office operational model, are given in a companion paper by Kitchen and Dickinson, in preparation.) Figure 1(b) shows the verifying analysis.

This ‘climate drift’ problem also impairs the ability of high resolution general circulation models (GCMs) to simulate accurately the observed climate. This in turn affects the model's ability to respond realistically to climate perturbations such as CO₂ doubling or to the local enhancement of sea surface temperature (this latter problem is discussed by Palmer and Mansfield (1986)). In Fig. 2(a) we show zonally averaged and time-averaged winds and temperatures from the last 60 days of a 90-day perpetual January integration of a version of the Meteorological Office 11-layer GCM (described in section 6), and in Fig. 2(b) an observed cross-section is shown. At all levels the flow is too strong.

* Present address: European Centre for Medium Range Weather Forecasts, Reading
Figure 1. (a) Time-mean sea level pressure field from days 26–30 of a wintertime forecast with the Meteorological Office operational model (prior to the inclusion of gravity wave drag). (b) The verifying analysis.
north of 40°N, and in high latitudes, especially in the stratosphere, temperatures are too low. By contrast the winds in the southern hemisphere are more accurately simulated. This contrasts with the behaviour of coarser resolution GCMs (see for example Pitcher et al. 1983) where the northern hemisphere mean sea level flow is well simulated, yet the southern hemisphere flow is seriously underestimated. The extent of the systematic error in other GCMs (past and present) and its dependence on resolution will be discussed later in the paper.

It is reasonable to expect an improvement in the northern hemisphere wintertime climatology of either the 11-layer GCM or the 15-layer operational model if, in mid-latitudes, the downward flux of westerly momentum to the land surface is increased. This

Figure 2. Zonal mean cross-sections of zonal wind (m s⁻¹) and temperature (K, dashed lines) for (a) days 31-90 of a January integration of the Meteorological Office 11-layer GCM (experiment C—see section 6); (b) January 1984, as derived from the Meteorological Office operational analyses.
suggests that friction over land may somehow be inadequate. A possible remedy may therefore be to increase the surface roughness length, $z_o$, over land. However, experiments with the 11-layer model have found that the general circulation is comparatively insensitive to changes in $z_o$ (Carson 1982). The reason for this is not hard to find. The flux of momentum to the surface from the first model sigma level is determined in the 11-layer model by the conventional bulk aerodynamic formula whose surface exchange coefficient is a function of low-level static stability. In stable conditions, for example over land in wintertime, the exchange coefficient and consequently the flux of momentum is very small and therefore insensitive to the degree of roughness of the land.

The parametrization of boundary layer friction in terms of bulk aerodynamic transfer formulae and diffusive mixing with coefficients determined by Prandtl's eddy mixing length theory, ultimately derives from studies of aerodynamic drag exerted by an homogeneous fluid on an object at high Reynolds number. Under these circumstances the drag on a fluid arises mainly from momentum transfer by eddies in the turbulent wake.

However, in stratified flow, the mechanism which produces drag can be quite different from that described above. If the fluid has buoyancy frequency $N$ and flows with a speed $U$ over a hill with a characteristic width $L$, then when the non-dimensional number $F = LN/U$ is of order unity or greater, momentum transfer from the hill to the fluid is possible by internal gravity wave activity, and a drag force can be imposed on the fluid without downstream boundary layer separation. With typical atmospheric values ($U \sim 10 \text{ m s}^{-1} ; N \sim 10^{-2} \text{ s}^{-1}$), $F \sim 1$, when $L \sim 1 \text{ km}$, implying horizontal wavelengths of about 6 km. Since the horizontal grid of NWP models currently has a spacing on the order of 100 km the effect of subgrid-scale momentum transfer by internal gravity wave modes cannot be dismissed a priori. Amongst others, Sawyer (1959), Lilly (1972) and Bretherton (1969) have presented evidence that such processes are important in the global momentum budget of the atmosphere. Indeed Lilly showed that a simple representation of gravity wave drag had a beneficial impact in GCMs.

There are two important properties of momentum transfer by gravity waves which distinguish it from momentum transfer by the conventional turbulent boundary layer processes described above. Firstly, under simple conditions of uniform flow, the downward flux of momentum due to flow over surface orography is directly proportional to $N$ (see section 4). This contrasts with the influence of $N$ on turbulent mixing lengths and transfer coefficients, which decrease as $N$ increases. Secondly, the upward flux of momentum may extend above the boundary layer, and the direct influence on the flow of the drag due to surface orography may be felt in layers well above the boundary layer. This effect of remote forcing by wave drag is reviewed clearly by McIntyre (1980). The theory of wave mean-flow interaction predicts that in steady conditions the wave drag will, to good approximation, occur only in regions where the wave is dissipating (though the effect of the drag may be felt over a much broader region).

Hence, in principle, we have a mechanism for reducing the westerly flow which would be effective over mountainous regions of the globe in statically stable conditions, and which could act directly on the flow above the boundary layer.

According to observational and theoretical evidence to be presented later in the paper, it is likely that a large fraction of this gravity wave drag will be felt in the lower stratosphere where tilting and overturning of isentropic surfaces leads to Kelvin–Helmholtz and convective instability. The induced ageostrophic mean meridional circulation will transmit the effect of this drag to the tropospheric winds, and increase polar temperatures by adiabatic descent. Hence the effect of this gravity wave drag may also help improve systematic errors in the thermal structure of a GCM's mean climate.
The plan of this paper is as follows. In section 2 we outline some of the theoretical framework for internal gravity wave excitation, propagation and dissipation. In section 3 we review observational evidence for the importance of orographically induced gravity wave drag. The expected influence of wave drag on the zonally averaged circulation is discussed in section 4. In section 5 we describe a parametrization of gravity wave drag in the Meteorological Office 11-layer general circulation model and present some results of the parametrization in section 6. In section 7 we discuss why the problem has only recently emerged, and why it is dependent on horizontal resolution.

The concept of envelope orography (Wallace et al. 1983) has also been developed as a possible means of reducing these systematic errors. In a second companion paper (Slingo and Pearson, in preparation) the effects both of envelope orography and gravity wave drag on multi-annual cycle integrations of the 11-layer model will be assessed.

2. Basic theory

The theory of internal gravity wave forcing and propagation is reviewed in many texts (see e.g. Scorer 1978; Smith 1979; Gill 1982; Fritts 1984). We give brief details of some relevant aspects below.

For stationary waves of constant amplitude, the vorticity, continuity and thermodynamic equations for two-dimensional adiabatic quasi-Boussinesq flow in the $x-z$ plane, linearized about some basic state flow $U(z)$ with Brunt–Väisälä frequency $N$, can be combined to give

$$\begin{align*}
\frac{\partial^2 \hat{w}}{\partial x^2} + \frac{\partial^2 \hat{w}}{\partial z^2} + l^2(z)\hat{w} &= 0 \\
\hat{w} &= \left\{ \frac{\rho^{1/2}(z)}{\rho^{1/2}(0)} \right\} w
\end{align*}$$

(1)

where $\rho$ is density, $w$ is vertical velocity, and

$$l^2 = N^2 / U^2 - (1/U) d^2 U / dz^2$$

is the Scorer parameter. Coriolis effects have been neglected consistent with our interest in horizontal wavelengths smaller than about 100 km.

From Eq. (1), vertical wave propagation associated with horizontal wavenumber $k = k_o$ requires $k_o^2 < l^2$. A sufficiently deep layer of low $l^2$ in the middle troposphere would act to trap gravity wave activity excited from flow over topography. This wave activity would then be swept downstream in the form of lee waves (Scorer 1978).

Selecting the purely upward propagating component, then the familiar WKBJ dispersion relation for stationary hydrostatic plane sinusoidal waves with horizontal wavenumber $k$ and vertical wavenumber $m$ (such that $k/m \ll 1$) is

$$m = N/U.$$  

(3)

Using this, and the continuity equation, the momentum flux $\tau = \rho \overline{uw}$ can be written as $\tau = -(1/\rho) mk \psi^2$ where the overbar is an average over a horizontal wavelength, and $\psi$ is a mass streamfunction.

The flow at the surface $z = h \sin kx$ satisfies the linearized boundary condition

$$w = U \frac{\partial}{\partial x} (h \sin kx) = \frac{1}{\rho} \frac{\partial \psi}{\partial x} \text{ at } z = 0.$$  

Hence the surface stress, $\tau_s$, is given by

$$\tau_s = \frac{1}{2} k \rho \psi U N h^2.$$  

(4)
More generally, if $\delta h$ is the amplitude of the vertical displacement of an isentropic surface, then

$$
\tau = \frac{1}{2} k \rho U N \delta h^2.
$$

(5)

From the Eliassen–Palm theorem (Eliassen and Palm 1961; McIntyre 1980), vertically propagating waves in the absence of transience and dissipation obey the condition $\tau = \tau_0$ at all levels.

Using the hydrostatic dispersion relation Eq. (3), the wave's impact on the local static stability and vertical shear can be written as

$$
N_{\text{total}}^2 = N^2 \{1 + (N \delta h/U) \cos \phi\}
$$

(6)

$$
\eta_{\text{total}} = \eta \{1 + R_i^{1/2} (N \delta h/U) \sin \phi\}
$$

(7)

where $\delta h$ is the amplitude of the displacement of the isentropic surface, $\phi$ the wave phase, $\eta = \partial U/\partial z$ and $R_i = N^2/\eta^2$ is the Richardson number. The subscript 'total' on the left-hand side of Eqs. (6) and (7) refers to the sum of the wave and background flow contributions.

Equations (6) and (7) suggest that a sufficiently large isentropic displacement could induce local Kelvin–Helmholtz instability. This forms the basis for our wave-breaking mechanism and incorporates ideas both from the billow instability mechanism discussed by Scorer (1978), and Lindzen's (1981) convective overturning parametrization for mesospheric gravity wave breaking. Further details are given in section 5.

It should be remarked that Eq. (7) can also be derived from a finite amplitude conservation equation formulated by Long (1953). Details of this derivation are given in Shutts (1986).

Figure 3. Mean observed profile of momentum flux over the Rocky mountains on 17 February 1970 (after Lilly and Kennedy 1973).
3. Observational evidence for gravity wave drag

In this section we review some of the direct and indirect evidence for the possible importance of orographically induced gravity waves on the large-scale flow. The direct evidence consists of aircraft measurements of wave momentum flux. The indirect evidence consists of microbarograph measurements of surface pressure over mountainous areas and estimates of residual terms in momentum and angular momentum budgets of the large-scale flow from operational or FGGE analyses. With appropriate caution, we shall interpret these residual terms as sources and sinks of large-scale momentum and angular momentum by scales of motion which cannot be resolved by the analyses.

(a) Aircraft observations

The most thorough study of gravity wave momentum flux using aircraft measurements has been conducted by Lilly and co-workers using data from a number of flights over the Rocky Mountains during the 1970s. Lilly and Kennedy (1973) reported measurements obtained flying in a mountain wave of ‘moderate amplitude’ and about 50 km horizontal wavelength. Figure 3 shows their estimate of mean observed momentum flux throughout the troposphere and lower stratosphere. The values are approximately constant with height in the troposphere with magnitude 0.6 N m\(^{-2}\). Between 15 and 20 km there is almost complete absorption of the upward flux of easterly momentum in a layer which Lilly and Kennedy describe as “a region of severe turbulence”. A potential temperature cross-section over the Rockies for this event is illustrated in Fig. 4. The wave undulations in the flow are readily apparent; the longer wavelength ones growing in amplitude to the tropopause.

![Figure 4. Potential temperature cross-section for the same case as Fig. 3. Solid lines are isentropes (K), dashed lines aircraft or balloon flight trajectories (from Lilly and Kennedy 1973).](image-url)
A more intense gravity wave event which led to a severe downslope windstorm in Boulder, Colorado, was reported by Lilly (1978). The displacement of the isentropes in the upper troposphere was similar to, but more intense than, that shown in Fig. 4. In the lower troposphere there are strongly undulating isentropes to the lee of the Rockies, indicating the presence of trapped waves. Lilly reports measured momentum fluxes of 1.2 N m\(^{-2}\) during this event.

The theory of downslope windstorms has been discussed by Klemp and Lilly (1975) and Peltier and Clark (1979), the latter emphasizing nonlinear interactions of the gravity waves with the mean flow. In particular, Peltier and Clark develop a time-dependent model in which upward propagating gravity waves break in the lower stratosphere, as a result of critical steepening of the isentropes. They conjectured that, as a result of wave breaking, a wave-induced critical layer exists which reflects downwards incident wave activity. Quasi-resonant amplification of the waves between this region and the ground leads to a further tripling of the surface wave drag.

A further set of twenty aircraft measurements of wave momentum flux over the Colorado Rocky Mountains was reported by Lilly et al. (1982). On average, fluxes were weaker for these flights than those previously reported by Lilly. Lilly et al. concluded that values between 0.05 and 0.1 N m\(^{-2}\) may be representative of a normal winter month, over the Rockies.

Aircraft measurements over Europe have also been reported in the literature. Hoinka (1984) describes aircraft measurements over the Pyrenees when the synoptic-scale flow was northerly in the mid and upper troposphere above the mountains. The observed momentum flux profile has a maximum of 0.7 N m\(^{-2}\) at 450 mb, and a minimum of 0.1 N m\(^{-2}\) near the surface and near the tropopause, with moderate turbulence reported. The minimum near the ground is not readily explicable, and the author suggests that it may be possible that the flight legs were not long enough to allow a clear separation of mean and fluctuating velocity. Hoinka (1985) also reports measurements of surface pressure drag (see section 3(b)) and momentum flux during a south-foehn event in the Alps. Above 5 km an upward flux with magnitude 0.3 N m\(^{-2}\) was measured, decreasing to zero at about 12 km.

Finally we report on a study by Brown (1983) over the British Isles. Momentum fluxes were measured from aircraft on five different occasions. On two of these occasions magnitudes were between 0.3 and 0.4 N m\(^{-2}\). On the other three days they were 0.1 N m\(^{-2}\) or less. Brown attempted to reconcile the observed horizontal wavelength with linear lee wave theory, and found that in four out of five cases the agreement was good. Largest values of momentum flux occurred for long horizontal wavelengths (~30 km) when the waves were only weakly trapped. It is not clear from Brown's study whether there were significant downstream fluxes of momentum due to trapped lee wave activity in the boundary layer when small momentum fluxes were measured by the aircraft. According to Brown, these occasions were associated with large values of the Scorer parameter in the lower troposphere.

A summary of these aircraft studies is given in Table 1.

**Surface observations**

The synoptic-scale surface pressure drag over large mountain ranges has been discussed by a number of authors using conventionally analysed pressure fields and a smoothed representation of orography. Whilst such drag can be comparable in magnitude with surface friction, much of it is resolved explicitly by GCMs such as the Meteorological Office 11-layer model, so that the corresponding momentum transfer is achieved through large-scale dynamics explicit in the model formulation.
On the other hand, measurements of pressure drag on scales up to one hundred kilometres or so are sparse. One well-documented study, however, is discussed by Smith (1978) who reports on a set of microbarograph measurements over the Blue Ridge mountain in the central Appalachians, taken over a two-week period in winter. These measurements provide particularly useful estimates of typical pressure drag over an almost two-dimensional (north–south) ridge with a width of about 5 km. Smith found that during the measurement period there were several occasions when pressure differences across the ridge exceeded 0.5 mb. During such periods the implied surface stress over the mountain would exceed 3 N m⁻². Over the two-week period pressure differences in excess of 0.2 mb were frequently observed.

Smith found that the pressure drag correlated well with the component of mountain top wind perpendicular to the ridge, and that during periods when the pressure drag was large, the surface flow at sites upstream of the ridge tended to be weak, with downslope flow comparable in magnitude with the mountain top wind. Hence, in Smith's study, downstream separation was not observed during periods of large form drag. Rather, upstream blocking was observed during such periods, of large low-level static stability, consistent with the excitation of gravity waves.

However, the typical values of stress (≈1 N m⁻²) implied by this study are much larger than those measured by aircraft over major mountain ranges (see Table 1). There are a number of possible reasons for this.

Firstly if the relatively stagnant air upstream of the ridge extends as far as the next upstream ridge in the (Appalachian) range, then a pressure gradient across the Blue Ridge could be associated with a cold stagnant air pool supported by the mountain slopes on both sides. Air could then flow over the valley between the two ridges without strong frictional coupling.

In agreement with Smith's discussion, however, a more likely explanation is that the mountain flow is exciting trapped lee wave activity whose momentum fluxes would not be measured by aircraft flying above the boundary layer. As discussed above, excitation of trapped lee waves could generate substantial orographic pressure drag.

Synoptic-scale pressure differences during the south-föhn event reported by Hoinka (1985, see above), suggested that the mountain drag should be between 1.6 and 6.7 N m⁻². Again, Hoinka attributed the discrepancy with aircraft observations to the existence of trapped convectively unstable gravity waves in the lower troposphere, though it should also be cautioned that since Coriolis effects are important on the synoptic scale, the aircraft measurements do not include the contribution $f v' \theta'/\theta_z$ (Eliassen and Palm 1961; Andrews and McIntyre 1976) to the total momentum flux, necessary to balance the synoptic-scale pressure drag. (From a Lagrangian viewpoint, the momentum flux corresponds to the pressure drag acting on material surfaces, e.g. isentropic surfaces in the steady state, defined by particle trajectories. The orographic pressure drag is then the

### TABLE 1. SUMMARY OF AIRCRAFT STUDIES

<table>
<thead>
<tr>
<th>Study</th>
<th>Number of measurements</th>
<th>Location</th>
<th>Average downward flux of horizontal momentum (N m⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lilly and Kennedy (1973)</td>
<td>1</td>
<td>Rockies</td>
<td>0.6</td>
</tr>
<tr>
<td>Lilly (1978)</td>
<td>1</td>
<td>Rockies</td>
<td>1.2</td>
</tr>
<tr>
<td>Lilly et al. (1982)</td>
<td>20</td>
<td>Rockies</td>
<td>0.08</td>
</tr>
<tr>
<td>Brown (1983)</td>
<td>5</td>
<td>British Isles</td>
<td>0.15</td>
</tr>
<tr>
<td>Hoinka (1984)</td>
<td>1</td>
<td>Pyrenees</td>
<td>0.7</td>
</tr>
<tr>
<td>Hoinka (1985)</td>
<td>1</td>
<td>Alps</td>
<td>0.3</td>
</tr>
</tbody>
</table>
special case in which the material surface coincides with the terrain surface.) Furthermore, it is not clear to what extent these pressure differences correspond to 'lift' rather than 'drag' forces on the flow.

Pressure forces of several tenths of a N m\(^{-2}\) on the Alps have also been reported by Davies and Phillips (1985).

(c) Zonal mean momentum budgets

(i) Latitude/height distribution of zonal mean momentum budget residual. Although direct measurement provides a very valuable means for assessing the importance (or not) of mesoscale gravity wave momentum transport, these studies are very localized, being confined to mountainous regions. Gravity wave drag exerted on more gentle variations in terrain elevation might be weaker but more widespread. It seems quite possible that under conditions of strong low-level static stability, even moderate-sized orographic ridges could accrue a substantial wave stress. It is clear that an integrated (overland) measurement of wave momentum transport is required to gauge the importance of gravity wave drag. One might hope that if the net decelerating effect of breaking gravity waves was important it would be revealed as a residual term in a zonal mean momentum budget. Holopainen et al. (1980) and Holopainen (1982) have carried out long-term momentum budgets for North America and Europe respectively. Both studies revealed large residual terms (attributable to subgrid-scale motion) the sense of which implies a substantial decelerating force over mountainous regions particularly at upper levels. Nurmi (1983) has carried out such a calculation for February 1979 using data from the first special observing period of FGGE. Global gridpoint analyses on a 1.875° lat./lon. grid were prepared using the ECMWF data assimilation scheme four times daily (00, 06, 12 and 18 GMT). Comparison of the zonally averaged terms in the momentum equation showed the expected tendency for cancellation in the horizontal momentum flux convergence term with the Coriolis torque. Even so the cancellation is not very close and a substantial residual term remains, implying, if unbalanced, a deceleration of typically 2 m s\(^{-1}\)/day near 200 mb. Peak deceleration rates of 4 m s\(^{-1}\)/day are attained near 30°N and generally the size of the residual is about half the size of the Coriolis torque.

It was decided to carry out a similar momentum budget for the three-month period from 1 December 1982 to 28 February 1983 using data from routinely archived uninitialized ECMWF analyses. As in Nurmi's study, data were obtained four times daily on a 1.875° lat./lon. grid—compatible with the ECMWF model at that time.

The residual \(\bar{R}_u\) is defined by

\[
\bar{R}_u = \frac{\partial u}{\partial t} + \frac{v}{\partial \phi} \frac{\partial u}{\partial \phi} + \frac{w}{\partial \rho} \frac{\partial u}{\partial \rho} - \tan \phi \frac{uv}{a} - f \bar{u} = \bar{R}_u
\]

where the double overbar represents a time and zonal mean and the terms on the left have their conventional meanings. Terms were evaluated as a vertical average for the layer between adjacent standard pressure levels so that, for example,

\[
\bar{R}_u = \frac{1}{2} \left( \bar{v} \frac{\partial u}{\partial \phi} \bigg|_{i, i+1} + \bar{v}_{i+1} \frac{\partial u}{\partial \phi} \bigg|_{i, i+1} \right)
\]

where \(\bigg|_{i, i+1}\) indicates the mean for the layer between levels \(i\) and \(i + 1\), and similarly for other terms except

\[
\bar{R}_u = \frac{1}{2} \left( \bar{w} (w_{i+1} - u_i) + u_{i+1} - u_i \right) / \Delta \rho.
\]
Layer-mean residuals were found using data supplied on the standard pressure levels 850, 700, 500, 400, 300, 200, 150, 100, 50 mb.

The resulting latitude/height cross-section (Fig. 5) shows a remarkably vertically-coherent pattern dominated by a mid-latitude region of deceleration with upper- and lower-level peaks. This low-level deceleration peak (850–700 mb mean) corresponds to the latitude of strongest westerlies, and might represent boundary layer friction and drag associated with trapped lee wave activity. It was decided not to carry out the budget calculation at levels lower than this, since much of the data would be below the surface (i.e. extrapolated). The dominant peak in the residual term occurs at about 35°N and 250 mb, close to the mean position of the subtropical jet and is negative, implying a drag force equivalent to 4 m s⁻¹/day—similar to Nurmii's estimate. Other notable features of the residual cross-section are a positive peak at 15°N 450 mb, which also is a drag force since the zonal mean wind is easterly there, and a narrow vertical strip of positive values near 60°N which constitutes an accelerating force since the zonal mean wind is westerly. However, if the supposed gravity wave drag causing this positive acceleration was absorbed locally (in longitude) in easterly winds the drag effect would still be positive in spite of the westerly zonally averaged wind at that latitude. It is interesting that Nurmii also picks up this narrow zone of positive forcing. This might reflect the fact that both budgets were carried out using ECMWF analyses and cannot be regarded as independent support for the calculation presented here. Nevertheless, Baldwin et al. (1985) find similar residuals using Goddard Space Flight Center analyses of FGGE SOP-1 data.

It is difficult to assess the accuracy with which the residual term represents the momentum forcing effect induced by subgrid-scale motion. Particularly in the latitude zone 25–40°N, which contains the residual peak, mountain complexes such as the Himalayas upset the analysis of wind and pressure. First-guess fields are generated by extrapolation to pressure surfaces of a six-hour forecast made on the ECMWF model's

![Figure 5. Residual term $\bar{\Pi}_w$ (m s⁻¹ d⁻¹) for zonal mean momentum budget calculated from uninitialized ECMWF analyses for December 1982 to February 1983 inclusive.](image-url)
hybrid vertical coordinate system (Simmons and Struufing 1983). This entails extrapolation over depths of several kilometres in the Himalayas and errors associated with this will spread laterally and vertically through the model’s optimal interpolation scheme (D. Shaw, personal communication). Even so, in the absence of error bars, the overall tendency for the momentum budget residual to imply a drag force should not be overlooked.

(ii) *Vertically integrated momentum budget.* Perhaps the biggest problem with the previous method of evaluating the subgrid-scale drag is the calculation of the mean meridional wind from real data. This can be circumvented by integrating the momentum budget vertically (with respect to pressure) so that the Coriolis torque vanishes due to the constraint of mass continuity. In its place appears the surface stress composed of a friction stress and mountain torque, neither of which can be determined accurately. Swinbank (1985) has made a detailed calculation of the zonally averaged mountain torque and surface friction during January and June 1979 using FGGE data and compared them with the total surface stress evaluated indirectly from the vertically integrated horizontal convergence of poleward momentum flux (Figs. 6, 7). Friction torque is calculated using the boundary layer scheme in the Meteorological Office GCM. A striking discrepancy is found in the total surface stress as calculated by the two independent methods in January 1979. In middle latitudes of the northern hemisphere the combined synoptic-scale mountain plus friction torque is a factor of two smaller than that inferred from the convergence of horizontal momentum flux. The poleward momentum flux is not likely to be an overestimate and Swinbank concluded that the effective surface ‘friction’ drag must be larger in reality. (A similar discrepancy exists in the tropics.) In contrast, the discrepancy in June 1979 is small and consistent with the fact that the systematic westerly error is a northern hemisphere winter phenomenon. It is also consistent with the notion that the ratio of the strength of boundary layer friction to gravity wave drag is strongly dependent on the low-level static stability over land. In winter, as discussed in the introduction, conventional boundary layer schemes predict small surface drag coefficients over land in association with high static stabilities, whilst wave drag theories suggest the

![Diagram](image_url)

Figure 6. Mountain torque and surface friction (per 2° latitude band) for January 1979, calculated from the Meteorological Office FGGE IIIa analyses. Also shown is the total torque inferred from the atmospheric angular momentum budget (from Swinbank 1985).
opposite. In summer the reverse is true so that if subgrid-scale wave drag were an important component of the total surface stress the discrepancy would be most obvious in winter.

(d) Other methods

We mention in passing that dual beam Doppler radar may in the future provide a powerful means for measuring momentum fluxes; indeed this method has been used for measuring such fluxes in the mesosphere (Vincent and Reid 1983; Fritts 1984).

On a more qualitative note, a casual glance at satellite imagery products readily shows the ubiquity of lee wave cloud and orographic cirrus, associated with the excitation of mountain waves.

4. Influences on the Zonally Averaged Circulation

In view of the weight of evidence presented in the last section supporting the idea that mesoscale gravity wave drag plays a fundamental role in the general circulation, it is desirable to try to understand what kind of qualitative effect it might have and how this could alleviate the systematic error. As far as the latitude/height cross-sections of zonal mean wind and temperature are concerned the systematic error can be regarded as composed of a surface wind and a thermal wind shear error. Although these two physical components of the error field are obviously not independent they are to some extent governed by different large-scale transport processes (i.e. vorticity and heat transfer respectively). For instance, the zonal mean eastward component of the surface stress is entirely determined by the vertically integrated horizontal momentum flux convergence in the time-mean. If the surface stress (including orographic contributions) were a monotonically increasing function of the mean surface zonal wind then, in the absence of changes in horizontal momentum transport, the inclusion of gravity wave drag would reduce the mean zonal surface wind no matter what the vertical distribution of the wave stress. In general, however, the destruction of thermal wind balance in the zonal flow by height-dependent wave stresses will induce mean meridional circulations

Figure 7. As Fig. 6, but for June 1979.
which can modify both the zonal mean wind and temperature distributions. This in turn would lead to changes in the flux of horizontal momentum associated with, say, unstable baroclinic eddies and stationary planetary wave radiation so that the dynamical consequences of any imposed change in the surface wind/stress relation are difficult to anticipate.

If, in accordance with the observational evidence cited earlier and theoretical evidence to be discussed later, much of the mesoscale gravity wave drag is transmitted directly to the upper troposphere or lower stratosphere it would be reasonable to expect the thermal wind as well as the surface wind to be reduced. In order to quantify this effect we consider in this section a simple model, on the sphere, of the mean meridional circulation induced by momentum sources, along the lines of Eliassen (1952). Details of the model are given in an appendix.

Consider a zonally averaged momentum sink \( \tilde{F} \) such that, with \( \mu = \sin \phi \),

\[
\tilde{F}(\mu, z) = \begin{cases} 
-16F_0 (1 - \mu) (\mu - \mu_c) & z_T > z \geq z_*, \quad \mu > \mu_c \\
0 & \text{elsewhere}
\end{cases}
\]

with \( F_0 = -5 \times 10^{-5} \text{ m s}^{-2} \) (about 5 m s\(^{-1}\) day\(^{-1}\)), \( \mu_c = 0.5 \) (see Fig. 8(a)) and where \( z_T \) is the height of the top of the model and \( z_* \) is the tropopause height. \( F_0 \) is typical of those diagnosed in the GCM runs in section 6. Choosing \( z_T = 25 \text{ km}, z_* = 10 \text{ km} \), \( H_o = 7.5 \text{ km} \) and

![Figure 8](image-url)

Figure 8. (a) Shape of momentum forcing \( \tilde{F} \) used in the model of the zonally averaged circulation. (b) Tendency of mean zonal wind \( \partial U/\partial t \) (ms\(^{-1}\) day\(^{-1}\)).
\[ N^2 = \begin{cases} 4 \times 10^{-4} \text{s}^{-2} & z_* \leq z \leq z_T \\ 1.5 \times 10^{-4} \text{s}^{-2} & 0 \leq z < z_* \end{cases} \]

(so as to model the different stabilities in the troposphere and stratosphere) leads to a mass streamfunction distribution as in Fig. 9. Figures 8(b) and 10 show the tendencies of mean zonal wind and potential temperature respectively. Figure 9 shows the induced circulation centred at 45°N 10 km and implying poleward (equatorward) mass flux above (below) this level. Figure 8(b) shows the zonal wind tendency is largest, naturally, in the region of largest forcing through \( \vec{F} \) (48°N). Even so its magnitude is about one half of \( \vec{F} \) there since the imposed drag is offset by the Coriolis torque associated with the induced meridional circulation. At this latitude the deceleration rate drops from about 2.5 m s\(^{-1}\)/day at 25 km to 1.1 m s\(^{-1}\)/day at 10 km. Below this, the flow decelerates also, even though \( \vec{F} \) is zero, because of the Coriolis torque implied by the equatorward drift there. A limited region of acceleration occurs in the stratosphere near 28°N where the poleward drift is not compensated for by the imposed drag. Figure 10 shows the distribution of adiabatic warming accompanying the induced circulation. Peak warming rates of \(-0.9 \text{K/day}\) are to be found between 60 and 70°N just above the tropopause. On the equatorial side of the imposed drag, adiabatic cooling rates of about \(-0.6 \text{K/day}\) occur near 33°N just above the tropopause.

The height-independent deceleration \( \vec{F} \) imposed in the model stratosphere implies a stress difference between the model tropopause and upper lid. Substituting values appropriate to our calculation and assuming \( \tau(z_T) = 0.0 \) gives \( \tau(z_*) = 0.11 \text{N m}^{-2} \) at \(-48°\text{N}\), which is of the order of the 'missing drag' in the vertically integrated momentum budget of Swinbank (1985).

These calculations show that if this 'missing drag' is delivered to the stratosphere, substantial adiabatic warming might be expected on the northern side of the drag maximum accompanying the overall decrease in zonal wind. The influence of this stratospheric drag is communicated to the troposphere through the induced mean meridional circulation and associated Coriolis torque. These combined effects work in the

![Figure 9](image_url). Mass streamfunction (kg m\(^{-1}\)s\(^{-1}\)) given by the momentum forcing \( \vec{F} \).
right sense to alleviate both the tropospheric and stratospheric westerly wind errors and warm the excessively cold polar stratosphere.

5. A PARAMETRIZATION SCHEME FOR SUBGRID-SCALE GRAVITY WAVE DRAG

It is apparent from even the simplest linear theories of stationary, orographically forced gravity waves (Sawyer 1960) that the character of flow over hills-mountains is sensitive to specific details of the upstream horizontal wind profile. Numerical studies of larger amplitude waves generated in more extreme circumstances suggest such a richness of dynamical behaviour and sensitivity to detail that parametrization of the dominant transport processes would seem to be out of the question. On the other hand, observational studies such as that of Brown (1983) (see section 3) indicate that under less extreme circumstances linear theory, judiciously interpreted, might form a useful basis for a parametrization scheme. Clearly, in devising a parametrization scheme we are not merely trying to mimic the momentum flux predicted by any specific theoretical model or measured in an observational study. The task is to represent the vertical flux of momentum due to subgrid-scale gravity waves by a parametrization scheme whose plausibility with respect to theory, observations and beneficial impact (in the GCM) are all maximized, though with economy of representation and computational efficiency paramount considerations. For future reference, we show in Fig. 11 the model sigma levels at which the model variables are stored, and the levels at which parameters used by the gravity wave scheme are calculated.

There are two parts to the parametrization scheme used here: first a formula for the surface pressure drag exerted on the subgrid-scale orography and second a separate computation of the vertical distribution of wave stress accompanying the surface value. To some extent the subgrid-scale pressure drag on the orography also helps mitigate effects of the inaccurate explicitly resolved component of drag on mountains of scale close to the two gridlengths. For instance, the entire European Alps cannot be adequately
resolved by either the 11-layer GCM or the 15-layer operational coarse mesh model in present use at the Meteorological Office.

Using Eq. (4), we represent the surface drag by the expression

$$\tau_s = \kappa \rho \nu \cdot \text{VAR}$$  \hspace{1cm} (8)

where \( \rho \) and \( \nu \) are evaluated on the first \( \sigma \) level (in layer number 11, see Fig. 11), and \( N \) is evaluated by a finite difference formula using values of \( T \) on the first two levels (in layers number 10 and 11). The quantity \( \text{VAR} \) is equal to the variance of the subgrid-scale orography, as determined by an orographic dataset (U.S.A. Navy) with a resolution of ten minutes of arc. (However, even a resolution as high as this might not be adequate. As discussed earlier, Smith (1978) finds a substantial drag on a mountain ridge which is only 5 km wide.) This variance was limited to \((400 \text{ m})^2\), since without this limit, very large subgrid-scale variances occur at the edges of mountain ranges, such as the south side of the Himalayas, and substantial numerical two-gridlength waves are generated. In subsequent experiments (not reported here) we have limited the variance by a Froude number criterion to a value of \((U/N)^2\). This corresponds to the notion that when \( Fr = hN/U > 1 \), for a mountain of height \( h \), stagnation of low-level air effectively redefines the orographic profile, so as to limit excitation of gravity waves. The constant \( \kappa \) is regarded as a legitimately 'tunable' parameter, constrained only to an order of magnitude by linear theory. We have chosen a value of \( 2.5 \times 10^{-7} \text{ m}^{-1} \), which (see section 6) gives reasonable values for the surface stress in our GCM integrations. This corresponds in linear theory to a horizontal wavelength of 250 km if the orographic variance is supposed to be sinusoidal. Since the subgrid-scale orography corresponds to scales between 20 and 200 km it is clear that our tuning constant is smaller than that suggested by a direct application of linear theory.

![Figure 11](image)

Figure 11. Vertical distribution of levels in the 11-layer model, indicating the sigma levels at which model variables are stored, and the levels at which parameters used by the gravity wave scheme are calculated.
Given the vertical profile of wind, potential temperature and density, the second part of the parametrization scheme predicts the vertical profile of wave stress $\tau$ as a function of height. The problem is formidable, apart from the principal difficulty associated with the modification of propagating gravity waves by small-scale turbulence in the breakdown phase, one is faced with the basic three-dimensionality of real gravity wave systems, transience, the turning of the wind with height, diabatic heating, internal reflection and numerous other fluid mechanical subtleties. It is doubtful whether a parametrization scheme could address all of these difficulties without being as computationally demanding as a numerical model of the motion system itself. We base our parametrization scheme for breaking gravity waves on the simple steady, two-dimensional model of stratified flow discussed in section 2 and invoke a saturation hypothesis similar to that of Lindzen (1981) to determine the stress distribution.

First of all we assume that the stress, $\tau$, at any level is parallel to the surface stress vector and can be written as

$$\tau = |\tau| = \kappa \rho U \delta h^2 \quad (9)$$

where $U$ is the magnitude of the component of the wind vector $u$ in the direction of the surface stress (i.e. $U = (u \cdot \mathbf{\tau})/|\mathbf{\tau}|$).

Secondly we combine Eqs. (6) and (7), which describe the gravity wave's influence on static stability and vertical shear, to form a 'minimum Richardson number'

$$Ri_{\text{min}} = Ri \frac{1 - (N\delta h/U)}{1 + Ri^{1/2} (N\delta h/U)^3} \quad (10)$$

which represents the smallest value the total Richardson number can achieve under the influence of the gravity waves.

For a monochromatic wave, Eqs. (6) and (7) show that regions of maximum vertical shear are $\pi/2$ out of phase with regions of minimum static stability. However, following the notion of parametrizing the effects of an ensemble of phase-incoherent subgrid-scale gravity waves, we have chosen to ignore the phase difference between Eqs. (6) and (7) in defining the compact expression for $Ri_{\text{min}}$ in Eq. (10). (See also the last paragraph of this section.) (In any case, in a three-dimensional wave system the vertical shear could be enhanced by tilting of isentropic layers in both the $x$--$z$ and $y$--$z$ planes, thereby giving a lower minimum Richardson number than anticipated by the two-dimensional theory.)

The condition that instability occurs when $Ri_{\text{min}} \leq \frac{1}{4}$ embodies a convective overturning criterion (numerator of Eq. (10) becomes small) (Lindzen 1981), and a billow instability mechanism (denominator of Eq. (10) becomes large) (Scorer 1978). Following Lindzen we will employ a 'saturation' hypothesis whereby the displacement amplitude $\delta h$, in regions where $Ri_{\text{min}} < \frac{1}{4}$ is reset so that $Ri_{\text{min}} = \frac{1}{4}$. One can, in principle, therefore determine the vertical distribution of $\tau$ within the region $Ri_{\text{min}} \leq \frac{1}{4}$, using the saturation hypothesis. Outside these regions the stress must be independent of height, in accordance with the Eliassen--Palm theorem.

Consider now the algorithm for applying the Richardson number scheme. First of all the surface stress $\tau_s$ is calculated (at each gridpoint) using Eq. (8). The Richardson number $Ri$ is then calculated at the next $\sigma$-layer boundary using values of $u$, $v$ and $T$ stored on the adjacent $\sigma$ levels. Using $\tau_s = |\tau_s|$, the displacement amplitude $\delta h$ is estimated from Eq. (9) (using the average value of $U$ on the adjacent $\sigma$ levels). This value of $\delta h$ is then substituted into Eq. (10) and $Ri_{\text{min}}$ calculated. If $Ri_{\text{min}} \leq \frac{1}{4}$ then $\tau$ at the $\sigma$-layer boundary is set equal to $\tau_s$. If $Ri_{\text{min}} < \frac{1}{4}$ then $Ri_{\text{min}}$ is put equal to $\frac{1}{4}$ and the saturation value $\delta h_{\text{sat}}$ calculated from Eq. (10). $\tau_{\text{sat}}$ is then estimated using Eq. (9), and $\tau$ becomes equal to $\tau_{\text{sat}}$ at the layer boundary.
The whole procedure is now repeated to estimate $\tau$ at the next layer boundary and so on until we reach the lower boundary of layer 1 ($\sigma = 0.06$). A value $\tau = 0$ is imposed at the upper boundary of layer 1 ($\sigma = 0$), implying that the waves must be dissipated at least somewhere in the atmosphere.

Hence the wave stress is known at each layer boundary and the wave-induced acceleration at the $\sigma$ levels can thus be calculated from the vertical gradient of the stress.

One point of practical importance should be mentioned here. It was found necessary to first apply the wave absorption technique at the boundary between layers 9 and 10, rather than 10 and 11, otherwise, in some cases, it was found that a large amount of drag acted on quite a shallow layer ($\sim 25$ mb) as a result of the weak winds (see below) in layer 11. This is quite reasonable since it does not make sense to use the scheme on layers shallower than the amplitude of the subgrid-scale orography. The stress at the boundary between layers 10 and 11 is calculated by assuming that the acceleration due to the wave drag is equal in the lowest two layers.

When invoking the saturation hypothesis to calculate $\delta h_{\text{sat}}$, Eq. (10) with $Ri_{\text{min}} = \frac{1}{2}$ is a quadratic in $\delta h_{\text{sat}}$ whose solution is given by

$$\varepsilon = Ri^{-1/2}(1 + 2Ri^{1/2})[2Ri^{1/4}(1 + 2Ri^{1/2})^{-1/2} - 1]$$  \hspace{1cm} (11)

where

$$\varepsilon = N \delta h_{\text{sat}} / U.$$  \hspace{1cm} (12)

From Eqs. (9) and (12)

$$\tau_{\text{sat}} = \varepsilon^2 \kappa \rho U^3 / N$$  \hspace{1cm} (13)

and the strong dependence of $\tau_{\text{sat}}$ on $U$ shows why wave breaking is strongly preferred in regions of weak flow. This is the reason why, if the wave absorption scheme was applied directly at the top of the lowest $\sigma$ layer, substantial wave absorption occurred.

As we go up through the model layers the decrease in density begins to play an important role. Perhaps of secondary importance in determining preferred locations for wave breaking is the dependence of $\tau_{\text{sat}}$ on $N$ and $\varepsilon^2$. The appearance of $N$ in the denominator shows that the stronger the static stability of the flow, all other variables kept constant, the smaller the value of $\tau_{\text{sat}}$ and hence the more readily waves will break. This is consistent with Scarf's (1978) remark that it is the tilting of the highly stable layers as air flows across hilly terrain which leads to the turbulence experienced by aircraft and glider pilots, and also with observations of shearing instabilities on the oceanic thermocline (Woods 1968; Phillips 1966).

On the other hand $\tau_{\text{sat}}$ is directly dependent on $\varepsilon^2$, which increases from zero for $Ri = \frac{1}{4}$ through a value of 0.21 for $Ri = 1$ to an asymptotic limit of 0.69 for $Ri \gg 1$. Since, from Eq. (10), purely convective instability requires $\varepsilon = 1$, wave breaking more readily occurs in our scheme than in a convective overturning scheme, even in the limit of large Richardson number.

If we put some rough figures into Eq. (13) we find that if $\varepsilon^2 = 0.5$ ($Ri \sim 10$) and $N = 2 \times 10^{-2} \text{s}^{-1}$ then, with $\kappa = 2.5 \times 10^{-5} \text{m}^{-1}$, a flux of 0.1 N m$^{-2}$ will saturate in the boundary layer ($\rho = 1 \text{ kg m}^{-3}$) if $U$ is less than about 5 m s$^{-1}$, in the mid troposphere if $U < 7$ m s$^{-1}$ and in the lower stratosphere ($\rho = 0.05 \text{ kg m}^{-3}$) if $U < 15$ m s$^{-1}$. Climatologically, values of $U \sim 5$ m s$^{-1}$ in the boundary layer and $U \sim 15$ m s$^{-1}$ in the lower stratosphere are not unreasonable in mid-latitudes and both of these regions are preferred locations for wave breaking (see Lindzen 1985). On the other hand mid-tropospheric winds will generally be strong enough to inhibit wave breaking.
As mentioned above, we apply the scheme using only the component of the model wind in the direction of the surface stress (or wind). When this component approaches zero the Richardson number becomes very small so that the effect of the scheme will be to drive $\delta h_{aat}$ to very small values and cause the stress to be absorbed. In this way the scheme represents critical-level absorption for stationary gravity waves though since the critical level invariably falls between two model levels it is necessary to enforce the condition $\tau = 0$ at all layer boundaries above the critical line (defined as the level at which the component of the wind in the direction of the surface stress vector changes sign).

In view of the sensitivity of the wave Richardson number to the magnitude of the undisturbed static stability and wind speed one must consider how real fine structure in the vertical profiles of these fields influences our interpretation of the parametrization of wave breaking. It has long been recognized from the analysis of radiosonde ascents that wind, temperature and humidity do not vary smoothly with height but exhibit layers of radically different wind shear, static stability and dryness (Danielsen 1959). Using raw radiosonde data, Danielscn plots vertical sections of observed potential temperature which show numerous shallow, highly stable layers sandwiched between layers of virtually neutral stability with horizontal continuity on the scale of thousands of kilometres. A variety of physical processes can contribute to the formation of a laminated vertical profile, such as the subduction of tongues of stratospheric air, frontogenesis, local mixing, cloud top radiational cooling and even inertia–gravity waves themselves (Barat 1983). The question that needs to be asked is, 'What happens when a vertically propagating gravity wave passes through a highly variable vertical profile of wind and static stability?' It is clear from the Richardson number scheme that the waves are likely to break in the thin stable layers, yet they are also likely to break in the nearly neutral intervening layers if the undisturbed Richardson number is less than $1/4$. However, if a vertically smoothed profile is taken then the parametrization scheme might imply no breaking at all. One can immediately appreciate that breaking could be taking place to a much greater extent than suggested by a too literal interpretation of the monochromatic theory of wave breaking.

6. RESULTS FROM MODEL INTEGRATIONS

(a) Introduction

In this section we present some results from GCM integrations to demonstrate the effect of the gravity wave drag parametrization. A much more extensive account using multi-annual cycle integrations will be given in a companion paper (Slingo and Pearson, in preparation). A control experiment is first described, to show how the model performs without the parametrization scheme. A second experiment included the gravity wave parametrization described above. Two further experiments were run using different idealized vertical stress profiles in order to gauge the sensitivity of the GCM climatology to the location of wave-breaking. The Meteorological Office 11-layer GCM (Slingo 1985a; Palmer and Mansfield 1986) has 2.5° latitude and 3.75° longitude horizontal resolution. Parametrizations of boundary layer processes and penetrative convection are included, together with a partially interactive radiation scheme (with fixed, zonally symmetric, clouds). See Slingo (1985b) for details of the parametrizations.

The model integrations were carried out in 'perpetual January' mode starting from initial data for 00 GMT 24 December 1983, taken from the Meteorological Office
Figure 12. Northern hemisphere distribution of mean sea level pressure (mb) for (a) the control experiment (C) and (b) mean January analysis for the years 1984 to 1986, from the Meteorological Office operational analyses.
operational analysis. The length of the integrations was in each case 90 days, and the results presented were calculated from the last 60 days of each experiment.

(b) Control integration

Figure 12(a) shows the simulated sea level pressure distribution for the northern hemisphere from the control experiment, i.e. the experiment without the gravity wave scheme (hereafter referred to as experiment C). This simulates the basic features of the atmospheric circulation but shows excessive surface westerlies in mid-latitudes, particularly across the continental land masses. The Icelandic and Aleutian lows are deeper than their climatological values of about 1000 mb, the Azores anticyclone is centred too far east and extends into the western Mediterranean with anomalously high pressure extending over the Mediterranean and Middle East with the Siberian high too weak and centred too far south. For comparison the average for Januaries 1984 to 1986, derived from Meteorological Office operational analyses, is shown in Fig. 12(b). At 500 mb (not shown) the polar vortex is too deep, consistent with excessive westerlies in middle and high latitudes at this level. A latitude–pressure cross-section of the mean zonal wind and temperature was shown in Fig. 2(a), with the January 1984 data in Fig. 2(b). The northern hemisphere subtropical jet is quite well positioned, if a little weak, but at higher latitudes the westerly winds are generally too strong. Consistent with this, the stratosphere is too cold at high northern latitudes. In the southern hemisphere the model simulation is better; the main jet is probably slightly too strong and too far north, and the low-level westerlies too weak. The simulated sea level pressure distribution has much less systematic error in the southern hemisphere (not shown) than the northern, although the circumpolar trough is probably not quite as deep as it should be.

Figure 13. As Fig. 12, but for experiment G, which included the gravity wave drag parametrization.
(c) Gravity wave integration

Experiment G was a similar integration but incorporating the parametrization of gravity wave drag. The simulated sea level pressure distribution is shown in Fig. 13. This shows some significant improvements over the control experiment, in particular a reduction in westerlies over the mid-latitude continents and some filling of the Icelandic and Aleutian lows. The simulation of the Azores and Siberian anticyclones is quite similar to the control experiment. Zonal mean temperature and westerly wind cross-sections are shown in Fig. 14 and the differences from the control (G minus C) in Figs. 15(a), (b). This experiment clearly gives an improved jet structure, with a stronger northern hemisphere subtropical jet and a weaker polar night jet. The difference cross-sections show a large reduction in westerlies in the lower stratosphere and a somewhat smaller reduction at lower levels. South of 40°N, however, the westerly winds are actually increased in strength, leading to a stronger subtropical jet. These wind changes are consistent with the temperature changes—a substantial warming of the polar stratosphere and some cooling further to the south. The temperature changes are probably directly attributable to the induced meridional circulation (see section 4), as is the increase in the westerly component between 25° and 30°N. In the simple model the latter results from the accelerating effect of the poleward induced meridional circulation.

Figure 16 shows the distribution of the surface gravity wave stress τ, again averaged over the final 60 days of the integration. The directions of the stress vectors are generally westerly, indicating a deceleration of the predominantly westerly atmospheric flow. The magnitude of the gravity wave stress is not inconsistent with the observational values given in Table 1: peak values of stress are typically 0.4 N m⁻² over the Rockies and Himalayas.

Figure 17 is a cross-section showing the time-mean zonal force per unit mass of the zonal mean flow directly due to the gravity wave drag (i.e. not taking into account the Coriolis torque of the induced meridional circulation). At middle and high latitudes in
Figure 15. Zonal mean cross-sections of the differences in (a) zonal wind (m s$^{-1}$) and (b) temperature (K) for experiment G minus experiment C.
Figure 16. Distribution of surface gravity wave stress $\tau$, (Nm$^{-2}$) for experiment G.
the northern hemisphere the gravity wave drag acts to reduce the westerlies at upper levels. There is some drag acting in the opposite sense between about 60°N and 70°N, attributable to low-level easterlies in some locations (e.g. Alaska—see Fig. 16). Because of the critical-level effect this drag acts at low levels. Similarly the low-level flow over northern Africa and other land areas between 30°N and 10°N is basically easterly, so any gravity wave drag from these regions acts at the critical level. The drag acting on the southern hemisphere flow is much less than in the northern hemisphere, primarily because of the comparative lack of mountainous areas. (Also the surface wind maximum lies well south of the main bulk of the Andes.)

As an example of how the gravity wave stress varies in the vertical, the 60-day mean vertical profile of stress for an area encompassing the Rockies is given in Fig. 18. The average surface stress of 0.1 N m⁻² is consistent with average values discussed in section 3. Figure 18(a) shows how the x component of stress varies with model σ level, while Fig. 18(b) shows values of Δτₓ/Δσ (proportional to the acceleration due to the stress) for each layer. At low levels the net x component of stress increases with height corresponding to a westward acceleration. This can be attributed to occasions when there was low-level easterly flow over orography (see above). The main effect of the drag is to decelerate the westerly flow at upper levels; the peak value of Δτₓ/Δσ of 0.8 N m⁻² in the second model layer corresponds to a deceleration rate of about 7 ms⁻¹ per day.

(d) Idealized experiments

In order to investigate the sensitivity of the model simulation to the vertical stress distribution, two further integrations were carried out from the same initial conditions. In these experiments the surface stress values were calculated in exactly the same way as in the experiment with the full parametrization scheme (G). In the first experiment (L) the gravity wave drag was forced to act in the lowest three model layers, i.e. ∂τ/∂z was uniform between σ = 1.0 and σ = 0.79 and τ was zero above σ = 0.79. In the second experiment (U) the gravity wave drag was forced to act on the uppermost three layers,
Figure 18. Vertical profile of gravity wave stress for an area encompassing the Rockies (60°N–35°N 127.5°W–105°W). (a) $x$ component, $\tau_x$; (b) vertical gradient, $\Delta \tau_v/\Delta \sigma$ (proportional to the acceleration due to the stress).
i.e. $\partial \tau / \partial z$ was uniform between $\sigma = 0.195$ and $\sigma = 0.0$. In both cases the critical-level effect was represented by setting $\tau$ to zero at a layer boundary where the wind (resolved along the direction of the surface wind) was zero or negative; the main effect of this was to ensure the gravity wave drag acted at low levels in the tropics in experiment U, as it had done in G. Thus U had a vertical stress distribution that was somewhat similar to that given by the full scheme, while that of L was quite different. In both experiments the surface stress distribution was similar to that found for G (Fig. 13).

Cross-sections showing the differences in zonal wind and temperature relative to experiment C are shown in Figs. 19 and 20, and can be compared with Fig. 15 for experiment G. This shows that the stress profile used in experiment U gave similar changes to the zonal mean flow, i.e. decreased westerlies, particularly at upper levels, north of about 40°N and increases further south, coupled with warming of the polar stratosphere. On the other hand, planetary wave structure was not as well simulated in U as in G. Experiment L gave much reduced changes in the stratosphere (although still generally in the same sense). In the troposphere, westerlies were reduced everywhere north of 30°N but the reduction in westerlies was smallest at 50°N, where the other experiments showed the largest changes. The control integration C showed a slight tendency to produce a double-jet structure, with a separate mid-latitude jet at 50°N (Fig. 2(a)): this is more marked in L, but is more typical of the southern hemisphere winter than the northern hemisphere. Both idealized experiments showed a positive impact on the simulation of sea level pressure and 500 mb height, although L did not simulate the higher levels as well as the other experiments. Further details from these experiments will be reported elsewhere.

Early work by Lilly (1972) showed little sensitivity to the level at which the drag acted. However, our results show that, in order to improve the simulation at upper levels and to reduce the tendency of the model to produce a 'double-jet' structure, it is necessary to have the gravity wave drag act at upper levels, as simulated by the full gravity wave parametrization. On the other hand, we have found that wave drag acting only at upper levels gives an inferior simulation to the full parametrization scheme.

7. Why has the problem only recently emerged?

With the advent of even higher resolution numerical weather prediction models (the current operational model at the Meteorological Office has a gridlength of $\sim 150$ km) there has been a noticeable increase in predictability in the range 2–6 days. As mentioned in the introduction vigorous baroclinic development, although well forecast, is frequently followed by an unrealistically slow period of decay. The time-integrated effect of the successive failure to fill deep cyclones between 50° and 60°N in the course of a long-term integration is to generate excessive westerlies in the zone 40° to 55°N—particularly over Europe. The problem therefore appears to be intimately connected with the more energetic transient eddies developed in the current generation of high resolution global models. Since it is mainly the efficiency of large-scale eddies in transporting horizontal momentum that determines the strength of the surface winds one might question the ability of high resolution models to reproduce this transport correctly. In fact, Green (1982) has suggested that baroclinic waves might be too efficient at transporting momentum polewards. (Efficiency is defined here in the sense of the ratio of eddy momentum transport to eddy kinetic energy.) Comparison of the momentum transport in our control experiment (to be described later) with observation shows, however, that this is probably not the case.
The earliest general circulation modelling studies (Manabe et al. 1965, 1970) tended not to be concerned with specific details of the predicted flow such as the zonal-mean sea level wind but concentrated on more robust diagnostics such as the position and intensity of the subtropical jetstream and the global distribution of temperature. Manabe et al. (1970) demonstrated the importance of resolution in determining the poleward rate of momentum transport though comparison with observation was made difficult by the use of annual mean radiation conditions in their long-term integrations. Wellick et al. (1971) compared integrations of the early NCAR GCM (without orography) for 10, 5 and 2.5 degree resolutions using a regular latitude/longitude grid. Whilst the 5° run looked quite reasonable in the northern hemisphere with a subtropical jet near 30°N and surface westerlies near 40°N, the 2.5° run had a deep barotropic jet at 45°N extending to the top of the model (~18 km) and very strong low-level westerlies near 50°N (~13 m s⁻¹ at 1.5 km). On the other hand, the southern hemisphere flow with 2.5° resolution was a great improvement on the 5° run. Wellick et al. attributed this difference in the zonal mean wind simulations to the substantially greater poleward eddy momentum flux at higher resolutions. Houghton and Chervin (1982) reach similar conclusions using 5° and 2.5° grid models when they examined the depth-integrated momentum transport.

This sensitivity to horizontal resolution can also be found in spectral GCMs. The NCAR Community Climate Model with a rhomboidal truncation at total wavenumber 15 has a good January simulation of the northern hemisphere climate, but excessively weak flow in the extratropical southern hemisphere (Pitcher et al. 1983). However, recent integrations with a rhomboidal truncation at wavenumber 30 have very similar systematic errors to the Meteorological Office 11-layer model (M. Blackmon, personal communication), with excessively strong surface flow in the northern hemisphere, and a much improved southern hemisphere climatology.

Kasahara et al. (1973) integrated a 12-level general circulation model starting from an isothermal state for 120 days under January conditions of ocean surface temperature, land and snow–ice albedo and solar radiation. The model had a 5° latitude/longitude grid with levels spaced at 3 km. Their computed zonal mean wind field for the period days 91–120, in an experiment which included a representation of the orography, shows a subtropical jetstream of realistic intensity though somewhat south of its observed position. Sea level zonal mean westerlies in the northern hemisphere are rather intense with a peak value of 5 m s⁻¹ at 50°N but are too weak in the southern hemisphere with a peak value of somewhat less than 5 m s⁻¹. These latter discrepancies are not in themselves very serious; however, an accompanying latitude/height cross-section in their paper reveals a gross underestimation of the poleward momentum transport due to stationary and transient eddies (combined) in mid-latitudes by about a factor of three as compared with observed values. As they point out, calculations by Wellick et al. suggest that the poleward momentum flux would be greatly improved using a 2.5° grid. It is clear, however, that trebling their modelled poleward momentum transport would almost certainly be accompanied by a substantial increase in surface wind speed in a 2.5° resolution integration. The implied (through geostrophy) mean sea level pressure distribution in the northern hemisphere would be quite unrealistic and one might also infer from Wellick et al. that the principal jetstream would shift northwards of its observed subtropical position. On the other hand, the southern hemisphere simulation might be improved by higher resolution through the greater intensity of poleward momentum transport.

These facts point directly to a failure in the modelled surface stress/wind relationship, in agreement with Swinbank (1985). Early model successes in simulating the observed northern hemisphere winter distribution of zonal mean winds seem to have resulted from
Figure 19. As Fig. 15, but for experiment L minus experiment C.
Figure 20. As Fig. 15, but for experiment U minus experiment C.
the cancellation of two modelling errors—the underestimation of poleward momentum transport due to insufficient resolution and too small an effective drag coefficient in the modelled surface stress/wind relationship for the northern hemisphere.

Further support for this notion is provided by the GLAS model reported recently by Randall (1983). The tropospheric simulation of the January zonal mean wind distribution in the northern hemisphere is very good but again there is evidence that the total eddy momentum flux convergence in mid-latitudes is too small by a factor of two. The southern hemisphere winter simulation of the surface zonal mean wind is poor with peak sea level wind speed at least a factor of two too small.

In order to test these ideas further, we have run another integration (C5) using a coarser version of the 11-layer model with 5° latitude and 7½° longitude horizontal resolution. Initial conditions were derived from the same analysis, and identical physical parametrizations were used. There was no parametrization of gravity wave drag in experiment C5, so it can be compared directly with experiment C. The simulated sea level pressure is shown in Fig. 21, and a cross-section of zonal wind and temperature in Fig. 22. In the northern hemisphere the zonal mean fields are simulated more accurately by the coarse resolution experiment (C5) than by the control experiment (C) (compare Figs. 22 and 2(a) with Fig. 2(b)). As Fig. 21 shows, C5 does not suffer from the excessive surface westerlies of experiment C (Fig. 14), and the Aleutian and Icelandic lows have more realistic central values. The subtropical anticyclones are, however, not well simulated and the Azores high has been displaced to the western Mediterranean. In the southern hemisphere (not shown) the simulation C5 is not satisfactory; the subtropical anticyclones are too weak and the circumpolar trough is too far north and not deep enough, leading to a very slack gradient in a latitude band where there should be strong

Figure 21. As Fig. 12, but for experiment C5, which used a coarse resolution version of the 11-layer model.
surface westerlies. The southern hemisphere zonal mean westerly winds and temperatures (Fig. 22) are also less well simulated by experiment C5 than by experiment C. Finally, graphs of the meridional eddy momentum flux at 250 mb of the two experiments are shown in Fig. 23. Results from experiment C agree well with observational values; however, results from C5 are only about half the observed values, consistent with findings

Figure 22. As Fig. 14, but for experiment C5.

Figure 23. Eddy momentum fluxes (m$^2$s$^{-2}$) at 250 mb for experiments C and C5, and also for January 1984, as indicated.
from other models. Thus in the northern hemisphere the two errors of insufficient poleward momentum flux and insufficient drag tend to cancel one another, while in the southern hemisphere, where the mountain wave effects are less important, the simulation is degraded with the coarse resolution model.

In conclusion, our answer to the question posed in the title of this section is that the systematic westerly wind error is serious only in the most recent very high resolution models which better represent the poleward flux of momentum by large-scale motion. Prior to this, models typically of 5° resolution frequently gave a reasonable simulation of the mean sea-level winds due to the compensation implied by underestimating both the surface drag and the poleward flux of momentum.

8. DISCUSSION

We have argued that the omission in GCM/NWP models of any representation of subgrid-scale pressure drag leads to unrealistically strong westerlies in middle and high latitudes. There is no single piece of evidence for this which is beyond contention and so we have had to muster all relevant facts, observational and theoretical, to make a convincing case. Perhaps the strongest indication of a deficiency in the way the surface stress/wind relationship is parametrized comes from Swinbank’s momentum budget calculation, which shows that the effective surface drag coefficient (embodying friction and pressure torque) in conventional parametrization schemes is about a factor of two too small in the northern hemisphere winter. The excessively strong polar night vortex in the model is at least consistent with the notion that this ‘missing torque’ is transmitted to the stratosphere where it forces meridional motion which can redistribute the associated momentum vertically. Our parametrization scheme naturally tends to absorb wave momentum fluxes in the lower stratosphere, and we have shown in an idealized model of this process that the decelerating effect of imposed stratospheric drag is felt throughout the troposphere and that adiabatic warming associated with the adjustment to thermal wind balance occurs to the north of the imposed drag. These influences result in a stronger subtropical jetstream correctly displaced equatorwards to near 30°N and clearly isolated from the model’s rather ill-resolved polar night jet. Indeed, the possibility exists that the lower stratospheric zonal wind minimum and reversed meridional gradient in temperature are directly forced by breaking orographically forced gravity waves (Tanaka and Yamanaka 1984; Lindzen 1985). The performance of the gravity wave parametrization scheme in a model with a better resolved stratosphere would certainly be of great interest.

Systematic errors in the European Centre for Medium Range Weather Forecasts (ECMWF) model have been greatly alleviated by the introduction of an enhanced large-scale orography (Wallace et al. 1983). Although the magnitude and geographical distribution of their error field is somewhat different from that of the Meteorological Office’s 11-layer model the tendency towards excessive westerly flow, particularly over Europe in winter, is similar. In an analysis of the physical mechanism by which ‘envelope orography’ improves the model forecast, Tibaldi (1987) finds that the surface pressure drag is increased (predominantly at the smallest resolved scales) and that a better long wave ‘climatology’ results indirectly from the reduced zonally averaged flow. Reduction of the stratospheric westerly error through orographic enhancement is harder to explain though it is quite conceivable that enhanced vertical propagation of planetary waves (generated by thermal or orographic forcing) would result—thereby playing the same role as gravity wave drag. In this respect the envelope orography and gravity wave parametrizations are similar though the former makes a direct impact on model vorticity
dynamics since an increased mountain volume results in greater vortex compression. For instance, increased mountain volume helps to boost the amplitude of the upper-level ‘Rockies ridge’ whereas gravity wave drag would not do this directly. To some extent the philosophy behind the use of an enhanced orography is complementary to, rather than competitive with, parametrized gravity wave drag. Both address specific physical aspects of flow over and around orography which need to be parametrized, and ultimately some combination of the two techniques may be optimum.

In this paper we have discussed only the impact of the gravity wave drag scheme in wintertime integrations. In summertime it is found (Slingo and Pearson, in preparation) that the scheme has little effect on what is already a realistic simulation. The reason for this is clear: in summer the weaker surface wind speed and lower static stability over land tend to reduce the surface stress (see Eq. (9)) to values small compared with those in wintertime. The effective ‘switching off’ of the parametrization in summer is, to some extent, seen as an advantage over envelope orography where effects, for example, of elevated heat sources, may give rise to spurious circulation features in summer (Hills 1979).

A simpler version of the gravity wave scheme has been used operationally at the Meteorological Office since December 1984. The scheme has a similar formula for calculating the surface stress but distributes it in the vertical so as to provide a height-independent force per unit mass. If a critical layer is present, a prescribed fraction of the surface stress is absorbed in the same way up to the critical level height. Kitchen and Dickinson (in preparation) have assessed the effect of this scheme on forecast error statistics in the 15-level operational model. Substantial reductions in the magnitude of the zonally averaged geopotential height and wind errors are found throughout the model’s troposphere and lower stratosphere. The failure of the earlier version of the model to adequately spin down low pressure systems over land in winter is also partially rectified.

The Canadian Climate Centre have independently developed a gravity wave drag parametrization scheme for use in their T21 spectral climate model (Boer et al. 1984; McFarlane 1986). As in our own experiments they find a substantial beneficial impact on the model’s climatology.

It is natural to wonder why the systematic excessive westerly bias has only recently emerged as a major modelling problem. The answer is no doubt more involved than the explanation put forward in the previous section though all the indications are that earlier low resolution GCMs underestimated the poleward momentum flux, thereby concealing the inefficiency of the parametrized earth/atmosphere momentum exchange. However, as further increases in computer power are made available to the forecaster the minimum resolvable scale of global models will fall below 100km and mesoscale phenomena will begin to be represented explicitly. It remains to be seen whether a scale will be reached where it is no longer necessary to parametrize gravity wave drag and sufficient momentum would be extracted explicitly from the model atmosphere by the resolved orographic scales.

ACKNOWLEDGMENTS

Our thanks to H. C. Davies, E. O. Holopainen, D. K. Lilly, M. Manton, P. J. Mason, M. E. McIntyre, M. J. Miller, S. Mobbs and R. S. Scorer for helpful discussions and correspondence; and to J. E. Kitchen and J. M. Murphy for carrying out some exploratory experiments with a preliminary version of the scheme.
APPENDIX

In this appendix we give some details of the zonally averaged model used to diagnose the meridional circulation, and the zonal wind and potential temperature tendency to gravity wave drag, described in section 4.

The linearized, zonally averaged Boussinesq equations of motion written in spherical, polar geometry are:

\[ \frac{\partial \bar{U}}{\partial t} - f\bar{v} = \bar{F} \]  
\[ \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (\bar{v} \cos \phi) + \frac{1}{\rho_o \partial z} (\rho_o \bar{w}) = 0 \]  
\[ \frac{\partial \bar{U}}{\partial z} = \frac{g}{f\theta_o \partial \phi} \]  
\[ \frac{\partial \bar{\theta}}{\partial t} + \bar{w} \frac{N^2}{g} = 0 \]

(zonally-averaged zonal component of the momentum equation)  
(thermodynamic equation)  
(continuity equation)  
(thermal wind equation)

where \( \phi \) is latitude, \( a \) is the radius of the earth, \( \bar{F} \) is a prescribed momentum sink, \( \theta_o(z) \) is the basic state potential temperature field, \( \rho_o(z) = \rho_o \exp(-z/H_o) \), with constant \( \rho_o \), \( H_o \) and all other symbols are defined as before.

A diagnostic equation can be obtained by eliminating the tendency terms, \( \partial \bar{U}/\partial t \) and \( \partial(\bar{\theta}/\theta_o)/\partial t \) from Eqs. (A1) and (A2) using the thermal wind equation (A4) so that

\[ \frac{N^2 \partial \bar{w}}{af \partial \phi} - f \frac{\partial \bar{v}}{\partial z} = \frac{\partial \bar{F}}{\partial z} \]  

Defining a mass streamfunction \( \chi \) consistent with the continuity equation (A3) such that

\[ \rho_o \bar{v} = -\frac{1}{\cos \phi} \frac{\partial \chi}{\partial z}, \quad \rho_o \bar{w} = \frac{1}{a \cos \phi} \frac{\partial \chi}{\partial \phi} \]

and substituting for \( v \) and \( w \) in Eq. (A5) gives

\[ \frac{N^2}{a^2 f \rho_o} \frac{\partial}{\partial \phi} \left( \frac{1}{\cos \phi} \frac{\partial \chi}{\partial \phi} \right) + \frac{f}{\cos \phi} \frac{\partial}{\partial z} \left( \frac{1}{\rho_o} \frac{\partial \chi}{\partial z} \right) = \frac{\partial \bar{F}}{\partial z} \]

which may be written as

\[ \frac{N^2}{4\Omega^2 a^2} \frac{(1 - \mu^2)}{\mu^2} \frac{\partial^2 \chi}{\partial \mu^2} + \rho_o \frac{\partial}{\partial z} \left( \frac{1}{\rho_o} \frac{\partial \chi}{\partial z} \right) = \rho_o \frac{\partial \bar{F}}{\partial z} \frac{(1 - \mu^2)^{1/2}}{2\Omega \mu} \]  

where \( \mu = \sin \phi \) and \( 2\Omega = f/\mu \).

Consider first a separable solution to the homogeneous version of Eq. (A6) such that \( \chi(\mu, z) = \Phi(\mu) Z(z) \).

Substituting in Eq. (A6) gives two ordinary differential equations coupled through a separation constant \( \gamma^2 \):

\[ \frac{\rho_o}{\bar{Z}} \frac{d}{dz} \left( \frac{1}{\rho_o} \frac{dZ}{dz} \right) = \gamma^2 \]

and
\[
\frac{1 - \mu^2}{\mu^2} \frac{d^2 \Phi}{d\mu^2} = -\lambda^2
\]

(A8)

where \( \lambda^2 = \gamma^2(2\Omega a)^2/N^2 \).

At the geographical poles, the mass streamfunction must vanish so that Eq. (A8) and \( \Phi(\pm 1) = 0 \) constitute a Sturm–Liouville problem for a set of orthogonal eigenfunctions \( \Phi_i \) with corresponding eigenvalues \( \lambda_i^2 \). These were found by expanding \( \Phi_i \) in a power series in \( \mu \)—the details of which are omitted. The horizontal variation of the forcing term on the right-hand side of (A6) is projected onto the derived eigenfunctions and the height-dependence is simply a Dirac delta function at \( z = z_a \). The vertical structure equations corresponding to each eigenfunction are then solved subject to the condition that \( \chi \) is zero at \( z = 0 \) and \( z_T \).

**REFERENCES**

Andrews, D. G. and McIntyre, M. E. 1976


A diagnostic study of eddy-mean flow interactions during FGGE SOP-1. *ibid.*, 42, 1838–1845

Barat, J. 1983


Boer, G. J., McFarlane, M. A., Laprise, R., Henderson, J. D. and Blanchet, J. P. 1984

The Canadian Climate Centre spectral atmospheric general circulation model. *Atmosphere–Ocean*, 22, 397–429

Bretherton, F. P. 1969


Brown, P. R. A. 1983

Aircraft measurements of mountain waves and their associated flux over the British Isles. *ibid.*, 109, 849–866

Carson, D. J. 1982

‘Comments on the sensitivity of numerical simulations to different parametrizations of the boundary layer properties and processes’. Proceedings of ECMWF workshop on Boundary Layer Parametrization, November 1981, pp. 119–153

Danielsen, E. F. 1959

The laminar structure of the atmosphere and its relation to the concept of a tropopause. *Arch. Met. Bioklim.*, 11, 293–332

Davies, H. C. and Phillips, P. D. 1985

Mountain drag along the Gotthard Section during ALPEX. *J. Atmos. Sci.*, 42, 2093–2109

Eliassen, A. 1952


Eliassen, A. and Palm, E. 1961


Fritts, D. C. 1984


Gill, A. 1982


Green, J. S. A. 1982

‘Zonal-mean winds and momentum budgets’. ECMWF workshop on Intercomparison of large-scale models used for extended range forecasts, pp. 47–51

Hills, T. S. 1979

Sensitivity of numerical models to mountain representation. ECMWF report of workshop on Mountains and numerical weather prediction. 20–22 June 1979, pp. 139–161

Hoinika, K. P. 1984

Observation of a mountain wave event over the Pyrenees. *Tellus*, 36A, 369–383

1985

Observation of the airflow over the Alps during a foehn event. *Quart. J. R. Met. Soc.*, 111, 199–224

Holopainen, E. O. 1982

Long-term budget of zonal momentum in the free atmosphere over Europe in winter. *ibid.*, 108, 95–102


Lindzen, R. S. 1985 Multiple gravity-wave breaking levels. *J. Atmos. Sci.*, 42, 301–305


Nurmi, P. 1983 ‘An analysis of the budgets of zonal momentum and kinetic energy in the northern hemisphere during the first special observing period of the FGGE’. Report No. 24. Department of Meteorology, University of Helsinki


Phillips, O. M. 1966 *The dynamics of the upper ocean*. Cambridge University Press


Scorer, R. S. 1960 Numerical calculation of the displacements of a stratified air-stream crossing a ridge of small height. *ibid.*, 86, 326–345

Scorer, R. S. 1978 *Environmental Aerodynamics*. Ellis Horwood Ltd.
ALLEVIATION OF BIAS IN NUMERICAL MODELS

Shutts, G. J.  

Simmons, A. J. and Struifing, R.  

Slingo, A.  
1985a Simulation of the earth's radiation budget with the 11-layer general circulation model. Met. Mag., 114, 121–141  

Smith, R. B.  

Swinbank, R.  
1985 The global atmospheric angular momentum balance inferred from analyses made during the FGGE. Quart. J. R. Met. Soc., 111, 977–992

Tanaka, H. and Yamanaka, M. D.  

Tibaldi, S.  

Vincent, R. A. and Reid, I. M.  

Wallace, J. M., Tibaldi, S. and Simmons, A. J.  

Wellick, R. E., Kasahara, A., Washington, W. M. and De Santo, G.  

Woods, J. D.  
1968 Wave induced shear instability in the summer thermocline. J. Fluid Mech., 32, 791–800