A comparison of a numerical model of radiation fog with detailed observations

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SUMMARY

A one-dimensional numerical model designed to study the formation and growth of radiation fog is described. The realism of the model in simulating the formation of fog is assessed by critical comparisons with detailed observations made on two nights. The comparisons show that the model is unable to reproduce the very light winds which occur near the surface in stable conditions. As a consequence the formation of radiation fog is inhibited in the model unless the afternoon relative humidity is very high. It is suggested that the discrepancy is caused by the use of a turbulence formulation tuned to match observations over an ideally flat site. The influence of turbulence, radiative cooling, advection and frost deposition on the formation of fog during the two nights is also discussed.

1. INTRODUCTION

The work described here is part of an investigation by the Meteorological Office into the physics of radiation fog. The results of an initial field study were described by Roach et al. (1976). To aid the interpretation of the field data a numerical model was developed (Brown and Roach 1976), hereafter referred to as I. The model supported many of the conclusions derived from the observations, for example the role of turbulent diffusion in inhibiting fog formation. Although the model predictions were in broad agreement with the observations, the model was deficient in several respects. Since it did not incorporate the momentum equations, turbulent transfer was parametrized using prescribed exchange coefficients which did not increase in response to the decrease in stability produced by a mature fog. This led to superadiabatic temperature profiles within the fog.

During a second field study, detailed measurements through deep fogs were made using a tethered balloon (Roach et al. 1982). These observations suggested that the presence of a mature fog modified the boundary layer wind profile. To confirm this hypothesis the one-dimensional momentum equations were introduced into the model and the exchange coefficients were made functions of the local Richardson number. These changes enabled the model to simulate the development of weak convection within a mature fog which was believed to be responsible for the observed behaviour of the wind. Some preliminary results indicating agreement between the model wind profile and that observed in a mature fog were described by Brown (1980).

This paper examines the potential of the extended model to describe and predict the formation of radiation fog. Although the model produced a realistic simulation of the growth of a mature fog, one would expect more difficulty in predicting fog formation. This is because the development of the mature fog is dominated by radiative cooling from the fog top, whilst the formation process depends upon a subtle balance between turbulent transport and radiative cooling. A small error in either of these could lead to a large error in the predicted time of fog formation. Particular difficulty should be anticipated in modelling the turbulent transport. The published turbulence formulations all contain empirical constants which are invariably estimated from observations over ideally flat sites. It is well known (but not apparently published) that such formulations overestimate the low-level wind speed at sites containing large roughness elements, e.g. hedges, trees. Other problems one would anticipate are the calculation of the surface temperature and in a one-dimensional model the absence of advection. These topics are
discussed in this paper in the light of the comparisons. Numerical experiments, performed to elucidate the factors influencing fog formation on the two nights studied, are also described.

2. DESCRIPTION OF THE MODEL

(a) The basic equations of the model

As in I the governing equations of the model are the one-dimensional continuity equations for heat, water vapour and liquid water:

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left( K_h \frac{\partial \theta}{\partial z} \right) + \frac{\theta}{T} \frac{L}{c_p} C + \frac{1}{\rho c_p} \frac{\partial F_{NL}}{\partial z} \tag{1}
\]

\[
\frac{\partial q}{\partial t} = \frac{\partial}{\partial z} \left( K_q \frac{\partial q}{\partial z} \right) - C \tag{2}
\]

\[
\frac{\partial w}{\partial t} = \frac{\partial}{\partial z} \left( K_w \frac{\partial w}{\partial z} \right) + C + \frac{\partial G}{\partial z} \tag{3}
\]

where

\( \theta \) = potential temperature of air

\( T \) = dry-bulb temperature of air

\( \rho \) = density of air

\( c_p \) = specific heat of air at constant pressure

\( F_{NL} \) = net long-wave radiative flux, positive downwards

\( z \) = height coordinate with origin at the earth’s surface

\( K_h, K_q, K_w \) = exchange coefficients for heat, water vapour and liquid water respectively (assumed equal)

\( L \) = latent heat of vaporization

\( C \) = rate of condensation per unit mass of air

\( q \) = humidity mixing ratio

\( w \) = liquid water mixing ratio

\( G \) = gravitational settling flux of liquid water, positive downwards.

Although these equations are not couched directly in terms of conservative quantities they still conserve the appropriate quantities during phase changes.

The temperature distribution within the soil is given by

\[
\frac{\partial T_s}{\partial t} = \frac{1}{\rho_s c_s} \frac{\partial}{\partial z} \left( k_s \frac{\partial T_s}{\partial z} \right) \tag{4}
\]

where \( T_s \) is the soil temperature and \( \rho_s, c_s, k_s \) are the density, specific heat and thermal conductivity of the soil respectively. Equation (4) differs from that used in I because the thermal properties of the soil are allowed to vary with depth. The values assigned to the soil properties are discussed later.

To these equations have been added the one-dimensional momentum equations

\[
\frac{\partial u}{\partial t} = f v + \frac{\partial}{\partial z} \left( K_m \frac{\partial u}{\partial z} \right) \tag{5}
\]

\[
\frac{\partial v}{\partial t} = f (U_s - u) + \frac{\partial}{\partial z} \left( K_m \frac{\partial v}{\partial z} \right) \tag{6}
\]
where \( u, v \) are orthogonal components of the horizontal wind parallel and perpendicular to the direction of the geostrophic wind, \( f \) is the Coriolis parameter; \( K_m \) is the exchange coefficient for momentum and \( U_g \) is the geostrophic wind, assumed constant with height.

The exchange coefficients are made functions of the local gradient Richardson number \((Ri)\) using the level-2 formulation of Mellor and Yamada (1974):

\[
K_m = l^2 \left( (\partial u / \partial z)^2 + (\partial v / \partial z)^2 \right)^{1/2} S_m \tag{7}
\]

\[
K_h = l^2 \left( (\partial u / \partial z)^2 + (\partial v / \partial z)^2 \right)^{1/2} S_h \tag{8}
\]

The functions \( S_m, S_h \) depend upon \( Ri \) and are given by Mellor and Yamada (in fog, \( Ri \) is defined so as to account for the effect of latent heat release on the stability). They contain several empirical constants which have been slightly revised by Mellor and Yamada in subsequent publications. The values used here are those given in Mellor and Yamada (1982); however, these changes have been found to make an insignificant difference to the model results. The level-2 formulation does not specify the mixing length \((l)\) but for consistency with its derivation it is necessary that \( l \rightarrow k z \) as \( z \rightarrow 0 \) \((k = 0.4)\). The mixing length has been taken to be of the form

\[
1/l = 1/kz + 1/l_o \tag{9}
\]

where \( l_o \) is an outer length scale which is set equal to one third of the boundary layer depth. (The top of the boundary layer is defined by \( Ri \) exceeding the critical value, see below.) This limitation is intended to recognize an upper bound to the scale of eddies in the turbulent boundary layer. In the stable boundary layer this limitation is secondary to the stability functions and critical \( Ri \). An important feature of the Mellor and Yamada (1974) level-2 formulation is that it predicts a critical gradient Richardson number \((R_{ic})\) of 0.19 to 0.24 (depending on the empirical constants used) at which turbulence ceases. Initial comparisons with the observations indicated that this condition was too restrictive. Yamada (1975) has demonstrated that the second-order closure model, from which the level-2 formulation was derived, does not produce a clear cutoff of turbulence, even when \( Ri \) exceeds the level-2 critical value. Therefore, following Yamada (1983), a small background value is retained in part of the stability functions when the flux Richardson number \((R_{if})\) exceeds 0.16. However, the modified formulation still predicts that turbulence ceases when \( R_{if} = 1 \) or \( Ri = 0.89 \). (The relationship between \( R_{if} \) and \( Ri \) is an integral part of the formulation.)

The finite difference analogues of Eqs. (4) to (6) and the exchange coefficient terms of Eqs. (1) to (3) (which represent turbulent flux components only in clear air) are solved using a fully implicit numerical scheme. Boundary layer models usually place the first grid point in air several metres above the surface. However, in the case of radiation fog it is believed that the important processes in fog formation take place below one metre, and that it is necessary to model them explicitly. The original fog model (1) placed the first grid point above the surface \((z_2)\) at 0.03 m \((z_1 = 0)\) and this specification is retained. Forty-eight grid points are defined from the surface up to 1400 m on a grid which expands away from the surface.

\( (b) \) Parametrization of the microphysics

The growth of the fog droplets is not calculated explicitly. After each diffusion time step (5 s) the temperature and water vapour mixing ratio are examined. If the air is supersaturated, water is condensed and latent heat released simultaneously until the air is just saturated. Liquid water in an unsaturated environment is evaporated and latent heat absorbed until the air is saturated or the water is all consumed.
The size distribution of the fog droplets is specified using a gamma distribution in order to compute the gravitational settling flux of liquid water and the long-wave transmissivity of the fog. In all integrations discussed here a droplet concentration of 100 cm\(^{-3}\) was specified together with \(r_v = 1.1 r_m\), where \(r_v\) is the volume mean radius and \(r_m\) is the mean radius. The latter condition is based on drop size distributions measured at Cardington.

(c) Calculation of the radiative fluxes

The two-band long-wave scheme used in I has been replaced by a modified version of the five-band scheme of Roach and Slingo (1979). They used simple analytic functions to fit the results of accurate calculations of molecular transmissivity as a function of absorber path. The functions adequately represented the calculated transmissivities down to a path length of 1 mb but became progressively less accurate for smaller path lengths. To cope with the higher vertical resolution of the fog model, the functions have been replaced by more exact look-up tables. The long-wave heating rates are recalculated every 90 s, but are applied to Eq. (1) every diffusion time step.

Since the dispersal of fog by solar radiation is not considered here, the computational expense of a short-wave scheme was avoided by considering only the solar radiation absorbed at the surface (S). This is parametrized using a modified form of the equation of Hoffert and Storch (1979):

\[
S = (1 - a)S_o \tau^{\sec \phi} \cos \phi
\]

(10)

where \(a\) is the surface albedo taken to be 0.2, \(S_o\) the solar constant, \(\tau\) the atmospheric transmission and \(\phi\) the solar zenith angle. For the comparisons with the case studies \(\tau\) was adjusted for optimum agreement with hourly mean values of the downward short-wave flux recorded at Cardington and had a value of about 0.7.

(d) Calculation of the surface temperature and soil heat flux

At the ground-air interface continuity of fluxes is assumed, i.e.

\[
F_N = F_H + F_L + F_S
\]

(11)

where \(F_H\) and \(F_L\) are the sensible and latent heat fluxes (positive upwards), \(F_S\) is the soil heat flux (positive downwards) and \(F_N (= F_{NL} + S)\) is the net radiative flux (long-wave + short-wave) incident at the surface. At each diffusion time step \(F_S\) is calculated from Eq. (11) as a residue of the other fluxes. The soil surface temperature \(T_{s1}\) is then calculated by approximating the soil surface temperature gradient by the gradient over the first grid interval in the soil:

\[
T_{s1} = T_{s2} + (z_{s1} - z_{s2})F_S/k_{s1,2}
\]

(12)

where subscripts s1 and s2 refer to the soil surface and the first soil grid level below the surface, and \(k_{s1,2}\) is the thermal conductivity of the uppermost layer in the soil. Equation (4) is then integrated subject to the boundary conditions \(T_s = T_{s1}\) at \(z = 0\) and \(T_s\) is constant at \(z = -2\) m. Twenty grid points are defined in the soil to this depth on a grid which expands away from the surface.

The model soil heat flux, surface temperature and temperature profile within the soil are sensitive to the soil properties chosen. However, no measurements of the soil properties were available for the observational area (Cardington). According to Eden (1982) the soil there is clay and is covered with a thin veneer of alluvial sands and gravels, the alluvial deposits being poor thermal conductors. Integrations using a homogeneous clay soil overestimated the soil heat flux and the minimum surface temperature and
underestimated the 5 cm soil temperature. This is because of the high thermal conductivity of the clay soil. In order to best match the observed soil heat fluxes, minimum surface temperature and 5 cm soil temperature it was necessary to specify an inhomogeneous soil structure. This had the thermal properties of clay below 10 cm, peat from the surface to 5 cm, with the properties varying linearly between these levels. There is some justification for this in the reported structure of the soil described above. The thermal properties for peat and clay used here are taken from Monteith (1975). Only results pertaining to the inhomogeneous soil prescription are discussed in any detail in this paper.

(e) Other boundary conditions

It is only possible to calculate the surface humidity mixing ratio \( q_1 \) explicitly by using sophisticated models of the transfer of water through the soil, e.g. Sievers et al. (1983). Such a complex treatment was not justified in the present study because of our limited knowledge of the state of the soil. In the model, \( q_1 \) is given by

\[
q_1 = f_p q_{sat}(T_1) + (1 - f_p) q_2
\]

(13)

where \( q_{sat}(T_1) \) is the saturated humidity mixing ratio at the surface \( (T_1 = T_{at}) \) and \( q_2 \) is the mixing ratio at the first grid point above the surface. The factor \( f_p \) is introduced to account for the fact that the rate of evaporation is usually less than the potential evaporation which occurs over a saturated surface. It is specified here using the concept of surface resistance (Monteith 1981) and is given by

\[
f_p = (z_2/K_{hi})/(z_2/K_{hi} + r_s)
\]

(14)

where \( r_s \) is the surface resistance. A value for \( r_s \) of 60 s m\(^{-1} \) has been used, following De Bruin and Holtslag (1982). For dew deposition, defined by \( q_{sat}(T_1) < q_2 \), the factor \( f_p \) is set to unity. The surface latent heat flux is given by

\[
F_L = \rho L f_p K_{q1} (q_{sat}(T_1) - q_2)/z_2 = \rho L K_{q1} (q_1 - q_2)/z_2.
\]

(15)

The liquid water mixing ratio in the air at the surface \( (w_1) \) is set to zero at all times because the surface is totally absorbing for fog droplets.

Despite the high resolution of the grid near the surface, the logarithmic profile shape implies that the gradients required to calculate \( K_{ml} \), \( K_{hi} \) and \( K_{q1} \) are not well defined numerically. Therefore for the bottom layer in the air the exchange coefficients are modified to make them equivalent to neutral drag coefficients. The neutral approximation is permitted because for all the integrations discussed \( z_2 \ll L_m \) where \( L_m \) is the Monin–Obukhov length scale. By requiring that \( K_{ml} u_2/z_2 = C_D u_2^2 \), where \( u_2 \) is the wind speed at \( z_2 \) and \( C_D \) is the neutral drag coefficient for momentum then

\[
K_{ml} = u_2 z_2 k^2/[\ln(z_2/z_o)]^2
\]

(16)

and

\[
K_{hi} = K_{q1} = u_2 z_2 k^2/[\ln(z_2/z_o) \ln(z_2/z_T)]
\]

(17)

where \( z_o \) and \( z_T \) are roughness lengths for momentum and heat (water vapour) transfer. It is assumed that \( z_o = 0.02 \) m and following Thom (1975) \( z_T = 0.2 z_o \).

At the top boundary of the model (1400 m) \( u = U_b, v = 0, w = 0 \) and \( q, \theta \) are held constant. The long-wave radiation scheme is integrated to the top of the (cloudless) atmosphere using a prescribed temperature and humidity profile above 1400 m.
3. Observations

The results from the model are compared with detailed observations made on two nights at Cardington, Bedfordshire (52°06'N 0°24'W, 29 m a.m.s.l.). The case studies will be referred to as A: 3–4 November 1976; B: 18 January 1973.

The type of observations made during case study B have been described previously in detail by Roach et al. (1976). Case study A is part of the later field project, details of the tethered balloon instrument package used are given in Roach et al. (1982). Data from BALTHUM ascents (operational tethered balloon soundings to 900 m, normally made every six hours) were also available. The BALTHUM balloon is not the same balloon as that which carried the instrument package. Temperature, humidity and wind speed measurements were also made from a 16 m mast. The surface energy balance was determined from measurements of the soil heat flux, the surface net radiation and the surface moisture deposition (from a lysimeter).

Soil temperatures were measured at depths of 5, 10 and 20 cm every three hours and at 30 cm and 1 m at 0900 GMT. Routine soil moisture deficit calculations from rainfall and evaporation rates from nearby Wellingborough suggested that the soil was near field capacity on both occasions. Field capacity refers to a soil in which all the available pore space, typically 40%, is filled with water, therefore the soil properties specified were for 40% moisture content.

The site used for the studies reported here is open and grassed, with boundaries about 200–500 m from the observational area. Consequently Cardington is not an ideal flat site but may be considered a typical rural site. Large sheds (60 m high, 250 m long) are situated 500 m to the north of the experimental site and they can affect wind measurements for a northerly flow. In the cases presented here the mean wind was not from that direction.

4. Simulation of case study A

(a) Synoptic background and observations

During the period of interest England lay beneath a ridge of high pressure which extended in a north-easterly direction from a high to the south-west of the United Kingdom. This led to clear skies and light winds over central England throughout the period from 12 h on 3 November to 06 h on 4 November. (Times are GMT throughout.) The ridge was moving steadily eastwards and its axis passed through Cardington about midnight. Thereafter the flow became more westerly and the pressure gradients tightened leading to an increase in wind speed by the end of the night. Although mist was reported from 20 h to 06 h, Cardington remained fog free with a minimum visibility of 4500 m. Mist patches were widespread throughout the south and east of England with a few reports of shallow fog. Various empirical forecasting rules gave contradictory results as to whether fog would develop, indicating that conditions were only marginally suitable for fog formation and so would provide a sensitive test of model performance.

Observations were made from the tethered balloon between 2130 and 0130 h and from the 16 m mast between 1730 and 0400 h. BALTHUM ascents were made at 05, 18 and 23 h on 3 November and 05 h on 4 November. The midday ascent on 3 November was cancelled.

(b) Initial conditions

The simulations were started at midday to allow the initial profiles to come into balance with the model equations before the development of a nocturnal boundary layer
(NBL). If the integrations had been started around sunset it is possible that fog could form spuriously as a result of the profiles adjusting to balance the equations. From the 05 and 18 h BALTHUM ascents, the midday Crawley (about 110 km SSE of Cardington) radiosonde ascent and the surface observations, an estimate was made of the midday profiles. However, the BALTHUM ascents showed that warming had occurred between 05 and 18 h, which represented a gain in energy of $\sim 3 \times 10^5$ J m$^{-2}$ below 900 m. A surface sensible heat flux of $\sim 60$ W m$^{-2}$ acting over a 9-hour period would be required to produce such a warming, even allowing for entrainment and solar absorption. In producing this estimate, the entrained flux was assumed to be 20% of the surface sensible heat flux (Carson 1973), with direct solar absorption contributing a further 30% (Moores 1982). Such a large sensible heat flux is incompatible with the surface energy balance measurements, suggesting that warm advection was partly responsible for the observed temperature increase. This was taken into account simply by increasing the initial temperatures used in the integration. This adjustment produced good agreement between the model and observed temperature profile at 18 h, Fig. 1.

The initial temperature and humidity profiles specified are also shown in Fig. 1. Above 900 m the profiles were based upon the midnight Crawley radiosonde ascent, which was also used in the long-wave flux calculations. The initial soil temperature profile specified was based upon the midday observations and is shown in Fig. 2. An initial wind profile, which was in balance with the model equations, was obtained by integrating the model for 15 hours without changing any external parameters. A constant geostrophic wind speed of 5.5 m s$^{-1}$ was specified, based on the surface pressure field and the BALTHUM winds.

![Figure 1](image)

Figure 1. Profiles of temperature and humidity mixing ratio for case study A: (---), initial profiles used for the model simulation; (--.--), model profiles at 18 h. Observed (BALTHUM) profiles at 18 h shown by squares.
(c) Development of model and observed boundary layers

The model and observed temperature and humidity profiles at 18h are shown in Fig. 1. The model temperature profile is similar to the observed profile, with both exhibiting a shallow surface-based inversion. The humidity profile does not agree so well with that observed; the model rapidly destroys the initial hydrolapse by turbulent mixing, whilst the observations show that the hydrolapse was maintained through the boundary layer. Underestimation of entrainment at the top of the model boundary layer is believed to be partly responsible for the elimination of the hydrolapse above 300 m.

The development of the NBL is illustrated in Fig. 3, which compares model and observed profiles of temperature, humidity and wind speed at 23h. The model ($Ri_c=0.89$) NBL is 112 m deep and the inversion depth is in reasonable agreement with the observations. An elevated humidity maximum is discernible in the model profile, just above the top of the NBL, the more pronounced maximum apparent at 18h having been eroded by dew deposition. A nocturnal jet with supergeostrophic wind speeds is also apparent at the top of the NBL in the model.

Also illustrated in Fig. 3 are the profiles resulting from a version of the model using the unmodified level-2 turbulence formulation with $Ri_c=0.19$. In this version the NBL is only 45 m deep and the inversions of temperature and humidity are stronger than were observed. Because of the shallower NBL the elevated humidity maximum is more pronounced and occurs lower down. Similarly, a strong nocturnal jet occurs closer to the surface, at variance with the wind profile observed. The pronounced humidity maximum causes fog to form at 45 m by 0245h, about four hours earlier than in the version with $Ri_c=0.89$. Because of the poor agreement between the model using the lower value of $Ri_c$ and the observations (also evident in other cases), only results using the modified formulation with $Ri_c=0.89$ are discussed subsequently.

(d) Comparison of model and observed surface energy balance

The observed surface fluxes are shown in Fig. 4(a), for comparison with the model fluxes in Fig. 4(b). The observed sensible heat flux was obtained as a residual from the other fluxes. The large negative peak around sunset is believed to be an artifact of this
Figure 3. Observed (BALTHUM, squares; tethered balloon, triangles; and 16 m mast, circles) and model NBL temperature, humidity and wind speed profiles at 23 h for case study A. The model profiles are shown for (-----) $R_l = 0.89$ and (-----) $R_l = 0.19$.

(a) Observations

(b) Model

Figure 4. (a) Observed and (b) model surface energy balance for case study A. The components of the energy balance are the surface net radiation $F_R$ (-----), latent heat flux $F_L$ (-----), sensible heat flux $F_H$ (-----) and 5 cm soil heat flux $F_S$ (-----).
procedure, it arises because the soil temperature gradient changes sign between 5 cm and the surface and it is not then possible to obtain an accurate estimate of the surface soil heat flux by extrapolation of the measured 5 cm and 10 cm values. (In Fig. 4 the 5 cm soil heat fluxes are shown.)

During the night the model surface energy balance is fairly realistic with $F_N$ and $F_S$ being the largest components. The model net radiative loss is about 20 W m$^{-2}$ greater than observed and this can largely be accounted for by the difference in surface temperatures. The grass minimum temperature (i.e. the minimum temperature measured at 1 cm, the height of the grass tips) was $-7.1^\circ$C, whilst the minimum surface temperature in the model was $-4.1^\circ$C. The model produces larger sensible and soil heat fluxes than were observed, which offset the additional radiative loss. The model latent heat flux is similar to that observed before 02 h but does not exhibit the decrease observed after this time, which was probably related to advective effects.

(e) Development of model and observed surface layer

Besides overestimating the minimum surface temperature, the model also significantly underestimates the temperature difference between screen level and the surface. In the model this varies from 1.5 degC at 1630 h to 0.9 degC at 0700 h. The observed grass minimum depression was 5 degC, a typical value for a clear night with light winds. The discrepancy arises because the model overestimates the wind speed near the surface, this results in enhanced turbulent mixing which reduces the intensity of the surface-based inversion. These differences are illustrated in Figs. 5(a) and (b), which show model and observed profiles of temperature and wind speed at 23 h and the resultant $Ri$ profiles. About half the $Ri$ profiles derived from the observations exhibited a shape similar to that in Fig. 5(b), with $Ri$ reaching a minimum value a few metres above the surface. In the model, $Ri$ always tends to zero at the surface, as in Fig. 5(b), a consequence of the

![Figure 5](image-url)  

Figure 5. (a) Observed (16 m mast, circles) and model temperature (——) and wind speed (——) profiles. (b) Observed (16 m mast, circles) and model (——) Richardson number profiles. (c) Calculated actual (---·---) and model (——) profiles of radiative heating. All the profiles are for 23 h for case study A. Observed values are 20-minute averages.
logarithmic temperature and wind speed profiles. The other observed \( Ri \) profiles showed \( Ri \) decreasing monotonically down to 0.5 m, with occasional values as low as 0.1. This suggests that intermittent turbulence was responsible for the transfer of heat and moisture to the surface on this night.

An important consequence of the model underestimating the difference in temperature between the surface and low-level air is that radiative cooling of the air is also significantly underestimated. This is shown in Fig. 5(c) which compares the model radiative cooling profile at 23 h with an estimate of the actual profile. The latter has been calculated by applying the long-wave scheme to profiles of temperature and humidity which were a synthesis of the mast, tethered balloon and BALTHUM data for 23 h, together with the midnight Crawley radiosonde ascent.

Another significant difference between the model and the observations emerges when the radiative and non-radiative heating rates are compared. At 23 h the model atmosphere at 1 m is cooling by about 0.5 degCh\(^{-1}\). Radiative cooling accounts for 0.3 degCh\(^{-1}\) and the divergence of the turbulent heat flux is responsible for the remaining 0.2 degCh\(^{-1}\). This is not consistent with the behaviour of the real atmosphere on this night. At 1 m the actual cooling rate around 23 h, although variable, was less than 0.5 degCh\(^{-1}\), Fig. 6(c), whilst the calculated radiative cooling rate was 1.8 degCh\(^{-1}\), Fig. 5(c). Non-radiative heating must account for the difference. This inference is consistent with results presented by Rider and Robinson (1951), who noted that the temperature change near the surface on a radiation night was the resultant small difference between gaseous radiative cooling and warming by turbulent diffusion and advection. From 18 to 23 h there was little evidence for warm advection, suggesting that convergence of the turbulent heat flux was primarily responsible for the non-radiative heating. After 23 h significant warm advection occurred; this is discussed later.

\( f \) The formation of fog

In the model simulation fog eventually formed at 07 h around 40 m and descended to the surface by 0730 h. At this level the model atmosphere was cooling by 0.5 degCh\(^{-1}\) to which radiative cooling contributed only 0.1 degCh\(^{-1}\). This suggests that the formation of fog aloft in the model was mainly due to turbulent mixing. The appearance of fog was not related to the effects of insolation since sunrise was at 0730 h.

Although the model appears nearly to have made a correct prediction that fog would not develop, the detailed comparisons suggest that agreement was largely by chance. The model did not form a surface-based fog because close to the surface the wind speed was too high and the radiative cooling too small. This is reflected in the slow approach of the model to saturation at 1 m compared with the observed screen relative humidity, Fig. 6(c). The question arises, why did a dense fog not form on this night? The observations do not provide a definite answer but suggest several reasons.

First, why did fog not develop aloft as in the model? Figures 6(a), (b) and (c) show model and observed time histories of temperature and relative humidity at 90 m, 35 m and 1 m (or screen level) respectively. The model temperature falls during the night at these levels and the relative humidity rises until the air at 35 m saturates around 07 h. The observations show that after 23 h, cooling at 35 and 90 m ceased and so the relative humidity did not continue to rise as it did in the model. Comparison of the 23 and 05 h BALTHUM ascents above the NBL showed a warming which increased with height e.g. 1 degC at 450 m, 2 degC at 900 m. Between 18 and 23 h the temperature changes above the inversion had been small (<-0.5 degC) and not of a consistent sign. Thus it appears that warm advection, possibly associated with the flow becoming more westerly, was responsible for the absence of local cooling in the NBL after midnight. Between 23 and
05h the humidity mixing ratio above 150 m increased by about 1 g kg\(^{-1}\). However, below 100 m it decreased; possibly the air reaching Cardington later in the night had followed a trajectory over colder ground and had been dried by dew deposition.

The advection of warmer, drier boundary layer air probably explains why fog did not develop after midnight. A few suggestions can be made on the absence of fog at the surface before midnight. The wind may have been just too strong, leading to sufficient dew deposition via intermittent turbulence, to prevent supersaturation of the air. The initial relative humidity may have been just too low since the empirical forecasting rules did not agree on whether fog would develop. Finally, deposition of frost may have enhanced the drying of the air (although the term ‘dew deposition’ has been used), since the grass minimum temperature was \(-7.1^\circ C\).

(g) Numerical experiments

Numerical experiments were performed to examine some of these ideas, especially the role of turbulence, gaseous radiative cooling and frost deposition. The results are
TABLE 1. RESULTS OF NUMERICAL EXPERIMENTS, CASE STUDY A

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Imposed conditions</th>
<th>Fog formation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>Turbulence suppressed after sunset (molecular diffusion retained)</td>
<td>By 07h, at 40 m</td>
</tr>
<tr>
<td>1</td>
<td>$U_g = 2 \text{ m s}^{-1}$</td>
<td>By 1730 h, at surface</td>
</tr>
<tr>
<td>2</td>
<td>$U_g = 2 \text{ m s}^{-1}$, gaseous radiative cooling removed after sunset</td>
<td>Fog did not develop</td>
</tr>
<tr>
<td>3</td>
<td>$U_g = 2 \text{ m s}^{-1}$, $q_1$ set to saturation value over ice when $T_1 &lt; 0^\circ \text{C}$</td>
<td>By 0015 h, at 18 m</td>
</tr>
</tbody>
</table>

summarized in Table 1 and should be compared with the reference integration described above. Experiment 1 indicates that turbulent mixing inhibits fog formation at the surface. In experiment 2, the near-surface wind speed was brought closer to the observed value by the simple expedient of reducing $U_g$ to 2 m s$^{-1}$. This resulted in a doubling of the maximum temperature difference between screen level and the surface, an increase in gaseous radiative cooling and significantly earlier fog formation than in the reference integration. Experiment 3 emphasizes the key role of gaseous radiative cooling in fog formation at the surface when the wind speed is low. Experiment 4 suggests that on this occasion deposition of frost could have delayed fog formation at the surface until midnight. In this experiment, fog formation was caused by an elevated maximum of radiative cooling, associated with the top of the NBL at 18 m, in conjunction with a humidity maximum. If the surface wind speed had been reduced by a method which did not affect the depth of the NBL, it is probable that fog formation would have been delayed longer.

5. SIMULATION OF CASE STUDY B

(a) Synoptic background and observations

This study was made on 18 January 1973 at the end of a generally foggy period associated with a stationary ridge extending from the Azores to Scandinavia. Fog from the previous night cleared by 11 h and the relative humidity only fell to 95% during the afternoon (cf. 70% for case A). A shallow fluctuating fog formed under clear skies around 1730 h. For a more detailed discussion of this night refer to Roach et al. (1976), case study C. The empirical forecasting rules were in agreement that fog would form on this night, on account of the high relative humidity.

(b) Initial conditions

A trial integration was performed using initial profiles based on the 11 h BALTHUM ascent. The temperature profile exhibited an inversion at 150 m, a consequence of fog from the previous night which had just cleared; using this profile the model failed to erode the inversion. However, the 17 h BALTHUM ascent showed that the inversion had been eroded and energy budget calculations, similar to those described for case study A, suggested that warm advection had occurred during the afternoon. Therefore hypothetical midday temperature and humidity profiles were specified which were designed to give reasonable agreement with the observations around 17 h, Fig. 7. A geostrophic wind speed of 4.5 m s$^{-1}$ was specified. Only 09 h soil temperatures at 30 cm and 100 cm were measured on this occasion, so the initial soil temperature profile (see Fig. 2) was
based on these and on the shape of the profile observed during case study A.

(c) Development of model and observed surface layers

Discussion of this case will concentrate on the surface layer, which was observed comprehensively. Figure 8(a) shows the screen temperature, 1 m mast and 1 m model temperatures as functions of time. The course of the model temperature with time is in reasonable agreement with the observations. The screen and mast temperatures differ by up to 2 degC, this may be partly due to the measurements being made about 100 m apart and at different heights. The minimum surface temperature in the model was \(-3.9^\circ\text{C}\) whilst the observed grass minimum temperature was \(-6.8^\circ\text{C}\). The observed hourly 10 m wind speed, mast wind speed at 8 m and model wind speed at 10 m are shown in Fig. 8(b). The model wind speed is again higher than that observed. The model relative humidity at 1 m, Fig. 8(c), is initially lower than the screen value but rises to a similar value by 16 h. Fog developed at Cardington at 1730 h but remained shallow and fluctuating for several hours, the model formed fog at the surface at 1830 h.

The structure of the surface layer is examined in more detail in Fig. 9(a), which shows model and observed temperature and wind speed profiles at 18 h. The observed fog was too shallow to be radiatively shielding the surface at this time and the profiles are typical of pre-fog conditions. As in case study A, the model overestimates the wind speed near the surface and underestimates the temperature gradient. The resulting Ri profiles, Fig. 9(b), show that the model again overestimates the level of turbulent mixing in the surface layer. This is supported by measurements from a turbulence probe at 8 m which gave mean values for 18-19 h of \(K_h \approx 4 \times 10^{-3} \text{m}^2\text{s}^{-1}\) and \(F_H = 0.4 \text{W m}^{-2}\). The corresponding values at 8 m from the model are \(K_h \approx 9 \times 10^{-2} \text{m}^2\text{s}^{-1}\) and \(F_H = 14 \text{W m}^{-2}\).
Figure 8. Case study B, time variation of (a) observed (screen, circles, and 16 m mast ——) and model (----) temperatures at 1 m. (b) Observed (10 m wind speed, circles, and mast wind speed at 8 m ——) and model (----) wind speed at 10 m. (c) Observed (screen, circles) and model (----) relative humidity at 1 m. The times of sunset and fog formation are indicated on the figure.

Figure 9. (a) Observed (16 m mast, circles) and model temperature (——) and wind speed (----) profiles. (b) Observed (16 m mast, circles) and model (——) Richardson number profiles. (c) Calculated actual (- - - - - - -) and model (——) profiles of radiative heating. All the profiles are for 18 h for case study B. Observed values are 20-minute averages.
Figure 9(c) shows that again the model radiative cooling rates in the surface layer are significantly less than those calculated from the observed profiles. The observations imply that, as in case study A, radiative cooling of the air near the surface was partially offset by convergence of the turbulent heat flux or by warm advection. In the model the surface layer is cooled by radiation and the divergence of the turbulent heat flux.

\(d\) Factors influencing the formation of fog

Numerical experiments were performed, similar to those for case study A. The results are summarized in Table 2, for comparison with the reference integration. Similar results to those for case study A are obtained, except that the effects of turbulence and radiative cooling in inhibiting and promoting surface fog formation respectively are less marked. This may be a consequence of the high initial relative humidity and explains why, despite the discrepancies in the surface layer structure, the reference integration produces fog only an hour later than was observed. Another possibility, suggested by experiment 3, was the absence of frost deposition in the reference integration.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Imposed conditions</th>
<th>Fog formation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td></td>
<td>By 1830h, at surface</td>
</tr>
<tr>
<td>1</td>
<td>Turbulence suppressed after sunset (molecular diffusion retained)</td>
<td>By 1630h, at surface</td>
</tr>
<tr>
<td>2</td>
<td>Gaseous radiative cooling removed after sunset</td>
<td>By 00h, at surface</td>
</tr>
<tr>
<td>3</td>
<td>(q_v) set to saturation value over ice when (T_1 &lt; 0°C)</td>
<td>By 2215h, at 35m</td>
</tr>
</tbody>
</table>

6. DISCUSSION AND CONCLUSIONS

The comparisons have revealed several areas in which the model predictions are at variance with the observations. The discrepancies are similar on each of the nights and thus indicate the inadequacy of the idealized structure of the model. A further cause of disagreement, not related to the model, is advection, especially that occurring over a period of a few hours.

\(a\) Behaviour of the wind near the surface

The significant differences in stability and radiative cooling between the model and observed surface layers follow from the inability of the model to reproduce the light winds observed near the surface. The latter defect is a characteristic of boundary layer models when applied to stable conditions and is believed to be due to the finite size of roughness elements. The site at Cardington, although open and grassed, is mainly surrounded by arable land with hedges, fences and occasional trees, to the north buildings predominate. The empirical constants in published turbulence formulations, such as the one used here, are derived from measurements made at ideal sites with flat uniform terrain. It is therefore not surprising that such a formulation cannot reproduce the low wind speeds observed near the surface at a typical rural site. Beljaars (1982) and Beljaars et al. (1983) have presented wind profiles obtained under non-uniform fetch conditions which deviate significantly from the flux-profile relationships for homogeneous terrain. Beljaars suggests that the deviation is caused by a slow relaxation in the exchange coefficients due to changing surface roughness. The results presented here suggest that
such effects should be represented in a model of radiation fog in order to simulate realistically the formation process.

One test of an improved formulation would be the ability to generate a convergence of sensible heat flux into the surface layer, as implied by the observations. Townsend (1967) suggests that the deceleration of the wind at the surface can lead to an elevated turbulent shear layer. One can envisage this resulting in the convergence of sensible heat flux in the layer beneath, which is only intermittently turbulent because of the low wind speed. In contrast, the model always produces a divergence of the sensible heat flux, probably because $Ri$ tends to zero at the surface, which is the last region in which turbulence decays.

(b) Prediction of the surface temperature

The model was unable to reproduce the low grass minimum temperatures observed, even when a soil with an insulating surface layer was specified, which indicates that the surface cooling rate was underestimated. This is a significant error because reductions in the surface cooling rate were found to delay fog formation at the surface. For example, in case study B, when the inhomogeneous soil was replaced by a homogeneous clay soil, fog formation was delayed by five hours.

Although the inhomogeneous soil prescription may be considered to represent the uppermost layers in the soil, i.e. the alluvial deposits or even the grass root system, it is still a considerable oversimplification. In reality the radiative surface temperature, the soil surface temperature and the surface temperature (from downwards extrapolation in the atmosphere) are not identical, although they are likely to be related. In the model they are all represented by a single temperature, which could lead to errors in the fluxes. An important effect, not considered in the model, is that the blades of grass are cooled by radiative loss to a temperature below that of the bare soil. As suggested in I, this would increase the radiative cooling of the air near to the surface and so encourage fog formation.

(c) Role of advection

In each case, use of initial conditions based upon midday observations did not lead the model to reproduce the temperatures observed after sunset. Analysis indicated that advection of warmer boundary layer air was at least partly responsible for the discrepancy on each occasion. It is possible that in both cases the role of advection was enhanced because fog from the previous night did not disperse until late in the morning. This may have led to temperature contrasts between foggy and clear areas. The analysis of case study A suggests that advection of warmer, drier air was probably responsible for preventing fog from forming after midnight on that occasion.

(d) Constructive results

The model reaffirms some of the results described in I. In particular a reduction in turbulence associated with lighter winds assisted the formation of fog at the surface. When gaseous radiative cooling was removed the model did not form fog at the surface unless the initial relative humidity was very high and then it formed much too late. When the model formed fog aloft, gaseous radiative cooling played only a minor role. The model bears some resemblance to results from recent field studies described by Jiusto and Lala (1983). They found that fog often formed aloft and descended to the surface, although sometimes this was related to the presence of a nearby river. Their observations also suggested that there are several stages in the formation of fog. Early in the night fog may develop in association with lulls in the wind speed, as described by Roach et al.
Later in the night, as the relative humidity increases through a deeper layer, then fog may develop without a lull in the wind. The results show that the model gives some support to this picture. The model also indicates that the deposition of frost may have played a role in delaying fog formation; however, the numerical results must be treated with some caution since frost deposition was modelled only crudely.

REFERENCES

Beljaars, A. C. M. 1982 The derivation of fluxes from profiles in perturbed areas. Boundary-Layer Meteorol., 24, 35–55
Carson, D. J. 1973 The development of a dry inversion-capped convectively unstable boundary layer. ibid., 99, 450–467
Jiusto, J. E. and Lala, G. G. 1983 ‘Radiation fog field programs—recent studies’. Atmospheric Sciences Research Centre—State University of New York at Albany, publication No. 869
Roach, W. T. and Slingo, A. 1979 A high resolution infrared radiative transfer scheme to study the interaction of radiation with cloud. ibid., 105, 603–614
Townsend, A. A. 1967 Wind and the formation of inversions. Atmos. Environ., 1, 173–175
Yamada, T. 1975 The critical Richardson number and the ratio of the eddy transport coefficients obtained from a turbulence closure model. J. Atmos. Sci., 32, 926–933
1983 Simulations of nocturnal drainage flows by a q^2 turbulence closure model. ibid., 40, 91–106