A study of the diurnal variation of stratocumulus using a multiple mixed layer model

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SUMMARY

A multiple mixed layer model of the cloud-topped boundary layer is developed to investigate the diurnal variation of stratocumulus. A simple representation of the microphysical properties of the cloud layer is included which enables high resolution, interactive radiation calculations to be made. A constraint on the buoyancy flux profile is introduced which permits the previously well-mixed layer to separate into two independently driven layers, whereby avoiding many of the unrealistic aspects of single-layer models and enabling the model to reproduce features seen in observational studies. The diagnosis of this decoupling, which generally occurs during the morning, is discussed and is shown to have a seasonal and latitudinal sensitivity due to its dependence on cloud layer short-wave absorption.

Comparisons of results with those obtained when separation is not permitted show significant differences. In particular, the cloud layer displays a much enhanced diurnal variation in thickness when separation is allowed, with the minimum occurring in the afternoon. These results are shown to produce much better agreement with observations. The likely consequences of separation for boundary layer evolution are discussed. The surface energy balance can be quite strongly affected, suggesting that it is important to resolve this diurnal variation, especially in areas where stratocumulus is a dominant climatological feature.

The sensitivity of the results to model assumptions are investigated and the limitations of the model are also assessed.

1. INTRODUCTION

The current interest in studying the cloud-topped boundary layer (CTBL) is a result of the increasing realization that low-level clouds are an important factor in a wide range of meteorological problems. The onset of cloud formation is associated with major changes in many physical processes (e.g. radiation, latent heat release, precipitation) which may result in the structure and energy transfers within the CTBL being very different from those found in cloud-free conditions. This paper examines the relationship between these various processes and their effect on convectively driven, horizontally extensive layer cloud (stratocumulus), by focusing attention on their diurnal cycle.

Previous studies (Fraval et al. 1981; Schaller and Kraus 1981; Hanson and Gruber 1982) have generally used single mixed layer models to investigate aspects of this diurnal variation. However, results from a recent series of observations made on flights around the United Kingdom (Nicholls 1984, referred to as N84; Nicholls and Leighton 1986, called NL) have brought important additional features to light which have not previously been considered. Most importantly, these observations have shown that under certain circumstances, the vertical distribution of heating and cooling within the CTBL may be incompatible with the central assumption of single mixed layer models.

As originally discussed in N84, certain processes, e.g. the absorption of insolation, entrainment at cloud top and small surface buoyancy fluxes, tend to promote the establishment of a more stable density stratification within the mixed layer. These are counteracted by destabilizing processes, e.g. cloud top radiative cooling and latent heat release at cloud base. However, the regions where these processes generate and consume buoyancy are vertically localized. For a well-mixed density profile to be maintained, the turbulence must redistribute buoyancy throughout the mixed layer to ensure that the density changes at the same rate at every level. This often requires that negative buoyancy fluxes be maintained in the subcloud layer (e.g. N84). The size of these negative fluxes
basically reflects the relative strengths of the opposing processes tending to stabilize and destabilize the layer and their vertical separation, i.e. they depend on the size and vertical location of the sources and sinks of buoyancy associated with the various physical processes. If a well-mixed layer is deemed to exist from the surface to cloud top at all times, there is no constraint on the size which these negative buoyancy fluxes may reach, although in reality limitations must be imposed through the turbulent kinetic energy (TKE) balance. Daytime observations in conditions with weak surface buoyancy fluxes typical of many marine boundary layers (N84; NL) have shown that little TKE appears to be exported from the regions of positive buoyancy generation near cloud top and the surface, so that only small negative fluxes can be supported. This implies that a single well-mixed layer from cloud top to the surface cannot always exist and that transport between the cloud and surface layers is inhibited. The conditions where this is likely to occur and the consequences for the subsequent development of the cloud and the remaining parts of the boundary layer are the central questions addressed by this paper.

One such consequence is that the boundary layer may develop an internal layered structure as the evolution of different sub-layers proceed largely independently. Structures of this kind are common features in observations made around the United Kingdom (e.g. NL), but data from other locations have not yet been examined with this in mind. This paper shows how the decoupling process and the subsequent boundary layer development can be predicted in a manner consistent with these observations and energetic considerations by incorporating an additional constraint on the layer-integrated buoyancy within a mixed layer model. Other types of model have not yet been used to study this process in detail, although a recent higher-order closure study of similar conditions by Bougeault (1985) has also predicted decoupled cloud layers. As shown below, disregarding this possibility can have serious consequences for the subsequent modelled evolution of the CTBL and the surface energy balance. Furthermore, ignoring cloud layer separation can also lead to unrealistic predictions from entrainment parametrizations, as shown by Nicholls and Turton (1986, called NT).

In addition, other features which have been determined to be important from the observational studies and which have previously been neglected have also been incorporated. A simple description of the cloud microphysics allows the droplet spectrum to vary with height and cloud depth in a realistic manner. This enables high resolution, interactive, long- and short-wave radiative transfer models to be used. The partitioning of the radiative cooling between cloud top and the adjoining inversion, which has proved controversial in previous models (e.g. see discussion by Nieuwstadt and Businger (1984)) is therefore calculated explicitly. Since observations have shown that water transport by gravitational settling of cloud drops is significant even in moderately thin (~400 m-thick) cloud, a crude parametrization based on observational data is also included. Finally, any one of four entrainment parametrization methods may be selected, again including one based on observational results.

Following a description of the model formulation, the effects of incorporating the features mentioned above are assessed by considering the diurnal response in simulated mid-latitude and subtropical conditions in both winter and summer. This enables the conditions in which decoupled cloud layers are most likely to form to be identified and the consequential effects on the cloud structure to be determined. The realism of the modelled features and their sensitivity to the assumptions made are discussed and compared with available observational evidence. The consequences of ignoring these effects on the surface energy balance are investigated. The limitations of the model and the relevance of 'entrainment instability' within the context of the model are also examined.
2. The Basic Model Description

Mixed layer models of turbulent boundary layers have had a long and productive history. They have been used to describe the atmospheric boundary layer both with and without clouds (e.g. Carson 1973; Lilly 1968) and also the ocean mixed layer (e.g. Kraus and Turner 1967). By assuming that the boundary layer can be described as a single slab or mixed layer within which turbulent mixing constrains the vertical variations of conservative quantities to be known functions of model variables (usually, and most simply, they are assumed to be height independent), a simple yet fairly complete model formulation can be developed. Such models are capable of predicting many aspects of boundary layer structure and are particularly suited to studying how this structure varies with time. Although mixed layer models are inherently more limited than techniques which predict details of the turbulent flow, e.g. higher-order closure models (Bougeault 1985) or large eddy simulations (Moeng 1984), they produce comparable results in many situations and have a number of advantages:

(i) Turbulent transport within the layer does not require explicit representation. The turbulent fluxes follow directly from heat and water budget considerations.

(ii) Vertical resolution is not restricted as in a model with a discrete vertical grid. The very large gradients usually associated with cloud top are well represented by discontinuous jumps.

(iii) The method is conceptually straightforward and computationally simple. Cause and effect are therefore easily identified. The technique offers a method of introducing cloudy boundary layer physics into large-scale models without the need for a large increase in the number of near-surface grid levels and the associated computational penalty (e.g. Suarez et al. 1983).

(iv) The relatively simple formulation enables such models to be easily converted to run in a diagnostic mode to facilitate comparisons with observations.

The main disadvantages of the mixed layer approach, apart from the limited set of quantities predicted, may be summarized as follows:

(i) It is one-dimensional, and therefore implicitly assumes horizontal homogeneity over a limited local area.

(ii) The model assumes a certain boundary layer structure. Furthermore, the assumptions linking the mean vertical gradients to the modelled variables remain valid only within a certain range of conditions. These restrict the range of applicability of the model.

(iii) The turbulent fluxes must be specified at the mixed layer boundaries. This is fairly straightforward at the surface, where tested parametrizations exist, but no consensus yet exists for parametrizing entrainment at the top of the CTBL.

For the studies described in this paper, the advantages of the mixed layer approach outweigh the drawbacks.

The structure of the boundary layer employed in the model is illustrated in Fig. 1. (The significance of the various levels shown in the figure is discussed later as they are introduced. A list of symbols and notation appears at appendix A.) Initially, the boundary layer is assumed to have a single mixed layer structure. Subsequently a simple test is applied to the buoyancy flux profile to check whether decoupling should occur. The rationale behind this procedure and the method of implementation are described in
Figure 1. Idealized boundary layer profiles of $\theta_e$ and $q_r$ for a single mixed layer structure (left) and for a multiple mixed layer structure (right), showing the additional levels.
sections 3. If such conditions are detected, two independent vertically separated mixed layers are allowed to form (see Fig. 1). The decoupled mixed layer (DML) is associated with the cloud layer and is driven primarily by cloud top radiative cooling. The surface mixed layer (SML) remains in contact with the surface and the two are separated by a stable layer or an inversion. The level at which decoupling occurs follows directly from the buoyancy flux constraint. The two mixed layers evolve largely independently although they may subsequently reconnect to reform a single mixed layer.

The remainder of this section deals with those parts of the model description which are common whether there is a single or a double mixed layer structure. The extensions necessary to describe the formation and evolution of an additional detached layer are covered in section 3. A summary of the actual computational procedure employed with references to the equations used is given in appendix B.

(a) The basic mixed layer equations

The central assumption made here is that vertical gradients of conservative quantities $\theta_e$ and $q_T$ remain negligible within a well-mixed layer. This alone probably limits the range of applicability of the model to convectively dominated conditions since the advent of significant turbulence production by shear may be associated with larger mixed layer gradients being maintained, as seen in data reported in NL. In convective conditions, these mixed layer gradients are generally found to be much smaller, e.g. NL found the mean measured cloud liquid water content was within 20% of the adiabatic value. Since the adiabatic liquid water content typically accounts for less than 15% of the total water content ($q_T$) at any level and the specific humidity in the cloud layer remains very close to its saturation value, $q_T$ changes at most by a few per cent throughout the depth of the cloud. The results presented below are therefore relevant to convective conditions. Furthermore, the relative importance of dry air entrainment at cloud top (which acts to promote larger gradients) as indicated by the ratio $w_{eU}/w_e$ (where $w_{eU}$ is the entrainment velocity at cloud top) is generally slightly less than the corresponding values reported by NL (~1%) in the results reported below. Consequently, the model-predicted liquid water content, which is constrained to be adiabatic, is likely to remain a sufficiently good approximation for use by both the radiation and rainfall parametrizations.

As explained in N84, heat and water budget considerations lead to the following expressions for the rates of change of $\theta_e$ and $q_T$ in the mixed layer.

$$d\theta_e/dt = \{(\bar{w}\theta_e)_L - (\bar{w}\theta_e)_U + R_L - R_U\}/(z_U - z_L) \tag{1}$$

$$dq_T/dt = \{(\bar{w}q_T)_L - (\bar{w}q_T)_U\}/(z_U - z_L) \tag{2}$$

where the subscripts U and L refer to values evaluated at the upper and lower mixed layer boundaries (see Fig. 1) and $R$ is the net radiative flux ($R = L \uparrow - L \downarrow + S \uparrow - S \downarrow$). The total water flux includes a contribution due to rainfall $\bar{w}_Rq_T$ (see N84) so that

$$\bar{w}q_T = \bar{w}q + \bar{w}_Rq_T \tag{3}$$

Provided that the fluxes at the mixed layer boundaries together with the vertical variation of $R$ and $\bar{w}_Rq_T$ are specified and that the cloud remains exactly saturated, the variation with height of the other turbulent fluxes (e.g. $\bar{w}\theta$, $\bar{w}\theta_v$, $\bar{w}q$ and $\bar{w}q_T$) throughout the mixed layer can be determined (see, e.g. N84). The upward movement of the mixed layer top is given by

$$dz_U/dt = w_{eU} - z_U D \tag{4}$$
where \( D \) is the horizontal divergence, assumed constant from the surface to \( z_U \). Similarly, at the mixed layer base

\[
dz_L/dt = w_L - z_L D
\]

where \( w_L \) is the rate of change of \( z_L \) due to the dynamics of the mixed layer. Clearly if \( z_L \) is at the surface, \( dz_L/dt = w_L = z_L = 0 \), otherwise both \( w_{eU} \) and \( w_L \) must be determined.

In situations where a multiple mixed layer structure exists (see Fig. 1), Eqs. (1), (2) and (4) are applicable to both layers, where the upper and lower boundaries of the surface mixed layer are \( z_X \) and \( z_o \) (the surface) respectively.

In all cases, it is necessary to specify the fluxes at, and the velocity of, the interface at every mixed layer boundary.

(b) Entrainment at cloud top

At cloud top the fluxes of conservative quantities are related to the entrainment velocity by the usual 'jump' relations:

\[
\begin{align*}
\bar{w} \theta_u & = -w_{eU} \Delta \theta_{eU} \\
\bar{w} q_v & = -w_{eU} \Delta q_{TV}
\end{align*}
\]

where the cloud top jumps, \( \Delta \theta_{eU} \) and \( \Delta q_{TV} \), are defined later (Eqs. (18), (19)). Many different ways of estimating \( w_{eU} \) have been suggested (e.g. Deardoff 1976; Kraus and Schaller 1978; Stage and Businger 1981) and brief details of each were given in NT, which also contained comparisons between the various predictions and observationally derived estimates for a number of particular cases. The predictions showed a fair degree of variability, but all tended to underestimate the values derived from measurements, so to compensate, a slightly different expression was derived following the suggestion made in NT that entrainment might be described by a relationship of the form

\[
w_{eU}/w_* = f(R_i, \Delta_m/\Delta \theta_{eU})
\]

where

\[
R_i = g(z_U - z_L) \Delta \theta_{eU}/\theta_w w_*
\]

\( \Delta \theta_{eU} \) is the cloud top jump in \( \theta_e \) and \( \Delta_m \) is a measure of the potential for evaporative cooling due to mixing at the interface. With no cloud, \( \Delta_m = 0 \), but as the potential for evaporative cooling increases, e.g. as the liquid water content at cloud top increases as the cloud thickens, or should the inversion become drier, \( \Delta_m \) is progressively reduced. Choosing an expression such as

\[
w_{eU}/w_* = a_1 R_i^{-1} \left( 1 + a_2 (1 - \Delta_m/\Delta \theta_{eU}) \right)
\]

contains the favoured inverse Richardson number dependence (e.g. Deardoff 1983) with the evaporative cooling effects contained within the curly brackets. Taking \( a_1 = 0.2 \) recovers the accepted behaviour for cloud-free mixed layers heated from below, and if \( a_2 = 60 \) Eq. (10) gives a reasonable fit to the values reported in NT. These values are listed in Table 1 together with those predicted by the three methods mentioned above, showing that Eq. (10) predicts generally higher values, which are in better accord with the observations.

Equation (10) can also be written in the form

\[
w_{eU} = 2.5 a_1 \frac{1}{(z_U - z_L)} \left[ 1 + a_2 (1 - \Delta_m/\Delta \theta_{eU}) \right]
\]

\[
\frac{\Delta \theta_{eU}}{\Delta \theta_{eU}}
\]
showing that \( w_{cU} \) is proportional to the buoyancy integral \( I \), where

\[
I = \int_{z_L}^{z_U} (\bar{w}\theta_v) \, dz. \tag{12}
\]

In this form it is apparent that the parametrization is identical to that proposed by Deardorff (1976) except that the term \( \Delta \theta_{cU}/[1 + a_2(1 - \Delta_{nc}/\Delta \theta_{cU})] \) replaces the quantity \( \Delta_2 \), defined by

\[
\Delta_2 = \beta \Delta \theta_{cU} - \theta \Delta q_{TU} \tag{13}
\]

which has been discussed at length by Randall (1980), Deardorff (1980) and others. Equation (10) therefore avoids one of the main drawbacks of the Deardorff (1976) scheme, in which \( w_{cU} \rightarrow \infty \) as \( \Delta_2 \rightarrow 0 \).

The model can be run using any of the entrainment parametrizations mentioned above.

<table>
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(c) Boundary fluxes at \( z_o \)

Since it is always assumed that \( z_o \) is below the cloud base \( z_B \), the fluxes across this boundary are specified if \((\bar{w}\theta)_o, (\bar{w}q)_o \) and \((\bar{w}\theta q)_o \) are known. The heat and water vapour fluxes are given by the usual bulk aerodynamic formulæ:

\[
\langle \bar{w}\theta \rangle_o = C_\theta V (\theta_S - \theta_o) \tag{14}
\]

\[
\langle \bar{w}q \rangle_o = C_q V (q_S - q_o). \tag{15}
\]

\( C_\theta \) and \( C_q \) are transfer coefficients for heat and water vapour, \( \theta_S \) and \( q_S \) are the potential temperature and saturation specific humidity corresponding to the sea surface temperature. The mixed layer temperature and humidity are given by the values in the appropriate mixed layer adjacent to the surface (i.e. \( \theta_o = \theta_x \) for a single layer structure, \( \theta_o = \theta_{x'} \) if a DML exists; similarly for \( q_o \)). The determination of \( (\bar{w}\theta q)_o \) is discussed in the following subsection.

(d) Cloud microphysics

To ensure that the radiation calculations respond realistically to changes in the cloud layer, it is necessary to parametrize certain features of the microphysical structure of the cloud. The drop-size distribution is represented by a gamma distribution (Levin 1958):

\[
n(r) = r^n (\alpha! \beta^{\alpha+1})^{-1} \exp(-r/\beta) \tag{16}
\]

where \( n(r) \) is the relative number of drops in the radius interval \( r \) to \( r + dr \). The properties
of this distribution are simple functions of $\alpha$ and $\beta$. Observational evidence (e.g. Noonkester 1984; N84; NL) shows that vertical variations of the total concentration, $N$, and dispersion, $\phi$, are typically small in stratocumulus, so they are assumed to be height-independent in the model and are held constant at prescribed values throughout a run. Given these, only the liquid water content is needed to specify the distribution. Since the model predicts the liquid water content as a function of height and time, other parameters, e.g. the mean volume radius, $r_v$, can likewise be obtained. Figure 2 shows some examples of drop-size distributions measured using an FSSP at three different levels in a mid-latitude stratocumulus cloud. The figure also shows distributions obtained using the above parametrization, calculated for the same levels. The parametrization reproduces the main feature of the observations, i.e. the increase in the mean size of the drops with height.

Another important process which is also strongly influenced by the cloud microphysical structure is the growth of precipitation-sized drops. Gravitational settling of drops leads to a significant transfer of water from the cloud layer to lower levels and is enhanced if the cloud is sufficiently thick to produce drizzle. Observations (e.g. N84)

![Figure 2](image-url)
show that the rainfall rate, \( \overline{\omega_r q_h} \), is approximately the same at all levels within the cloud but decreases steadily below cloud base down towards the surface (at a rate \( \sim 10^{-5} \text{m s}^{-1} \text{km}^{-1} \)) due to evaporation. This is represented in the model by defining a single value representative of the cloud layer, which is used at all in-cloud levels, and assuming a fixed rate of decrease beneath cloud base (0-86 \times 10^{-5} \text{m s}^{-1} \text{km}^{-1})

The drizzle may therefore all evaporate before reaching the surface in certain conditions. The in-cloud rainfall rate is determined from the expression

\[
\overline{\omega_r q_h} = -3g(r_v^*) \left\{ \int_{z_B}^{z_U} q_i dz \right\}^{1/2} 10^{-5} \text{m s}^{-1}
\]  

(17)

where

\[
g(r_v^*) = \begin{cases} 1 & \text{if } r_v^* \geq 10 \mu\text{m} \\ \left( \frac{r_v^*}{10} \right)^3 & \text{if } r_v^* < 10 \mu\text{m}. \end{cases}
\]

Here \( r_v^* \) is the maximum value of the mean volume radius, i.e. the value at cloud top where the liquid water content is also a maximum (\( q_i^* \)). The expression fits the limited data available, reported in NL, to within the estimated accuracy of the measurements (±40%, NL). The parameter \( g \) reflects some of the consequences of the cloud microphysical structure: if the cloud is sufficiently thin or the droplet concentration is sufficiently high that \( r_v^* < 10 \mu\text{m} \), the possibility of there being sufficient numbers of large droplets present to initiate drizzle production by coalescence decreases strongly with \( r_v^* \).

If \( r_v^* \geq 10 \mu\text{m} \), the drizzle rate is proportional to the square root of the vertically integrated liquid water path, which in this model is approximately proportional to cloud thickness (or maximum liquid water content, \( q_i^* \)).

While this parametrization is admittedly crude, the limited information currently available means there is little justification for a more elaborate approach. It does, however, ensure that realistic levels of water transport due to precipitation are included in the model. The drizzle fluxes and the rates of evaporation below cloud base are assigned values which are close to those which have been observed. Including drizzle in the model affects the evolution of the cloud layer in two ways. First, the evaporation of drizzle beneath cloud base leads to larger negative buoyancy fluxes in the subcloud layer, which tends to promote separation. Secondly, the drizzle flux has an important influence on the boundary layer water budget, because it is similar in magnitude to the other turbulent water fluxes, as inserting typical values into Eqs. (15) and (17) will confirm.

The subsequent evolution of the cloud (which also strongly influences the buoyancy fluxes) is very sensitive to the water budget and drizzle should be included to prevent too rapid moistening. Drizzle therefore exerts an important influence on the energetics of the whole system both through the water budget and cloud thickness, and its effect on the buoyancy flux profile.

\( e \) Radiative transfer calculations

As the model assumes local horizontal homogeneity, the two-stream approximation is used, the short-wave fluxes being calculated by the Slingo and Schrecker (1982) scheme and the long-wave fluxes by the Roach and Slingo (1979) method. The temperature and humidity structure of the atmosphere well above the cloud layer is specified as an initial condition (as described later in section 4) and is assumed to remain constant. Above 200 mb, the relevant seasonally averaged data from McClatchey \( et al. \) (1972) is imposed.
The relevant cloud microphysical description is introduced to both schemes (from Eq. (16)). The short-wave surface albedo is assumed to be constant at 5% for the mid-latitude case and 9% for the subtropical case (Hanson 1982) and the emissivity is taken as unity. Both the long- and short-wave schemes have been shown to agree well with measurements in the types of situations for which the model has been designed (e.g. Slingo et al. 1982; N84; NL).

The production of turbulence by buoyancy within the cloud is strongly dependent on the vertical distribution of radiative heating and cooling, so the main features of the radiative flux profiles must be adequately resolved. This is achieved by performing calculations at six levels within the cloud layer, \( z = 0, 0.3, 0.6, 0.8, 0.95 \) and \( 1.0 \) where \( z = (z - z_0)/(z_U - z_B) \), i.e. \( z = 0 \) at cloud base and \( 1 \) at cloud top. The levels are concentrated near cloud top where gradients tend to be largest, and vary to follow the changing cloud geometry. Beneath cloud base, the fluxes are assumed to vary linearly to the surface, a good approximation since the flux gradients are small.

Even though the clear air within the inversion immediately above cloud top is usually very dry, the long-wave flux divergence remains substantial (varying between 5% and 20% of the maximum in-cloud value, according to calculations in NL) because of the relatively large temperature difference (usually \( \sim 5-15 \) K). As discussed by NL, this represents a significant cooling of the inversion, although most of this effect is confined to a relatively thin layer immediately adjacent to cloud top. In order that this can be properly represented, the fluxes are also calculated at a distance \( \delta \) above cloud top at \( z_t \) (see Fig. 1). A fixed value for \( \delta \) of 30 m is currently specified, which is sufficiently large that approximately 80% of the inversion layer cooling is contained within it, yet small enough that certain properties of the cloud top interface can reasonably be expected to be described by an average across it (as described in the following subsection). Note that if conditions were such that \( \delta \) was significantly larger, the model geometry, which assumes a sharp cloud top interface, would no longer be valid. By defining this extra layer, the radiative cooling occurring in the vicinity of cloud top can be partitioned between the mixed layer and the inversion rather more realistically than has been possible in many previous models and is essentially the approach recommended by Nieuwstadt and Businger (1984). Cloud top radiative cooling therefore has two effects in this model: promoting buoyancy generation within the mixed layer by cooling from cloud top and less importantly, cooling the inversion layer just above cloud top thereby tending to enhance entrainment indirectly by reducing the stability of the inversion.

\((f)\) The inversion above cloud top

As indicated in the previous subsection, the choice of a value for \( \delta \) was only partially based on the results of radiation calculations. Another requirement was to select a value comparable with the vertical scale of local cloud top irregularity or corrugation typically observed in convective, stratiform cloud layers. This aspect is not described by the model, but account can be taken of the fact that entrainment of air from anywhere within this region is possible by defining the jumps at cloud top in terms of layer-integrated quantities \( \theta_e \) and \( q_T \). The effective cloud top jumps are then defined by

\[
\Delta \theta_{eU} = \tilde{\theta}_e - (\theta_{e})_{U-}
\]

and

\[
\Delta q_{TU} = \tilde{q}_T - (q_T)_{U-}
\]
values $\tilde{\theta}_e$ and $\tilde{q}_T$, by the values at level $z_1$ and by the lapse rates ($\gamma_\theta$ and $\gamma_q$, which are fixed at all times) above $z_1$. Since $\delta$ is fixed

$$d(\theta_e) / dt = w_v \gamma_\theta \theta_e = w_v (\gamma_\theta + L_v \gamma_q / c_p)$$

(20)

and the inversion layer heat budget is given by

$$d\tilde{\theta}_e / dt = (w_v / \delta) \{(\tilde{\theta}_e) - \theta_e\} + (1 / \delta) \{R_{U+} - R_I\}.$$  

(21)

Equations (20) and (21) enable the rates of change of $(\theta_e)_I$ and $\tilde{\theta}_e$ to be determined in terms of known quantities. Similar expressions may be derived for $(q_T)_I$ and $\tilde{q}_T$ except that they are, of course, not affected by radiative effects.

However, as the radiation schemes used here require thermodynamic data at the levels where the fluxes are calculated, it is necessary to define parameters at level $z_{U+}$. Since the schemes assume a linear variation between successive calculation levels, a consistent definition is obtained if

$$(\theta_e)_{U+} = 2\tilde{\theta}_e - (\theta_e)_I$$  

(22)

$$(q_T)_{U+} = 2\tilde{q}_T - (q_T)_I$$  

(23)

which preserves the layer averages used above. This construction is also used in the illustrations of the results presented below and is shown dashed in Fig. 1.

3. Extension of the model to include separation

A summary of the procedures described in this section appears in appendix B together with a flow chart showing the way in which the calculations are organized and clarifying the interrelationships.

(a) The diagnosis of separation

The model is initialized with conditions which are known to be consistent with the existence of a single well-mixed layer. At each subsequent timestep, the ratio $I_s/I_c$ is monitored, where

$$I_s = \int_{z_L}^{z_B} (w \bar{\theta}_e) \, dz$$

(24)

and

$$I_c = \int_{z_B}^{z_U} (w \bar{\theta}_e) \, dz$$

(25)

(for a single mixed layer $z_L = 0$). If

$$I_s / I_c < C,$$

(26)

where $C$ is a negative constant representing the maximum allowable energy loss against negative buoyancy forces, it is argued that the losses incurred in working against buoyancy in the subcloud layer are too large to be sustained by transport from the cloud layer, that the strong coupling between them is thereby inhibited and that the two layers become decoupled as an internal density step develops. Or alternatively, the distribution of sources and sinks of buoyancy becomes such that the rate of working against buoyancy necessary to maintain a single mixed layer, a state of maximum internal potential energy,
becomes too great. If this rate of working decreases, the internal potential energy must also decrease, i.e. an internal density gradient will develop. It is shown below that Eq. (26) is consistent with the development of an internally stratified structure such as that shown in Fig. 1 and that mixed layer methods coupled with Eq. (26) are capable of modelling the formation and development of internally stratified cloudy boundary layers.

A consideration of layer-integrated energetics shows that $C$ must lie between 0 and $-1$. $C = 0$ is equivalent to assuming that there is no transport of TKE from the cloud to the subcloud layer to sustain a net loss against buoyancy there. However, it is a feature of convective layers that TKE is transported from regions of net production to maintain regions with a net loss, so $C = 0$ represents an upper bound. If $C = -1$, the layer-integrated buoyancy production ($I_b + I_c$) is zero, which represents the maximum sustainable loss in the subcloud layer. But since buoyancy is the only source of TKE present in the model and as there must always be dissipation, $C = -1$ must represent a lower bound. Although any choice of $C$ between these limits is possible, other more specific conditions, including comparisons with observations and the requirements of internal consistency, constrain the range still further, as discussed in section 5(a). In most of the results presented below, a value of $C = -0.4$ is used. It should be emphasized that although the details of actual simulations are sensitive to the value chosen for $C$ (as they are to the choice of entrainment parametrization employed or to small changes in the boundary conditions), the character of the solutions is unchanged. This is also discussed further in section 5(a).

Once the criterion expressed by Eq. (26) is satisfied, it remains to determine the level at which separation will occur, $z_L$. This requires one further assumption: that the turbulent fluxes at the base of the separated layer are initially zero. This is consistent with the idea that the maximum available amount of TKE is then being used to maintain

![Figure 3](image_url)

**Figure 3.** Cloud top entrainment velocity, $w_{eu}$ (using Eq. (10)) and buoyancy integral ratio, $I_b/I_c$, as a function of subcloud layer depth, $z_b - z_L$, for a typical example. The solution for $z_L$ when separation is diagnosed is shown by the dashed line.
mixing down to level $z_L$ and therefore that none is available to support entrainment fluxes at this level. The value of $z_L$ is then uniquely determined, as illustrated in Fig. 3, which shows how $z_L$ varies with $I_s/I_c$ in a typical example. Although a value $C = -0.4$ was chosen, the criterion (Eq. (26)) was first satisfied when $I_s/I_c = -0.42$ ($=C'$), giving $z_L \approx 390$ m. (The small difference between $C$ and $C'$ reflects the fact that the radiation calculations were performed at longer intervals (15 min) than the basic model timestep of five minutes. Increasing the frequency of these calculations would decrease this difference but does not significantly affect the results.) Figure 3 also shows how $w_{ew}$ (parametrized by Eq. (10)) varies with $z_L$, the predicted value being 0.34 cm s$^{-1}$. Since $I_s/I_c$ is dependent on $z_L$ in a complex way, the solution for $z_L$ must be obtained iteratively.

Note also that separation can occur only if

$$ (d\theta_v/dt)_L \geq (d\theta_v/dt)_X \quad (27) $$

where the subscripts L and X refer to the base of the DML and the top of the SML respectively, otherwise the subsequent configuration would be statically unstable. However, the conditions expressed by Eqs. (26) and (27) are clearly closely related since both reflect the tendency for the mixed layer to become stably stratified. In all tests it was found that if Eq. (26) was satisfied with $C = -0.4$, this was sufficient to ensure that Eq. (27) was also true, although the complexity of the model prevents an explicit proof being derived.

Figure 4 shows the buoyancy flux profiles for the same example just before and just after separation. This clearly illustrates the reduction in the negative values in the subcloud layer which accompany the development of an internal stratification. However, while the ratio $I_s/I_c$ is held constant during separation, the sum total $I = I_s + I_c$ is not. Therefore both $I$ and $w_{ew}$ (which is related to $I$) are discontinuous at this time although the change in $w_{ew}$ is relatively small, as will be seen in subsequent examples. The main source of buoyancy production just beneath cloud top, which largely reflects the effects of radiative cooling and entrainment, is therefore little affected. The main changes occur near cloud base and are associated with the reduced fluxes at $z_L$.

(b) The development of the DML

Once a DML has been diagnosed, its subsequent evolution is determined by assuming that the maximum amount of mixing which can be supported beneath cloud base is the same as that occurring at separation, i.e.

$$ I_s/I_c = C' \quad (28) $$

(for consistency, the actual value at separation, $C'$, is used subsequently rather than $C$). To clarify the discussion in the remainder of this section, consider a situation in which the divergence $D = 0$ at all levels, i.e. there is no mean vertical velocity. On separation, the DML is usually warming slightly and becoming drier (due to continuing entrainment, losses by rainfall and the loss of the upward water flux from the SML). Cloud base therefore rises and in this situation $z_L$ must also rise to prevent $I_s/I_c$ falling below its minimum value ($C'$). A new value of $z_L$, which enables Eq. (28) to be satisfied can be found iteratively, while the fluxes at $z_L$ remain zero for the same reasons as at separation. In this case, $dz_L/dt$ is determined directly, rather than $w_{ew}$. As $z_L$ rises, it leaves behind a slightly stable layer whose stratification is governed by $dz_L/dt$ and $(d\theta_v/dt)_L$.

As the evolution of the DML proceeds, the radiative fluxes respond to changes in the cloud layer structure and also vary according to the diurnal cycle. In particular, the absorption of short-wave radiation may be significantly reduced as the cloud thins during
the afternoon. The buoyancy fluxes respond to these changes and eventually \( I_s/I_c > C' \). This indicates that the cloud layer is capable of supporting greater mixing in the subcloud layer, some of which may be available to promote downward growth of the lower DML boundary by entrainment (i.e. \( w_L < 0 \)). This allows a value of \( w_L \) to be determined which again renders the effective buoyancy flux at \( z_L \), \((w\theta_v)_L^*\), just sufficient to hold \( I_s/I_c \) at a prescribed value. The condition where the maximum energy is used for entrainment is \( I_s/I_c = C' \). However, it may be more likely that only a fraction \( f_e \) of this additional energy is available, in which case

\[
I_s/I_c = f_e C' + (1 - f_e) (I_s/I_c)_{\text{enc}}.
\]

The parameter \( f_e \) therefore influences the rate at which the DML grows downwards into the stable layer. Maximum downward entrainment occurs when \( f_e = 1 \). When \( f_e = 0 \), no entrainment occurs at the DML base, \((w\theta_v)_L^* = 0\), \( I_s/I_c = (I_s/I_c)_{\text{enc}} \), and downward growth (in the absence of mean vertical motion) occurs only as a result of encroachment. With


\((\overline{w \theta}_v)_L^*\) fixed by the constraint on \(I_v/I_c\), \(w_L\) and the fluxes at the interface can be given by jump relations similar to those at cloud top. However, because the density difference across this boundary, \(\Delta \theta_{vL}\), could be small, a relationship such as

\[
\frac{w_L}{(w \theta_v)_L^*/\Delta \theta_{vL}}
\]

may not be valid because a significant fraction of the available energy might be used to impart TKE to previously quiescent fluid (TKE storage) instead of working against buoyancy. In the extreme case as \(\Delta \theta_{vL} \to 0\), entrainment is completely limited by TKE storage rather than by losses to buoyancy forces. An expression which takes this into account was derived by Zilitinkevich (1975) by considering the parametrized TKE budget of a layer at the entrainment interface (see also the discussion by Driedonks (1982)). This yields

\[
w_L = \frac{(w \theta_v)_L^*/\Delta \theta_{vL}}{(1 + R_{iL}^{-1})}
\]

where

\[
R_{iL} = g(z_U - z_L)\Delta \theta_{vL}/\theta_v w_2^*
\]

and \((w \theta_v)_L^*\) is the buoyancy flux which could be sustained at the interface if storage losses were zero. With this storage correction, the fluxes at the interface are given by

\[
(w \theta_v)_L = w_L \Delta \theta_{vL} = \frac{(w \theta_v)_L^*}{(1 + R_{iL}^{-1})}
\]

with similar expressions for \((w \theta_e)_L\), \((\overline{w \theta})_L\) and \((\overline{w q})_L\), e.g.

\[
(w \theta_e)_L = w_L \Delta \theta_{eL}
\]

Equation (33) shows that as \(R_{iL}\) decreases, the buoyancy flux at \(z_L\) is progressively reduced from the maximum available (i.e. \((w \theta_v)_L^*\)) by the factor \(1/(1 + R_{iL}^{-1})\), the remainder being used for TKE storage. While \(R_{iL} > 10\), this correction has little effect, but as \(\Delta \theta_{vL} \to 0\), it can be shown from Eq. (31) that \(w_L\) tends towards a limiting value given by

\[
w_L = \frac{(w \theta_v)_L^*}{g(z_U - z_L)/\theta_v w_2^*}.\]

In this case, \((w \theta_v)_L = 0\) and all the available energy is used for TKE storage (at cloud top, where \(R_{iL} \approx 100\), these considerations are negligible).

When \((w \theta_v)_L^* \to 0\), \(w_L\) and the fluxes also tend to zero because there is no energy available for entrainment. However, a minimum downward growth rate is necessary to maintain static stability, a process usually referred to as encroachment. This minimum value is given by

\[
(w_L)_{encr} = \frac{(d \theta_v/dt)_L}{(d \theta_v/dz)_{SSL}}
\]

where SSL refers to the underlying slightly stable layer. In this case, the fluxes at the interface remain zero.

(c) **The development of the SML**

On separation, the SML extends from the surface to level \(z_L\). The method used to calculate its future development is basically identical to that used in the DML, except that the lower boundary is always assumed to be at the surface. The description employed in this region is therefore very similar to many previous schemes (e.g. Driedonks 1982). The rates of change of \(\theta_v\) and \(q_v\) within the layer are given by equations identical to Eqs. (1) and (2) except that the upper and lower boundaries are now those of the SML,
referred to as \( z_X \) and \( z_o \) (the surface) respectively (see Fig. 1). The surface fluxes, \( (\overline{w\theta})_o \) and \( (\overline{wq})_o \), are determined by Eqs. (14) and (15). The rate of growth of the SML is given (cf. Eq. (4)) by
\[
dz_X/dt = w_{eX} - z_X D \tag{37}
\]
where \( w_{eX} \) (the SML entrainment velocity) is parametrized using the same method employed in the DML and the single mixed layer. Here further simplification is possible because all the entrainment parametrizations mentioned in section 2(b) (including Eq. (10)) reduce to the same formulation in the absence of cloud if \( a_1 = 0 \). Then
\[
(\overline{w\theta}_v)_X = -a_1 w_* \Delta \theta_{vX} Ri_X^{-1} \tag{38}
\]
where \( \Delta \theta_{vX} \) is the jump in \( \theta_v \) at the top of the SML,
\[
Ri_X = gz_X \Delta \theta_{vX}/\theta_v w_*^2 \tag{39}
\]
and \( w_* \) now refers to the SML value (which is different from the DML value). However, since the inversion above the SML is likely to remain weak, the storage-modified relation (e.g. Eq. (33)) is again employed, so that
\[
(\overline{w\theta}_v)_X = -w_{eX} \Delta \theta_{vX} = (\overline{w\theta}_v)_X^*/(1 + Ri_X^{-1}) \tag{40}
\]
and from Eqs. (38) to (40),
\[
w_{eX} = a_1 w_*/(1 + Ri_X). \tag{41}
\]
As with Eq. (31), \( w_{eX} \) tends towards a limiting value \( a_1 w_* \) as \( \Delta \theta_{vX} \to 0 \) and \( (\overline{w\theta}_v)_X \to 0 \). When \( Ri_X \gg 1 \), then \( w_{eX} = - (\overline{w\theta}_v)_X^*/\Delta \theta_{vX} \).

All the fluxes vary linearly within the SML (the net radiation fluxes vary linearly between \( z_X \) and the surface) unless the rainfall rate falls to zero somewhere within the layer, when an additional computational level is employed.

\( d \) The region separating the SML and the DML

Following decoupling, when the base of the DML is rising faster than the top of the SML, levels \( z_L \) and \( z_X \) become vertically separated and since Eq. (27) is always satisfied, this region is always stably stratified. Just below level \( z_L \), at the top of this region (at level \( z_{L-} \)), the stratification is given by
\[
(d\theta_v/dz)_{L-} = (1/w_L)(d\theta_v/dt)_L \tag{42}
\]
but since both \( w_L \) and \( (d\theta_v/dt)_L \) change with time, the actual \( \theta_v \) structure within the region between the DML and the SML may be quite complicated. To avoid such complication a simpler description is used in which only values at the boundaries of the region (denoted \( z_{L-} \) and \( z_{X+} \)) are defined and a linear variation between them is assumed.

Within this stably stratified layer, turbulent transport is assumed to be negligible, so expressions for quantities at levels \( z_{L-} \) and \( z_{X+} \) may be defined as follows (taking \( \theta_v \) as an example). If \( w_L > 0 \), values at the upper boundary are simply set equal to those in the DML as the base rises, i.e.
\[
(\theta_v)_{L-} = (\theta_v)_L. \tag{43}
\]
At the lower boundary, \( (\theta_v)_{X+} \) changes as entrainment by the SML erodes the base of the layer and in response to the effects of radiation and rainfall evaporation (layer-averaged values are used) so that
\[
(d\theta_v/dt)_{X+} = w_{eX}(d\theta_v/dz)_{X+} + (d\theta_v/dt)_{RAD} + (d\theta_v/dt)_{EVAP}. \tag{44}
\]
Other quantities may be predicted similarly.

If \( \dot{w}_L \leq 0 \), the value at the lower boundary is still given by Eq. (44). The upper value is then also defined by an identical equation, since entrainment is eroding the layer from the top, except that \( \dot{w}_L \) replaces \( \dot{w}_{eX} \) (the derivatives on the r.h.s. are assumed constant throughout the layer).

If there is no diabatic heating of the layer nor any entrainment at either boundary, then \( \theta_v \) remains unaltered at both boundaries and the stable layer is merely advected adiabatically by the large-scale motion.

(e) Reconnection of the DML and the SML

Eventually, the upward growth of the SML and the downward growth of the DML may consume the stably stratified region separating the two layers, leaving an inversion between them. The situation in which two turbulent layers are separated by a density interface has been investigated in laboratory experiments (see Turner 1973). These showed that turbulence was still strongly damped at the interface so that the two layers continued to coexist, but that the interface moved away from the more turbulent layer until the entrainment rates on the two sides balanced. By considering the conservation equation for \( \theta_v \), integrated across the interface as the depth of the interfacial region tends to zero, it can be shown that the fluxes must be discontinuous across the interface (cf. Eqs. (6) and (7)) and that

\[
\frac{dz_L}{dt} = \frac{dz_X}{dt} = \frac{\langle w \theta_v \rangle_L - \langle w \theta_v \rangle_X}{(\theta_{vL} - \theta_{vX})} - z_L D. \tag{45}
\]

Here, the subscripts L and X refer to values just above and just below the interface. Since

\[
\langle w \theta_v \rangle_L = w_L (\theta_{vL} - \theta_{vX}) \tag{46}
\]

and

\[
\langle w \theta_v \rangle_X = - w_{eX} (\theta_{vL} - \theta_{vX}) \tag{47}
\]

Eq. (45) may be written

\[
\frac{dz_L}{dt} = \frac{dz_X}{dt} = w_L + w_{eX} - z_L D. \tag{48}
\]

Equations (45) and (48) show that the fluxes are continuous across the interface only when the entrainment rates in each layer are exactly opposed, then in the absence of mean vertical motion \( (D = 0) \), the interface would be stationary.

If the storage correction terms are assumed to be negligible, since both layers on either side of the interface are turbulent,

\[
\langle w \theta_v \rangle_L = \langle w \theta_v \rangle_L^* \quad \text{and} \quad \langle w \theta_v \rangle_X = \langle w \theta_v \rangle_X^*. \tag{49}
\]

The fluxes \( \langle w \theta_v \rangle_{L,X}^* \) are determined by the same procedures which apply when \( z_L > z_X \) as discussed earlier, i.e. \( \langle w \theta_v \rangle_L^* \) is adjusted until Eq. (29) is satisfied and \( \langle w \theta_v \rangle_X^* \) is given by Eq. (38). Then \( w_L \) and \( w_{eX} \) can be found from Eqs. (46) and (47). The other fluxes at the interface are determined from similar expressions and the velocity of the interface is given by Eq. (48).

The DML and SML are treated as separate entities with transport between them until the virtual potential temperature difference at the interface, \( \Delta \theta_{vX} = (\theta_{vL} - \theta_{vX}) \), becomes sufficiently small that the two layers are assumed to reconnect and reform a single well-mixed layer. An expression for the minimum value of \( \Delta \theta_{vX} \) consistent with the maintenance of a separate DML and SML may be defined by considering the
deformation of the interface due to convective motions in either layer. If the interface consists of a sharp density step where no mixing occurs, a parcel with an initial velocity \( w_* \) directed towards the interface will be brought to rest in a distance \( d \), where

\[
d = \frac{\theta_* w_*^2}{2g \Delta \theta_{\text{in}}} = \frac{h}{2 \text{Ri}_x}. \tag{50}
\]

Here \( \text{Ri}_x = gh \Delta \theta_{\text{in}}/\theta_* w_*^2 \) and \( h \) is the depth of the appropriate mixed layer. However, the interface can be defined as such only if \( d/h < 1/f \) where \( f \gg 1 \), which (using Eq. (50)) implies

\[
\Delta \theta_{\text{in}} > f \theta_* w_*^2/2gh \quad \text{or} \quad \text{Ri}_x > f/2. \tag{51}
\]

The parameter \( f \) is therefore related to the minimum density difference between the separate layers below which it is unreasonable to expect them to remain distinct. If Eq. (51) is not satisfied in either layer, reconnection is deemed to occur. Varying \( f \) therefore has some effect on the time at which reconnection occurs, although the primary influence is still the thermodynamic evolution in the two layers. Choosing \( f = 5 \) gives satisfactory results (then \( \Delta \theta_{\text{in}} \sim 0.15 \) K if \( h \sim 500 \text{m} \) and \( w_* \sim 1 \text{ m s}^{-1} \)) and is used in the results presented below. Note that neither \( f \) nor \( f_c \) are strictly necessary for solutions to be obtained, but they provide a means for influencing the exchanges between the DML and SML. The reasons behind the choice of values and the sensitivity of the results to them are discussed in section 5(a).

Mixing between the layers is assumed to be sufficiently efficient to redistribute conservative quantities (whose total integrated values are conserved) back to a well-mixed configuration within a single timestep. For quantities which are significantly different in the two layers just prior to reconnection, the return to single mixed layer values implies the existence of very large flux gradients during the readjustment period (N.B. this is not true for the buoyancy flux, since \( \Delta \theta_{\text{in}} \) is necessarily small, which might otherwise prove energetically inconsistent). This is consistent with the entrainment rates (Eqs. (46) and (47)), and therefore the exchange between the layers, becoming very large as \( \Delta \theta_{\text{in}} \to 0 \). A rapid adjustment is therefore implied, although in reality it would probably take longer to render the resultant single layer homogeneous again.

4. Results

The model has been used to simulate the development of stratocumulus-topped boundary layers in two different climatic regions. One is representative of a mid-latitude ocean location and is based on aircraft observations made on 16 November 1983 off NW Scotland as reported by NL (flight 624). The other is characteristic of subtropical conditions found off the coast of California and is also based on aircraft data obtained on 27 June 1981 and reported by Hanson (1982, 1984a). For the radiation calculations, the aircraft measurements are supplemented by upper air data from nearby radiosondes. Many of the initial values are listed in Table 2. In both cases, the mixed layer is initially about 1 km deep with cloud occupying the upper half. In all the examples to be presented, the horizontal wind speed (\( V \)), horizontal divergence (\( D \)) and sea surface temperature (\( T_s \)) are held constant throughout the run since it is not intended to attempt to model cloud evolution along particular trajectories, but to study the diurnal variation. In the absence of better information, a value of \( D \) of \( 3.0 \times 10^{-6} \text{s}^{-1} \) was chosen to limit the rise of the mean cloud top over a diurnal cycle. The cloud microphysical parameters, \( N \) and \( \phi \), were matched to observations in the mid-latitude case (see NL) and the same values (\( N = 240 \text{cm}^{-3} \), \( \phi = 0.17 \)) were also used in the subtropical runs because similar
DIURNAL VARIATION OF STRATOCUMULUS

Figure 5. Diurnal variation of (top) cloud layer (shaded); (middle) cloud top entrainment velocity, $w_{ct}$, buoyancy integral, $I$ and $w_{ct}/w_{*}$; (bottom) $\theta_e$ and $q_r$. From the integration for mid-latitude stratocumulus in winter (using Eq. (10)).

observational data were not available. (However, these values are also typical of Californian stratocumulus, e.g. Noonkester (1984).)

In both cases, the surface buoyancy fluxes are small and positive, a common characteristic of both regions, but despite this the Monin–Obukhov length, $L_m$ (see appendix A), is limited to about a hundred metres because the relatively light winds over the ocean produce little stress. Since surface-shear-induced mixing is generally considered to become small at heights $z > -L_m$, the assumption that it may be neglected is valid for the cloud layer without even considering the much larger in-cloud buoyancy production. The possible relevance of shear production at lower levels is discussed in section 5(a).

In each example, the model is initialized with the appropriate single mixed layer structure at midnight and is run forward through one complete diurnal cycle using a timestep of 5 minutes, although the radiation calculations are only updated every 15 minutes. Values of $C = -0.4$, $f_e = 0.5$ and $f = 5$ have been used unless otherwise indicated.

(a) The mid-latitude case (56°N)

(i) Winter. Results from an integration with conditions appropriate to 1 January and with entrainment parametrized by Eq. (10) are shown in Fig. 5. Little diurnal variation is apparent in any of the quantities because of the weak solar flux. Decoupling of the cloud
layer is not diagnosed and a single mixed layer persists throughout the period. In the mixed layer, \( \theta_e \) increases slowly since the combined surface and cloud top entrainment fluxes into the layer (5.5 and 1.3 \( \text{K cm}^{-1} \text{s}^{-1} \) respectively) are greater than the net radiative loss (about 4.8 \( \text{K cm}^{-1} \text{s}^{-1} \)). The total water content, \( q_T \), remains fairly steady since the surface vapour flux (1.9 in units of \( 10^{-5} \text{m s}^{-1} \)) is balanced almost exactly over the period by losses due to entrainment (1.3 in the same units) and rainfall at the surface (0.6).

The use of different entrainment parametrizations does not lead to significantly different behaviour. Figure 6 shows that other methods produce a comparable diurnal variation, but predict less entrainment than Eq. (10), which makes a decoupled cloud layer even less likely.

(ii) \textit{Summer}. Figure 7 shows results from the same model (i.e. using Eq. (10)) and initial conditions except that the radiation calculations are now performed for 1 July. This time the model predicts a decoupled cloud layer, so to assess the consequences, the simulation was repeated but separation was not allowed to occur. The results are also shown in Fig. 7.

The variation in cloud thickness in the case without separation is again very small, similar to the winter case. The result of the enhanced short-wave absorption in the cloud is to drive the layer-integrated buoyancy production, \( I \), to zero around midday. When \( I = 0 \), no entrainment can be supported and \( w_{eU} \) must also become zero (Fig. 7). As NT have pointed out, this is unrealistic on at least two counts. Firstly, the assumption that a mixed layer can be maintained is incompatible with a layer-integrated production of zero, since there must be dissipation. Secondly, the buoyancy production term remains locally quite large and positive about 30 m below cloud top, even during the day, because
Figure 7. Diurnal variations in mid-latitude stratocumulus in summer when separation is (left) and is not (right) allowed. (Top) cloud layer (shaded) and stable layer (hatched); (middle) cloud top entrainment velocity, \( w_{eU} \), buoyancy integral, \( I \) and \( w_{eU}/w^* \); (bottom) \( \theta_e \) and \( q_T \). From model integrations using Eq. (10).
Figure 8. Profiles of (top left) $\theta$, (top right) $q_T$, (bottom left) $\overline{w\theta}$, and (bottom right) water substance fluxes, $wq$, $wq_1$, and $\overline{wT_1}$ at 10 h, from the integration for mid-latitude stratocumulus in summer (using Eq. (10)), after separation has occurred.
of the very strong localized radiative cooling from cloud top. With this source of TKE being maintained adjacent to the entrainment interface it seems unreasonable to expect entrainment to cease. Figure 7 shows that the essential difference which results from including separation is that the excessive diurnal variation of the entrainment rate is reduced and replaced by an enhanced variation in cloud thickness. Although an identical entrainment parametrization is used, neither \( w_{\text{el}} \) nor \( I \) fall to zero when decoupling is allowed, which would therefore appear to be a much more realistic approach.

When separation is allowed, the results are identical until decoupling is diagnosed at 07h, about 3h after sunrise. Note that the jump in \( w_{\text{el}}/w_{\text{e}} \) at this time is primarily due to the change in \( w_{\text{e}} \), which refers to the DML only after separation. Increasing short-wave absorption and continued entrainment result in the DML warming slightly while the total water content decreases steadily because the effects of entrainment and rainfall loss are no longer balanced by a vapour flux from below. The cloud and the DML therefore become shallower during the morning, leaving a slightly stable layer beneath, into which the SML grows by entrainment, as shown in Fig. 7.

Figure 8 shows some of the model-predicted profiles for 10h in more detail. The top panels show profiles of \( \theta_e \) and \( q_v \), illustrating the structure of the DML and SML and also the stable layer separating them. The bottom panels show profiles of the buoyancy and water substance fluxes. As the DML is thinning (i.e. \( w_{\text{el}} > 0 \)) the turbulent fluxes at level \( z_L \) are zero, although the SML is growing upwards by entrainment at level \( z_X \). All the rainfall evaporates above 230m, before reaching the surface, so the fluxes are nonlinear at this level. In the body of the cloud layer the fluxes \( w\theta_v \) and \( wq_v \) are both upward and are of similar size.

Eventually by 11h the short-wave heating within the DML is reduced sufficiently, due to the rapidly thinning cloud, for the DML to begin growing downwards by entrainment at its lower boundary, i.e. \( w_L < 0 \), this is further promoted during the afternoon as the sun goes down. The top panels in Fig. 9 show profiles of \( \theta_e, \theta_v \) and buoyancy flux \( w\theta_v \) at 12h while the DML is growing downwards. There is a small, negative buoyancy flux at \( z_L \) of 0.04 Kcm/s due to entrainment at this time, and the stable layer separating the DML and SML has almost disappeared. By 13h the stable layer separating the DML and SML is consumed and an inversion is formed as the layers come into contact. Transport across this interface moistens the DML and the cloud layer slowly thickens. The lower panels in Fig. 9 show profiles of \( \theta_e, \theta_v \) and \( w\theta_v \) at 18h, before reconnection, where the DML and the SML are separated by a weak inversion. Levels \( z_L \) and \( z_X \) are now coincident and are moving downwards because downward entrainment by the DML is greater than the upward growth of the SML, as reflected by the sign of the \( w\theta_v \) jump at this level.

Just after sunset, the difference in \( \theta_v \) between the DML and the SML becomes sufficiently small for reconnection to be diagnosed and a single well-mixed layer is re-established. This configuration is once again stable and the cloud continues to thicken slowly through the remainder of the evening as shown in Fig. 7.

The results presented above were obtained with entrainment modelled using Eq. (10); however, tests using alternative methods produce very similar solutions. If separation is not allowed, all the techniques predict zero or near zero values of \( w_{\text{el}} \) around noon, as shown in Fig. 10 (cf. Fig. 6 for the winter case). This, it has been argued, indicates that separation should have occurred. When this possibility is reintroduced, decoupling is diagnosed in every case regardless of which entrainment parametrization is used, although the precise details change. This is illustrated in Fig. 11 which shows results from simulations with identical conditions but different entrainment specifications. The run using Eq. (10), which gives the most entrainment, separates earliest, stays
Figure 9. Profiles of (left) $\theta_e$, (middle) $\theta_v$, and (right) $\overline{w\theta_v}$ at 12 h (top) and 18 h (bottom). From the integration for mid-latitude stratocumulus in summer (using Eq. (10)), after separation has occurred.
decoupled for the longest time and therefore predicts the thinnest cloud. The model with the Deardorff (1976) scheme decouples about two hours later and reconnects about two hours earlier. The Stage and Businger (1981) method predicts the least entrainment (see also NT) and does not separate until after 10 h. However, under certain conditions this method is incompatible with the way in which the evolution of the DML is formulated in that introducing a negative flux \((\overline{w \theta_v})_\tau\) at \(z_L\) can actually result in \(L_d/L_c\) becoming less, rather than more, negative. This occurs in this example and consequently solutions cannot be determined after 1145 h. When the Kraus and Schaller (1978) scheme is used \(w_{eU}\) falls to zero by 0745 h, at which time the ratio \(L_d/L_c\) is still positive (0.18). This occurs because the scheme is very sensitive to negative subcloud buoyancy fluxes, as shown in NT, and indicates that this scheme is incompatible with the diagnosis of separation made in this example.

If rainfall is neglected in the model this results in larger estimates for \(w_{eU}\). Despite this increased entrainment, separation is delayed by about three hours (when Eq. (10) is used to predict \(w_{eU}\)), thus illustrating the effect of evaporation of rainfall beneath the cloud in promoting decoupling.

(b) The subtropical case (29°N)

Similar investigations were carried out using the second set of conditions listed in Table 2 with summertime radiation (27 June). Results, again obtained using Eq. (10),
both with and without separation allowed, are shown in Fig. 12. These are qualitatively very similar to the mid-latitude summer behaviour, where including the decoupling process reduces the diurnal variation of $w_{eU}$, but enhances cloud thickness variations. Figure 12 also illustrates another important consequence of cloud layer separation which is that the thinner cloud during the day is much more transmissive at short wavelengths, but still optically thick at longer wavelengths. The net long-wave loss from the surface $(L_{\uparrow} - L_{\downarrow})_o$ therefore hardly changes, while the net short-wave flux received $-(S_{\uparrow} - S_{\downarrow})_o$ increases significantly. When averaged over a 24h period, the net surface energy input (radiation + sensible + latent heat) is changed from 7 W m$^{-2}$ to 102 W m$^{-2}$ when separation is considered, quite a large change, which could be important in regions where stratocumulus sheets are significant climatological features.
Figure 12. Diurnal variations in subtropical stratocumulus in summer when separation is (left) and is not (right) allowed. Top: cloud layer (shaded) and stable layer (hatched); middle: cloud top entrainment velocity, $w_{eu}$ and $w_{eu}/w_*$; bottom: surface short-wave, $(S - S \uparrow)_0$ and long-wave, $(L - L \downarrow)_0$, radiative fluxes (note the different scales). From model integrations using Eq. (10).
\begin{table}
\centering
\begin{tabular}{lcc}
\hline
Quantity & Mid-latitude & Subtropical \\
\hline
$z_U$ (m) & 1120 & 950 \\
$z_B$ (m) & 580 & 500 \\
$P_b$ (mb) & 955 & 954 \\
$\theta_c$ (K) & 294.6 & 313.0 \\
$q_T$ (g kg\(^{-1}\)) & 5.4 & 9.5 \\
$\Delta \theta_c$ (K) & 1.1 & -3.0 \\
$\Delta q_T$ (g kg\(^{-1}\)) & -3.5 & -7.1 \\
$\gamma_a$ (K km\(^{-1}\)) & 7.3 & 9.8 \\
$\gamma_B$ (g kg\(^{-1}\) km\(^{-1}\)) & 2.9 & -4.0 \\
$T_5$ (K) & 284.5 & 291.1 \\
$C_0$ & $1.3 \times 10^{-3}$ & $1.1 \times 10^{-3}$ \\
$C_a$ & $1.0 \times 10^{-3}$ & $1.1 \times 10^{-3}$ \\
$V$ (m s\(^{-1}\)) & 7.0 & 7.3 \\
$(w\theta)_a$ (m s\(^{-1}\) K) & $13.7 \times 10^{-3}$ & $4.4 \times 10^{-3}$ \\
$(w\theta)_B$ (m s\(^{-1}\)) & $1.9 \times 10^{-3}$ & $2.5 \times 10^{-5}$ \\
$L_m$ (m) & -68 & -119 \\
\hline
\end{tabular}
\end{table}

Short-wave absorption is also sufficient to cause decoupling in a winter (1 January) simulation, although the time at which separation occurs (0815 h) is later and the reconnection time (1630 h) earlier than the summer case. The relationship between cloud short-wave absorption ($\Delta S = (S \uparrow - S \downarrow)_B - (S \uparrow - S \downarrow)_U$) and decoupling is not straightforward because other factors are also acting simultaneously to modify the layer-integrated buoyancy flux. For example, their relationship is also dependent on $\xi$, the ratio (subcloud layer depth/mixed layer depth). Nevertheless, the relative importance of these processes can be gauged for a particular situation. Figure 13 shows how the ratio $I_s/I_c$ would change, for the conditions used to initialize the subtropical case, if the surface sensible heat flux and $\Delta S$ were varied ($\Delta S$ was altered by changing the solar zenith angle). Conditions lying to the right of the line labelled $-0.4$ have $I_s/I_c > C (= -0.4)$ and thus remain coupled, while to the left of the line $I_s/I_c < C$ where decoupling would be predicted. In the shaded area $I_c < 0$ and the model is inappropriate, although such conditions are unlikely to occur in reality. The figure shows that for this set of conditions ($\xi = 0.53$) a surface heat flux $> -4 \text{ W m}^{-2}$ is sufficient for a single well-mixed layer to be maintained at night (when $\Delta S = 0$), as seen in Fig. 12 where the initial surface heat flux was actually $5 \text{ W m}^{-2}$. These conditions correspond to the point $X$ in Fig. 13. As the sun rises and $\Delta S$ increases, the relevant point moves up the line as indicated (provided $(w\theta)_a$ remains unchanged) and decoupling would be predicted when $\Delta S > 25 \text{ W m}^{-2}$. In fact in the example shown in Fig. 12 decoupling occurred at 0630 h when $\Delta S \sim 15 \text{ W m}^{-2}$ because evolution during the intervening period ($\xi$ having increased to 0.62) has rendered the conditions represented by Fig. 13 no longer relevant. Figure 13 also illustrates the importance of small surface heat fluxes to the mechanism of cloud layer separation. The slope of the line labelled $-0.4$ shows that increasing the surface heat flux by a certain amount has approximately the same effect on determining whether or not the criterion for decoupling ($C < -0.4$) is satisfied as decreasing the short-wave absorption by three times that amount. With the maximum value of $\Delta S$ calculated to be $\sim 90 \text{ W m}^{-2}$ when the sun is directly overhead, decoupling would never be diagnosed for this set of conditions if the surface heat flux were to exceed $30 \text{ W m}^{-2}$. However, a flux of this
Figure 13. Buoyancy integral ratio $L/L_c$ as a function of surface heat flux $u_0$, and cloud radiative flux divergence, $R_u - R_v$ (or cloud short-wave absorption, $\delta S$), from model using Eq. (10) for subtropical case initial conditions. A prediction of decoupling based on the observed surface heat flux is also shown.

magnitude is quite large for a marine boundary layer, especially one containing stratocumulus.

While cloud layer decoupling is predicted to occur quite often, the consequential modification of the horizontally averaged vertical thermodynamic structure remains fairly small, e.g. the maximum differences between the DML and the SML in the example shown in Fig. 12 are $-0.5$ K ($\theta_v$) and $-1.7$ g kg$^{-1}$ ($q_T$). Detecting these differences observationally is hampered by sampling limitations, horizontal gradients and the diurnal variation. For example, the composite thermodynamic profile data from which the initial conditions for this example were derived (from Hanson 1984a) were obtained over a period of about 4 h and distances of several tens of kilometres. Data averaged in this way will appear ‘nearly well-mixed’ whether separation has occurred or not. A more detailed analysis of individual profiles and turbulence data is necessary to determine whether decoupling has taken place.

Features of the cloud layer diurnal variation seen in Fig. 12 are also in fairly good agreement with observations made from geostationary satellites. Figure 14 shows the results of an analysis by Minnis and Harrison (1984) of the diurnal variation of low-level, subtropical, stratiform cloud over a 250 km$^2$ region during summer. Hourly mean cloud amount and cloud top temperature were derived using a bispectral technique from
Figure 14. Hourly average low cloud (<2 km) amount (-----) and cloud top effective black-body temperature (----) in the eastern Pacific derived from GOES-E data during November 1978 (from Minnis and Harrison 1984).

GOES-E visible and infrared data. The results in Fig. 14 are a composite for November 1978 at a location off the west coast of S America (21°S 86°W) although typical of a much larger area. The strongest diurnal variation across the whole GOES region was observed in these low-level clouds, with the maximum amount occurring around sunrise associated with the coldest tops and a strong decrease during the late morning to minimum cover during mid-afternoon. The model does not predict cloud amount, but if, as seems reasonable, the thickness of the modelled cloud is associated with some measure of cloud amount over a wider region, a comparison of Figs. 12 and 14 shows that they are correctly in phase.

The modelled effective black-body temperature of cloud top varies by ±0.45 K with a minimum at 08 h and a maximum at 19 h. The observed variation is similar during the night, but increases by about 2 K during the day to a maximum in mid-afternoon which the model does not reproduce. Since the modelled cloud remains optically thick in the infrared, modelling such a variation would require the cloud top temperature to vary by the same amount. This could be achieved only if there were either a much larger diurnal variation of the entrainment rate, which seems unlikely given the constraints on the energetics in the model, or a diurnally varying subsidence velocity which increases by ~1 cm s⁻¹ during the daytime. However, a more likely explanation is that the cloud becomes thin or broken on sub-pixel scales which cannot be resolved in the satellite data. This would lead to increased radiances and hence higher effective temperatures and is consistent with the maximum effective cloud top temperature occurring at the same time as both the minimum observed cloud amount and the minimum modelled cloud thickness.
This avoids the need to postulate other large diurnal variations to account for a significant actual cloud top temperature increase in the afternoon.

In either case, the larger diurnal variation in cloud thickness predicted by the model when separation is allowed is in better agreement with the observations.

5. Discussion

(a) Sensitivity to choice of model parameters

A central question for this model is the value assigned to the parameter \( C \) in Eq. (26) which essentially determines the maximum sustainable rate of working allowed against negative buoyancy. As discussed in section 3, this must lie in the range \(-1 < C < 0\), but other factors reduce this choice further.

One constraint is provided by the requirement that the initial conditions listed in Table 2 should be consistent with a single mixed layer configuration at night. This was confirmed by observations in the mid-latitude case, and implies \( C < -0.17 \). For the subtropical example \( C < -0.25 \), although observational evidence that a single well-mixed layer existed is not available. Other data imply similar values.

As \( C \) is increased, decoupling is diagnosed progressively later and lasts for a shorter period, as illustrated in Fig. 15. Eventually, with \( C \) sufficiently negative, decoupling is not diagnosed at all and the effect is identical with disallowing separation, as seen in Fig. 7. However, in this situation, \( w_{eu} \) becomes zero during the day, which it has been argued is unrealistic. A lower bound on \( C \) is therefore implied if \( w_{eu} \) is to remain positive at all times. This value depends on the entrainment closure method selected. For the Deardoff (1976) and Eq. (10) closures, this is achieved as long as \( C > -1 \), but the other methods imply less negative values which depend on the particular situation.

In the absence of a better understanding of the TKE balance under these conditions, it is not possible to estimate \( C \) more rigorously and a value of \( C = -0.4 \) was selected as the best compromise which satisfied all these various constraints in most simulations. However, it should be noted that the results are as sensitive to the way in which entrainment is specified (e.g. Fig. 11) or to variations in the boundary conditions as to changes in \( C \) (e.g. Fig. 15).

Observational studies of decoupled mixed layers reported in NL imply values of \( C \) which are rather less negative (\( C > -0.35 \)). Although this might imply deficiencies in the criteria used to determine the onset of separation in the model, it is unlikely that these values are directly comparable. The model imposes a mixed layer solution as an approximation to the observed structure, subject to constraints on the layer-integrated energetics, while in the observational results, the mixed layer boundaries were diagnosed directly from the measurements.

Once \( C \) has been fixed, the time when the DML and SML reconnect depends, to some extent, on the values chosen for \( f \) and \( f_e \). The value for \( f \) in Eq. (51) determines the minimum density difference between the two layers below which they cannot be regarded as separate. The value of \( f_e \) in Eq. (29) influences the rate of exchange between the layers (and hence the thickening of the cloud layer during the afternoon) and therefore the rate at which the density difference is reduced. The possible ranges of values for \( f \) and \( f_e \) are limited and these constraints are related to the value of \( C \). If reconnection occurs too early (\( f, f_e \) too large), the ratio \( I_e/I_c \) immediately after reconnection may still be less than \( C \). This results in repeated separation and reconnection on subsequent timesteps until \( I_e/I_c > C \), which is unrealistic. If reconnection occurs too late (\( f, f_e \) too small), then on re-establishing the single layer \( I_e/I_c \) may be much greater than \( C \). This
results in a sudden change in the dynamics of the layer and implies that separation should have been diagnosed sooner. Within these constraints, the solutions are not particularly sensitive to the actual choice of these parameters. For example varying \( f_e \) between 0.5 and 1 has a smaller effect on the results than does changing \( C \) from −0.4 to −0.5 (as was shown in Fig. 15). In most of the simulations, \( f = 5 \) and \( f_e = 0.5 \) represent a reasonable compromise and avoid the above objections.

Although particular solutions are sensitive to the values chosen for \( C, f_e \) and \( f \) as indicated above, it should be noted that the main aim of the paper is not to produce a definitive forecast in particular situations (where the accurate specification of the boundary conditions is just as important to consider), but to investigate the character of the general class of solutions which are obtained in a variety of conditions when an energetically consistent model is used. While the details of the results may vary if these parameters are altered within the limits suggested, the general behaviour does not.
(b) Entrainment instability

If air being entrained at cloud top is sufficiently dry, the subsequent evaporative cooling can cause the resulting mixture to be negatively buoyant under certain conditions ($\Delta_2 < 0$; Randall (1980); Deardorff (1980)). It has been suggested that this would lead to a rapid breakup of the cloud layer because of a positive feedback mechanism between the production of buoyancy at cloud top and the entrainment rate. More recently, a number of arguments have been put forward which suggest that the criterion $\Delta_2 < 0$ is not sufficient to predict the onset of such an instability (Mahrt and Paumier 1982; Hanson 1984b; Randall 1984; NT). In the context of mixed layer models, Stage and Businger (1981) pointed out that if $\Delta_2$ is only slightly negative, mixing entrained air throughout the whole layer may consume more energy than it produces. They showed this to be the case if $\Delta_2 > \Delta_3$ where

$$\Delta_3 = -\xi^2 \Delta_1/(1 - \xi^2)$$

$$\Delta_1 = \Delta \theta_{eU} - (L_v/c_p - \psi') \Delta q_{TU}. \quad (52)$$

$$\Delta_1 = \Delta \theta_{eU} - (L_v/c_p - \psi') \Delta q_{TU}. \quad (53)$$

If $\Delta_2 > \Delta_3$, an increase in $\theta_{eU}$ causes the layer-integrated buoyancy production, $I$, to decrease. While the entrainment parametrization in use relates $\theta_{eU}$ to $I$ (or an equivalent), a solution for $\theta_{eU}$ can still be found by the methods described earlier and the model continues to function normally. A solution is unobtainable only if $\Delta_2 \leq \Delta_3$ or if the entrainment parametrization is expressed in an unsuitable form (e.g. $\theta_{eU} \propto I/\Delta_2$). In the case of a vanishingly thin cloud layer, $\xi \rightarrow 1$ and $\Delta_3 \rightarrow -\infty$ so solutions are always possible. Only when cloud fills the entire mixed layer and $\xi \rightarrow 0$ and $\Delta_3 \rightarrow 0$ can solutions not be found if $\Delta_2 < 0$.

Conditions where $\Delta_3 < \Delta_2 < 0$ were maintained during most of the subtropical example. Figure 16 shows that $\Delta_2 < 0$ after 01 h but never falls below $\Delta_3$. During this period, entrainment results in $\theta_{eU}$ at cloud top being positive, which therefore makes a contribution to the production of TKE in the cloud layer. However, this and therefore $I$, $\theta_{eU}$ and other related quantities, are affected to a much greater extent by the variation of other processes, e.g. radiative effects and changing cloud or subcloud layer depth. The entrainment rate, $\theta_{eU}$, is related to a number of quantities, including $\theta_{e}$ and

![Figure 16](image_url)

Figure 16. Diurnal variation of cloud top jump-related values $\Delta_2$ and $\Delta_3$, from the integration for subtropical stratocumulus in summer (using Eq. (10)) when separation is allowed.
inversion parameters which respond to the sum of all the processes included in the model. Thus although $\Delta_2 < 0$ after 01 h and decreases until 15 h in the example shown in Fig. 16, this is not reflected in the variation of $w_{eU}$ (see Fig. 12, separation allowed) and provides no indication of its value.

If $\Delta_2 \lesssim \Delta_3$, it is implied that $w_{eU}$ will increase rapidly until presumably being limited by TKE storage requirements. This may be associated with entrainment instability, although in such conditions it is likely that many of the model assumptions, negligible vertical gradients in mixed layers, well-defined interfaces between layers and horizontal uniformity, will break down and that a different type of model would be required.

(c) Other comparisons with observations

One of the main reasons for developing this model was to provide an energetically consistent picture of the way in which individual physical processes might combine to affect the structure and dynamics of stratocumulus layers and thereby provide a unified description of the various features observed on different research flights. In this respect, the model appears to be successful. Although the details of individual cases reflect particular combinations of conditions and previous history, the overall behaviour of the model is broadly consistent with the observations. At mid-latitude locations, the tendency for more active (in terms of $w_e$), single mixed layers to exist during the winter or at night, while less active, decoupled cloud layers are commonly observed during summer days is apparent in the observations (NL).

The model is also capable of reproducing a type of boundary layer structure which is very common around the United Kingdom. If the large-scale subsidence remains near zero, on average, over the course of a few days, as might happen along a trajectory around the edges of an anticyclone, the cloud top will continue to rise due to entrainment driven by cloud-related processes largely independent of the surface fluxes. This is illustrated in Fig. 17, which shows the predicted evolution over a period of three summer days from the mid-latitude initial conditions with the horizontal divergence $D$ (and the above-cloud lapse rates $\gamma_B$ and $\gamma_D$) set to zero. After about 30 h, the changing geometry of the layers results in the cloud layer becoming more or less permanently decoupled by

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**Figure 17.** (Left): mixed layer evolution over three days, the cloud layer is shaded, stable layers are hatched. (Right): predicted $\theta_e$ and $q_r$ profiles at 15 h on day 3. From the integration for mid-latitude stratocumulus in summer (using Eq. (10)) with zero subsidence and with zero lapse rates above cloud top.
an extensive stable layer. Although the cloud layer becomes progressively drier as it rises, adiabatic cooling means that the cloud thins only slowly and therefore entrainment continues. By the afternoon of day three, the double mixed layer structure is clearly evident, as also shown in Fig. 17, with differences in $\theta_v$ and $q_v$ of about 2 K and 1 g kg$^{-1}$ across the 775 m-thick intervening stable layer. Structures of this type have been reported by Nicholls et al. (1983), Taylor et al. (1983) and NL. Although the above description is not the only way that this kind of structure could be produced, it does seem a likely candidate given the prevalence of low-level layer cloud in this area (Nicholls 1984).

Other points highlighted by this example are:

(i) Large-scale subsidence is an important factor in maintaining a cloudy boundary layer in a well-mixed state since it restricts the layer depth.

(ii) The initial formation and subsequent variation of the DML depth is largely governed by the absorption of incident solar radiation, as seen in Fig. 17. However, this diurnal restratification is also modulated on a seasonal timescale, as indicated by the summer/winter results discussed earlier. In this respect, the response of the DML to solar input has many similarities to the response of the upper layers of the ocean to the same quantity (Barkmann and Woods 1986).

(iii) The existence of a pronounced diurnal variation emphasizes the importance of a properly designed sampling strategy in any field experiment. Methods which do not resolve this structure are liable to misinterpret the nature of the transport process.

(d) Model limitations

Although the model has been shown to simulate many of the observed features of the CTBL, the two main limitations (apart from uncertainties associated with parameterizations) are the assumption that vertical gradients of conserved variables are negligible and the implied horizontal uniformity, i.e. small-scale inhomogeneities are not considered. The approach taken here assumes that the decoupling process can be adequately described by introducing two well-mixed layers, in which the assumptions are separately valid, rather than one poorly-mixed layer. While this offers a consistent framework and has a base in observational fact, the extent to which this description remains accurate is unknown. This must await further observational and modelling studies. Perhaps the situation where this description is least likely to pertain occurs during the afternoon, before reconnection, when the DML and SML are in contact but are assumed to stay separate although the density difference between them is small. A single poorly-mixed layer encompassing both the DML and the SML with limited transport between them might then be more realistic, although the current model does permit limited transport and could well be an adequate description.

The assumption that shear generation of TKE is negligible is also a limitation. If it was sufficiently large to sustain significantly larger losses against negative buoyancy, decoupling might be prevented, but in the conditions modelled here, the assumption should be a good one in the DML for the reasons given in section 2(a). However, in moderate winds, $z \sim -L_m$ (see Table 2) so mechanical production is likely to remain an important factor in the SML. As argued by NL, this might be expected to lead to larger vertical gradients in $\theta$ and $q$ within the SML than in a convectively dominated situation with the same boundary heat and vapour fluxes. Even in lighter wind conditions where shear is negligible, the ratio $w_{x+}/w_*$ is usually found to be quite large ($\sim 2\%$) in the SML and this relatively large entrainment could also cause significant vertical
gradients. In either case, negligible vertical gradients are less likely within the SML than within the DML, but because precise modelling of the SML is not a central concern of this paper, the simpler description is retained. Nevertheless, these effects would have to be accounted for if a model with predictive capability was required; and the SML model would also need the capability of dealing with stable surface stratification, a common feature beneath stratocumulus when relatively warm air blows over a cooler ocean surface.

The assumption that conditions are locally horizontally uniform means that the model can describe only complete or zero cloud cover. This becomes progressively less realistic as the thickness of the cloud decreases since observations indicate that convective motions may produce thickness variations on kilometre scales which are of the same order as the total cloud depth. Once the cloud is less than, say, ~100 m deep, the assumption of horizontal homogeneity becomes difficult to justify and the cloud must probably be regarded as broken.

Both of these limitations are again exposed if the variation of the relative humidity at the top of the SML (i.e. at level $z_{x-}$) in the mid-latitude summer case is plotted (Fig. 18). The variation reflects the changing depth of the SML in which $q_T$ increases steadily (cf. Fig. 7) and $\theta$ is nearly constant. The maximum value, which is greater than 100%, occurs in the early afternoon. Although a peak value < 100% could have been arranged in this example merely by reducing the surface vapour flux, there are two possible interpretations.

(i) That cumulus should form within the SML and rise into the overlying strato-
    cumulus layer. Once the lifting condensation level lies within the SML, the layers above
    are unconditionally unstable to moist ascent since $d\theta/dz < 0$ (see, e.g., Fig. 9).

(ii) That $\theta$ and $q$ should not be evenly vertically distributed within the SML, but
    have significant gradients as suggested above. Only small values would be necessary to
    reduce the mean relative humidity at $z_{x-}$ below 100% (e.g. in the mid-latitude case
    $d\theta/dz \approx 1.5 \text{ K km}^{-1}$ or $dq/dz = -0.4 \text{ g kg}^{-1} \text{ km}^{-1}$) and the compensating changes at the
    surface would also act to reduce the surface fluxes.

Neither explanation can be implemented within the model since cumulus is essentially horizontally non-uniform and vertical gradients are not allowed. However, because no action is taken to augment transport between the SML and the DML when this condition occurs, it is tantamount to selecting option (ii) above. To include a proper representation of either process would require a more sophisticated model.

Figure 18. Diurnal variation of the relative humidity at the top of the SML, from the integration for mid-
latitude stratocumulus in summer (using Eq. (10)).
6. Conclusions

This paper has considered how the dynamics of an unbroken sheet of stratocumulus may be described by a mixed layer model. Including a description of the cloud layer microphysical properties enables detailed, interactive long- and short-wave radiation calculations to be made from which the local heating and cooling rates can be determined and also enables the effects of droplet gravitational settling to be crudely parametrized. In contrast to all previous models of this type, a further constraint is placed upon the buoyancy flux beneath cloud base to limit the amount of work which can be done against negative buoyancy. Once a threshold value is reached, the layer which was previously well mixed from the surface to cloud top is allowed to separate into two independently driven layers, as suggested by observational studies. The evolution of the two separate layers is followed independently although there may subsequently be transport between them and they may eventually recombine to reform a single layer. The addition of this extra constraint and the accompanying decoupling process enable the results to remain energetically consistent throughout a complete diurnal cycle. The model therefore retains much of the simplicity of the mixed layer approach without compromising energy balance constraints and leads to a different class of solutions characterized by an internal layered structure.

The inclusion of separation is shown to lead to a stronger diurnal variation in the cloud structure, with the cloud thinning during the morning after decoupling and thickening later in the afternoon. The character of the results is essentially independent of the choice of entrainment closure or of the values attached to model parameters, although the precise details of the solutions do vary. These results show similar features to those obtained using a higher-order closure model (Bougeault 1985) and are also consistent with satellite observations of the diurnal variation of low-level stratiform clouds (Minnis and Harrison 1984). The model is capable of reproducing features similar to those observed on a series of research flights (NL) and provides a consistent framework within which these and other results may be interpreted.

When separation is not allowed, the cloud thickness varies only very slowly and the main diurnal variation is transferred to the entrainment velocity, which may become small or zero around noon. This, it is argued, is physically unrealistic and suggests that separation ought to have occurred.

In a given situation, the tendency for the layers to become decoupled is shown to be promoted by significant short-wave absorption by the cloud and by small surface buoyancy fluxes. This implies a seasonal dependence, decoupling being more likely in summer than in winter at mid-latitudes, although the sun may remain sufficiently high in the subtropics for it to remain a possibility at any time of year. However, no simple, universal criterion exists. The likelihood of separation is also related to the geometry of the mixed layer (as expressed by the ratio $\xi$) which is as strongly influenced by the large-scale divergence as it is by the local conditions.

The onset of separation, which is essentially a restratification process, may have a number of consequences for mixing within the cloud-topped boundary layer. The enhanced variations in cloud thickness can radically alter the surface energy balance suggesting that it is important to resolve the diurnal variation, especially in areas where stratocumulus is a dominant climatological feature.

The model has two main limitations: the inability to deal with broken cloud and the implicit assumption that the effects of shear-generated mixing may be neglected in comparison with those produced by convection. While observations show that this is often a reasonable approximation in mid-latitude marine boundary layers, the degree to
which this remains true in other areas awaits the results of further observational and theoretical work.

APPENDIX A

Notation: Symbols not explicitly defined in the text

- $c_p$: specific heat of air
- $g$: acceleration due to gravity
- $L_m$: Monin–Obukhov length scale ($= -\theta_*u_*^3/gk(\overline{w\theta}_*)$, where $k$ is von Kármán’s constant, assumed = 0.4)
- $L_e$: latent heat of evaporation
- $L \uparrow$, $L \downarrow$: upward and downward long-wave fluxes
- $q$: specific humidity
- $q_l$: specific liquid water content
- $q_T$: total water content ($= q + q_l$)
- $S \uparrow$, $S \downarrow$: upward and downward short-wave fluxes
- $u_*$: surface friction velocity
- $V$: horizontal 10 m wind speed
- $w_*$: convective velocity scale ($= \{2.5(g/\theta_*) \int_{z_L}^{z_U} \overline{w\theta}_* dz\}^{1/3}$)
- $z$: height
- $\beta$: a slowly varying function of temperature (see N84, Eq. (27))
- $\psi$: ratio of molecular weights of dry air and water vapour, minus one ($= 0.61$)
- $\theta$: potential temperature
- $\theta_e$: equivalent potential temperature ($= \theta + L_e q/c_p$)
- $\gamma_{e*}$, $\gamma_{qT}$: lapse rates of $\theta_e$ and $q_T$ above the mixed layer.

Subscripts denoting model levels U, L, X, etc. may also be qualified by + or − if there is a discontinuity in the value at that level, e.g. $(q_T)_U^+$ is the value just above cloud top and $(q_T)_U^−$ the value just below, i.e. within cloud (see Fig. 1).

APPENDIX B

Computational procedure

A summary of the main computational procedure is outlined briefly below and a condensed summary of the main sequence is illustrated in the flow chart in Fig. B1. Numbers in brackets refer to equations in the main text.

(i) The mixed layer boundaries $z_L$, $z_0$ ($z_L$, $z_X$ if appropriate), $\theta_e$ and $q_T$ within the mixed layer(s) are known. Cloud base $z_B$, cloud liquid water content and the cloud microphysical parameters including the drizzle fluxes can then be diagnosed (16, 17). The radiative heating rates may then be calculated.

(ii) Single mixed layer, $z_L = 0$. $(\overline{w\theta})_0$, $(\overline{wq})_0$ given by (14), (15). $w_eU$ can then be determined using an entrainment parametrization scheme, e.g. (10). $(\overline{w\theta}_e)_U$, $(\overline{wq_T})_U$ follow from (6), (7). The buoyancy integral ratio $I_e/I_e$ may then be determined. If $I_e/I_c \geq C$ a single mixed layer configuration is permitted. Then $dz_U/dt$ is given by (4), $dz_L/dt = 0$. Go to (vi). If $I_e/I_c < C$ then separation is implied, (iii).

(iii) Separated layers. If at separation, or $I_e/I_c < C$ in a DML, then $z_L$ is not known. Set $(\overline{w\theta})_L$, $(\overline{wq})_L = 0$ and assuming a value for $z_L$ ($0 < z_L < z_B$) find $w_eU$ (10) and hence
Figure B1. A simplified flow chart of the main computational procedure. Numbers in brackets refer to equations in the main text. The abbreviation (FLX) is used as follows: (FLX)\(L\) = \((w\theta)_L\), (w\(q\))\(L\); (FLX)\(x\) = \((w\theta)_x\), (w\(q\))\(x\); (FLX)\(L,x\) = \((w\theta)_L,x\), (w\(q\))\(L,x\).
Calculate $I_u/I_c$. Using an iterative procedure, successively adjust $z_L$ until $I_u/I_c = C'$ (28). Then $w_L$ is given directly by the change in $z_L$ since the last timestep, or is zero if separation has just occurred. $dz_U/dt$, $dz_L/dt$ are given by (4), (5) respectively and values at $z_{L-}$ by (43) and similar. Go to (iv).

If $I_u/I_c \geq C$ in a DML then guess an initial value for $(\overline{w\theta})_L^*$. Find $w_L$ and hence the fluxes at $z_L$ using (49, 46 and similar) if $z_L = z_X$ and storage terms are neglected or (31, 33, 34 and similar) if $z_L > z_X$ where storage is included. The entrainment velocity and fluxes at $z_U$ are given by (10, 6, 7) as before. Iteratively adjust $(\overline{w\theta}_L)_L^*$ until $I_u/I_c$ has the value given by (29), thus defining $w_L$, $w_{eU}$ and the boundary fluxes. Note that a minimum value for $w_L$ is set by the encroachment value (36). $dz_U/dt$ is given by (4), $dz_L/dt$ by (48) if the DML and SML are in contact ($z_L = z_X$) or by (5) if there is an intervening stable layer ($z_L > z_X$) when $d(\theta_e)_L/dt$ etc. are given by equations similar to (44). Go to (iv).

(iv) Surface mixed layer. Surface fluxes are given by (14, 15). If $z_L > z_X$, storage terms are included, $w_{eX}$ is given by (41), the fluxes at level $z_X$ by (40) and similar, $dz_X/dt$ by (37) and the rates of change of parameters at level $z_X$ by (44) and similar. Go to (iii) via (vi). If $z_L = z_X$, then $(\overline{w\theta}_L)_L^* = (\overline{w\theta}_L)_X^*$ given by (38), $w_{eX}$ is found from (47) and the other fluxes from similar expressions using $w_{eX}$. $dz_X/dt$ is given by (48). Go to (v).

(v) Reconnection. If the density difference between the DML and SML when they are in contact is so small that (51) is not satisfied in either layer, combine the layers conserving layer-integrated quantities and return to (ii) via (vi). Otherwise return to (iii) via (vi).

(vi) Update $\theta_e$, $q_T$ in (each) mixed layer together with the boundary levels $z_U$, $z_L$, $z_X$ (where appropriate) using the known rates of change (1, 2, 4, 5, 37, 48), new intermediate values (where appropriate), e.g. $(\theta_e)_L^*$, $(\theta_e)_X^*$ (44), and new above-cloud values (18–23). This is referred to as ‘update parameters’ in Fig. 19. New cloud base, cloud physics, radiation calculations are then performed as at (i).

1 Since $\Delta\theta_{a3}$ is usually small, a correction term $-w_L\delta(\overline{w\theta}_L/dt)_{a3}/2$ is added to it in (31–33) to take account of changes occurring during a timestep $\delta t$. A similar correction is also made to $\Delta\theta_{a3}$ in (38–40).

REFERENCES


Dearorff, J. W. 1976 On the entrainment rate of a stratocumulus-topped mixed layer. ibid., 102, 563–582


Hanson, H. P. and Gruber, P. L.


Kraus, H. and Schaller, E.


Kraus, E. B. and Turner, J. S.

1967 A one-dimensional model of the seasonal thermocline. II: The general theory and its consequences. *ibid.*, 19, 98–106

Levin, L. M.


Lilly, D. K.


Mahrt, L. and Paumier, J.

1982 Cloudtop entrainment instability observed in AMTEX. *J. Atmos. Sci.*, 39, 622–634

Minnis, P. and Harrison, E. F.


Moeng, C-H.

1984 A large eddy simulation model for the study of planetary boundary layer turbulence. *J. Atmos. Sci.*, 41, 1588–1600

Nicholls, S.


Nicholls, S. and Leighton, J.


Nicholls, S. and Turton, J. D.


Nicholls, S., Brummer, B., Fiedler, F., Grant, A., Hauf, T., Jenkins, G., Readings, C. and Shaw, W.


Nieuwstadt, F. T. M. and Businger, J. A.


Noonkester, V. Ray


Randall, D. A.

1980 Conditional instability of the first kind upside down. *ibid.*, 37, 125–130


Roach, W. T. and Slingo, A.


Schaller, E. and Kraus, H.


Slingo, A. and Schrecker, H. M.


Slingo, A., Nicholls, S. and Schmetz, J.

1982 Aircraft observations of marine stratocumulus during JASIN. *ibid.*, 108, 833–856

Stage, S. A. and Businger, J. A.


Sauer, M. J., Arakawa, A. and Randall, D. A.


Taylor, P. K., Grant, A. L., Gunther, H. and Olbruck, G.


Turner, J. S.

1973 Buoyancy effects in fluids. Cambridge University Press

Zilitinkevich, S. S.