Relevance of conditional symmetric instability in the interpretation of wide cold frontal rainbands. A case study: 20 May 1976

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SUMMARY

Comparisons are made between the characteristics of rainbands observed in a frontal system using a C-band Doppler radar and dynamical mechanisms relevant on the mesoscale (~100 km).

The orientation of these rainbands, nearly aligned with the thermal wind, suggests that symmetric instability may play a role in their formation. This possible explanation of the precipitation is discussed.

1. INTRODUCTION

It is well known that regions of heavy precipitation in frontal systems are often organized on the mesoscale in the form of rainbands (Elliott and Hovind 1964, 1965; Nozumi and Arakawa 1968; Browning and Harrold 1969; Kreitzburg and Brown 1970; Houze et al. 1976; Hobbs 1978 Hobbs et al. 1980; Matejka et al. 1980; Herzegh and Hobbs 1980, 1981; Houze et al. 1981; Hobbs and Persson 1982; Wang et al. 1983; Parsons and Hobbs 1983a, b, c; Rutledge and Hobbs 1983). A great variety of physical mechanisms has been proposed to explain these bands, including ducted gravity waves (Lindzen and Tung 1976), conditional symmetric instability (Bennetts and Hoskins 1979; Emanuel 1983a, b) and symmetric wave CISK in a baroclinic flow (Emanuel 1982).

This paper presents a case study on the mechanisms relevant on the mesoscale (~100 km) which may play a role in the formation of these rainbands.

The rainbands that provide the observational basis for this paper were associated with a cold front which moved east-south-eaeward over France during 20 May 1976.

Several aspects of this case have been brought out by Testud et al. (1980). They describe the three-dimensional circulation of air in the vicinicy of the front as observed from a C-band Doppler radar. They used a modified VAD analysis (Browning and Wexler 1968) based on a least-square-fitting numerical method for data analysis to provide this description of the wind before, during and after the frontal passage.

In this paper, comparisons between these observations and the theories proposed to explain the initiation of mesoscale precipitation are presented. After a brief description of the precipitation deduced from the Testud et al. analysis, the observed properties of the rainbands are compared with the characteristics predicted by the theories.

2. MESOSCALE ORGANIZATION OF THE PRECIPITATION

The observed reflectivity field (Fig. 1) is organized in three parallel bands 60 km apart. Using the classification of rainbands proposed by Houze et al. (1976), Hobbs (1978) and Matejka et al. (1980), these rainbands correspond to the type labelled "wide cold frontal rainbands (U3b)". These bands described as "approximately 50 km in width oriented parallel to the cold front" occur where lifting above the cold front is enhanced to several tens of cm s⁻¹. The observed rainbands differ from this description by a slight
Figure 1. Isopleths of radar reflectivity at the level \( h = 1500 \) m observed from the sequence at 1657 GMT. The position of the cold front observed at the surface and the location of the VAD (⊙ at 1430, 1720, 1840 GMT) and of the radiosounding (RS, at 12 GMT) are indicated. The mean wavelength between the rainbands is about 60 km. Dashed lines (marked 1, 2, 3) show the shear vector direction for the different VAD.

Figure 2. Trappes sounding at 12 GMT 20 May 1976. Solid line shows temperature; thick dashed line, dewpoint temperature. Thin dashed lines are pseudo-moist adiabats. The atmosphere appears saturated and nearly neutral or stable for vertical ascent.
but significant angle of $25 \pm 15^\circ$ between the bands and the cold front. In fact the orientation of these rainbands is determined (as shown in section 4) by the direction of the winds, which are nearly in geostrophic balance (see section 4(c) of Testud et al. (1980)). The region of saturation which produced the entire envelope of the bands appears as the result of frontogenetic forcing (see Fig. 7 from Testud et al.).

The motion of the bands perpendicular to their main axis appears close to that of the front with a propagation speed of $8 \text{ m s}^{-1}$ toward the south-east. Indeed, the reflectivity pattern is stationary in the frame moving with the front from 16 GMT to 19 GMT (Fig. 13 from Testud et al.). Moreover, the cross-front velocity null in the north-eastward warm air flux indicates that these bands travel with the mean flow in which they are embedded. This warm air flux appears saturated and nearly convectively neutral (see Fig. 2).

3. Comparisons between observations and theories

Several mechanisms have been proposed to account for the generation of this kind of rainband.

Bennetts and Hoskins (1979) and Emanuel (1983a, b) have suggested symmetric instability in saturated air as an explanation for band organization. This instability manifests itself as helical roll perturbations with axes parallel to the thermal wind. The latent heat released within the rising air, in the saturated frontal region associated with the lifting above the cold frontal surface, assists the symmetric instability (called conditional symmetric instability or moist symmetric instability or slantwise convection) which develops when the vorticity on a wet-bulb potential temperature surface becomes negative. This mechanism leads to the generation of regions of conditional inertial instability along moist isentropic surfaces where previously there was none, and to convective rainbands which travel with the mean flow in which they are embedded.

Emanuel (1982) proposed another mechanism, called symmetric baroclinic wave CISK where the symmetric destabilization is assisted by the release of convectively available potential energy, which means this instability can grow only in regions that are already convectively unstable. He shows that the associated rainbands would propagate relative to the mean flow.

Lindzen and Tung (1976) consider that these rainbands are the result of ducted mesoscale gravity waves. They show that a stable moist layer can act as a duct for gravity waves provided there is (i), a stable layer beneath it to prevent interaction with the ground surface; (ii), a potentially unstable layer above it to act as a reflector; and (iii), a flow speed close to the ducted mode speed. The associated rainbands move slowly relative to the winds.

The non-existence of a potentially unstable layer (which is necessary to duct the gravity waves and to feed the vertical convection) on the soundings taken just before (see Figs. 1 and 2) and well after (not shown) the cold front passage, and moreover the fact that the relative phase speed of the rainbands (or the relative horizontal phase velocity of the waves) is very small, tend to eliminate the Lindzen and Tung process as an explanation of the presently observed rainbands.

On the other hand, the characteristics of these rainbands are consistent with those expected from moist symmetric instability theory. The bands of precipitation are oriented approximately parallel to the vertical shear vector (see section 4); they grow in saturated regions that are nearly convectively neutral or stable (see Fig. 2); they move with the mean speed of the environment in which they are embedded; and the observed spacing has a mesoscale character ($\sim 50 \text{ km}$).
4. MOIST SYMMETRIC INSTABILITY—A POSSIBLE INTERPRETATION

To assess the relevance of the CSI (conditional symmetric instability) theory to the occurrence of these rainbands, the theoretical growth of the instability must be evaluated. Following Bennetts and Hoskins (1979), this growth rate for a small amplitude disturbance is given by

\[ \sigma^2 = -\frac{q_w \theta_o}{g(\partial \theta_w / \partial Z)} \]  

(1)

where all quantities refer to the mean state of the flow; \( \partial \theta_w / \partial Z \) is the lapse rate of the wet-bulb potential temperature; \( q_w \) is the vorticity on a wet-bulb potential temperature surface, defined by

\[ q_w = (fg/\theta_o)(\xi \cdot \nabla \theta_w) \]

where \( \xi \) is the absolute vorticity vector, \( f \) the Coriolis parameter, \( g \) the acceleration due to gravity and \( \theta_o \) a reference temperature.

Moist symmetric instability occurs when \( \sigma^2 > 0 \).

If we introduce the basic flow frequencies \( F \) (inertial frequency), \( S \) (baroclinic frequency), for dry air, \( S_w \), and \( N_w \) (Brunt–Väisälä frequency) for a moist atmosphere with

\[ S^2 = (g/\theta_o) \partial \theta / \partial X = f \partial V / \partial Z \]
\[ S_w^2 = (g/\theta_o) \partial \theta_w / \partial X \]
\[ N_w^2 = (g/\theta_o) \partial \theta_w / \partial Z \]
\[ F^2 = f(\partial V / \partial X) \]

then Eq. (1) may be written

\[ \sigma^2 N_w^2 = -q_w = -(F^2 N_w^2 - S^2 S_w^2). \]

In the region of large-scale saturated ascent above the cold frontal surface and within the precipitating area \( S^2 \) may be given by \( S_w^2 \). Indeed \( S^2 / S_w^2 \) is unity for saturated air near the surface (\( \theta \approx \theta_w \)) and typically in this case 1.2 near 700 mb. \( q_w \) will be slightly overestimated.

Then it is seen that if \( N_w^2 > 0 \) (vertical stability), moist symmetric instability will occur when

\[ q_w = F^2 N_w^2 - S_w^4 < 0. \]  

(2)

Equation (2) shows that the regions of saturated ascent most subject to moist symmetric instability correspond to regions with small \( N_w^2 \) and with large zonal shear, \( \partial V / \partial Z \).

In the observed case, two regions correspond to this description: (i) the sheared cold frontal surface which separates the two homogeneous flows, cold air below and warm air above (region B in Fig. 3); (ii) the cyclonic sheared zone ahead of the cold frontal surface (region A in Fig. 3).

Figure 4 gives the hodographs deduced from VAD analyses at: 1430 (all times GMT) (1) in the cyclonic shearing zone, located ahead of the frontal discontinuity (denoted A in Fig. 3); 1720 (2) and 1840 (3) at the rear of the discontinuity at the ground and in the frontal surface (denoted B) (see Fig. 1). In each hodograph the direction of the shear is indicated (in the lower layer for 1430 h and in the frontal surface for 1720 h and 1840 h).
In region A, ahead of the frontal discontinuity, the orientation of the band appears quite identical with the direction of the maximum shear layer 0–500 m. Behind the discontinuity in region B, and in the frontal surface located in the 1200–1700 m layer at 1720 h and in the 1500–2100 m layer at 1840 h, the bands of precipitation are also oriented parallel to the shear vector (see Figs. 1 and 4). In fact, the observed shear in this case must reflect both the thermal wind and the vertical shear of the frontally-forced ageostrophic flow which should be across the bands from the warm to the cold side. Moreover, friction must influence the observed winds in region A.

Thus the theory fails in this case: it does not take into account the effects of the steady components of the ageostrophic winds and friction is not included in the present theory. Yet comparison between theory and observation will be made.

$q_\kappa$ may be evaluated with reasonable accuracy within these zones using the results from Doppler radar analysis (Testud et al. 1980).

(a) Region A ahead the frontal surface

\( \partial V / \partial X \) and \( \partial V / \partial Z \) (deduced from VAD analysis), and \( \partial \theta_w / \partial Z \) (given by the radiosounding at 1200 h) may be written \( \Delta V_x / \Delta X, \Delta V_z / \Delta Z \) and \( \Delta \theta_w / \Delta Z \). The observed
Figure 4. Hodographs deduced from VAD analyses at 1430 GMT (1), 1720 GMT (2), 1840 GMT (3). The direction of mean shear is indicated and the frontal system motion is given by the cross on the X axis.
values of $\theta_w$, $\Delta \theta_w$, $\Delta V_X$ and $\Delta V_Z$ are

$$\theta_w = 283 \, \text{K} \quad \Delta \theta_w = 0.2 \, \text{K} \quad \text{for} \, \Delta Z = 1000 \, \text{m};$$

$$\Delta V_X = 6 \, \text{m s}^{-1} \quad \text{for} \, \Delta X = 100 \, \text{km}; \quad 4 < \Delta V_Z < 9 \, \text{m s}^{-1} \quad \text{for} \, \Delta Z = 1000 \, \text{m}.$$

With these values, we obtain

$$N_w^2 = 6.2 \times 10^{-6} \, \text{s}^{-2}$$

$$F^2 \approx 1.6 \times 10^{-8} \, \text{s}^{-2}$$

$$4 \times 10^{-7} < S_w^2 < 9 \times 10^{-7} \, \text{s}^{-2}$$

and

$$-0.05 \times 10^{-12} < q_w < -0.70 \times 10^{-12} \, \text{s}^{-4}.$$

This region appears as a region of generation of moist symmetric instability.

(b) Region B in the frontal surface

In this region, located just behind the cold frontal surface at the ground, no soundings are available. We will use the following method to estimate the value of $\partial \theta_w / \partial Z$. The shallow zone of strong shear which is systematically observed in the mesoscale region of VAD analysis ($\sim 50 \, \text{km}$) between 1700 and 2100 h appears as an homogeneous region which may be considered as the basic state in which the small perturbations (filtered by the analysis) grow. $\partial V / \partial X$, $\partial V / \partial Z$ and $\partial \theta_w / \partial Z$ may be written $\Delta V_X / \Delta X$, $\Delta V_Z / \Delta Z$ and $\Delta \theta_w / \Delta Z$ with $\Delta \theta_w$ the jump of the potential temperature $\theta_w$ across the sheared zone, $\Delta V_X$ (along $X$ axis) and $\Delta V_Z$ (along $Z$ axis) the jumps in velocity.

The ratios of the accelerations and the Coriolis forces in the $X$ and $Y$ directions are given by

$$\alpha = \frac{D U / D t}{f V} = \frac{U^2 / l}{f V} \sim \left( \frac{U}{V} \right)^2 \frac{V}{fl}$$

$$\beta = \frac{D V / D t}{f U} = \frac{U V / l}{f U} \sim \frac{V}{fl}$$

if we assume that $D / D t \sim U / l$.

In the frontal surface, the maximum values are $U = 4 \, \text{m s}^{-1}$, $V = 10 \, \text{m s}^{-1}$, $l = 100 \, \text{km}$ and yield $\alpha = 0.1$, $\beta = 1$. Thus we may use the geostrophic balance in the cross-front direction $X$ and estimate

$$\Delta \theta_w \approx (f \theta_o / g) \cdot \Delta V_Z \cdot S_c^{-1}$$

where $S_c$ is the slope of the sheared zone (see Testud et al. 1980).

The observed mean values of $\theta_o$, $\Delta V_Z$ and $S_c$ being 299.5 K, 9 m s$^{-1}$ and 1.1%, respectively, it follows $\Delta \theta_w = 2.5 \, \text{K}$ in the frontal surface with a thickness of $h = 500 \, \text{m}$.

This value of $\Delta \theta_w$ is in good agreement with that observed from radiosounding data taken well after the frontal passage (section 4(b) from Testud et al.). $\Delta V_X$ is more difficult to estimate. Its value lies between 4.5 and 5 m s$^{-1}$ for $\Delta X = 50 \, \text{km}$.

Using these values we obtain

$$S_w^2 = 1.8 \times 10^{-6} \, \text{s}^{-2}$$

$$N_w^2 = 1.64 \times 10^{-4} \, \text{s}^{-2}$$

$$1.9 \times 10^{-8} < F^2 < 2 \times 10^{-8} \, \text{s}^{-2}$$

and

$$-12 \times 10^{-14} < q_w < 4 \times 10^{-14} \, \text{s}^{-4}.$$
Figure 5. Cross-front section of the horizontal wind component parallel to rainbands at three altitudes (1 km, 2 km and 3 km) and of the mean ascent in the layer 0–4000 m. The dimensions of each rectangle represent velocity uncertainty and time resolution. The mean value of the horizontal wind (indicated in the upper left corner) has been subtracted.
Thus the bands are observed in an atmosphere nearly neutral for slantwise convection and seem to persist for many hours with this condition of neutrality.

This result agrees very well with those reported by Emanuel (1983b) for a case of banded precipitation in Oklahoma and by Sanders and Bosart (1985) for the snowbands observed during the storm of 11–12 February 1983.

Emanuel (1985) suggests that such a form of slantwise moist convective adjustment in the initially slantwise unstable baroclinic atmosphere when it is lifted to saturation, may explain these observations. The presence of active frontogenesis (as shown by Thorpe and Emanuel (1985)) with this condition of neutrality may maintain the band structure which can then persist for long periods.

Bennetts and Hoskins (1979) show that the most unstable mode is a case of inertial instability on $\theta_w$ surfaces. For small amplitude disturbances, the wavelength between rolls may be expressed (Emanuel 1979) $\lambda = h(N_w^2/S_w^2)$. The observed values of $h$, $N_w^2$ and $S_w^2$ give $\lambda = 46$ km, which is in good agreement with that deduced from the radar reflectivity field (Fig. 1). The equation of temporal evolution for the kinetic energy of this mode is

$$\frac{\partial}{\partial t} \left\{ \frac{1}{2} (u'^2 + v'^2) \right\} = -u'v' \frac{\partial V}{\partial X} - v'w' \frac{\partial V}{\partial Z}$$

(see Bennetts and Hoskins) where $\frac{\partial V}{\partial X}$ and $\frac{\partial V}{\partial Z}$ are the horizontal shear and the vertical shear of the basic wind, $u'$, $v'$, $w'$ the perturbation velocities.

Without horizontal shear and with $\frac{\partial V}{\partial Z} > 0$, this equation implies that the kinetic energy of the bands comes partly from the kinetic energy of the basic flow through a negative correlation between $v'$ and $w'$.

Figure 5 gives the cross-front section of the horizontal wind component relative to the mean value of this component (at each altitude), parallel to rainbands and of the mean vertical ascent in the layer 0–4000 m. It shows that the predicted negative correlation between the fluctuations of the horizontal wind and of the vertical velocity is indeed well observed. The contribution to the vertical velocity of the warm air ascent along the cold frontal surface which appears clearly between $-25$ km and $25$ km (see Fig. 5) perturbs this correlation slightly. On the other hand, the maximum vertical velocity is consistent with that expected from CSI theory. We can obtain an estimate of this theoretical vertical velocity by making the approximation $w'/u' \sim \tan \phi$ where $\tan \phi$ is the slope of the moist updraught. Along $\theta_w$ surfaces, it is given by

$$\tan \phi = \frac{S_w^2}{N_w^2} \sim 0.01.$$

Then with the observed fluctuations of $U$ (~5 m s$^{-1}$) $w'$ must be equal to about 5 cm s$^{-1}$.

5. SUMMARY AND CONCLUSION

We have compared observations on wide cold frontal rainbands with the moist symmetric instability mechanism for their formation.

This case was particularly suitable for this study (deduced from a single Doppler radar analysis). The 2-dimensional and the stationary hypothesis being nearly true in this case helped us to calculate the potential vorticity to diagnose the stability of the flow.

This study shows that CSI may explain many of the observed features of those rainbands which grow in the saturated warm air, neutral for vertical convection, which lies ahead of the surface cold front. The cyclonic shear region ahead of the cold frontal
surface appears conditionally unstable according to the CSI. The rainbands associated with the shearing zone of the frontal surface are observed in an environment neutral for slantwise convection as observed previously by others.

This study is limited by the impossibility of observing the 3D airflow associated with rainbands and of assessing the synoptic forcing and its impact on the mesoscale frontal dynamics.

An experiment such as the Mesoscale Frontal Dynamics project, FRONTS 87, associating synoptic observations, several Doppler radars and clear-air radars (wind profiler), etc., may overcome these limitations (Browning et al. 1986).

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Comments on ‘The statistical properties of the general atmospheric circulation: Observational evidence on a minimal theory of bimodality’, by R. Benzi et al. (112, 661–676)

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In the study by Benzi et al. (1986) a reexamination of the so-called ‘multiple equilibrium’ theory originally proposed by Charney and DeVore (1979), hereafter CV, is presented. Observational evidence for a bimodal distribution in the frequency of occurrence of high amplitude wave