Representativeness errors for radiosonde observations

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SUMMARY

Data from the United Kingdom operational upper air network, as well as from special radiosonde trials, have been analysed to provide statistics of spatial and temporal atmospheric variations. These statistics are applied to a study of the network performance and of representativeness errors associated with the use of the data in synoptic analysis. The results also have application to comparisons between measurements from radiosondes and those from other observing systems. For example, it is demonstrated that minimum collocation errors associated with radiosonde temperature measurements located at the centre of a satellite radiometer scan spot of radius 55 km are <1 degC for levels in the troposphere and lower stratosphere.

1. INTRODUCTION

Radiosonde measurements are essentially point or line samples along a slant path through the atmosphere. The samples have two undesirable characteristics, namely:

(i) Owing to the limited sample volume the data contain information concerning atmospheric variations on time or space scales which are inappropriate to their use in synoptic meteorology. The extent of the contamination depends not only upon the spectrum of atmospheric variations, but also on the radiosonde sampling characteristics. The importance of the contamination depends upon the temporal and spatial resolution of the meteorological analysis in which the radiosonde data are incorporated.

(ii) Radiosonde observations are usually ascribed to the launch site location, whereas measurements in the upper troposphere and stratosphere are typically displaced several tens of kilometres horizontally from that point.

The primary aims of the work described here are to quantify the impact of the sampling characteristics upon the representativeness of the observations and upon the ability of the United Kingdom radiosonde network to extract information on the spatial and temporal gradients in the atmospheric fields. In order to meet these aims, statistics of atmospheric variations on scales from a few tens to several hundred kilometres and from a few hours to a few tens of hours were generated. This was accomplished by the analysis of routinely available radiosonde data from stations in the United Kingdom. In addition, data have been analysed from a number of special radiosonde trials which were performed in 1984/5; these trials have provided radiosonde ascent series with smaller time and space separations than is routinely available from the United Kingdom operational network.

The methods of analysis used here follow the example of Hawson (1970) who performed an exhaustive study of variability data in order to determine optimum performance criteria for radiosondes. Recent improvements in the reproducibility of radiosonde measurements (see Nash and Schmidlin 1987) enable significant advances to be made over this earlier work. Almost all the measurements analysed in this paper are from Meteorological Office RS3 radiosondes. The reproducibility of the measurements from this sonde was measured directly by Edge et al. (1986) and so the effect of measurement uncertainty upon the atmospheric variability statistics can be properly assessed. The high reproducibility also enables accurate assessments to be made of atmospheric variability on smaller time and space scales than was previously possible using
radiosonde data. The wider applicability of the variability statistics is also investigated by comparing some key results for United Kingdom stations with those from a selection of stations in Central and North America (latitudes 9° to 80°N).

In section 2, the database of radiosonde observations is described; the method by which atmospheric structure functions are derived is outlined in section 3. Section 4 contains an analysis of the measurement errors. The characteristics of the derived structure functions are described in section 5 and the functions are used to determine the magnitude of observational errors associated with the quality control of radiosonde data within numerical analysis models (section 6). Collocation errors associated with the comparison of radiosonde measurements with data from other sources (such as aircraft and satellites) are discussed in sections 7 and 8. The applicability of the results to other latitudes is discussed in an appendix.

In the published literature, it is the variability in the horizontal wind field that has received closest attention to date; see, for example Durst (1954), Hutchings (1955) and Ellsaesser (1969a) and more recent work by Brown and Robinson (1979), Gage (1979) and Jasperson (1982). In contrast, there appear to be remarkably few published statistics of temperature variations. Bruce et al. (1977) utilized radiosonde data to estimate collocation errors for radiosonde/satellite temperature sounding comparisons and noted the paucity of readily available variability statistics.

2. THE RADIOSONDE OBSERVATIONS

The United Kingdom synoptic upper air network consists of eight radiosonde stations which routinely make full temperature, humidity and wind (TEMP) observations at the nominal hours 00 and 12 GMT (see Fig. 1). Additional wind only (PILOT) ascents are performed at 06 and 18 GMT. Separation distances between stations in this network are in the range 220–370 km and the average distance between each station and the nearest two neighbours is 310 km. Another four radiosonde stations make observations in support of military test range operations on demand. Ascents from this group of stations are concentrated in the morning period.

Archived TEMP and PILOT messages from the above stations have provided the majority of the source data for this work. Important supplementary data have come from the WMO radiosonde comparison held at Beaufort Park, Bracknell, in June and July 1984 and also a trial (primarily of wind-finding radars) held in November and December 1985, also at the Beaufort Park site. The dataset from the former trial consisted of a series of 100 ascents with time separations ≥4 hours whereas time separations (for wind measurements) in the latter trial were as low as 0·25 hours, with again about 100 ascents in total.

The RS3 radiosonde systems used at all United Kingdom stations are identical. The radiosonde reports values of temperature every 2 seconds and pressure every 8 seconds. The time constant of the RS3 temperature sensor is extremely small, ≲1 s in the troposphere and ≲1 s at altitudes up to 30 km (see Nash et al. 1985). Temperature measurements at the standard pressure levels are essentially point samples. Geopotential heights are computed using every available data sample. For wind-finding, all stations use a Cossor type 353D radar which measures the azimuth, elevation and range of a radar target attached to the flight rig. Horizontal wind measurements attributed to the standard levels represent averages over a period of about 64 seconds, equivalent to a line average over a vertical depth of about 300 m.
Figure 1. Map of the British Isles showing the locations of radiosonde stations in the United Kingdom upper air network. , Stations in the synoptic network; Stations at military sites; The experimental site is shown by a broken circle.

Two datasets were assembled which consisted of observations at standard pressure levels: one set covers the winter half year from October 1983 to March 1984 inclusive and the other covers the summer half year from April to September 1984 inclusive. The 6-month periods were chosen to generate statistically significant datasets whilst retaining some information upon seasonal differences. The availability of data from the synoptic network is extremely high with only a few ascents missing from each dataset, and more than 90% of ascents reached the 50 hPa level.

Spatial variations across the synoptic network are analysed by comparing the observations from Crawley with those from the other stations in the synoptic network, and temporal variations from consideration of the time series observations from Crawley. Differences between Crawley and Beaufort Park and between Crawley and Larkhill (see Fig. 1) are used to extend the analysis to smaller spatial separations. The availability of these data is more limited than that for data from the synoptic network. Variability over shorter time intervals is examined using data from Larkhill and the special trials at Beaufort Park.

The RS3 humidity sensor (goldbeaters skin) suffers from serious systematic errors and has a time constant of response which is less than ideal for the sampling of humidity structure (Nash et al. 1985). Therefore humidity measurements are only considered briefly in this paper and data from the Väisälä RS80 radiosondes are utilized. This radiosonde has a more responsive capacitive humidity sensor. Time series of ascents using RS80 radiosondes were available from the two experimental trials at Beaufort Park. About 100 RS80 ascents were made in June/July 1984 and about 50 in November/December 1985.
3. ANALYSIS METHOD

Consider two radiosonde observations of a given parameter at the same pressure level. Then the difference between the measurements ($\Delta x$) can be written as (see e.g. Morgan 1984)

$$\Delta x = x_i + x_s + x_e$$  \hspace{1cm} (1)

where $x_i$ is the difference associated with the separation in time of the observations, $t_i$; $x_s$ is the difference associated with the horizontal separation between the observations, $s$; and $x_e$ is that part of the difference due to the measurement errors. Squaring and taking the mean over a large set of differences:

$$\langle \Delta x^2 \rangle = \langle x_i^2 \rangle + \langle x_s^2 \rangle + \langle x_e^2 \rangle + 2(\langle x_i x_s \rangle + \langle x_i x_e \rangle + \langle x_s x_e \rangle).$$  \hspace{1cm} (2)

The individual differences $x_i$, $x_s$ and $x_e$ can all be expressed as the sum of a mean and random term:

$$x_e = \bar{x}_e + x'_e.$$  \hspace{1cm} (3)

Each of the cross-terms in Eq. (2) can therefore be expanded as in the following example:

$$\langle x_i x_e \rangle = \langle \bar{x}_i \bar{x}_e \rangle + \langle \bar{x}_i x'_e \rangle + \langle x'_i \bar{x}_e \rangle + \langle x'_i x'_e \rangle$$

$$= \bar{x}_i \bar{x}_e + \bar{x}_i \langle x'_e \rangle + \bar{x}_e \langle x'_i \rangle + \langle x'_i x'_e \rangle$$

$$= \bar{x}_i \bar{x}_e + \langle x'_i x'_e \rangle \quad \text{since} \quad \langle x'_i \rangle = \langle x'_e \rangle = 0.$$  \hspace{1cm} (4)

Only datasets containing observations from identical radiosonde systems will be considered, so it can be assumed that $\bar{x}_i = 0$. Also, it is assumed that the measurement errors are not correlated with the differences $x_i$ and $x_s$ so $\langle x'_i x'_e \rangle = 0$. This assumption is not universally true, due to residual systematic differences between day and night temperature measurements (caused by imperfect radiation corrections); however, it can readily be shown that there is a negligible impact of the residual errors on the calculations of atmospheric variability: $\langle x_i x_s \rangle$ is likely to be small but not zero. However, the use of Eq. (2) will be confined to situations where $x_i \gg x_s$ or $x_i \ll x_s$, i.e. differences due to spatial separation will dominate those due to temporal separation or vice versa.

2$\langle x_i x_s \rangle$ is then small compared with either $\langle x_i^2 \rangle$ or $\langle x_s^2 \rangle$ and will be neglected.

Thus

$$\langle \Delta x^2 \rangle = \langle x_i^2 \rangle + \langle x_s^2 \rangle + \langle x_e^2 \rangle.$$  \hspace{1cm} (5)

Simplifying the notation, for a given parameter $x$,

$$\Delta X^2 = \tau^2_s(t) + S^2_s(s) + 2E^2_x$$  \hspace{1cm} (6)

where $\tau_x$ is the r.m.s. difference associated with the time separation, $S_x$ is the r.m.s. difference associated with the space separation and $2E^2_x$ is the mean-square-difference associated with the random components of the radiosonde errors. $E_x$ is therefore the uncertainty associated with each of the radiosonde measurements.

$S_x(s)$ will be derived from the r.m.s. difference between nominally simultaneous radiosonde observations at the same pressure level by radiosondes launched from two stations separated by distance $s$. The r.m.s. difference can be expressed as

$$\Delta X^2 = \tau^2_s(\delta t) + S^2_s(\delta s) + S^2_x(\delta s) + 2E^2_x$$  \hspace{1cm} (7)

where $\delta t$ is the small time difference between the nominally simultaneous data due to variations in the launch time and ascent rate of the balloons. $\delta s$ is the difference between
the actual horizontal separation of the observations and the station separation. For the range of $s$ considered here $\delta s \ll s$ so it is assumed that $S_x(\delta s)$ can be neglected as being small compared with $S_x(s)$ in Eq. (7). For normal operational practice in the United Kingdom $\delta t < 0.25$ hours. In section 5 it is shown that $\tau_x(\delta t)$ is small compared with $S_x(s)$ for vector wind variations and it is assumed that this is also true for temperature and geopotential variations.

$\tau_x(t)$ will be derived from a time series of radiosonde observations from a single station. In this case, Eq. (6) becomes

$$\Delta X^2 = \tau_x^2(t) + S_x^2(\delta s(t)) + \tau_x^2(\delta t) + 2E_x^2.$$  

(8)

$\delta t$ is here the difference in the actual time separation from the nominal launch time separation, again caused by minor variations in launch time and balloon ascent rates. Changes in the wind-field occurring during the period $t$ between ascents in a time series will introduce a horizontal displacement $\delta s(t)$ between the observations. Comprehensive statistics of $\delta s(t)$ for United Kingdom conditions are not presently available but analysis of a limited sample of radar tracking data suggests that r.m.s. values of $\delta s(t = 24$ hours) are of the order of 2 km at the 850 hPa level, 5 km at 500 hPa, 15 km at 250 hPa and 20 km at both 100 and 50 hPa. It is shown in section 5 that $\tau_x(t) > S_x(\delta s(t)$) and for the range of $t$ considered here $t \gg \delta t$ and $\tau_x(\delta t)$ may be neglected.

Equations (7) and (8) therefore reduce to:

for nominally simultaneous ascents

$$\Delta X^2 = S_x^2(s) + 2E_x^2$$  

(9)

for time series of ascents

$$\Delta X^2 = \tau_x^2(t) + 2E_x^2$$  

(10)

and $S_x(s)$ and $\tau_x(t)$ can thus be obtained from measurements of $\Delta X$ and $E_x$.

Equation (9) assumes conditions of horizontal homogeneity and isotropy so that $S$ is a function of $s$ only. Gandin (1970) suggests that for upper air observations separated by up to 2000 km, isotropy and homogeneity may be assumed without significant error. The largest $s$ considered here is 1000 km but there is evidence of directional variations affecting a small number of the functions $S_x(s)$ (see section 5). The standard deviations of the half-yearly time series of observations from each synoptic radiosonde station show no evidence of serious spatial inhomogeneities.

The standard deviations of the standard pressure level observations from Crawley within the summer and winter half year datasets ($\sigma_r$) were computed. $\sigma_r$ is related to the structure functions $\tau_x(t)$ as follows (see e.g. Ellsaesser 1969b):

$$\tau_x^2(t) = 2\sigma_r^2[1 - r_x(t)]$$  

(11)

where $r_x(t)$ is the temporal correlation coefficient. As $r_x(t) \to 0$, at large time separations, then $\tau_x(t)$ should tend towards $\sqrt{2} \sigma_r$ so values of $\sqrt{2} \sigma_r$ are plotted down the right-hand margins of Figs. 4, 7 and 10.

4. RADIOSONDE MEASUREMENT ERRORS

Each measured parameter is identified subsequently by the symbols: $V$ for vector wind, $p$ pressure, $T$ temperature, $\Phi$ geopotential, $H$ geopotential increment, and $U$ relative humidity.
For radiosonde observations at the standard pressure levels, the term \( E_x \) has three components:

\[
E_x^2 = (\varepsilon_p \frac{dX}{dp})^2 + \varepsilon_x^2 + R_x^2. \tag{12}
\]

\( \varepsilon_p \frac{dX}{dp} \) represents the r.m.s. errors introduced by small errors in the radiosonde pressure measurements. \( \varepsilon_x \) represents the r.m.s. errors introduced by the radiosonde sensing system other than by mis-location in the vertical. \( R_x \) is the r.m.s. uncertainty associated with rounding errors in the reported data. The values of the first two terms on the r.h.s. of Eq. (12) for \( p, T \) and \( \Phi \) can be derived from the results of experiments performed by Edge et al. (1986) in which simultaneous data from two RS3 radiosondes flown side-by-side on the same flight rig were compared. The reproducibility of horizontal vector wind measurements from Cossor 353D radars (\( \varepsilon_V \)) was measured in a separate experiment described by Edge et al. The experimentally determined values of the terms in Eq. (12) are shown in Table 1. To obtain values of \( E_x \) relevant to information in operational TEMP messages, values of \( R_x \) are derived based upon the resolution of the reported values. However, for the time series ascent data from the special trials at Beaufort Park (see section 2), the coded message output was not used and for these data, the \( R_x \) (and hence \( E_x \)) terms are smaller. The different values of \( E_x \) are inserted into Eqs. (9) and (10) as appropriate.

For wind-only ascents which produce PILOT messages, the first term in Eq. (12) takes the form \((\varepsilon_h \frac{dV}{dh})^2\) where \( \varepsilon_h \) is the r.m.s. difference between the actual height of the wind measurement \( (h) \) and the true height of the pressure level to which it is assigned. In practice, it is found that any differences between \( E_V \) evaluated from TEMP and PILOT data do not result in significant differences in the values of \( \Delta V \).

<table>
<thead>
<tr>
<th>Pressure level (hPa)</th>
<th>( \varepsilon_p ) (hPa)</th>
<th>( \frac{dV}{dp} ) (m s(^{-1}) hPa(^{-1}))</th>
<th>( \varepsilon_V ) (m s(^{-1}))</th>
<th>( R_x^* ) (m s(^{-1}))</th>
<th>( E_x^* ) (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>850</td>
<td>1.0</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>500</td>
<td>0.9</td>
<td>0.2</td>
<td>1.0</td>
<td>0.5</td>
<td>1.1</td>
</tr>
<tr>
<td>250</td>
<td>0.7</td>
<td>0.2</td>
<td>1.0</td>
<td>1.0</td>
<td>1.4</td>
</tr>
<tr>
<td>100</td>
<td>0.4</td>
<td>0.2</td>
<td>1.5</td>
<td>0.1</td>
<td>1.5</td>
</tr>
<tr>
<td>50</td>
<td>0.3</td>
<td>0.2</td>
<td>1.5</td>
<td>0.1</td>
<td>1.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Geopotential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure level (hPa)</td>
<td>{\varepsilon_T^2 + (\varepsilon_p \frac{dT}{dp})^2}(^{1/2})</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>850</td>
<td>2.5</td>
</tr>
<tr>
<td>500</td>
<td>4.9</td>
</tr>
<tr>
<td>250</td>
<td>0.11</td>
</tr>
<tr>
<td>100</td>
<td>0.06</td>
</tr>
<tr>
<td>50</td>
<td>0.15</td>
</tr>
</tbody>
</table>

* Values are estimates.
† These errors refer to analysis of TEMP messages only; smaller values apply to the data from special radiosonde trials.
Geopotential height increment, or the thickness of a layer bounded by two pressure levels, is proportional to the mean virtual temperature of the atmospheric layer. The uncertainty in its derivation from radiosonde geopotential measurements is estimated using the equation

$$\varepsilon_H = \frac{\bar{H}}{\bar{T}_v}$$

(13)

where $\bar{H}$ is the mean increment and $\bar{T}_v$, the mean virtual temperature of the layer. $\varepsilon_H^2$ is the r.m.s. bias in the radiosonde temperature measurements averaged over the layer, and for these purposes, can be considered as being approximately equal to $\varepsilon_T$.

5. ATMOSPHERIC VARIABILITY STATISTICS

(a) General points

The statistics of vector wind variations are presented in Figs. 2, 3, 4; of temperature in Figs. 5, 6, 7; of geopotential in Figs. 8, 9, 10; of geopotential increment in Fig. 11; and of relative humidity in Fig. 12.

In Figs. 2, 5, 8, 11 vertical profiles of $S_v(s)$ are presented for $s$ equal to the minimum separation distance for stations in the synoptic radiosonde network (220 km) and for the minimum value of $s$ from all available data (52 km). Similarly profiles of $\tau_v(t)$ were constructed, again for characteristic of synoptic observations (12 h) and for the minimum $t$ for which statistics were available. For comparison, profiles of the measurement error term $E_r$ have been included in these diagrams.

In a few cases at the smallest time and space separations considered, $2E_r^2$ was greater than half $\Delta X^2$. In these cases, reliable values of $S_v(s)$ and/or $\tau_v(t)$ could not be computed and hence have been omitted from the diagrams. For example, the rounding of geopotential heights of levels above 500 hPa reported in the TEMP messages is the cause of some missing data in Fig. 8.

The spatial structure functions for vector wind, temperature and geopotential are shown for 850, 500, 250, 100 and 50 hPa in Figs. 3, 6 and 9; temporal structure functions are shown in Figs. 4, 7 and 10. These levels were chosen for illustration because they are located in characteristically different atmospheric layers and the main features of the vertical profile of variability statistics are well represented by consideration of these levels. Results based upon the more limited supplementary data from Beaufort Park and Larkhill are joined by dashed lines in the diagrams.

The magnitude of the term $\delta s(t = 24$ hours$)$ was estimated to be about 20 km at most (see section 3) in the derivation of Eq. (10). From examination of Figs. 3 and 4, it is concluded that $S_v\{\delta s(t)\}$ is less than $0.5\tau_v(t)$ and thus we have been justified in assuming that $\tau_v^2(t) > S_v^2\{\delta s(t)\}$. A similar argument also holds for temperature and geopotential statistics on the basis of the results presented in Figs. 6, 7, 9, 10.

The vector wind data for the separations $t = 0.3$ hours from the 1985 (November-December) Beaufort Park experiment (see section 2) are not included with the main body of data in Fig. 4, because of the small amount of data from only 10 ascent pairs. However, the significance of the statistics for the Beaufort Park data is increased by considering all measurement levels, not just the standard pressure levels. The individual vector differences can thus be grouped into height bands and the results are given in Table 2. The very low values of $\tau_v(t = 0.3$ hours$)$ suggest that the contribution to $S_v(s)$ arising from variations in launch time in the synoptic radiosonde network of less than 0.25 hours can be neglected, as $\tau_v(\delta t)$ is only about 0.5$S_v(s)$ even for the smallest values of $s$ considered (52 km) (see section 3 and Eqs. (7) and (8) in particular). Equivalent statistics
TABLE 2. THE R.M.S. DIFFERENCE BETWEEN VECTOR WIND OBSERVATIONS SEPARATED IN TIME BY ABOUT 0.3 HOURS, ALONG WITH ESTIMATES OF THE MEASUREMENT ERRORS ASSOCIATED WITH THE COMPARISON

<table>
<thead>
<tr>
<th>Height band (km)</th>
<th>$R_N$ (m s$^{-1}$)</th>
<th>$\varepsilon_N$ (m s$^{-1}$)</th>
<th>$\sqrt{2 \cdot \varepsilon_N}$ (m s$^{-1}$)</th>
<th>$\tau_N(t = 0.3)$ (m s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–2</td>
<td>0.14</td>
<td>0.5</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>2–4</td>
<td>0.14</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>4–6</td>
<td>0.14</td>
<td>1.0</td>
<td>1.4</td>
<td>1.4</td>
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<tr>
<td>6–8</td>
<td>0.14</td>
<td>1.0</td>
<td>1.4</td>
<td>1.8</td>
</tr>
<tr>
<td>8–10</td>
<td>0.14</td>
<td>1.0</td>
<td>1.4</td>
<td>1.4</td>
</tr>
</tbody>
</table>

of temperature and geopotential variations on time scales of about 0.25 hours are not presently available. Thus it is difficult to assess the contribution of $\tau_N(\delta t)$ and $\tau_N(0)$ to $S_T(s)$ and $S_N(s)$ for the lowest values of $s$ plotted in Figs. 6 and 9.

Some of the salient features of the variability profiles and structure functions in Figs. 2–12 are described below.

(b) Vector wind (Figs. 2, 3, 4)

(i) Maximum variability on the synoptic scale is observed (much as expected) at the 300 hPa level, which is also the level of maximum mean wind speed. However, as Fig. 2 shows, for the smallest separations considered, $S_N(s = 52 \text{ km})$ and $\tau_N(t = 0.3 \text{ hours})$, there is little evidence of a peak in variance at the 300 hPa level.

![Figure 2](image-url)  

Figure 2. Profiles of the r.m.s. difference between vector wind observations at standard pressure levels separated by distance $s$ or time interval $t$.

(a) $S_N(s = 52 \text{ km})$, summertime; $S_N(s = 220 \text{ km})$, summer half year; $S_N(s = 220 \text{ km})$, winter half year; $\times \times \times \sqrt{2 \cdot \varepsilon_N}$ (the measurement error associated with the derivation of these statistics from TEMP and PILOT message data).

(b) $\Delta \Delta \Delta \tau_N(t = 0.3 \text{ h})$, wintertime; $\sigma \sigma \sigma \sigma \sigma \tau_N(t = 2 \text{ h})$, summer half year; $\bullet \bullet \bullet \bullet \tau_N(t = 12 \text{ h})$, summer half year; $\bullet \bullet \bullet \bullet \tau_N(t = 12 \text{ h})$, winter half year.
Figure 3. Root mean square differences between measurements of vector wind separated by a distance $s$ (spatial structure functions) at five standard pressure levels. (a) Winter half year •—850 hPa level; ○—500 hPa; ▲—250 hPa; △—100 hPa; ■—50 hPa. (b) Summer half year for the same pressure levels.

Figure 4. Root mean square differences between measurements of vector wind separated by a time interval $t$ (temporal structure functions). (a) Winter half year •—850 hPa; ○—500 hPa; ▲—250 hPa; △—100 hPa; ■—50 hPa. (b) Summer half year for the same pressure levels. Values of $\sqrt{2} \sigma_v$ are plotted down the r.h.s. of the graph outside the axes.
(ii) Significant differences between winter and summer half years in the troposphere are evident in temporal but not in spatial variations. In the stratosphere, the wintertime increase in variability is evident both spatially and temporally.

(iii) For Figs. 3 and 4, the functions $S_V(s)$ and $\tau(t)$ can be approximated, at least over the range of scales 200–500 km and 6–24 h, by a power law relationship of the form

$$S_V(s) = as^\beta, \quad \tau_V(t) = bt^\gamma.$$  \hspace{1cm} (14)

Estimates of the constants $\beta$, $\gamma$ are listed in Table 3. For all levels, $\beta > \gamma$ and most values of $\gamma$ are larger than the widely quoted 1/3 for temporal variations in the troposphere (see e.g. ELLSASSER 1969b). A value of $\gamma = 1/3$ results from assumptions that the similarity theory of isotropic turbulence is applicable to these large time scales but as ELLSASSER (1969b) has pointed out, larger slopes may be expected in mid-latitudes where kinetic energy generation on the synoptic scale is taking place.

DURST (1954) analysed time series of balloon ascents (for each season) from Larkhill in a similar way to that described here. In general, his estimates of $\tau_V(t)$ are slightly larger than those in Fig. 4. For example, at the 500 hPa level in summertime, $\tau_V(t = 6 \text{ hours}) = 7.5 \text{ m s}^{-1}$ and $\tau_V(t = 24 \text{ hours}) = 13.5 \text{ m s}^{-1}$, whereas the corresponding figures for the summer half year from Fig. 4 are 6.2 and 12.1 m s$^{-1}$ respectively. These differences are typical of those between the datasets. In Durst’s work, it appears that no allowance was made for measurement uncertainties. Values of $\gamma$ calculated from Durst’s data are between about 0.4 and 0.5 for all levels between 700 and 200 hPa, similar to the values in Table 3.

**Table 3. Estimates of structure function slopes over specified ranges of $s$ and $t$ (see Eq. (14))**

<table>
<thead>
<tr>
<th>Pressure level (hPa)</th>
<th><strong>Summer half year</strong></th>
<th><strong>Vector wind</strong></th>
<th><strong>Winter half year</strong></th>
<th><strong>Temperature</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta$ 200–500 km</td>
<td>$\gamma$ 6–24 h</td>
<td>$\beta$ 200–500 km</td>
<td>$\gamma$ 6–24 h</td>
</tr>
<tr>
<td>850</td>
<td>0.5</td>
<td>0.4</td>
<td>0.6</td>
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<td>0.7</td>
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<td>250</td>
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<td>0.6</td>
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</tr>
<tr>
<td>100</td>
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<td>0.6</td>
<td>*</td>
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<tr>
<td>50</td>
<td>0.3</td>
<td>*</td>
<td>*</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td><strong>Geopotential</strong></td>
<td>12–24 h</td>
<td>12–24 h</td>
<td></td>
</tr>
<tr>
<td>850</td>
<td>1.0</td>
<td>0.7</td>
<td>1.0</td>
<td>0.7</td>
</tr>
<tr>
<td>500</td>
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<td>0.7</td>
<td>1.0</td>
<td>0.7</td>
</tr>
<tr>
<td>250</td>
<td>0.9</td>
<td>0.7</td>
<td>1.0</td>
<td>0.7</td>
</tr>
<tr>
<td>100</td>
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<td>1.0</td>
<td>0.7</td>
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<tr>
<td>50</td>
<td>*</td>
<td>0.2</td>
<td>*</td>
<td>0.5</td>
</tr>
</tbody>
</table>

* Structure functions not well defined.
(iv) If the slopes of the structure functions, $\beta$ and $\gamma$, are of similar magnitude, this would suggest an equivalence between atmospheric space and time variability, i.e. $S_{xy}(\bar{u}t) = \tau_x(t)$ (the Taylor transformation, see e.g. Gage (1979)). Although $\beta \neq \gamma$ (Table 3), the data presented here suggest that the relationship is approximately valid over limited ranges of $x$ and $t$. Values of $\bar{u}$ appropriate to a spatial separation $\bar{u}t = 300$ km are of order $10$ m s$^{-1}$ (for temperature and geopotential, as well as for vector wind); that is, the variability observed at $300$ km spacing was approximately the same as that for $10$ hours time separation. $\bar{u}$ is slightly higher for the winter half year than for the summer half year. Since spatial gradients in vector wind are of similar magnitudes in summer and winter half years, it appears that higher rates of advection in winter cause the significant increase in wintertime variability noted in (ii) above.

(v) There is evidence of directional variations in $S_{xy}(s)$ in wintertime at the higher levels. In Fig. 3, the Crawley – Camborne differences ($s = 370$ km) are significantly lower than for Crawley – Aughton ($s = 330$ km) at 100 and 50 hPa. This is not attributable to mean vector wind differences between these pairs of stations but reflects a real difference in wind variability. For levels in the troposphere, differences between pairs of stations lying approximately east–west ($s = 52, 115, 370$ km) are compatible with the data from pairs aligned closer to north–south; indicating no directional dependence for the values of $\alpha$ and $\beta$ in the troposphere.

(vi) $\tau_x(t)$ in the summertime stratosphere does not increase smoothly with increasing $t$, e.g. $\tau_x(t = 12$ hours) $< \tau_x(t = 6$ hours), as shown in Fig. 4. This is almost certainly due to the modulation of the wind field by inertial oscillations of short vertical wavelength (see e.g. Kitchen and Tolworthy 1987; Cot and Barat 1986). In the region of the wind minimum in the summertime lower stratosphere, these oscillations, which have an amplitude of the order of $5$ m s$^{-1}$, are a significant source of variability on short time scales. The minimum in $\tau_x$ at $t = 12$ hours is simply a consequence of the period of the waves being close to the inertial period, which is about $14$ hours at these latitudes. There is also some evidence, as shown in Fig. 4, that there is a similar, but relatively small, impact on the relative magnitudes of $\tau_x(t = 6$ hours) and $\tau_x(t = 12$ hours) in wintertime.

(c) Temperature (Figs. 5, 6, 7)

(i) An example of a set of radiosonde temperature soundings in the upper troposphere/ lower stratosphere which are closely spaced in time is presented in Fig. 13(a). These were obtained from five Väisälä RS80 radiosondes launched within a 6-hour period from the Beaufort Park site on 10 December 1985. The data which are plotted for each sounding in Fig. 13 were sampled at a rate of about 50 samples per minute, and are unsmoothed.

This set of ascents serves to illustrate some of the typical characteristics of temperature profiles measured by radiosonde ascents in mid-latitudes. The temperature profiles in the lower stratosphere are characterized by oscillations on small vertical scales (i.e. less than a few kilometres), which are up to several degrees Celsius in amplitude. Significant changes in this small-scale structure occur on time scales of a few hours (see e.g. Fig. 13(a) around the 100 hPa pressure level). The temperature differences between the profiles in the upper troposphere were smaller than in the lower stratosphere, but increased with height towards the tropopause. Note also that the changes which occurred in the upper troposphere were largely due to a progressive change in the mean temperature, not to small-scale perturbations.
(ii) The profiles of temperature variability presented in Fig. 5 exhibit similar properties to those described above. The maximum variability in temperature occurs at the 200 hPa pressure level, i.e. above the average position of the tropospheric wind maximum and above the level of maximum variability in vector wind, where a relative minimum in temperature variability is observed. A secondary maximum in temperature variability occurs around the 500 hPa pressure level.

(iii) There is little difference between $S_T(s = 220 \text{ km})$ in the winter and summer half years between the surface and the 200 hPa pressure level, whereas $\tau_T(t = 12 \text{ hours})$ is clearly larger in winter than in summer at most levels (Fig. 5).

(iv) $S_T(s)$ and $\tau_T(t)$ can also be approximated by relationships similar to those for vector wind (and over a similar range of scales). The range of slopes $\beta$ and $\gamma$ (Table 3) and transformation velocities are similar to those for vector wind.

Directional differences in the temperature variability in the wintertime lower stratosphere are similar to those in the vector wind statistics described in (b) above. Crawley – Camborne differences ($s = 370 \text{ km}$) are significantly lower than for Crawley – Aughton ($s = 330 \text{ km}$), as shown in Fig. 6.
Figure 6. As Fig. 3 but showing spatial structure functions for measurements of temperature at the five standard pressure levels. (a) Winter half year ––850 hPa; O—500 hPa; ▲—250 hPa; △—100 hPa; ■—50 hPa. (b) Summer half year for the same pressure levels.

Figure 7. As Fig. 4 but showing temporal structure functions for temperature measurements. (a) Winter half year ––850 hPa; O—500 hPa; ▲—250 hPa; △—100 hPa; ■—50 hPa. (b) Summer half year for the same pressure levels.
Figure 8. As Fig. 5 but for r.m.s. differences in geopotential measurements. (a) \( S_\theta(s = 52\, \text{km}) \), summertime; \( S_\theta(s = 220\, \text{km}) \), summer half year; \( S_\theta(s = 220\, \text{km}) \), winter half year; \( \sqrt{2} \cdot E_\phi \). Two points are plotted for \( S_\theta(s = 220\, \text{km}) \) joined by a horizontal line. The lower values of each pair are those derived after additional quality control was applied to the data.

(b) \( \tau_\phi(t = 4\, \text{h}) \) summertime; \( \tau_\phi(t = 12\, \text{h}) \) summer half year; \( \tau_\phi(t = 12\, \text{h}) \) winter half year.

(d) Geopotential and geopotential increment (Figs. 8–11)

(i) The vertical profiles of geopotential variability in Fig. 8 are of similar shape to those of vector wind (Fig. 2) in as much as the geopotential structure functions do not increase monotonically with increasing height but reach a maximum in the upper troposphere. Except for spatial variations in the winter stratosphere, values of \( S_\theta(s) \) and \( \tau_\phi(t) \) at 100 and 50 hPa are similar to those in the lower troposphere. These profiles show that changes in the mean layer temperature of the atmosphere above the level of the peak in geopotential variability (i.e. about 300 hPa, see Fig. 8) generally compensate for changes occurring in the lower troposphere.

Effective quality control of the dataset is essential to obtain reliable statistics of geopotential differences for small spatial separations. Any isolated errors in the radiosonde temperature calibrations could have a significant impact on the geopotential differences for levels in the stratosphere. To investigate this possibility, a further level of quality control was introduced whereby the individual differences in geopotential between Crawley and Hemsby were examined and all differences more than 3 standard deviations from the mean of the distribution were removed; the mean and standard deviation of the differences were then recomputed. The effect of this process upon the profiles of \( S_\theta(s = 220\, \text{km}) \) can be seen in Fig. 8, where two values of \( S_\theta(s = 220\, \text{km}) \) are plotted for each level and joined by a horizontal line. The lower values of each pair are
Figure 9. As Fig. 6 but for geopotential rather than temperature. (a) Winter half year ●—850 hPa; ○—500 hPa; ▲—250 hPa; △—100 hPa; ■—50 hPa. (b) Summer half year for the same pressure levels.

Figure 10. As Fig. 7 but for geopotential. (a) Winter half year ●—850 hPa; ○—500 hPa; ▲—250 hPa; △—100 hPa; ■—50 hPa. (b) Summer half year for the same pressure levels.
Figure 11. Root mean square differences between geopotential increment (thickness) measurements separated in space by a distance \( s \) or in time by an interval \( t \). The differences are plotted as equivalent temperatures. The statistics of geopotential increment are plotted in histogram form to delineate the atmospheric layers over which the increments were calculated.

(a) \( \Phi^H(s = 52 \text{ km}) \), summertime; \( \Phi^H(s = 220 \text{ km}) \), summer half year; \( \Phi^H(s = 220 \text{ km}) \), summer half year, from Fig. 5; \( \sqrt{2} E^H \).

(b) \( \tau^H(t = 8 \text{ h}) \), summertime; \( \tau^H(t = 12 \text{ h}) \), summer half year; \( \tau^H(t = 12 \text{ h}) \), summer half year, from Fig. 5.

The results after the additional quality control has been applied. Thus it is demonstrated that the high values of \( S_\Phi(s = 220 \text{ km}) \) at 30 hPa in the winter half year are not due to a small number of excessively large differences. Note that the effect of the quality control on the summer half year results is relatively larger.

(ii) The slopes of the structure functions for geopotential (Figs. 9 and 10) and also for geopotential increment (not shown) are generally greater than those for temperature (Figs. 6 and 7).

(iii) Structure functions for geopotential increment \( S_H(s) \) and \( \tau_H(t) \) can be expressed as structure functions for equivalent mean layer temperatures \( S^H_H(s) \) and \( \tau^H_H(t) \). The values are converted using a relationship analogous to Eq. (13). Vertical profiles of \( S^H_H(s = 220 \text{ and 52 km}) \) for the summer half year are plotted in Fig. 11 alongside \( S_\Phi(s = 220 \text{ km}) \) for comparison. For the pressure bands 500–400, 300–200, 70–50 and 50–30 hPa, \( \sqrt{2} E^H > S^H_H(s = 52 \text{ km}) \). Estimates of \( S^H_H(s = 52 \text{ km}) \) for these bands were therefore considered unreliable and have not been plotted in Fig. 11. As anticipated \( S^H_H(s = 220 \text{ km}) < S_\Phi(s = 220 \text{ km}) \) at all levels, and averages of temperature over vertical depths 1–4 km in the stratosphere are less variable by up to 50% than temperatures measured at a pressure level.
(e) Relative humidity (Figs. 12 and 13(b))

(i) As stated in the introduction, humidity data were only available for two periods of two months. The maximum variability in June/July on time scales of several hours occurred not at the lowest levels but around the 500 hPa level, as shown in Fig. 12. For the November/December dataset, the maximum is at the 700 hPa level. Due to the high reproducibility of the RS80 radiosonde humidity measurements (see Antikainen and Hyvonen 1983), the observations appear to be easily capable of resolving the atmospheric variability on these time scales.

(ii) The temporal structure functions (not shown here) are markedly different from those for the other parameters. For the June/July dataset, in which the minimum time separation is four hours, the average slopes of the structure functions are very low (about 0.2). $\tau_{\theta}(t = 4\text{ hours})$ was in the range 11–25% in the troposphere and clearly a significant fraction of the temporal variations on isobaric surfaces occurs on time scales shorter than four hours. Some information on shorter time scales is available from the November/December dataset. Even $\tau_{\theta}(t = 2\text{ hours})$ was >10% at the 850 and 500 hPa levels.

(iii) From examination of time series of humidity profiles, it is apparent that the concentration of relative humidity variance on time scales of a few hours or less is partly due to the existence of strong vertical gradients in humidity commonly associated with stable layers (see Fig. 13(b)). Small variations in the heights of these layers can cause large changes in relative humidity at a specific pressure level.

It is clear from the above that point measurements of humidity at fixed levels are often representative of only the fine-scale structure. The effect of averaging in the vertical upon the representativeness of the measurements has yet to be examined.

(iv) Arpe (1985) noted that radiosonde relative humidity measurements separated by synoptic-scale distances are poorly correlated compared with other parameters. He concluded that the operational humidity data provided by radiosondes are less representative on the scales of interest to synoptic meteorology than the measurements of horizontal wind and temperature.
(f) The efficacy of the synoptic radiosonde network

The values of $S_z(s)$, and $\tau_z(t)$ associated with space and time separations characteristic of the synoptic observing network are listed for the summer half year in Table 4. The results demonstrate a very good matching between the space and time separations of the measurements from the network; i.e. spatial and temporal atmospheric gradients are resolved to a similar precision. Also, the listed values are all much larger than the magnitude of $\sqrt{2}.E_z$ for equivalent pressure levels (see Table 1) which demonstrates
that measurement errors impose minimal limitations on synoptic analysis. The values of \( \tau_s(t) \) in Table 4 are in general much smaller than the corresponding values of \( \sqrt{2 \cdot \sigma_t} \), indicating that a large fraction of the total variance in each parameter (computed over a 6-month period) is resolved. However, at the 250 hPa level in the summer half year, \( \tau_T(t = 12 \text{ hours}) \) is 55% of the value of \( \sqrt{2 \cdot \sigma_T} \) (Fig. 7) which implies a correlation between consecutive synoptic observations of about 0.7 (from Eq. (11)). This is the lowest correlation found for any parameter or pressure level.

**Table 4. Tabulated Values of Differences Between Spatially and Temporally Adjacent Synoptic Network Observations for the Summer Half Year**

<table>
<thead>
<tr>
<th>Pressure level (hPa)</th>
<th>( S_x(s = 300) ) (m s(^{-1}))</th>
<th>( \tau_x(t = 6) ) (m s(^{-1}))</th>
<th>( S_T(s = 300) ) (degC)</th>
<th>( \tau_T(t = 12) ) (degC)</th>
<th>( F_0(t = 300) ) (gpm)</th>
<th>( \tau_0(t = 12) ) (gpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>850</td>
<td>6.6</td>
<td>5.1</td>
<td>2.3</td>
<td>2.1</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
<td>500</td>
<td>8.5</td>
<td>6.1</td>
<td>2.1</td>
<td>2.4</td>
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<td>38</td>
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<tr>
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<td>59</td>
</tr>
<tr>
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<td>3.8</td>
<td>1.6</td>
<td>1.4</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
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<td>4.0</td>
<td>1.4</td>
<td>1.1</td>
<td>27</td>
<td>27</td>
</tr>
</tbody>
</table>

\( s \) in km; \( t \) in hours
6. **Assessment of Observational Errors to be Used in Numerical Analysis**

In order to make optimum use of observations in numerical analyses, the total uncertainty associated with the observation (the observational error) should be known. The observational error is a combination of the measurement error and a representativeness error which in turn is a function of the scales of atmospheric variations which can be resolved in the process of analysis.

In the analysis process used in the Meteorological Office operational global forecasting model, the observations are compared with the value from the model ‘background field’ at the observation point. The significance of the difference is decided according to the magnitude of the observational error and the uncertainty in the background field.

The observational error \( O_x(d) \) is defined by

\[
O_x^2(d) = I_x^2(d) + E_x^2
\]

(15)

where \( I_x(d) \) is the representativeness error appropriate for the model analysis; \( d \) is defined below. It is possible to derive values of the observational error for the RS3 sonde appropriate for use in this quality control from the structure functions \( S_x(s) \) and \( \tau_x(t) \). Consider an observation situated at the centre of a model grid-box. In the vicinity of the United Kingdom, the model grid-boxes have dimensions of approximately \( 130 \times 170 \) km; for the purpose of estimating the representativeness error, it is assumed that the boxes are squares of side \( d = 150 \) km. Then the error associated with interpolation to the observation point from values of the model background field at the grid points is given by (see Gandin 1970, Eq. (32))

\[
I_x^2(d) = S_x^2(d/\sqrt{2}) - S_x^2(d/4) - S_x^2(\sqrt{2}.d)/8.
\]

(16)

Note that here errors in the model background field are considered to be zero at the grid points because they do not contribute to the observational error as defined above. Also, if values of \( S_y^2 \) are substituted into the r.h.s. of Eq. (16), then the l.h.s. is strictly \( (I_u^2 + I_v^2) \) (where \( u \) and \( v \) are the wind components) rather than \( I_y^2 \). The difference between \( (I_u^2 + I_v^2) \) and \( I_y^2 \) is assumed to be small for the purpose of estimating observational errors. From Eqs. (15) and (16), estimates of the observational error may be obtained. However, such estimates would only apply to radiosonde observations located precisely at the point in space and time to which they are attributed in the model. In practice, two other factors need to be considered, namely: (i) Radiosonde observations are attributed to the position of the launch site in the analysis, rather than the location of the observation. (ii) It is a feature of the particular numerical model considered here that observations with nominal times of up to \( \pm 3 \) hours from the analysis time are all considered to have been made at the analysis time. This factor is of particular importance to data from those stations outside the synoptic network which make ascents on an irregular basis (see section 2).

The above factors are incorporated into Eq. (15) by the addition of two extra terms:

\[
O_x^2(d) = I_x^2(d) + S_x^2(\epsilon_x) + \tau_x^2(\epsilon_t) + E_x^2.
\]

(17)

\( \epsilon_x \) and \( \epsilon_t \) here represent the displacement in space and time from the observation location assigned in the analysis. Three sets of \( \epsilon_x \) and \( \epsilon_t \) were chosen: case 1 corresponding to ‘ideal’ radiosonde observations \( (\epsilon_x = 0, \epsilon_t = 0) \); case 2 corresponding to extreme time displacement \( (\epsilon_x = 0, \epsilon_t = 3 \) hours\); and case 3 corresponding to large spatial displacement typical of ascents through jet streams in the United Kingdom \( (\epsilon_x = 50 \) km at 250 hPa, 100 km at 100 and at 50 hPa; \( \epsilon_t = 0 \)\). Equation (17) was evaluated for cases 1–3 and \( d = 150 \) km using the summer half year structure functions. In Table 5, the results are
TABLE 5. Estimates of the observational errors associated with the use of RS3 radiosonde measurements in the United Kingdom numerical forecasting model.

<table>
<thead>
<tr>
<th>Pressure level (hPa)</th>
<th>$O_v$ (m s$^{-1}$)</th>
<th>$O_T$ (deg C)</th>
<th>$O_p$ (gpm)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 3</td>
</tr>
<tr>
<td>850</td>
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</tr>
<tr>
<td>500</td>
<td>2.4</td>
<td>4.8</td>
<td>3.1</td>
</tr>
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<td>3.2</td>
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</tr>
<tr>
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</tr>
<tr>
<td>50</td>
<td>3*</td>
<td>5*</td>
<td>5*</td>
</tr>
</tbody>
</table>

* The structure functions are not well defined, values are best estimates but should be treated with caution.
† Exact figure depends upon an assessment of temperature data quality from individual radiosonde stations. Most U.K. synoptic stations fall within the band indicated.

Values are presented for each of the three cases defined in the text and may be compared to the actual observational error levels currently in use in the model.

compared with $O_v$ and $O_T$ which are in current operational use in the Meteorological Office model. These operational values are derived from studies of the observation minus background fields and are applied to all radiosonde observations, not just those from the United Kingdom network (see Bell 1985).

The magnitudes of $O_v$ and $O_T$ currently used in observation quality control for the model generally agree to within a factor of 2 with our experimentally derived values.

The differences between the results from the three cases are significant. The effect of large horizontal displacements (case 3) is to increase the temperature errors by about 50% over that of the ideal (case 1) in the stratosphere. This result suggests that a further reduction in the model values of observational errors could be achieved by ascribing the radiosonde observations to their correct position. It is recognized that the structure functions $\delta_t(s)$ used in this analysis are derived from data for displacements randomly orientated with respect to the gradients at a particular level in the atmosphere, whereas the balloon displacements are always along wind. Thus, observational errors for case 3 may be overestimated here. The effect of the largest time displacement is similar, increases over case 1 being at least 50%.

7. COLLOCATION COMPARISONS BETWEEN RADIOSONDES AND OTHER POINT OBSERVATIONS

Radiosonde observations are often considered as a means of verifying observations made by other systems. Atmospheric variability statistics can be used to determine the collocation errors associated with such comparison exercises.

Examples of observing systems (other than radiosondes) which provide point or line samples are aircraft sensors which provide measurements of horizontal wind and temperature, either through conventional AIREP reports or the newer automated systems such as ASDAR (Aircraft to Satellite Data Relay). Each ASDAR report represents the output through an exponential filter which operates on the raw instrumental output with a half width of 10 seconds (equivalent to about 2.5 km), and ASDAR data can be considered to be point samples. Designed overall accuracies of the ASDAR system are a few tenths of a degree Celsius for temperature and a few m s$^{-1}$ vector error for horizontal wind. Therefore a verification exercise should aim to confine collocation errors to less than say 0.3 degC and 3 m s$^{-1}$. 
From the data plotted in Figs. 3, 4, 6, 7, it is readily seen that for both temperature and wind, these conditions impose maximum separations between ASDAR and radiosonde measurements of no more than a few tens of kilometres and one or two hours. This suggests that the amount of suitable collocation data obtained from the operational radiosonde network will be very small.

8. **Collocation Comparisons between Radiosondes and Area-Average Observations**

Temperature profiles inferred from satellite radiances measurements represent averages over the radiometer scan-spots. For example, the HIRS and MSU sounding instruments on the TIROS N series of polar orbiting satellites have scan-spots at the sub-satellite point which are described approximately by circles of radius \( z = 15 \) and 55 km respectively. Consider an RS3 radiosonde observation located at the centre of the spot; the r.m.s. difference \( (A_x) \) between the measurement and the true value of the areal average is given (see Bruce et al. 1977) by

\[
A_x^2 = \frac{1}{\pi z^2} \int_0^z S_x^2(s) 2\pi s \, ds + E_x^2. \tag{18}
\]

Substituting for \( S_x^2(s) \) from Eq. (9) and approximating the functional form of \( \Delta X^2 \) using a power law \( (\Delta X^2 = a s^b) \) then

\[
A_x^2 = \frac{2}{z^2} \int_0^z \Delta X^2(s) s \, ds + E_x^2 \tag{19}
\]

and

\[
A_x^2 = 2az^b/(\beta + 2) - E_x^2. \tag{20}
\]

\( A_x \) calculated from Eq. (20) represents the very minimum collocation error that can possibly be achieved. Realistic satellite/radiosonde collocation experiments involve comparison with radiosonde observations perhaps up to 100 km from the centre of the scan-spot and from within a period of a few hours either side of the overpass time. Additional terms must be added to the r.h.s. of Eq. (20) to include the contribution to \( A_x \) from displacement (in time and/or space) of the radiosonde soundings from the ideal, in a similar way to that described in section 6.

Collocation errors were calculated for three cases: (i) Case 1 for the ideal ascent at the centre of a sub-satellite MSU scan-spot at the overpass time. (ii) Case 2 for an ascent displaced by 150 km in space and 3 hours in time from the ideal. These figures were chosen because they correspond to the collocation criteria currently used in the Meteoro logical Office operational HIRS+MSU sounding bias correction procedures. (iii) Case 3, which is similar to case 2 except the displacement is reduced to 75 km.

Table 6 gives the results of the calculation: For case 1, temperature collocation errors are confined to <1 degC (note that these errors refer to radiosonde measurements of temperature at a given pressure level) and geopotential height errors to a few metres for an MSU scan-spot. For case 2, the errors are a factor of 3–4 larger. This result highlights the importance of properly chosen collocation criteria because the space and time mis-match errors tend to dominate the total error associated with the collocation. The measurement error for the RS3 radiosonde makes a relatively small contribution. The 150 km/3h criteria in case 2 probably do not represent an appropriate mid-latitude matching of space and time criteria in order to minimize collocation errors. For a given
TABLE 6. Estimates of the errors associated with a comparison between satellite measurements of temperature (and derived measurements of geopotential) which are areal averages over an MSU scan-spot, and RS3 radiosonde ascents.

<table>
<thead>
<tr>
<th>Pressure level (hPa)</th>
<th>Case 1</th>
<th></th>
<th>Case 2</th>
<th></th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
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<td>(A_T) (degC)</td>
<td>(A_\phi) (gpm)</td>
<td>(A_T) (degC)</td>
<td>(A_\phi) (gpm)</td>
<td>(A_T) (degC)</td>
</tr>
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<td>1.7</td>
<td>24</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Three different collocation criteria are considered. For case 1 \(z = 55\) km, \(\epsilon_r = 0\); for case 2 \(z = 55\) km, \(\epsilon_r = 150\) km, \(\epsilon_r = 3\ h\); for case 3 \(z = 55\) km, \(\epsilon_r = 75\) km, \(\epsilon_r = 3\ h\).

number of collocations the criteria should be such that the error due to spatial mis-match is equal to that from temporal mis-match. From Figs. 6 and 7, a better matched choice would be perhaps 75 km/3 h (case 3). In Table 6, it can be seen that case 3 does offer a significant reduction in maximum collocation errors at some levels; however, adoption of more stringent criteria is likely to have a large impact on the data capture. To achieve collocation errors represented by case 1 on a regular basis for any location would inevitably involve synchronizing radiosonde ascent times with satellite overpasses.

The satellite sounder channel radiances represent temperature measurements which are weighted averages with significant contributions from atmospheric layers of order a few kilometres deep (see e.g. Smith et al. 1979). The retrieved temperature profiles therefore do not contain the level of detail on small vertical scales which are resolved by radiosondes. In order to improve the representativeness of the radiosonde temperatures, Phillips et al. (1979) and McMillin et al. (1983) compared satellite and radiosonde mean layer temperatures using atmospheric layers a few kilometres deep. Adoption of such a procedure also tends to improve the horizontal representativeness of the radiosonde temperatures, as can be seen from the differences between \(S_T\) and \(S_H\) in Fig. 11. Note that \(S_H\) was computed here for the same pressure bands as used in McMillin et al.

\(A_H\) was computed from Eq. (20) for five geopotential increments and converted to equivalent values of \(A_H\). Comparing \(A_H\) in Table 7 with \(A_T\) in Table 6, the reduction in collocation errors for a selection of layers a few kilometres deep over fixed levels is significant in the stratosphere for case 2. This is also the result that could be expected on the basis of the \(S_T\) and \(S_H\) profiles in Fig. 11.

TABLE 7. Similar to Table 6 but for a comparison with radiosonde measurements of mean layer temperatures (derived from geopotential increments).

<table>
<thead>
<tr>
<th>Pressure band (hPa)</th>
<th>Case 1</th>
<th></th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A_H) (degC)</td>
<td>(A_H) (gpm)</td>
<td>(A_H) (degC)</td>
</tr>
<tr>
<td>850–700</td>
<td>0.5</td>
<td>3</td>
<td>1.4</td>
</tr>
<tr>
<td>500–400</td>
<td>0.7</td>
<td>4</td>
<td>2.0</td>
</tr>
<tr>
<td>300–200</td>
<td>0.4</td>
<td>5</td>
<td>1.7</td>
</tr>
<tr>
<td>100–70</td>
<td>0.9</td>
<td>9</td>
<td>1.3</td>
</tr>
<tr>
<td>50–30</td>
<td>0.5</td>
<td>7</td>
<td>0.9</td>
</tr>
</tbody>
</table>
9. CONCLUSIONS

(i) The high reproducibility of the UK RS3 radiosonde ensures that measurement errors are confined to an acceptably low level when considering analysis of temperature, geopotential and wind fields on the synoptic scale.

(ii) The total observational error associated with the use of RS3 radiosonde data in synoptic analysis could be reduced in some cases if the observations were attributed to the correct location rather than the launch site. Similarly, the use of asynoptic radiosonde data with a nominal ascent time of up to 3 hours from the analysis time increases the observational error level significantly.

(iii) In the design of satellite/radiosonde collocation experiments, the errors associated with the comparison can be properly assessed only from a knowledge of atmospheric variability on the appropriate scales. For typical collocation criteria and modern radiosonde designs, the radiosonde measurement errors make only a small contribution to the total error of comparison. Minimum collocation errors for radiosonde measurements of temperature located at the centre of an MSU scan-spot are <1 degC at all pressure levels. To achieve these error levels for routine calibration of satellite soundings would require special radiosonde soundings to be made which are synchronized with satellite overpasses.

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APPENDIX

In this paper, all the statistics of atmospheric variability were deduced from observational data obtained within the United Kingdom. In order to investigate whether the results have wider applicability, some key parameters were also computed from radiosonde data covering a range of latitudes. Twenty-two widely separated upper-air stations in the U.S.A. (including Alaska) and Canada were selected for this comparison, along with one in Puerto Rico and one in the Panama Canal Zone. All the stations were known to use radiosondes of similar VIZ design. The choice excluded stations in very mountainous areas.

The measurement errors of the VIZ radiosonde are not presently known as precisely as for the RS3 sonde. Therefore, all the statistics described here are uncorrected for measurement uncertainty. Fractional errors in these statistics, due to differences in the measurement errors or observing practices between stations in the United Kingdom and America, should be small.

The standard deviation of the vector wind \( \sigma_V \), computed over the summer half year, shows a well defined variation with latitude (Fig. A1). The same quantity computed from the Crawley data is very close to the values from similar latitudes in North America. Note that \( \sigma_V \) is very low in the tropical troposphere compared with mid-latitudes. \( \Delta V(t = 24 \text{ hours}) \) has a similar dependence on latitude to \( \sigma_V \) (Fig. A2), but in this case, the values from Crawley appear to be about 2 m s\(^{-1}\) lower than from American sites at the three chosen levels in the troposphere. For temperature variations, larger fractional
Figure A1. The standard deviation of the vector wind in the summer half year for stations in Central and North America and for Crawley plotted as a function of latitude. The Crawley values are ringed. (a) $\Delta - 100$ hPa; $\times - 50$ hPa. (b) $\bullet - 850$ hPa; $\bigcirc - 500$ hPa; $\blacktriangle - 250$ hPa.

Figure A2. The r.m.s. difference between measurements of vector wind separated in time by 24 hours for the summer half year. Values for stations in Central and North America and Crawley (ringed points) are plotted as a function of latitude. (a) $\Delta - 100$ hPa; $\times - 50$ hPa. (b) $\bullet - 850$ hPa; $\bigcirc - 500$ hPa; $\blacktriangle - 250$ hPa.
Figure A3. As Fig. A1 but for temperature measurements rather than for vector wind.

Figure A4. As Fig. A2 but for temperature measurements.
differences were found between United Kingdom and North American data at similar latitudes. Figure A3 shows that $\sigma_T$ computed over the summer half year at 850 and 500 hPa was about 1.5 degC lower at Crawley, whereas at the 250,100 and 50 hPa levels, the agreement was to within a few tenths of a degree. For $\Delta T(t = 24$ hours) the differences between United Kingdom and North American data around 50°N were similar (see Fig. A4). The structure of variations in the tropics is in marked contrast to that at mid-latitudes, with very low values of $\sigma_T$ and $\Delta T$ in the troposphere.

The four key statistics compared in Figs. A1 to A4 provide a context in which the applicability of the United Kingdom results to latitude bands may be judged. For example, on the basis of the data in Figs. A1 and A2, there is clear evidence that the vector v.i.d structure functions derived from the United Kingdom data are similar (to within a few metres per second) to those for low-level sites over a latitude band 35–60°N in the troposphere. More care is required in the generalization of the United Kingdom temperature statistics to other areas.

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