A composite diagnosis in sigma coordinates of the atmospheric energy balance during intense cyclonic activity

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SUMMARY

Following the recent trend for composite analysis of the energetics of cyclonic systems, a rapidly deepening Saharan depression is studied within this framework. This case refers to a cyclone which was the prevailing feature in the synoptic analyses over the Mediterranean for several days in mid March 1981. This system was marked with a range of interesting phenomena over the central and eastern Mediterranean, such as very strong surface winds, rising and transportation of desert dust to large distances, and coloured precipitation. The synoptic dynamics of the evolution of the system is briefly presented and a reasoning for its unusually rapid deepening is provided.

The forms of energy considered here are kinetic energy, and sensible and latent heats. The diagnostic analysis of the budgets of these forms of energy has been carried out using the sigma coordinate system. Bearing in mind the explosive nature of the cyclogenesis studied here, the analysis is separated into two parts: one associated with the deepening phase of the cyclone and the other with its filling phase. The results are presented and discussed as time–height cross-sections of the energy contents; as time-averaged vertical distributions of budget components; and as vertically averaged budgets. The analysis revealed contrasting differences in the energy budgets in each phase of the development of the system.

1. INTRODUCTION

Numerous investigations have shown that mid-latitude cyclonic systems are very active components of the general circulation as far as its energetics is concerned. Studies on the energetics of extratropical cyclones have been reviewed by Smith (1980). The major emphasis in most of these diagnostic studies is to evaluate the role played by cyclonic disturbances in generating, transforming and transporting or dissipating large-scale kinetic energy (Palmén 1958; Sechrist and Dutton 1970; Petterssen and Smeybe 1971; Smith 1973a, b; Vincent and Chang 1975; Kung and Baker 1975; Ward and Smith 1976; Chien and Smith 1977; Kung 1977; Fuelberg and Scoggins 1978; Michaelides 1983).

The budgets of other forms of atmospheric energy in the vicinity of individual mid-latitude cyclones have not been evaluated thoroughly. Regarding the available potential energy (Lorenz 1955), only a single component of its budget, namely the generation of available potential energy, has been given special attention (Danard 1966; Widger 1969; Bullock and Johnson 1971; Vincent et al. 1977). The budget of latent heat in the atmosphere during a period of cycloic development over North America has been presented by Palmén and Holopainen (1962).

More effort has recently been made in investigating the budgets of more than one form of mid-latitude cyclone atmospheric energy simultaneously (Robertson and Smith 1983; Smith and Dare 1986; Michaelides 1987). This composite approach is certainly more effective in the analysis of the energetics, because a more complete picture of the flow of energy of such disturbances may be attained. The present article forms a further endeavour within this framework.

The purpose of the study is to present an analysis of the kinetic energy, sensible heat and latent heat budgets from 00 GMT 17 March to 12 GMT 20 March 1981 over the central Mediterranean. During this period, this region experienced cycloic activity of unusual intensity. From a synoptic point of view, this particular system deserves special attention, as will be seen in the synoptic discussion. Because of the large pressure falls
noted (22 hPa in 24 hours) the depression we study falls in a class of cyclonic systems termed 'bombs' (see Sanders and Gyakum 1980; Karacostas and Flokas 1983; Conte 1986).

The budgets are expressed in a coordinate system with $\sigma$, the vertical coordinate, $\sigma = p/p_s$, where $p$ is the pressure and $p_s$ the surface pressure (Phillips 1957). The properties of this system are discussed by Holton (1979) and Simmons and Burridge (1981).

2. ENERGY BALANCE RELATIONSHIPS

In the $\sigma$-coordinate system an expression for the continuity equation is given by (see Haltiner 1971)

$$\frac{\partial p_s}{\partial t} = - \{ \nabla_\sigma \cdot (p_s V) + \sigma \partial (p_s \hat{\sigma})/\partial \sigma \}$$

where $\nabla_\sigma$ is the lateral gradient on a constant $\sigma$ surface (symbols are defined in the appendix).

The equation of motion is given by

$$\frac{\partial V}{\partial t} + (V \cdot \nabla_\sigma) V + \sigma \partial V/\partial \sigma + \nabla_\sigma \Phi + \sigma \alpha \nabla_\sigma p_s + f k \times V + F = 0.$$  

(2)

Taking the dot product of $V$ with each term in this equation, multiplying the resulting relationship by $p_s/g$ and then adding Eq. (1) multiplied by $k/g$, an expression for the kinetic energy balance is obtained:

$$\frac{\partial}{\partial t} \left( \frac{p_s k}{g} \right) = - \left\{ \nabla_\sigma \cdot \left( \frac{p_s k V}{g} \right) + \frac{\partial}{\partial \sigma} \left( \frac{p_s k \hat{\sigma}}{g} \right) \right\} - \frac{p_s V}{g} \cdot (\nabla_\sigma \Phi + RT \nabla_\sigma \ln p_s) - \frac{p_s V \cdot F}{g}$$

where $k = \frac{1}{2} V \cdot V$ is the kinetic energy per unit mass. Using the diagnostic relation (see Savijärvi 1982)

$$- \left\{ \nabla_\sigma \cdot (\Phi p_s V) + \frac{\partial}{\partial \sigma} (\Phi p_s \hat{\sigma}) + \frac{\partial}{\partial t} \left( \frac{\Phi \sigma \partial p_s}{g} \right) \right\}$$

$$= -p_s V (\nabla_\sigma \Phi + RT \nabla_\sigma \ln p_s) - (p_s \alpha \omega)$$

we may rewrite Eq. (3) as follows

$$\frac{\partial}{\partial t} \left( \frac{p_s k}{g} \right) = - \left\{ \nabla_\sigma \cdot \left( \frac{p_s k V}{g} \right) + \frac{\partial}{\partial \sigma} \left( \frac{p_s k \hat{\sigma}}{g} \right) \right\} -$$

$$- \left\{ \nabla_\sigma \cdot \left( \frac{\Phi p_s V}{g} \right) + \frac{\partial}{\partial \sigma} \left( \frac{\Phi p_s \hat{\sigma}}{g} \right) + \frac{\partial}{\partial t} \left( \frac{\Phi \sigma \partial p_s}{g} \right) \right\} - \frac{p_s \alpha \omega}{g} - \frac{p_s V \cdot F}{g}$$

(5)

This balance equation states that changes in kinetic energy ($KE = p_s k/g$) are due to the total flux convergence of $KE$ (first term on the r.h.s., denoted symbolically by BKE), the generation of $KE$ by the pressure work (second term, denoted by GKE), the adiabatic conversion of sensible heat into $KE$ (third term, denoted by CSK) and the dissipation of $KE$ by frictional processes (last term, denoted by DKE).

The first law of thermodynamics may be written as

$$\frac{\partial}{\partial t} (c_p T) + \nabla_\sigma \cdot (c_p T V) + \frac{\partial}{\partial \sigma} (c_p T \hat{\sigma}) = \alpha \omega + Q$$

(6)

where $c_p T$ is enthalpy, commonly referred to as sensible heat, as opposed to latent heat.
Multiplying this relationship by $p_s/g$ and adding the continuity equation multiplied by $c_p T/g$, leads to the balance equation for sensible heat:

$$\frac{\partial}{\partial t} \left\{ \frac{c_p p_s T}{g} \right\} = - \left\{ \nabla \cdot \left( \frac{c_p p_s T V}{g} \right) \right\} + \frac{\partial}{\partial \sigma} \left\{ \frac{c_p p_s T \sigma}{g} \right\} + \frac{p_s \alpha \omega}{g} + \frac{p_s Q}{g}. \quad (7)$$

This relation states that changes in sensible heat ($SH = c_p p_s T/g$) are due to the flux-convergence of sensible heat (first term on the r.h.s., denoted by BSH), the adiabatic conversion of KE into SH (second term, denoted by CKS) and contributions from diabatic processes (last term, denoted by DSH). The diabatic processes which are considered here are radiation, conduction and release of latent heat. Contributions to sensible heat from frictional processes are quite small and are not considered here.

Based on conservation principles, an equation for the water vapour balance in the atmosphere may be written as

$$\frac{\partial q}{\partial t} + V \cdot \nabla q + \sigma \frac{\partial q}{\partial \sigma} = M \quad (8)$$

where $M$ represents sources or sinks of water vapour in the atmosphere. Multiplying this equation by $p_s \lambda/g$ and adding Eq. (1) multiplied by $\lambda q/g$, the resulting balance equation for latent heat takes the form

$$\frac{\partial}{\partial t} \left\{ \frac{p_s \lambda q}{g} \right\} = - \left\{ \nabla \cdot \left( \frac{p_s \lambda q V}{g} \right) \right\} + \frac{\partial}{\partial \sigma} \left\{ \frac{p_s \lambda q \sigma}{g} \right\} + \frac{p_s \lambda M}{g}. \quad (9)$$

This equation states that changes in latent heat ($LH = p_s \lambda q/g$) originate from the flux convergence of LH (first term on r.h.s., denoted by BLH) or from the net contribution from sources and sinks of water vapour (last term, denoted by DLH). Such local sources and sinks of moisture are identified as evaporation and condensation, respectively.

The balance equations (5), (7) and (9) are written in symbolic form as

$$\frac{\partial KE}{\partial t} = BKE + GKE + CSK + DKE \quad (10a)$$

$$\frac{\partial SH}{\partial t} = BSH + CKS + DSH \quad (10b)$$

$$\frac{\partial LH}{\partial t} = BLH + DLH. \quad (10c)$$

For the adiabatic conversion terms it can easily be verified that CSK = −CSK. These terms describe an adiabatic and reversible process by which SH is transformed into KE ($\alpha \omega < 0$, CSK>0) and vice versa ($\alpha \omega > 0$, CSK>0). The set of equations (10a, b and c) are diagrammatically presented in Fig. 1.

3. DATA AND COMPUTATIONAL PROCEDURES

The data required for calculating the component parts of Eqs. (10a, b and c) are taken from the archives of the European Centre for Medium Range Weather Forecasts. These data refer to the 00 GMT and 12 GMT synoptic times within the period from 00 GMT 17 March to 12 GMT 20 March 1981.

The results of an analysis involving computations of the kind required by Eqs. (10a, b and c) are certainly dependent upon the data source (see Kung and Tanaka 1983). Bengtsson et al. (1982) describe the sources of the original data and the assimilation procedure used at the ECMWF which makes use of a three-dimensional multivariate optimum interpolation and a nonlinear mode initialization scheme (see Temperton and Williamson 1981; Williamson and Temperton 1981).
The energy balance calculations in the present study are area averages over the computational region shown in Fig. 2. This region is bounded by meridians 13-125° and 30°E and by latitude circles 30° and 43-125°N. The ECMWF grid used is a regular latitude–longitude grid with 1-875° resolution, the Greenwich meridian and the poles defining a grid-line. Therefore, meteorological fields over the computational region are determined at 80 regularly spaced grid-points. The finite difference schemes used for the
calculations of terms in Eqs. (10a, b and c) are described by Burridge (1980). They are based on a staggered Arakawa C scheme in which the zonal wind component points and the meridional wind component points are displaced with respect to the other parameter points in the east–west and north–south directions, respectively.

The instantaneous values of all the terms in Eqs. (10a, b and c) except DKE, DSH and DLH have been estimated at each of the synoptic times in the period under consideration. Such instantaneous values are area averages taken over the 80 grid-points of the computational area. It can be shown by Gauss’ theorem that for horizontal flux divergences this gives the same result as the line integral along the boundary of the area (see Savijärvi 1981). The GKE term has been calculated using Eq. (4). Rates of change of KE, SH and LH have been estimated as ratios of finite differences of energy contents at consecutive times over the 12 h period. The fluxes and conversion terms have been estimated as arithmetic means at consecutive times. Finally, DKE, DSH and DLH have been estimated as residuals to each of the respective energy balance relationships.

The \( \sigma \) levels referred to in the present study are shown in Table 1, together with the weights assigned to each level, which have subsequently been used to calculate the vertical means of energy components. Such a vertical mean is determined as the sum \( \sum X_i w_i \) \( (i = 1 \text{ to } 15) \) where \( X_i \) denotes the calculated value of an energy component at the \( i \)th \( \sigma \) level, and \( w_i \) its respective weight.
TABLE 1. SIGMA LEVELS AND THEIR ASSOCIATED WEIGHTS

<table>
<thead>
<tr>
<th>i</th>
<th>$\sigma$</th>
<th>$w_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.025</td>
<td>0.0504889</td>
</tr>
<tr>
<td>2</td>
<td>0.077</td>
<td>0.0531852</td>
</tr>
<tr>
<td>3</td>
<td>0.132</td>
<td>0.0579259</td>
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<td>0.0640000</td>
</tr>
<tr>
<td>5</td>
<td>0.260</td>
<td>0.0706963</td>
</tr>
<tr>
<td>6</td>
<td>0.334</td>
<td>0.0773037</td>
</tr>
<tr>
<td>7</td>
<td>0.414</td>
<td>0.0831111</td>
</tr>
<tr>
<td>8</td>
<td>0.500</td>
<td>0.0874074</td>
</tr>
<tr>
<td>9</td>
<td>0.588</td>
<td>0.0894815</td>
</tr>
<tr>
<td>10</td>
<td>0.678</td>
<td>0.0886222</td>
</tr>
<tr>
<td>11</td>
<td>0.765</td>
<td>0.0841185</td>
</tr>
<tr>
<td>12</td>
<td>0.845</td>
<td>0.0752593</td>
</tr>
<tr>
<td>13</td>
<td>0.914</td>
<td>0.0613333</td>
</tr>
<tr>
<td>14</td>
<td>0.967</td>
<td>0.0416296</td>
</tr>
<tr>
<td>15</td>
<td>0.996</td>
<td>0.0154370</td>
</tr>
</tbody>
</table>

Area-averaged values of the surface pressure, $p_s$, have been calculated and are presented in Table 2. Using these values, 12 h pressure tendencies have been calculated and are also presented in Table 2. Negative tendencies indicate an overall pressure fall. On the basis of these pressure tendencies, the period under consideration has been separated into two sub-periods: the first from 00 GMT 17 March to 00 GMT 19 March, referred to as the deepening phase; and the second from 00 GMT 19 March to 12 GMT 20 March, referred to as the filling phase. Composite time-averages of the energy balance terms associated with each of the two phases of the present cyclonic development have been formed and are discussed separately.

TABLE 2. AREA-AVERAGED SURFACE PRESSURE AND 12h-MEAN PRESSURE TENDENCIES

<table>
<thead>
<tr>
<th>Day/Time(GMT)</th>
<th>$p_s$(hPa)</th>
<th>$\frac{\partial p}{\partial t}$(10$^{-2}$hPa s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17/00</td>
<td>997.21</td>
<td>-2.08</td>
</tr>
<tr>
<td>17/12</td>
<td>996.31</td>
<td>-9.00</td>
</tr>
<tr>
<td>18/00</td>
<td>992.42</td>
<td>-13.61</td>
</tr>
<tr>
<td>18/12</td>
<td>986.54</td>
<td>-6.57</td>
</tr>
<tr>
<td>19/00</td>
<td>983.70</td>
<td>-2.71</td>
</tr>
<tr>
<td>19/12</td>
<td>984.87</td>
<td>15.09</td>
</tr>
<tr>
<td>20/00</td>
<td>991.39</td>
<td>8.54</td>
</tr>
<tr>
<td>20/12</td>
<td>995.08</td>
<td></td>
</tr>
</tbody>
</table>

4. SYNOPTIC DISCUSSION

The cyclonic system studied in the present analysis is initially identified as a Saharan depression. This term has usually been reserved for cyclonic disturbances originating in the area south of the Atlas range (HMSO 1962; Prezerakos 1985; Zohdy 1986).

The developmental phases of the system can be followed in the sequence of synoptic analyses depicted in Figs. 3 to 10. A surface low to the south of the Atlas range is identified at 00 GMT 17 March 1981 (Fig. 3(a)). The cold and warm fronts express a distinct separation of air masses in the area of the low, as can be identified in the 850 hPa analysis of Fig. 3(b), which reveals a marked low-level baroclinic zone. In the 500 hPa analysis shown in Fig. 3(c) an upper trough is seen to approach the area from the west. From a synoptic point of view the Saharan depression is a rather insignificant feature at this stage, associated with no important weather phenomena. The surface low becomes
activated when the upper trough approaches the area close enough so that an area of positive vorticity aloft is advected over the region of warm air advection. The resulting intensification of the circulation at mean sea level may be seen in Fig. 4. The central pressure of the cyclonic system fell about 8 hPa in 12 hours. The intensification of the circulation resulted in a sharp increase of the wind at the surface which caused extensive rise of desert dust. At 00 GMT 18 March the centre of the depression is found over the
Mediterranean waters, south of Sicily (see Fig. 5(a)). The movement of the depression to the north-east is in the direction of the thermal advection, clearly demonstrated in Fig. 5(b), and in the direction of positive vorticity advection aloft. From a stationary front situation at 00 GMT 17 March, a wave-type frontal depression developed within 24 hours. The prevailing feature in the upper levels is an extensive diffluent trough (see Fig. 5(c)) which is found at the southernmost part of the eastern flank of an omega

Figure 5. As Fig. 3 but for 18 March.

Figure 6. As Fig. 4 but for 18 March.
blocking pattern over the north Atlantic. The southward transfer of kinetic energy by the jet-stream on the eastern flank of the omega blocking results in an increase of the mean regional absolute vorticity (see Palmén and Newton 1969). Also, colder mid-tropospheric air is advected over the warmer desert air, leading to a rapid destabilization of the air mass in the vicinity of the depression and therefore to an enhancement of the available potential energy in the area (Lorenz 1967). Explosive rejuvenation of the surface low occurs where the baroclinicity is more pronounced. At 00 GMT 19 March the depression appears to have two distinct centres: one over Greece, 984 hPa, and the other over central Yugoslavia, 992 hPa (Fig. 7). The latter should correspond to the original centre from the Sahara and the former to a new development in response to the appearance of the jet-streak and the short wave superimposed on the eastern flank of the omega blocking. Cyclogenesis over the Mediterranean would have occurred even in the absence of the pre-existing depression of Saharan origin. However, this depression contributed both to the enhancement of the thermal contrast and the destabilization of the air.

The rejuvenation phase of the original system takes place in the 24-hour interval from 00 GMT 18 March to 00 GMT 19 March (Figs. 5, 6 and 7). Duststorms continue to be observed within the warm air. A marked rain band forms in the cold air ahead of and close to the old centre and in the region where the new centre forms. Thunderstorms of very local character are reported in the colder air, mainly near the new centre and to the west of it. No marked precipitation occurs in the warm and dry air mass.

Figure 7. As Fig. 3 but for 19 March.
Filling of the cyclone commences after 00 GMT 19 March, with a further propagation of the system to the east. The last stages of the development of the system appear in Figs. 8, 9 and 10. During this phase, precipitation of lesser intensity continues near the remaining centre and the occluded front. Dust carried upwards and subsequently advected north-eastwards is responsible for the hazy conditions and the coloured rain observed over the central and eastern Mediterranean for many days.
5. ENERGY CONTENTS

A time–height cross-section of KE is shown in Fig. 11. At $\sigma = 0.193$ KE takes its maximum value soon after the development of the surface cyclone but with the filling of the system, a displacement of the maximum towards lower levels is observed. At 12 GMT 19 March all tropospheric levels attain their maximum KE.

A time–height analysis for SH is presented in Fig. 12. During the first 24 hours little change in the SH content is observed at any level. However, soon afterwards a gradual cooling is noted at low and medium tropospheric levels. At 12 GMT 19 March these levels attain their lowest SH content. It is worth noting that this is the time at which the KE content of these levels is a maximum, as discussed above. There is little variation with time of the SH content at upper levels.

The temporal variation of the LH content is shown in Fig. 13. At all times, the amount of water vapour is highest near the ground and decreases upwards. Above $\sigma = 0.414$ LH decreases sharply. Starting from 00 GMT 17 March, a tendency for increasing
atmospheric water vapour at all levels is observed. At low levels a maximum is reached 12 hours later. Between 00 and 12 GMT 18 March a peak in LH content is attained at all levels. After this time there exists a distinct decrease in LH content at all levels, ascribed primarily to the condensation of water vapour which is subsequently precipitated to the surface.

In a case study of a cyclone over north America, Danard (1964) found that the release of latent heat by condensation had an effect on vertical velocities. Amplification of the vertical velocity was induced by an increase of low-level convergence and high-level divergence. He also states that the modifying effects of the release of latent heat are considered to play a role after development starts. In the present case, soon after its formation the cyclone deepened further with no significant precipitation being produced. This supports the above view that latent heat release played no important role in the initiation of cyclogenesis.

6. Energy balance

(a) Deepening phase

The balance of KE in the deepening phase as determined by Eq. (10a) and at all 15 $\sigma$ levels is shown in Fig. 14. From small increases within the boundary layer, $\delta KE/\delta t$ increases upwards to a maximum of 6.23 W m$^{-2}$ at $\sigma = 0.845$. A secondary maximum is noted at about the level of the polar front jet-stream. The stratospheric layers experience a considerable decrease in KE.

From very small negative values within the boundary layer, BKE becomes positive and increases upwards, reaching its maximum, 68.53 W m$^{-2}$ at $\sigma = 0.193$. This implies that the lower atmospheric layers export small amounts of KE, whereas the upper layers are quite dependent on imports of KE.

The generation of KE by pressure work, GKE, is a considerable sink of KE throughout the atmosphere. This sink increases from the surface upwards, up to a maximum $-1787.17$ W m$^{-2}$ at the highest tropospheric levels.

Production of KE by CSK occurs at essentially all levels. Actually CSK is the most important of all sources of KE. Kung (1966), studying a number of cyclonic disturbances,
concludes that kinetic energy augmentation through this conversion process takes place in the troposphere with a maximum in the mid-troposphere. We must recall here that $\omega = -\frac{p_\omega}{g}$ and that conversion of SH into KE occurs when, on the large scale, $\omega < 0$. This process has customarily been identified as one of sinking of colder and rising of warmer air (Margules 1903), although, as Lorenz (1955) states, this interpretation cannot be defended in a rigorous manner. Bearing this in mind, on the average, $\omega < 0$ at almost all levels, indicating that an overall sinking of colder and rising of warmer air takes place in this phase.

Although DKE has commonly been referred to as the dissipation term (Fuelberg and Browning 1983), the way it is calculated here implies that frictional processes are not the only ones represented by DKE: this term also represents the accumulated analysis error of all the other terms in the energy balance equation as well as transfer of KE from scales of motion not resolved with the grid used, to scales of motion adequately resolved, and vice versa. Because of problems in calculating this term directly, the procedure for estimating DKE as a residual in a kinetic energy balance relationship has been followed in general. This appears to be a convenient way of estimating DKE without reference to specific theoretical aspects of frictional processes (Kung 1966; Kung and Smith 1974). Indeed, Holopainen (1973) attempted to isolate frictional processes from balance requirements in what he terms “a frustrating experiment”.

The shape of the vertical profile of DKE is characterized by two maxima: one near the surface and the other at the jet-stream level. This dual maximum of dissipation appears to be a common characteristic of mid-latitude storm regions (Kung and Baker 1975; Kung and Tsui 1975; Tsui and Kung 1977).

The kinetic energy generation term, GKE, largely opposes the conversion term CSK. Indeed, most of the SH converted into KE goes to compensate the losses due to the action of pressure forces and only a small part of it is directed towards enhancing the local tropospheric wind field. Moreover, within the stratospheric layers, the losses due to the pressure forces are dominating the conversion sources and in spite of a net input
of kinetic energy, the local flow is seen to decelerate. The residual term, DKE, is quite small at all levels except within the boundary layer and at the jet-stream level. The behaviour of the above balances is ascribed to the fact that the large-scale flow over the computational area is nearly quasi-geostrophic (see Savijärvi 1981).

The vertical distribution of the components of the sensible heat budget is shown in Fig. 15. There is a net cooling at all levels from the surface to $\sigma = 0.260$. A small net warming is observed in the layers above, up to $\sigma = 0.132$. The rate of change of SH becomes negative in the uppermost layers.

Export of SH takes place only at the lowest and the uppermost layers. Above the boundary layer, sensible heat is seen to be imported with a maximum at the jet-stream level. Despite this appreciable import of SH, the rate of change of SH is negative, as discussed above. This is due to the large conversion of SH into KE taking place at almost all levels, with a maximum in the upper troposphere. Kung and Baker (1975), investigating a number of mid-latitude cyclonic disturbances, also found a maximum of adiabatic conversion at the jet-stream level.

The term DSH represents all non-frictional diabatic sources and sinks of sensible heat. Within the lower atmospheric layers DSH acts as a sink of SH. This sink is most likely accounted for by a net flux of heat from the lower tropospheric warmer air to the underlying relatively cooler Mediterranean waters. A substantial source of SH is observed in the uppermost layers, which is most likely to be radiative.

The terms in Eq. (10c) as distributed in the vertical are shown in Fig. 16. The rate of change of LH reveals a reduction of water vapour from the surface up to about $\sigma = 0.678$ with a maximum in the lowest layers. Within the boundary layer the decrease in LH is almost equal to an export of LH from these layers. Above $\sigma = 0.845$ the role of BLH changes and it becomes a source of LH with a maximum in mid-troposphere. Palmén and Holopainen (1962), investigating a cyclonic disturbance over North America, deduced a similar water vapour accumulation within the area occupied by the cyclone.

The effect of DLH is negative at levels from the surface to $\sigma = 0.193$ and nearly zero above. The large negative values of DLH, mainly needed to compensate for the flux convergence of water vapour in the middle and upper troposphere, are ascribed to
condensation of water vapour occurring within the tropospheric layers during this phase. This noticeable condensation is certainly associated with the widespread and intense precipitation produced by the disturbance soon after the deepening of the surface cyclone commenced. Within the boundary layer DLH is negative, but of considerably minor importance for balance, compared with the large values in the mid-troposphere. The interpretation of this is that condensation of water vapour by lifting of air takes place primarily above the boundary layer. Also, evaporation of water vapour from the earth’s surface pumped into the atmosphere through the boundary layer acts as a source of water vapour.

The mean vertical budgets for KE, SH and LH during the deepening phase of the cyclone are shown in Fig. 1(a). There is a KE increase at a mean rate of 3.28 W m\(^{-2}\) in this phase. A strong conversion of SH into KE is noted which, however, is almost offset by an intense sink of KE due to the pressure work, namely GKE. The net result of both these processes is a loss of KE amounting to 3.16 W m\(^{-2}\). The role of KE transfer into the area is therefore of considerable importance during this phase. Indeed, BKE is a source of KE at a mean rate of 17.09 W m\(^{-2}\). As expected from frictional considerations, DKE dissipated KE at a rate of 10.65 W m\(^{-2}\).

The mean temperature of the atmosphere in the area under study showed a net decrease, thus the SH is changed at a mean rate of -32.6 W m\(^{-2}\). This decrease is largely accounted for by the difference of a net import of SH amounting to 54.6 W m\(^{-2}\) and a net conversion of SH into KE amounting to 88.7 W m\(^{-2}\). The role of diabatic processes is one order of magnitude smaller than all the other terms in the balance of SH. Indeed, DSH is a mere 14.19 W m\(^{-2}\). This implies that once cyclogenesis is initiated

![Figure 16. As Fig. 14 but for latent heat.](image-url)
on a baroclinically unstable airflow, the role of diabatic heating in the energetics of the system is of minor importance. On the contrary, the dynamical processes are the most important components of the energy balance.

The mean balance of LH shows a net decrease of LH, at a mean rate of $-38.14 \, \text{W m}^{-2}$. This net loss of moisture is accounted for by the difference between the high rates in DLH sinks and BLH sources. The mean import of LH into the area is at a rate of $112.13 \, \text{W m}^{-2}$. As discussed previously, DLH changes by evaporation of water into the area of the cyclone or by condensation of water vapour. There exists a mean net loss of moisture by DLH amounting to $150.27 \, \text{W m}^{-2}$. It is therefore seen that condensation much exceeds evaporation in this phase.

(b) Filling phase

From the vertical distribution of the components of the KE balance during the filling phase in Fig. 17, it is seen that the tropospheric layers experience a decrease in KE. A marked increase in KE is also observed at the jet-stream level. The decrease in the lower and middle tropospheric layers is partly accounted for by an export of KE.

The generation of KE by pressure work, GKE, is negative in the lowest layers and the upper troposphere, and positive in the middle troposphere and the stratosphere. This term largely opposes the conversion term, CSK. This conversion is a source of KE at low levels but changes sign above $\sigma = 0.845$. Conversion of SH into KE is re-established in the upper tropospheric levels but a reversal is observed in the stratosphere.

The dissipation term, DKE, takes considerably larger negative values in the near-surface layers. Bearing in mind the decrease in KE observed during this phase, the behaviour of DKE is interpreted as a manifestation of the drastic braking process imposed by friction near the surface. The dissipation increases sharply away from the surface and becomes positive at $\sigma = 0.845$ and 0.765. In the rest of the troposphere DKE remains a sink of KE. However, DKE is positive at $\sigma = 0.193$ and 0.132, presumably responding to the large increase in KE observed at these levels. Apparently, such positive values are

![Figure 17. Vertical profiles of the area- and time-averaged terms of the kinetic energy balance during the filling phase. Units: W m$^{-2}$.](image-url)

in contradiction to the role ascribed to dissipation, which is considered to act as a sink of KE. However, as discussed above, the computationally deduced DKE may contain contributions from subgrid-scale motions too. It is therefore noted that these unresolved motions, acting as a source of KE, overcompensate frictional losses of KE at levels where large increases in KE take place. Similar results have been obtained by many other investigators (Smith 1973b; Vincent and Chang 1975; Kung and Tsui 1975; Kung and Baker 1975; Tsui and Kung 1977; Chien and Smith 1977).

The vertical profiles of components of the sensible heat balance are shown in Fig. 18. The rate of change of SH is positive at almost all levels. The contribution of the flux convergence term in the SH balance is rather erratic in the vertical. The most important features of the BSH vertical profile are a considerable import of SH at about the level of maximum increase in SH and a considerable export of SH just below and above the level of maximum wind.

The conversion term CKS presents a primary maximum at $\sigma = 0.678$ and a secondary maximum at the level of maximum wind. At $\sigma = 0.334$, CKS reaches its minimum, $-205.68 \, \text{W m}^{-2}$. The diabatic processes represented by DSH lead to a net heating of almost all layers, which is more pronounced near the surface. With the breaking of the cloud deck and the further clearing of the hazy conditions produced by dust storms, more short-wave radiation reaches the surface and is thereafter transferred as sensible heat upwards through the boundary layer.

The profiles of the terms of the LH balance equation are shown in Fig. 19. A marked net decrease in water vapour is noted within the lower tropospheric layers. At levels from the surface up to $\sigma = 0.678$ there exists a substantial export of LH, with the
maximum of this export occurring at the surface. A pronounced addition of water vapour is calculated at near-surface levels, which is ascribed to increased rates of evaporation from the surface. This interpretation is supported by the fact that the maximum of DLH is observed at the surface. Condensation at low rates continues to take place at middle and upper tropospheric levels.

The column-mean energy budgets are shown in Fig. 1(b). In this period, KE decreases at $-3.95 \text{ W m}^{-2}$. The only source of KE is GKE. The other three components of the KE balance act as sinks of KE. Through the adiabatic conversion process, denoted by CSK, it is seen that the sensible heat reservoir depletes energy from the comparatively smaller reservoir of kinetic energy. Kinetic energy is also lost through the transport term BKE at a rate of $7.07 \text{ W m}^{-2}$. The dissipation term DKE destroys KE at a mean rate of $9.40 \text{ W m}^{-2}$.

All terms in the SH balance equation operate as sources of SH. The adiabatic conversion term has been dealt with above. The transport term BSH is seen to supply small amounts of SH into the area. The diabatic term DSH appears to be the major source of SH, amounting to $207.53 \text{ W m}^{-2}$. The overall effect of CKS, BSH and DSII is a large increase in the heat content over the area. Thus the filling of the cyclone is accompanied by an overall warming of the atmosphere.

The LH balance shows a net decrease in water vapour resulting from the behaviour of both BLH and DLH. Contrary to what was taking place in the deepening phase, water vapour is now seen to be exported from the area at a rate of $59.47 \text{ W m}^{-2}$. However, condensation of water vapour still predominates over any evaporative sources.
7. DISCUSSION

The various processes attending the energy budgets studied exhibit varying levels of temporal and spatial involvement. The marked increase in KE of the tropospheric layers in the deepening phase is followed by a more pronounced decrease in the filling phase (Figs. 14 and 17). This reversal of the rate of change of KE is accounted for by the general weakening of the wind field and the migration of the tropospheric strong winds to the east of the upper trough axis, as may be visualized by contrasting the analyses of Figs. 5 and 9. In the former the entry of the sub-synoptic low into the area marks the appearance of the jet-streak, thus increasing KE over the area, whereas in the latter the strongest wind is found to the east of the low and mostly outside the region under study.

At low levels the role of the transport term BKE is small, on average, becoming increasingly important at higher levels. A significant import enhances KE in the deepening phase, whereas in the filling phase an export of KE takes place. The respective maxima of import and export of KE occur at about the level of maximum wind.

The adiabatic conversion term is by far the most important source or sink of KE at almost all levels (see Figs. 14 and 17). In the deepening phase the role of CSK in generating KE is predominant, that is to say, strong ascending motion takes place in the warmer air and strong descending motion takes place in the colder air. The behaviour of the conversion term is essentially changed in the following phase (Fig. 17). On average, sinking of colder air and rising of warmer air is still observed at low and upper tropospheric levels, but in the middle troposphere warmer air appears to sink and colder air to rise, thus the reservoir of SH depletes energy at the expense of KE. This process has been found to be associated with anticyclonic circulations (Kung and Baker 1975).

Although the dissipation term DKE cannot be considered as a frictional process in the true thermodynamic sense, it is not unrealistic to relate DKE to friction within the boundary layer (see Kung and Tsui 1975). It can be seen from Figs. 14 and 17 that, near the surface, dissipation is more than doubled in the filling phase compared with the deepening phase. Therefore, it can be inferred that frictional processes within the boundary layer become more important in destroying KE during the final stages of the development of the cyclone.

The vertical profiles of the terms in the SH balance differ considerably between the two phases (see Figs. 15 and 18). A substantial cooling in the troposphere during the deepening phase is followed by a comparative warming during the filling phase. This change in behaviour is also depicted in the mean vertical budgets of Figs. 1(a) and (b).

Import of SH at tropospheric levels is the most important source of SH in the deepening phase. It is thought that this is accomplished primarily by horizontal advection of warmer desert air over the area. The role of BSH changes dramatically in the filling phase. Sensible heat continues to be infed at mid-tropospheric levels and at the uppermost stratospheric levels but sensible heat is depleted elsewhere.

Until 00 GMT 18 March, warm advection is predominant over the region under consideration, at almost all tropospheric levels (see Figs. 3 to 5). However, after this time cold advection starts taking place over western and north-western parts of the region. This cold advection is continuously enhanced at a rate which decreases from north-west to south-east (see Figs. 7 and 9). Soon, the north-western part of the area is subject to a pronounced cold advection whereas in the rest of the area the temperature field is subject to much less change. These conditions lead to the establishment of a very intense temperature gradient with a significant component oriented from west to east and thus to the generation of eddy available potential energy, which under suitable conditions may be converted into KE of the cyclone. As seen above, such conditions
soon become present and SH is converted to KE at high rates during the deepening phase.

The rate of change of LH reveals a tendency for loss of atmospheric water vapour (see Figs. 1(a) and (b)). On the average, this loss is more than doubled in the filling phase. In the deepening phase the water vapour balance is largely dependent upon imports of water vapour but in the filling phase there exists a reversal in the performance of BLH. In both phases BLH acts as a sink of LH at low levels, this effect being maximum at the surface (see Figs. 16 and 19). Although not possible to quantify in the present study, such losses may be accounted for by the observed strong convection which carries water vapour from low levels upwards.

The average losses of water vapour expressed by DLH (see Figs. 1(a) and (b)) are certainly related to the widespread and intense precipitation occurring, as mentioned above, largely in the colder air. Evaporative contributions are probably more important near the earth’s surface. The hot and dry tropical continental air mass has a high evaporative demand so when it is advected over the Mediterranean waters evaporation from the sea into the lowest atmospheric layers takes place at high rates.

The findings in the present study reflect that in the deepening phase the mean vertical motion over the computational area is one of ascent and induces a net cooling of the air mass and a large import of sensible heat from the surroundings. This cooling is partially offset by latent heat release. From the joint kinetic energy and sensible heat budgets, it is inferred that the cooling of the air mass results primarily as a response to the working of the pressure forces round the boundary of the chosen region. These terms therefore largely arise from the choice of a region in which there is mean ascent. In the filling phase the roles of the vertical motions and the pressure working forces are changed so that, although much smaller, they both operate in the opposite sense. In this latter phase there is latent heat release at a mean rate of 37 W m$^{-2}$, associated with a diabatic warming of 207 W m$^{-2}$. Assuming heating of the atmospheric volume by conduction proceeds at negligibly small rates, the above imply a rate of radiative warming of 170 W m$^{-2}$ (i.e. about 1.5 degC per day). In the deepening phase, however, there is an implied radiative cooling of 126 W m$^{-2}$ (i.e. about 1 degC per day). Presumably this difference, assuming errors are small, is indicative of the effect of more cloud cover during the deepening phase.

8. CONCLUDING REMARKS

The method for carrying out the budget analysis in the present study is based on an Eulerian computational area. This fixed area was taken large enough to contain the recognizable pattern of the surface cycloonic circulation studied. Bearing in mind that the associated upper cyclonic circulation plays a primary role in the energetics of the system, its inclusion in the area of analysis is also essential. However, this Eulerian method of analysis suffers from a shortcoming which stems from the fact that, although the fixed area is dominated by a well recognized synoptic-scale system, one can also identify other disturbances partially existing in the area (see also Kung 1977). Utilization of an Eulerian system moving with the synoptic-scale eddying motion recognizable at the surface has been presented by Robertson and Smith (1980). However, defining the area occupied solely by the system under study on strict and objective grounds is not an easy task.

Kinematically estimated fields of large-scale vertical motion are derived from computed divergence fields. A net convergence or net divergence computed from the wind field leads to an erroneous local mass imbalance. To maintain mass balance, vertically integrated divergence should be zero or at least negligibly small. Such imbalance should be avoided in diagnostic studies and carefully removed from the data used. The ECMWF
initialized analyses have a small or negligible local mass imbalance (Savijärvi 1982), giving the diagnostic results more credibility.

A difficulty in the interpretation of results in $\sigma$ coordinates arises because $\sigma$ surfaces do not generally coincide with conventional pressure or height surfaces. In particular, pressure and sigma surfaces would coincide if there were no variations in $p_s$. However, this is not the case in general and transformation of results obtained in the $\sigma$-coordinate system into the more comprehensive pressure system is not a straightforward affair (see Mahlam and Moxim 1976).

In the deepening phase the cyclone is quite dependent upon imports of KE (see Fig. 14). This finding is in agreement with a remark by Palmén and Newton (1969) that it is possible that kinetic energy may be imported into the area of the cyclone, this import being quite significant in the early stages of its development. Similar results have been obtained by Michaelides (1987). From an energetics point of view, it is through similar studies that the extent of dependence of individual synoptic-scale systems upon other atmospheric systems can be established (see Smith and Horn 1969).

The role of CSK (or CKS) is very important throughout the development of the system. However, it is very interesting to note that the ageostrophic component of the wind acts in different ways in the deepening and the filling phases. In the former phase an intense conversion of SH into KE is established, whereas in the latter phase the opposite occurs, though at a lower rate.

The balance of water vapour reveals an important characteristic of the cyclone we study, regarding the availability of water vapour in the atmosphere. The net effect of decreasing water vapour content by condensation and increasing water vapour content by evaporation, namely DLH, is to create a deficit, or, equally, that local sources of water vapour play no important role in the balance of water vapour, being largely offset by condensation and precipitation. The transfer of water vapour during the deepening phase within the atmospheric column under study is by far the most important source of water vapour.

The residual terms DKE, DSH and DLH depend very much on divergent related quantities. Therefore, their values calculated in this study depend on the quality of divergent winds in the set of data used (see also Savijärvi 1981).

With regard to their energetics, it is now clear that cyclonic disturbances exhibit different properties during their developmental phases (see Petterssen and Smebye 1971; Smith 1973b; Vincent and Chang 1975; Ward and Smith 1976; Michaelides 1987). The present study provides further evidence supporting this postulate.

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APPENDIX

List of symbols

c_p specific heat of dry air at constant pressure
f Coriolis parameter
g magnitude of the acceleration of gravity
k = \frac{1}{2}V \cdot V, kinetic energy per unit mass
\mathbf{k} unit vector directed upward
p, p_s pressure, surface pressure
q specific humidity
t time
F frictional force
M rate of water vapour supply or loss
Q rate of diabatic heating or cooling
R gas constant for dry air
T temperature
V horizontal velocity vector
\alpha specific volume
\lambda latent heat of condensation
\sigma, \dot{\sigma} = \frac{p}{p_s}, \frac{d\sigma}{dt}
\omega = \frac{dp}{dt}
\Phi geopotential
\nabla_\sigma horizontal gradient on a surface of constant \sigma.

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