The equatorial Pacific Ocean prior to and during El Niño of 1982/83—a normal mode model view

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SUMMARY

An equatorial ocean model with multiple vertical and horizontal modes is developed for the Pacific Ocean, and run for the five years 1979 to 1983. In the year prior to the warm event, or El Niño, of 1982/83 the model shows an unusually strong and delayed seasonal Kelvin wave generated in the western Pacific. By analogy with the ocean–atmosphere coupling mechanism postulated by several recent authors for explaining the El Niño/Southern Oscillation cycle, it is suggested that this wave could ultimately have been instrumental in causing the subsequent El Niño to be so strong. The course of events during El Niño is described in terms of equatorial normal modes and related to the evolving wind field. The strength and importance of the second vertical mode during the event is emphasized. Evidence for a coupled system decay mechanism is also found. The model is compared with several types of observations, but principally the expendable bathythermograph (XBT) data of the Pacific Ship of Opportunity Programme. These data are analysed from a normal mode perspective which is related to that of the model, supporting the main conclusions derived from it. Such data have not been analysed previously in this manner, converting observed variables directly to theoretical equatorial wave amplitudes. The adequacy of the data for such analysis is discussed.

1. INTRODUCTION

The warm event, or El Niño, in the tropical Pacific during 1982/83 was exceptionally strong (Rasmusson and Wallace 1983), and better observed than previous such climatic anomalies. It generated a large amount of scientific interest and endeavour in order to try to understand, and predict, such quasi-periodic events. The theory of equatorial waves (e.g. Cane and Sarachik 1981 and Gill 1982) went some way to explaining the development of El Niños, and models related to this theory (e.g. Blundell and Gill 1983, Busalacchi and Cane 1985 and Inoue and O’Brien 1986) were successful in reproducing integrated variables such as sea level. Primitive equation ocean models (e.g. Philander and Seigel 1985 and Latif et al. 1985) gave rather better descriptions, but not an elucidation of the physical processes behind the El Niño/Southern Oscillation (ENSO) cycle. The development of models coupling the ocean and atmosphere and their interpretation (e.g. Cane et al. 1986, Schoepf and Suarez 1988, Battisti 1988 and Graham and White 1988) has led to many new ideas on why the ENSO cycle occurs. Wave theory, both equatorial and off-equatorial, remains as a pivotal component of several of these ideas.

In this paper we attempt to analyse the expendable bathythermograph (XBT) data of the Pacific Ship of Opportunity Programme (White et al. 1985) in such a way as to obtain some quantitative information on equatorial wave processes. To our knowledge, XBT data have not been so analysed previously. We also use a model similar to that of Busalacchi and Cane (1985) to examine the El Niño through a dynamical representation involving multiple normal vertical, and horizontal, wave modes. We run our model both before and after the El Niño peak, forcing it with Florida State University analysed winds (e.g. Inoue and O’Brien 1984b), and use the information provided directly by the model equations to examine the modal structure of the warm event, and its prelude and postlude. Some previous workers, like Tang and Weisberg (1984), have used idealized wind forcing fields for such studies but this cannot reproduce ocean variables during such a complicated period as 1981–83. The numerical model is compared with various observational studies such as the Pacific Ship of Opportunity Programme (White et al. 1985) already mentioned,
and observations from Pacific tide gauge stations (Wyrtki 1984, 1985). Interest will be focused on whether the theory of equatorial modes, both horizontal and vertical, is of use in describing the course of events in the tropical Pacific in the early 1980s, and whether support for ideas linking the El Niño prelude and decay to wave processes is found.

A major tool for interpreting the model results, and for investigating the ocean, is the XBT data referred to above. A preliminary study has analysed these data for equatorial horizontal mode signals (Gill and Bigg 1985) and provided suggestions of equatorial Kelvin and Rossby wave activity in 1981 and 1982. A relation between the multimode model's structure and the horizontal mode formulation for the XBT data can be found. This enables comparison between the modal amplitudes of the model and data. Other fields derived from these data, such as surface elevation, are also presented. A hitherto unremarked event revealed by the model, and supported by the data, is the passage of a first mode Kelvin wave, and its reflected Rossby waves, across the entire Pacific in 1981. Such a clear signal of a Kelvin wave and its reflected Rossby waves is not found at any other time during the period 1979–1983.

The model and data can also be used to investigate the mechanism proposed by Battisti (1988) for the decay of El Niño. From analysis of a simple idealized coupled ocean–atmosphere model he found that El Niño events ended when oceanic Rossby waves, created by the initial westerly wind anomaly, were reflected at the western boundary of the ocean basin with the impact of the resulting Kelvin waves slowly leading, via ocean–atmosphere interaction, to the decay of the wind anomaly. The present model is ideal for validating such a suggestion. The wave event of 1981 may also be relevant to such ideas (see also Schopf and Suarez (1988) and Graham and White (1988)), as it is suggested that this event led to a pre-ENSO weakening of the tropical easterlies and so perhaps contributed to the strength, if not the initiation, of the 1982/83 El Niño.

The plan of the paper is as follows. The analysis of the XBT data is described in the next section and then the theory of the multimode model is given in section three. Section four deals with the model results and section five with the various data comparisons. We then discuss the conclusions to be drawn from the model runs.

2. XBT DATA ANALYSIS

Temperature records between 20°N and 20°S from the Pacific Ship of Opportunity XBT programme (White et al. 1985) were used to examine the horizontal normal mode structure of the ocean. The maximum depth of the profiles is about 450 m (although a substantial fraction fail short of this); this depth is insufficient for the vertical modes to be resolved. The data were supplied by the Scripps Institute of Oceanography and had already undergone a degree of quality control. In addition, profiles not reaching 300 m depth were rejected from the analysis. The records for June 1979 to the end of 1983 were available. Data within the 5 cross-equatorial strips depicted in Fig. 1 were used; these strips correspond to regular merchant ship routes, and are the only well-sampled areas of the ocean. These tracks were divided into boxes of 1° meridional extent, and monthly averages of various quantities were computed. Any gaps in space or time were filled by linear interpolation. Few data points (typically two or three) went into each monthly average. Figure 14 illustrates the frequency of data-free boxes for the combined bands defined below. Clearly, there are insufficient data for a temporally complete and detailed modal expansion. We simply wish to demonstrate that our model results are consistent with a modal interpretation of the sparse observations.
Plots of the variation of selected isotherm depths with time are shown in Fig. 2. In this figure tracks 3 and 4 have been combined, as have tracks 1 and 2. The depths have been averaged over the range 5°N to 5°S (i.e. about 2 Rossby radii on either side of the equator).

In Fig. 2(a) the western Pacific isotherms (band 1, see Fig. 1) show relatively small perturbations until August 1982 when the temperature of the top 300 m falls dramatically and stays low until March 1983. It should be noted, however, that a slow cooling began earlier, about March or April 1982. This is most evident around 200 m. The flatness of the record in late 1983 is due to a gap in the data available to us rather than a uniformity in the temperature. The central Pacific record, shown in Fig. 2(b), is somewhat different. In mid 1981 there is a shallowing of the thermocline of more than 30 m (a larger isotherm shift than at any time in Fig. 2(a), the western Pacific, prior to mid 1982). During 1982/83 there are two disturbances. The first of these, around May 1982, is relatively small and confined to the upper 150 m. The second is massive and occurs through most of 1983, beginning in the October of the previous year. It is seen throughout the top 300 m, although stronger above 200 m. Turning to the band-3 record for the eastern Pacific (Fig. 2(c)) we find a very different picture. Once again there is little disturbance in the temperature field before 1982, but from October 1982 to January 1983 there is a very large temperature rise throughout the upper water column, as the thermocline deepens drastically. During 1983 the thermal field returns to normal, albeit not smoothly.

In this paper, interest is directed towards equatorial normal mode theory rather than a description of the XBT data set. This set has been well covered in a number of papers including, among others, Meyers et al. (1983), Rebert et al. (1985), White et al. (1985) and Pazan et al. (1986). Readers interested in the ocean climatology during the early 1980s are referred to these studies. In order to investigate the normal modes, the dynamic height, $\eta$, and geostrophic zonal velocity, $u$, need to be derived from the temperature records. The salinity corrections required for the dynamic height calculations were taken from the $T$-$S$ climatology of Emery and Dewar (1982), as few salinity measurements are available for the tropical Pacific during 1979–1983. Kessler et al. (1985) have incorporated
Figure 2. Plots of isotherm depth variation with time in (a) the western Pacific (band 1); (b) the central Pacific (band 2); and (c) the eastern Pacific (band 3).
these few observations into their analysis, producing dynamic height fields varying slightly from ours. As the height calculations of this paper are relative to 300 m, in order to maximize the number of XBT records that can be used, while Kessler et al.'s results are relative to 450 m, it is difficult to compare them directly. However, the gradients are similar, implying similar modal amplitudes. Use of our 450 m records changes the dynamic heights only slightly and gives a much sparser coverage. It should be noted that there is evidence for large surface salinity changes during El Niño (Donguy and Eldin 1985), and that this will introduce significant errors in the computation of $\eta$ and $u$ during 1982 and 1983 (Cooper 1988).

This error can be seen in Fig. 3, which compares dynamic height and sea level anomaly records along or near the three ship bands. The anomaly fields for each of the 5 ship tracks and the sea level were formed by subtracting an average over the 3-year period June 1979 to May 1982. This period was chosen to include as many complete seasonal cycles as possible whilst excluding most of the El Niño. For tracks 1 and 2, and tracks 3 and 4, the anomalies were then averaged to form single bands in the eastern and central Pacific respectively. It is these anomalies that are depicted in Fig. 3.

In Fig. 3(a), depicting the western Pacific sea level comparisons, there is general agreement between the tide gauge results and the XBT dynamic heights, even during the large drop in sea level during 1982/83. The central Pacific band comparisons, shown in Fig. 3(b), also demonstrate this feature. Agreement is reasonable until mid 1982 when in the inter-tropical convergence zone (ITCZ) heavy rainfall is associated with a rise in sea level so that the XBT dynamic heights are underestimating the real effect. Finally, the eastern Pacific dynamic heights, shown in Fig. 3(c), are again consistent with the sea level measurements in general. Unfortunately, there are no islands in the northern end of this band and so the only available comparison is with a coastal station, in this case Buenaventura. This station is some distance from our band 3 and is also subject to a large annual signal in the sea level. The latter, largely confined to the coastal region, is due to the seasonal fluctuation in the position of the ITCZ (Bigg and Gill 1986). Thus comparison with the off-shore XBT heights is poor. Note, however, that the magnitude of El Niño's peak is similar.

To calculate equatorial wave amplitudes, the geostrophic zonal velocity, $u$, is also needed. For the long-wave approximation to the shallow water equations, given in Heckley and Gill (1984), this can be calculated from

$$yu = -g\beta^{-1/2}\partial f/\partial y$$

(1)

where $\beta$ is the gradient of $f$. Close to the equator, where Eq. (1) breaks down, the second-order form

$$u = -g\beta^{-1/2}\partial^2 f/\partial y^2$$

(2)

is used. It should be noted that $\eta$ and $u$ now denote anomalies.

Gill (1982, chapter 11) shows how a direct representation of the equatorial long-wave amplitudes may be found from the two physical quantities $\eta$ and $u$, which can be derived as above from the XBT temperature records. A more formal discussion will be given in the next section, when the theory of the multimode model is given; here it will suffice to define

$$W = g\eta/c_0 + u = \sum_{n=0}^{\infty} W_n(x,t)D_n(y^*)$$

(3)

where $D_n(y^*)$, the equatorial wave eigenmodes, are parabolic cylinder functions of order $n$ (see Gradshteyn and Ryzhik 1980, section 9.24), and $y^* = y/a_0$ is the latitude non-
dimensionalized by the equatorial Rossby radius $a_0 = (c_0/2\beta)^{1/2}$. We take $c_0$, the equatorial Kelvin wave speed, to be $2.8\,\text{m}\,\text{s}^{-1}$ (Wunsch and Gill 1976), giving $a_0 \approx 2.2\,\text{°C}$. $W_0$ then gives the amplitude of an eastward-propagating Kelvin wave, $W_1$ is the mixed planetary–gravity wave and $W_n (n \geq 2)$ correspond to the westward-propagating planetary or Rossby wave modes. The coefficients $W_n$ follow from an orthogonality condition as

$$W_n = \frac{1}{n! \sqrt{(2\pi)}} \int_{-\infty}^{\infty} WD_n(y^*) dy \quad n = 0, 1, 2, \ldots.$$  (4)
Note that we have no information about the vertical mode structure in this formulation. There is an assumption of first vertical mode behaviour in the scaling, but it can be shown (appendix A) that this representation and the full modal structure obtained from the model are related. Though our bands are not meridional, for the purposes of the analysis we took them to be so (we emphasize that this is not ideal but the data force the choice on us). This assumes that the zonal gradients are smaller than the meridional ones on the scale of the slope of the bands. This should be true for the western and central Pacific bands, which are nearly meridional, but is less certain for the eastern Pacific band which slopes appreciably. This slope will tend to lead to underestimates of up to a month in the time required for Kelvin wave propagation across the Pacific. Initially one might think that the multimode model could be used to examine the effect of having a sloping band, but in practice the model’s accuracy is seriously eroded beyond 8–9° from the equator so that any simulation of the XBT data over the selected range of ±15° would be problematical. Also, as can be seen in Fig. 1, the width of band 1 is so large that, given the paucity of XBT data, the data from any particular month may well represent paths varying from near-meridional in the equatorial region to an extremely oblique transect.

The noisiness and sparsity of the data set make evaluation of the integral in (4), truncated to ±15° about the equator, useful only for n = 0, 1 and 2. Long-wave theory leads us to expect W1 to be zero and indeed the data have that characteristic. Therefore only the Kelvin wave amplitude W6 and a crude estimate of the first planetary wave Ws can be obtained from the XBT data. Time series of W6 and Ws, relative to the mean state of June 1979 to May 1982, are shown in Figs. 12 and 13 respectively. A detailed comparison with the model results will be given in section 5.

3. MULTIMODE MODEL THEORY

The numerical model is based on the well-developed theory of equatorial normal modes outlined in Gill (1982, sections 6.10 and 11.11) and Gill (1984), among many other places. Observational evidence for such modes is given in, for example, Wunsch and Gill (1976) and Weisberg et al. (1979); they are also a computationally economical and flexible representation of equatorial processes. The model is similar in design to that of Cane (1984), Cane and Patton (1984) and Busalacchi and Cane (1985), with the significant difference that their model kept a finite difference formulation in both x and y while the present model uses the horizontal mode structure of the theory to reduce the computation to a strictly one-dimensional, that is, x, form. This has the advantage of being simpler and faster. It does, however, have the drawback of not representing the coastal boundaries well, as they become merely meridional lines. The results suggest that this is not a strong drawback of the model except close to these meridional boundaries. It should also be noted that the state variables in what follows are anomalies relative to a mean which, for comparison with the XBT analysis, is taken to be the average of the period June 1979 to May 1982. The mean states of the model and XBT data are similar over ±5°; see White et al. (1985) for an idea of the mean XBT field.

Since \( \tau^v \) is generally small within ~10° of the equator, and we are focusing on processes in a near-equatorial strip, we consider only the zonal component, \( \tau^x \), of the wind stress (we note here that Battisti (1988) found that off-equatorial winds do not make much difference to equatorial wave processes, which we can confirm). In any case, the introduction of \( \tau^v \) mostly affects the meridional current, which we do not consider, and the mixed planetary–gravity wave, which tends to be a local phenomenon.
If it is also assumed that this forcing acts only over the mixed layer, then in the long-wave approximation the shallow water equations on an equatorial beta plane become

\[
\begin{align*}
&u_t - \beta y v + P_x = X(x, y, t)S(z) - ud \\
&h_{zt} + u_x + v_y = 0 \\
&\beta y u + P_y = 0 \\
&N^2 h + P_z = 0 \\
\end{align*}
\]

(5)

where \(N(z)\) is the buoyancy frequency, \(P\) is the pressure perturbation divided by the density, \(h_t = w, d\) is the Rayleigh friction coefficient and

\[
\begin{align*}
X(x, y, t) &= \tau^2 / (\rho_0 H_{\text{mix}}) \\
S(z) &= \begin{cases} 1 & \text{in the mixed layer} \\ 0 & \text{below.} \end{cases}
\end{align*}
\]

(6)

Neglecting the barotropic mode, we can expand the baroclinic component of (5) in vertical normal modes (Pollard 1970; Gill 1984) through the expressions

\[
(u, v, P) = \sum_{m=1}^{\infty} (\tilde{u}_m, \tilde{v}_m, \tilde{h}_m)p_m(z)
\]

(7)

\[
h = \sum_{m=1}^{\infty} \tilde{h}_m h_m(z)
\]

(8)

\[
S(z) = \sum_{m=1}^{\infty} \sigma_m p_m(z)
\]

(9)

where the variables with the tilde are functions of \(x, y\) and \(t\) only. Note that with a suitable normalization of the eigenfunctions as used in our model, \(\sigma_m\) can be directly interpreted as a measure of the amount of the forcing that goes into the \(m\)th vertical mode. The vertical normal mode eigenfunctions \(h_m\) and \(p_m\) satisfy, for \(m \geq 1\),

\[
dp_m/ \frac{dz}{dz} = -N^2 h_m
\]

(10)

\[
c_m^2 \frac{d h_m}{dz} = p_m
\]

(11)

where \(c_m\) is the eigenvalue for the \(m\)th mode. It has the dimensions of speed.

Substituting the expansions (7)–(9) into (5), and letting

\[
\begin{align*}
mq &= \tilde{h}_m / c_m + \tilde{u}_m \\
m\rho &= \tilde{h}_m / c_m - \tilde{u}_m
\end{align*}
\]

(12)

gives a set of equations for \(mq\) and \(m\rho\) which, upon further expansion in terms of horizontal equatorial modes (cf. (3)) via

\[
mq = \sum_{n=0}^{\infty} m q_n(x, t) D_n \{y(2\beta / c_m)^{1/2}\}
\]

(13)

\[
X(x, y, t) = \sum_{n=0}^{\infty} m X_n(x, t) D_n \{y(2\beta / c_m)^{1/2}\}
\]
yields the much simpler set of equations

\[
\left[ \frac{\partial}{\partial t} + c_m \frac{\partial}{\partial x} \right] m q_0 = m X_0 \sigma_m - d_m m q_0 \quad (14)
\]

\[
m q_1 = 0 \quad (15)
\]

\[
\left[ (2n + 1) \frac{\partial}{\partial t} - c_m \frac{\partial}{\partial x} \right] m q_{n+1} = (m m X_{n+1} - m X_{n-1}) \sigma_m - d_m m q_{n+1} \quad n \geq 1. \quad (16)
\]

The parameter \( d_m \) is a friction coefficient for the \( m \)th vertical mode and \( d_m = d/(c_m \beta)^{1/2} \). Equations (14) and (16) are just simple forced wave equations for the variables \( m q_n \), \( n = 0, 2, 3, \ldots \). However, the direction of wave propagation reverses between (14) and (16) so that (14) describes the motion of an eastward-propagating equatorial Kelvin wave for the \( m \)th vertical mode, while (16) depicts the behaviour of a westward-propagating equatorial Rossby wave of order \( n \) for the \( m \)th vertical mode. Equation (15) shows that the long-wave approximation to the equatorial shallow water equations forces the mixed planetary–gravity wave to be a merely local phenomenon. Following Gent (1985), the damping can be varied as a function of the vertical mode by letting

\[
d_m = c_m^{-\sigma} \quad (17)
\]

Gent states that a value of \( Q = 0.5 \) corresponds to constant \( d \) in (5) while \( Q = 2.5 \) assumes that the coefficient of diffusion is inversely proportional to \( N^2 \). \( Q = 1.5 \) was used by Gent et al. (1983). We use a \( Q \) of 2.5, although its value does not alter our results significantly, except during late 1982 and early 1983, when the higher modes were more important. Even then the changes do not alter the observed course of events discussed in the next section. This damping is such that each vertical mode decays exponentially with the same time scale.

The set of equations (14)-(16), integrated along characteristics, are the basic equations to be solved numerically. At the eastern boundary there is a condition of no zonal flow, which implies

\[
m q_{n-1} = (n + 1) m q_{n+1} \quad (18)
\]

and, from (15), that \( m q_{2n+1} \) are zero at this boundary. At the western boundary we impose a condition of no net zonal flux (Cane and Sarachik 1981, see (B5) for the resulting wave amplitude equation). Once the equations have been solved, by reversing the expansion procedure outlined above one can compute physical quantities such as velocity and dynamic height, using Fejer modification (Lanczos 1966) to reduce spurious oscillations introduced by truncating the vertical mode expansions. Note that this modification is applied to the wind stress before it is used in (14) and (16). The variables \( q \) are of interest in their own right as they correspond to the amplitudes of the equatorial waves. Thus, \( m q_0 \) are the amplitudes of the \( m \)th vertical mode Kelvin waves and \( m q_{n+1} \) are the amplitudes of the \( n \)-th order Rossby waves for the \( m \)th vertical mode.

Section 6.10 of Gill (1982) has a discussion of normal mode theory and details of the eigensolution problem. In order to close the system a vertical profile for the buoyancy frequency is required. Following Gill (1984) we choose

\[
N(z) = \frac{s}{H + H_{\text{virt}} - z} \quad 0 \leq z < H - H_{\text{mix}} \quad (19)
\]

where \( H_{\text{mix}} \) is the depth of the mixed layer and \( H_{\text{virt}} \) is the above-surface asymptote. In the mixed layer we take \( N = 0 \). This profile gives analytical eigenmodes. Good agreement
with observed profiles at depth (e.g. the Hawaii-Tahiti shuttle equatorial profile of Lukas and Firing (1985)) can be obtained by choosing the parameters in (19) to be \( s = 2.8 \text{ ms}^{-1} \), \( H = 5000 \text{ m} \), \( H_{\text{mix}} = 50 \text{ m} \) and \( H_{\text{surf}} = 150 \text{ m} \). It needs to be remembered that \( N \) can be quite variable in space and time and so this model will somewhat distort wave propagation across the basin. For instance, in the eastern Pacific the \( N \) profile is very different to that in the central Pacific, having a much shallower mixed layer (Levitus 1982). Abnormal rainfall, such as is associated with El Niño conditions, will also introduce temporal variations of \( N \) in certain areas. The effect of \( N \) on the problem is seen from (10) and (11), which relate the two sets of eigenfunctions. The practical effect of changes to the buoyancy frequency profile was tested by running the model with different \( H_{\text{mix}} \) values (which alters other factors in the problem as well, see Gill (1984)). For instance, halving \( H_{\text{mix}} \), to give values more like those in the east Pacific, increases the amplitude of the model solutions by 20%, through changes in the \( \sigma_m \) terms. The net effect on the phases was, however, only small. It is therefore considered that our model should be qualitatively correct.

Applying a rigid lid boundary condition, the \( c_m \) can be found from the eigenvalue equation for \( h_m \) resulting from (10) and (11) being combined. In our case this gives \( c_1 = 2.74 \text{ ms}^{-1} \), \( c_2 = 1.51 \text{ ms}^{-1} \), \( c_3 = 1.02 \text{ ms}^{-1} \), \( c_4 = 0.77 \text{ ms}^{-1} \), etc. These speeds are those of the \( m \)th vertical mode Kelvin waves. The corresponding forcing coefficients \( \sigma_m \) are \( \sigma_1 = 0.25 \), \( \sigma_2 = 0.24 \), \( \sigma_3 = 0.18 \), \( \sigma_4 = 0.12 \), etc. The speed \( c_1 \) of the first mode in the model is thus very nearly the \( c_n \) used in the modal analysis of the XBT data in section 2. The link between the two analyses is given in appendix A.

The meridional boundaries of the model are set at 140°E and 80°W. It was run with various numbers of vertical and horizontal modes using pseudo wind stress data supplied by J. O'Brien, an air density of 1.2 kg m\(^{-3}\) and a drag coefficient of \( 1.5 \times 10^{-4} \). The forcing components \( m X_n \), defined in (13), were computed from data between ±15° of the equator. Land intrudes only very slightly into the model domain, near the meridional boundaries. The forcing was here taken to be zero. Our \( x \) resolution was 5° and the time step a quarter of a month. The monthly mean stress was interpolated to permit this; the results are robust to changes in the interpolation. The most straightforward numerical scheme introduced a strong degree of numerical diffusion so a method described by Noye (1986) was adapted to our requirements. It is discussed in appendix B.

### 4. Model results

The results to be presented here are those for the numerical model run with ten vertical modes, each with ten horizontal modes. Provided there are at least five vertical modes each with at least five horizontal modes, the model is relatively insensitive to the addition of more modes, either vertical or horizontal. The model is also quite insensitive to the magnitude of the damping: we have chosen this to give a decay time of 4 years, which is much greater than the time for the more important higher vertical or horizontal mode waves to cross the Pacific. The quantities in the model are anomalies with respect to the mean state from June 1979 to May 1982, for ease of comparison with the XBT anomalies.

To test the reproducibility of the model it was run for successively longer periods, increasing from the two years 1982 and 1983, to the final run of five years from 1979 to 1983. Extending the initialization of the model does not significantly affect the main features, once a spin-up time of about six months is passed. Also, when running the model with the average forcing for our mean state it was found that effective equilibrium was reached after less than two years. The description that follows is strictly an interpretation of the model and should therefore not be construed as a definitive reconstruction.
of the course of events in the real ocean. It simply presents the model's behaviour to a realistic wind forcing. How the model compares with reality is discussed in the next section. The model's contribution towards our understanding of the 1982/83 event is discussed in section 6. The emphasis of our discussion of the model is on the modal wave features it reveals. Previous models of this event employing such a methodology have not dealt with this aspect of the El Niño directly and so we will not include discussion of these models in this paper. Most of the models, however, agree with the gross features in sea level changes that are described here (see, for instance, Fig. 4 of Busalacchi and Cane (1985)). The inadequacies of our type of model are brought out in section 6 also. Such a simple formulation cannot hope to reproduce the full state of the ocean but we believe that it does reveal useful insights into possible mechanisms for change in the tropical Pacific.

In Figs. 4 and 5 we show the surface elevation and zonal surface current anomalies respectively, averaged over four Rossby radii or, roughly, the latitude range 5°S to 5°N. There is a large rise in sea level in the eastern Pacific during late 1982 and the first half of 1983, and a corresponding fall in the western Pacific. Figure 4 also shows evidence of possible wave activity in 1981, most clearly seen in the region of positive anomaly.

![Figure 4](image)

Equatorial average (over ±2 Rossby radii) surface elevation anomaly for the multi-mode model. The contour interval is 5 cm, with \(---\) : \(<0\); and \(\ldots\ldots\) : \(>0\).
propagating westward from the eastern boundary in the boreal spring, and reaching the western boundary by the end of the year. Further wave activity beginning at the eastern boundary in early 1982 and propagating with a similar westward phase speed meets the main (eastward-propagating) El Niño signal in the central Pacific in mid-late 1982. The evidence for the postulated wave activity is less clear in the zonal surface current anomaly shown in Fig. 5. The main features in this diagram are the strong anomalies spreading over the Pacific in 1982, seemingly propagating eastwards, and then westwards in the first half of 1983.

The speed with which the zonal velocity anomalies move during El Niño is reminiscent of that of first vertical mode Kelvin waves, whereas the surface elevation anomalies of 1981 have characteristics similar to Rossby waves. It is instructive, therefore, to turn to the basic elements of our model, namely the various modal wave amplitudes. In Fig. 6 the Kelvin wave amplitude at the central longitude of each of the three XBT ship of opportunity bands, for each of the first three vertical modes, is shown, while in Fig. 7 we give the first planetary wave response for the first two vertical modes. It has often been postulated that vertical mode-1 Kelvin waves force the seasonal cycle sea level in
the Pacific (e.g. Kindle 1979; Busalacchi and O'Brien 1980) and that El Niño events are initiated by such waves as well (e.g. McCreary 1976; Hurlburt et al. 1976; Gill 1983). Figure 6 indeed shows that there is much activity on a seasonal time scale prior to 1982 in the first vertical mode, but also shows a significant amount in mode 2. The westward-propagating first planetary waves, \( nq_2 \), show less evidence of activity on seasonal time scales, except for \( q_2 \) during 1981 (marked \( A' \) on Fig. 7(a)) and possibly 1982 (marked \( B' \)). All of the modes displayed show significantly increased amplitude during the El Niño.
Figure 7. Time series of (a) $\eta_2$; and (b) $\eta_3$ for the same longitudes, and same line styles, as in Fig. 6.

We will now discuss the pattern of events from 1981 onwards, as found in the multimode model. Figure 8 shows the zonal wind field used to force the model from 1981–1983, averaged over a cross-equatorial strip. This illustrates those features of the wind field that will principally affect the model, because of its focus on equatorial processes; for a more detailed discussion of the wind field in general see Inoue and O'Brien (1984a, 1986). A rather late beginning to the seasonal cycle in February–March 1981 (due to zonal wind anomaly A in Fig. 8) produces a vertical mode-1 Kelvin wave, seen in Fig. 6(a) as event A, propagating without loss of signal across the Pacific and reaching South America in April. This initiates the return passage of a vertical mode-1 gravest planetary wave (see event A' in Fig. 7(a)), which increases in strength as it traverses the Pacific until reaching the western Pacific around November–December. The stronger-than-normal Trades that prevailed over much of the central and west Pacific between $\pm 15^\circ$ during mid and late 1981 (see our Fig. 8 and Inoue and O'Brien (1984b)) would have contributed to this strengthening. The existence of this wave event was postulated in the discussion of the surface elevation (Fig. 4). The arrival of the wave coincides with the beginning of the next seasonal cycle in December (see anomaly B in Fig. 8). Note that although a seasonal Kelvin wave is then set off (B, Fig. 6(a)), it decays significantly before reaching the east coast in February or March owing to the blocking of the enhanced south-east Trades (Fig. 8 and Inoue and O'Brien (1984(b))). The ideas of Schopf and Suarez (1988) and Battisti (1988) relating oceanic Kelvin waves to the decay of atmospheric wind anomalies suggest that this wave may have been instrumental in initiating the decay of the easterlies that eventually led to El Niño. The associated
Figure 8. The zonal pseudo wind stress forcing for the model along the equator for 1981 to 1983: anomalies with respect to the XBT averaging period of June 1979 to May 1982. Key anomalies mentioned in the text are indicated by letters from A to J. The contour interval is 10 m$^2$s$^{-2}$, with ————: $<0$; ........: $=0$; and ————: $>0$. 
returning planetary wave (B', Fig. 7(a)) initially behaves similarly to that of 1981, but is suppressed between the central and western Pacific. The suppressing agent is the burgeoning El Niño anomaly. A precursor to this anomaly seems to occur in the far western Pacific in March 1982 (anomaly C in Fig. 8) and has significant energy partitioning between the first three vertical modes. Indeed, little disturbance is found in $q_0$ while in $q'_0$ and $q''_0$ there are small peaks (C in Figs. 6(b), (c)). This precursor may also have contributed to the collapse of the Trades through the air–sea interaction mechanism suggested by Schopf and Suarez (1988) and Battisti (1988).

The next principal feature of the wind field is the onset of the El Niño, namely a westerly anomaly appearing in July–August (D in Fig. 8) which moves eastward over the next 12 months with subsidiary maxima F and G. The three maxima (D, F, G) trigger individual first mode Kelvin waves (D, F and G in Fig. 6(a)). The eastward movement of the wind anomaly gives rise to a more continuous production of a second mode disturbance (D in Fig. 6(b)), especially once the anomaly establishes itself firmly in October (E in Fig. 8). This disturbance has a secondary maximum, due to wind anomaly G, which reaches the eastern boundary in April 1983. The second mode generation continues until the westerly wind anomaly ceases in the boreal spring of 1983. Note that as this anomaly moves eastward the continuing production of the second mode disturbance is best seen ‘downstream’ in the eastern Pacific. The easterly anomaly near 110°W (ending at H in Fig. 8) appears to have only a slight effect in modifying the propagation of the above waves.

While the strong westerly anomaly was generating the Kelvin waves just described, it was also causing the continuous emission of westward-propagating Rossby waves (labelled J in Figs. 7(a), (b)). These were reflected at the western boundary as negative Kelvin waves which were unable to penetrate through the region covered by the westerly anomaly. The reflected Kelvin waves do, however, keep $q_0$ and $q'_0$ below zero throughout early 1983 in the western and central Pacific (Figs. 6(a), (b)). Battisti (1988), from studies of idealized warm events using a simple coupled ocean–atmosphere model, suggests that such a process should eventually lead to the decay of the westerly anomaly, and this did indeed happen. As the westerly anomaly, extending in mid 1983 across the entire Pacific, dies away the equatorial wave action decreases but the sudden arrival of strong easterly anomalies between 170°E and 160°W in August–September 1983 (I, Fig. 8) results in large negative Kelvin waves in all modes moving towards South America in the last few months of the integration (I in Figs. 6(a), (b), (c)) and similar westward-propagating Rossby waves (I in Figs. 7(a), (b)). This removes all elements of El Niño and introduces a climate anomaly of opposite sign. Our model thus supports Battisti’s idea (these ideas are also implicit in Schopf and Suarez (1988)).

5. COMPARISON WITH OBSERVATIONS

Having constructed a model picture for the course of events prior to and during the warm event of 1982/83 it is now necessary to consider the consistency of these model results with the real world. This is done using two basic data sets: island sea level records and the XBT data set discussed in section 2. This latter set gives us a means of comparing the model equatorial wave amplitudes with an estimate of those of reality via Eqs. (A3) and (A5). Some current mooring records are also used to study the validity of the model.

Six tide gauge stations of the many employed to check the model’s sea levels are used for illustration here. They consist of two in each of the eastern, central and western Pacific regions. Figure 9 compares the monthly average tide gauge sea levels with those
of the model. The western Pacific stations of Ponape (7°N 158°E) and Honiara (9.5°S 160°E), shown in Fig. 9(a), demonstrate qualitative agreement in both phase and amplitude between the model and island sea levels. They probably, however, suffer somewhat from being some distance from the equator (see below) and from end-effects, as our model assumes a western boundary at 140°E. This is closer to these stations than the real rim of the Pacific basin.

In the central Pacific (Fig. 9(b)), at Christmas Island (2°N 158°W) and Funafuti (8.5°S 179°W), there is rather better agreement, especially at Christmas Island. Funafuti's sea level is less well predicted during the warm event as our model approximations start to break down some ±10° away from the equator. At Pago Pago, a station used in Fig. 3(b) for comparison with the XBT-derived sea levels, the latitude is far enough south (14°) for the model to collapse completely. We show Funafuti here to illustrate the beginning of this process—it is not shown in Fig. 3(b) as the island lies outside our central
Pacific XBT band. The decreasing amplitude of the model away from the equator is shown well in Figs. 10(b) and 11(b).

Considering the eastern stations (Fig. 9(c)), namely La Libertad (2°S 81°W) and Santa Cruz (0°5S 90°W), we find consistency between the model and tide gauge observations in both amplitude and phase. This agreement is somewhat better at Santa Cruz than at La Libertad; this latter station is very close to the model boundary as it lies on the coast of mainland South America. The coastal station is also strongly influenced by the local coastal winds, which are largely meridional (Bigg and Gill 1986).

In addition to points raised above, another possible reason for the discrepancies in the western and central Pacific during the El Niño is that the $N^2$ profile is assumed to be fixed for the whole integration. As mentioned in section 2, there was a large rainfall anomaly associated with the El Niño, causing a change of more than one part per thousand in the surface salinity (Donguy and Eldin 1985) near the equator, in a pattern
that moves east with time. Such an alteration to the salinity will cause $N^2$ to alter both in space and time, and, as discussed in section 3, the amplitudes predicted by the model will be only qualitatively correct.

Turning now to comparisons with the ship of opportunity XBT data, we have three different tests to consider. From the XBT data we calculate dynamic heights, zonal currents and the equivalent equatorial wave amplitudes $W_0$ and $W_2$, as described in section 2. It needs to be kept in mind in these comparisons that the time history of the XBT observations is very variable, with some months sampled distinctly better than others (see Fig. 14). This will be particularly important in the discussion of $W_0$ and $W_2$.

All three of the XBT bands give similar agreement with the results of the model along the longitude corresponding to the equatorial crossing of the band, including the strongly sloping eastern band, so we will discuss the dynamic height and zonal current anomalies for the central Pacific strips only (see Fig. 1). In Fig. 10 the anomalous height field for the XBT data from this band is contrasted with that of the model. There is general similarity between the two fields out to 10° from the equator, particularly in the
Figure 11. Zonal surface current anomaly along the central Pacific ship track over ±15° using (a) XBT data; and (b) the model. The contour interval for (a) is 20 cm s⁻¹, ranging from −50 cm s⁻¹ to +50 cm s⁻¹ with currents of greater magnitude not contoured. For currents >0: ———; and <0: ———. The contour interval for (b) is 20 cm s⁻¹ with the line style as for Fig. 10.

timing of major events. In Fig. 11 the zonal current anomalies are compared. The nature of the calculation of this variable from the XBT data (Eqs. (3) and (2)) makes it a very noisy field. Also, the neglect of nonlinear terms in the model's momentum equation, and the localized mixed planetary–gravity waves, means that successful velocity predictions are unlikely. Figures 10(a) and (b) therefore appear to be rather different. Some important events can, however, be detected in both, for example the eastward (positive) anomalies south of and near the equator around July 1981 and July 1982, and the westward (negative) anomalies north of the equator in 1983 and on the equator in late 1983.

In addition to velocity computations from the XBT data some current meter records from equatorial moorings, showing very distinctive anomalies, are available for this time (inferences regarding the North Equatorial Counter Current at around 9–10°N by Meyers and Donguy (1984) are not testable using this model because of the degradation of its
accuracy as one moves away from the equator). Halpern et al. (1983) reported a strong (80 cm s\(^{-1}\)) westward near-surface current in January 1983 in the eastern equatorial Pacific. Figure 5, showing the variation of model surface zonal equatorial current (averaged over roughly 5\(^{\circ}\)S-5\(^{\circ}\)N) with time, has only a slight westward anomaly then at 100\(^{\circ}\)W, suggesting the magnitude of the current anomaly was related to local forcing (see feature H in Fig. 8). However, comparison of Halpern's (1984) zonal current time series at 110\(^{\circ}\)W with Fig. 5 shows strong similarities about this date in the timing and strength of the prevailing eastward currents both before and after this striking short-lived westward flow.

Another current meter series which can be used to test the model is that of the Line Islands profiling of Firing et al. (1983), at 159\(^{\circ}\)W. A strong (120 cm s\(^{-1}\)) eastward surface jet was observed during October and November 1982 at this equatorial site. Reference to Fig. 5 again shows that the model gives peak equatorial eastward flow at this longitude during the last quarter of 1982, with anomalies of more than 100 cm s\(^{-1}\). If this agreement is more than coincidence, then Fig. 5 suggests that the jet was a progressive feature, the
peak flow moving from 170°W in September to 140°W in December. It should be noted, however, that the reversal of flow at 159°W, demonstrated in Firing (1984), is not captured by the model, although the strength of the anomaly is substantially reduced (the averaging in Fig. 5 should not be neglected here).

From the several zonal velocity comparisons it appears that the model, given its linear, non-thermodynamic, equations, performs reasonably well in hindcasting surface equatorial currents. Sub-surface features observed at Firing's (1984) and Halpern's (1984) moorings are less well given, particularly the amplitude. The constancy of $N^2$ implicit in the model is one reason for this degradation.

The last model–data comparison is via the equivalent equatorial wave amplitudes

![Comparison of WO series](image)

(a)

![Comparison of WO series](image)

(b)

![Comparison of WO series](image)

(c)

Figure 12. Equivalent equatorial wave amplitudes (in m s$^{-1}$) $W_0$ for (a) the western Pacific; (b) the central Pacific; and (c) the eastern Pacific. Model: ———; XBT estimate: ———.
$W_0$ and $W_2$, shown in Figs. 12 and 13 respectively. Graphs of the number of meridional bins lacking data in each of the three XBT bands are also shown in Fig. 14, as the less data in a given month the less reliable is the computation of these quantities from the derived $\eta$ and $u$. However, time-lag correlation analyses for $W_0$ and $W_2$ between two XBT bands (finishing in March 1983) presented in Gill and Bigg (1985), and reproduced here in Fig. 15, show that the features visible through the noise are a Kelvin wave moving at a speed comparable to $c_0$ (or $c_1$ of section 4) in early 1981 and an eastward-propagating disturbance in late 1982, moving at a reduced speed suggestive of contributions from higher vertical modes. The correlation analysis for $W_2$ also shows a statistically significant signal for the return Rossby wave associated with the 1981 event.

The $W_0$, or Kelvin wave, amplitude of Fig. 12 is first examined. The data and the model share many of the main signals. Looking at particular features we study first the western Pacific band (Fig. 12(a)). The double peak in November 1979-January 1980 is visible in both, and the behaviour during 1980 and early 1981 is well matched. The data lack the model's peak of late 1981, but Fig. 14 shows that this was a period of particularly poor sampling. The model does not reproduce the events of late 1982 well, and once early 1983 is reached the quantity of data drops drastically. The difference in late 1982 may be due to poor data, the western boundary representation or the freshening phenomenon described in section 2. The central Pacific amplitudes (Fig. 12(b)) also agree reasonably well from the early 1980 peak onwards. In particular, the two-peak structure of late 1980/early 1981 appears in both, albeit with the model's first peak of lesser magnitude. Some 30% of the required data was absent at this time however, as is also
Figure 14. Plot of the number of meridional boxes lacking XBT data for each month, for each of the three ship tracks (see Fig. 1). Line style for ship track as for Fig. 13. (Note that band 3 has a maximum of 28, rather than 40, boxes.)

Figure 15. Running (12-month) correlations of equatorial wave amplitudes. (a) Between $W_0$ of band 2 and band 3 of Fig. 1, with band 2 leading. Two-month correlation: ——; one-month correlation: ———. (b) Between $W_2$ of band 2 and band 3, with band 3 leading. Three-month correlation: ———; two-month correlation: ———.
true in January 1982 when the trough in the data record is more pronounced than in the model. Both records also have a two-peak structure in late 1982 and show a trough at the same time in late 1983. The signal in the spring and early summer of 1983, however, is somewhat different.

In the eastern Pacific (Fig. 12(c)) we also see a qualitatively similar picture from 1980 onwards. Early 1981 is a time of poor sampling and so the data do not show as clear a peak then as the model suggests. This is also true in late 1980 and early 1982. At the end of 1982 both $W_0$ plots show high values in October but then the model equivalent amplitude continues strengthening until January 1983 while the data regard this peak as a second, subsidiary, maximum. The real ocean presumably has more second vertical mode signal than the model allows. The contribution of local winds in the eastern equatorial Pacific to events, and the consequent formation of surface jets in this region (Halpern et al. 1983), will also be important because of modal interaction and thermocline deepening.

Both XBT and model $W_0$ time series show large peaks in April 1983 and drop away a little unsteadily, to a trough at the end of the year. It is to be noted that the model expects a much larger negative Kelvin wave at the end of 1983 than the data suggest. It is possible that the magnitude of the easterly wind anomaly of the autumn of 1983 was overestimated in the wind data (this has also been noted by E. Harrison, personal communication). This is consistent with a decrease in the number of observations in the equatorial Pacific during much of 1983 found in an analysis of the Meteorological Office Pacific ship wind data set, the results of which (although not the above-mentioned one) were reported in Whysall et al. (1987).

Turning finally to $W_2$ (Fig. 13) we find a very noisy signal from the XBT-derived $W_2$ plot (Fig. 13(a)), because it is much harder to resolve this mode with the limited amount of data available. There is some qualitative similarity in the timing of the principal peaks of the model (Fig. 13(b)) and data curves. The western Pacific does not possess the clear model peak of late 1981 and the structure of late 1983 owing, probably, to sampling inadequacy (see Fig. 14). The eastern Pacific band also suffers from poor sampling and has an ambiguous signal of the 1981 wave. It has, however, qualitative agreement with El Niño features, as has the central Pacific band, which also shows the 1981 event. The most important similarity between model and data is that both show some evidence of Rossby wave propagation in 1981. The 1981 Rossby wave event is also seen in satellite observations of sea surface temperature in the eastern Pacific in June/July (Pullen et al. 1987). During the warm event, the western Pacific had little data from October 1982 to the end of 1983 (see Fig. 14) and so evidence for propagation of Rossby waves then is difficult to find. There is, however, similarity between the model and data $W_2$ in the central Pacific band during this period suggesting that the data may have a record of the Rossby waves generated in the model by the westerly anomaly.

6. Discussion and conclusion

We have demonstrated qualitative agreement between the model and several different data sets, both before and during the warm event of 1982/83. Apart from some problems near the artificial western boundary and the decreasing validity of our basic equations as we move away from the equator, major faults of the model are in its velocity field and its stronger response than the data suggest in late 1983. This latter difficulty is probably due to poor estimates of the wind field at that time. The former requires more detailed examination of the model physics (see below).

We have tried not so much to simulate the detailed state of the equatorial Pacific
during 1979–83 as to construct a model which may help us to understand the processes occurring in the ocean and atmosphere during that period. Given the limitations of the model, and the scarcity of data with which to check its results, we feel that the agreement is sufficient for us to offer the interpretation of events below as a constructive hypothesis which is worth pursuing further.

The year prior to El Niño 1981, saw a pronounced equatorial first vertical mode wave event associated with a late beginning to the seasonal cycle that appears to feed back into this cycle. Whether the arrival of the event’s Rossby wave back in the western Pacific at the same time as the beginning of the 1982 seasonal cycle contributes via ocean–atmosphere coupling to the initiation of the warm anomaly, or to its subsequent strength, is problematical but suggestive in view of the coupled ocean–atmosphere models of Schopf and Suarez (1988) and Battisti (1988). Such a feedback would not occur in a normal year as the wave would return too early, but the birth of the seasonal Kelvin wave was later than usual in 1981 allowing this possible feedback mechanism to arise. It is also noteworthy that this is the only instance in the five-year integration when a Rossby wave is clearly seen returning all the way across the Pacific, and preliminary results from a 23-year integration suggest the same conclusion holds for 1961–1983, a period containing several Los Niños. It is also the only time in the XBT correlation analysis of Gill and Bigg (1985) that a significant positive correlation for Rossby-wave-type propagation was found in oceanographic data. This intriguing possible mechanism for triggering or strengthening an El Niño warrants additional investigation.

Whatever is responsible for the initiation of the westerly wind anomaly, it builds up and moves east throughout the second half of 1982 and early 1983 (after a precursor anomaly at the beginning of 1982), emitting equatorial waves of various vertical modes continuously. This direction of energy into more than one mode is not prominent in earlier years and is due to the strength and latitudinal extent of the anomaly. The second mode in particular plays a very important role in the progress of the warm event. The variability in this strength produces the two-peak sea level structure found in the eastern Pacific. There is evidence of (westward) Rossby wave generation from the anomaly, and, from Battisti (1988), the resultant Kelvin waves from reflection at the western boundary lead, eventually, to the collapse of the westerly anomaly in the boreal autumn of 1983. This collapse, rather more dramatic in the subjectively analysed wind product than would appear to be the case in reality, then produces a substantial first mode Kelvin wave (and Rossby waves) that moves the ocean towards, and beyond, its climatology to a cold anomaly.

How does the above construction link with other theories of El Niño formation, given the model’s inadequacies? Firstly the model’s failings. It is linear and does not contain any thermodynamics. It cannot therefore be said to be able to reconstruct the ocean circulation during an El Niño (let alone the strong 1982/83 event, when nonlinearities will have played a larger role than usual). The absence of the nonlinear terms in the momentum equations ensures that there is no modal interaction (this comes about only in the boundary conditions of the present model), and also means that current predictions are unlikely to be quantitatively accurate, even if qualitatively consistent with observations. The absence of a thermodynamic equation means that the surface heat (and precipitation) fluxes are neglected and the effect of thermocline anomalies is less direct. The assumption of a constant \( N^2 \) is also a source of error, as are the near-equatorial dynamical expansions. Another problem arises from there being significant differences between the surface wind products of various agencies, which could be of importance in any model calculations. These different wind products have been tested on general circulation models of the Philander and Seigel (1985) genre by E. Harrison
and E. Sarachik (personal communication) and their results suggest that in such models the different datasets produce noticeably different ocean circulations. An intercomparison is beyond the scope of the present paper, but we believe that the existing oceanographic data are not inconsistent with the results presented. It is also true that the key XBT data for the modal surmises are barely adequate (some, no doubt, claim inadequate) for checking the model results. We believe, nevertheless, that this agreement, and the model's hindcasting of sea level and surface currents, are sufficiently good to make our hypotheses worth considering.

Secondly, how do the results of this model simulation relate to other theories of El Niño events? The model can tell us little about the full ENSO cycle, as it does not couple the ocean and atmosphere, and this is a vital ingredient in studying ENSO (Graham and White 1988). It may, however, give some hints to how the warm anomaly part of the cycle occurs. The recent work of Schopf and Suarez (1988) and Battisti (1988) on coupled ocean–atmosphere modelling of idealized ENSO events made a case for equatorial waves playing an important role in ENSO. The present work, from both the modelling of an actual El Niño and an analysis of XBT data, lends support to these ideas and suggests some details of the practical processes involved.

Other theories relating to the ENSO cycle can also be brought forward. It has been suggested that stochastic westerly wind bursts in the western or central Pacific, perhaps associated with a coincidence of tropical cyclones on opposite sides of the equator (e.g. Luther et al. 1983; Lau 1985), could be responsible for El Niño initiation. This may have been what began the coherent equatorial wave activity in the model in early 1981 (see Fig. 8). Graham and White (1988) have presented a theory where the coupling of the ocean and atmosphere has a key pathway through slow moving (non-equatorial) oceanic Rossby waves travelling outside of the equatorial wave guide. This provides the timescale of the quasi-periodic ENSO cycle. The present model is not capable of examining such ideas, because of its restricted latitude range of validity. It remains possible therefore that some linking of the various theories will provide a comprehensive construction of the ENSO cycle (which Graham and White go some way towards proposing) with the off-equatorial Rossby waves feeding into the equatorial processes to provide a basic cycle, while the randomness of atmospheric forcing determines the timing, and intensity, of resulting Los Niños.

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APPENDIX A

Relating XBT and model modes

As the XBT data come from a limited depth range they cannot be used to produce a full decomposition into vertical and horizontal modes. The decomposition into horizontal modes discussed in section 2 assumed that only one vertical mode, the first, was present
(Eq. (3)). The model, on the other hand, gives a full decomposition. To compare an important facet of both the model results and the XBT data a link between these two formulations is required. Here \( W_0 \) and \( W_2 \) (section 2’s Kelvin and first Rossby wave variables) are expressed in terms of the full complement of normal modes.

(i) The Kelvin wave. From section 2

\[
W_0 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left( \frac{\gamma}{c_0} + u \right) D_0(y^*) \, dy^*.
\]

(A1)

Expanding the integrand of (A1) in normal modes, using the changes of variable (14) and (15), and the relation between \( q_n \) and \( r_n \) (Heckley and Gill 1984) we find

\[
W_0 = \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{\infty} dy^* D_0(y^*) \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} D_n(y_m) \times \left\{ \left( 1 + \frac{c_m}{c_0} \right) m q_n - \left( 1 - \frac{c_m}{c_0} \right) (n+2) m q_{n+2} \right\}
\]

(A2)

where \( y_m = y^*(c_0/c_m)^{1/2} \). Writing the parabolic cylinder functions in terms of Hermite polynomials (Gradshteyn and Ryzhik (9.253), 1980) and making the change of variable \( t^2 = y^2(1 + c_0/c_m)/4 \) one can evaluate the resulting integral using the asymmetry of odd order Hermite polynomials and Gradshteyn and Ryzhik (7.373.2) to obtain

\[
W_0 = 1 q_0 + \frac{1}{2} \sum_{m=2}^{\infty} \left[ \frac{c_m}{c_0 + c_m} \right]^{1/2} \times \left\{ \left( 1 + \frac{c_m}{c_0} \right) m q_0 - \left( 1 - \frac{c_m}{c_0} \right) \sum_{n=1}^{\infty} \frac{(2n-2)!}{(n-1)!} \left( \frac{c_0 - c_m}{2(c_0 + c_m)} \right)^n n q_{2n} \right\}.
\]

(A3)

(ii) The first order Rossby wave. Using the same manipulations as produced (A2), the XBT data first-order Rossby wave amplitude can be written as

\[
W_2 = \frac{1}{4\sqrt{2\pi}} \int_{-\infty}^{\infty} dy^* D_2(y^*) \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} D_n(y_m) \times \left\{ \left( 1 + \frac{c_m}{c_0} \right) m q_n - \left( 1 - \frac{c_m}{c_0} \right) (n+2) m q_{n+2} \right\}.
\]

(A4)

By making the same change of variable as above, representing the parabolic cylinder functions in terms of Hermite polynomials and using Gradshteyn and Ryzhik (7.374.4), \( W_2 \) may be written, after some simplification, in terms of the full modal solution as

\[
W_2 = 1 q_2 + \frac{1}{2} \sum_{m=2}^{\infty} \left[ \frac{c_m}{2(c_0 + c_m)} \right]^{1/2} \times \left\{ m q_0 (\frac{c_m}{c_0} - 1) + \sum_{n=1}^{\infty} m q_{2n} \frac{(2n-2)!}{(n-1)!} \left[ \frac{c_0 - c_m}{2(c_0 + c_m)} \right]^{n-1} \frac{c_0}{c_0 + c_m} \left[ 8n (\frac{c_m}{c_0} - 1 - (\frac{c_m}{c_0})^2 \right] \right\}
\]

(A5)
APPENDIX B

The numerical scheme

The basic equations we have to solve, Eqs. (14) and (16), are forced wave equations. The boundary conditions introduce the complication that the various horizontal modes are coupled in a different manner at either end. Conventional forward time differencing has a considerable amount of artificial diffusion that is not desirable in our problem, where the fundamental interest is in wave propagation. Various schemes exist to combat this difficulty (e.g., Noye 1986); however, they use no, or periodic, boundary conditions. After study of these schemes we found that the so-called 'box' method of Noye, while not the least diffusive, was able to transmit pulses well and could be adapted to our boundary conditions. It is a two-time, two-space step procedure for the wave equation

$$\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} = 0$$  \hspace{1cm} (B1)

and can be written in finite difference form at the \((n+1)\)th time level as

$$Q_{j+1} = Q_{j} + \frac{1 - \nu}{1 + \nu} (Q_{j+1} - Q_{j})$$  \hspace{1cm} (B2)

where \(\nu = c\Delta t/\Delta x\). It is stable, marching from \(x = j\) to \(j + 1\), for all \(c > 0\).

In our problem we have, for a given vertical mode \(m\), a Kelvin wave \(m\eta_0\) moving to the east and a series of planetary waves \(m\eta_n\ (n > 1)\) moving west, with \(m\eta_1 = 0\). The eastern boundary condition

$$m\eta_{n-1} = (n + 1) m\eta_{n+1} \quad \text{at} \ x = 0$$  \hspace{1cm} (B3)

means that given \(m\eta_0(0)\) all the other \(m\eta_n(0), \ n \geq 1\), are known. Similarly, the western boundary condition

$$\int_{-\infty}^{\infty} \bar{u}_n dy = 0, \quad x = -X$$  \hspace{1cm} (B4)

which, using the relationship between \(m\eta_n\) and \(m\tau_n\) (Heckley and Gill 1984), gives

$$\sum_{n=0}^{\infty} (m\eta_n - (n + 2) m\eta_{n+2}) = 0$$  \hspace{1cm} (B5)

and implies that \(m\eta_0(-X)\) is related to a sum of the higher horizontal modes for this given vertical mode \(m\). Therefore, starting from rest at \(t = 0\) and letting \(m\eta_0(-X)\) be some, as yet unknown, constant \(\Gamma\), the Kelvin wave stepping can be taken across the ocean, the result (containing \(\Gamma\)) fed into the higher horizontal modes, and these sent back the other way. Then at the western boundary a linear expression for \(\Gamma\) will be found from (B5) and so the \(m\eta_n\) for that time step determined. This is then repeated at each time step. The extra computational cost, in our problem, was not significant and wave shapes are transmitted much more accurately (see Blundell and Bigg 1986). One disadvantage of the scheme is that it produces two-grid-length waves ahead of and behind a pulse which, given our boundary conditions, quickly destroy the solution. However, use of a two-pass Shapiro filter (Shapiro 1970) at each time step easily eliminates this problem.

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