Baroclinic adjustment in a zonally varying flow

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SUMMARY

In agreement with previous published results, based on a 2-layer model measure of baroclinicity, the extratropical northern hemisphere zonal mean atmosphere is found to be close to marginal stability in both winter and summer of the FGGE year. In winter, however, considerable zonal variations are present. Results of some numerical model runs, with both zonally symmetric and varying thermal forcing, are presented, and it is found that for sufficiently short forcing time scales a state close to marginal stability, and a storm-track-like region, can be produced. Large steady eddies do not seem to result so readily from the thermal forcing. Time-lagged correlations provide evidence of forcing of the high frequency (periods less than 10 days) eddy heat flux by the baroclinicity, and it is suggested that the low frequency eddy heat flux may be a result of a nonlinear cascade from the baroclinic scales.

1. INTRODUCTION

Baroclinic adjustment is the (assumed) process whereby "over long periods of time baroclinic waves adjust the [meridional temperature] gradient so as to keep it from being appreciably supercritical" (Stone 1978). The main purpose of this paper is to examine baroclinic adjustment in the context of a zonally varying flow. The results presented by Stone (1978) were for the zonal mean, and he did not consider how zonal variations might affect the concept of baroclinic adjustment. In the northern hemisphere, especially during the winter, there are important zonal variations. Transient eddy activity of the kind associated with baroclinic instability is strongly concentrated in the storm tracks found just downstream and poleward of the Asian and North American jet cores (see for example Blackmon et al. 1977; Hoskins et al. 1983). Thus there is a clear difference in behaviour between the storm tracks and the other regions, and it is not obvious why Stone's zonal mean results should hold. In this paper we do consider the zonal variations in baroclinicity and eddy heat flux, and two separate questions naturally arise. Firstly, to what extent is it true that the atmosphere on seasonal time scales is in a state of marginal stability? Secondly, how can the level of atmospheric stability be explained? We examine the first question using both FGGE northern hemisphere data and numerical model runs, with zonal variations forming an important part of this work. The second question is touched upon but not fully explored.

In devising simple climate models (see for example Stone 1972; North 1975; White and Green 1982) the problem of how to parametrize eddy heat fluxes arises. Since a significant proportion of mid-latitude eddy heat flux is thought to be associated with baroclinic instability most people have been guided by the linear theory of such instability

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when parametrizing the eddy heat fluxes. As a part of these fluxes, especially of the steady eddy fluxes, will not be a result of baroclinic instability, this might be expected to lead to errors, and the importance of considering the different components of the heat flux separately will be emphasized later.

The commonest solution to the parametrization problem is to use a mixing-length-type formulation where it is assumed that the eddy heat flux is proportional to a positive power of temperature gradient. These formulations have been attempted with varying degrees of sophistication, with the parametrized eddy heat flux determined usually by some linear model of baroclinic instability with its amplitude determined from energy considerations. The fixing of the amplitude effectively takes account of the important role played by nonlinear developments.

Baroclinic adjustment presents an alternative to such approaches. Eddy heat fluxes are assumed to prevent the state of the atmosphere from becoming supercritical (that is, unstable according to simple theoretical criteria) so that the state can be calculated without regard to the eddy heat fluxes and then adjusted suitably to avoid supercriticality. (This is analogous to the treatment of upright convection by assuming convective equilibrium.) Examples of this approach can be found in Stone (1978) and Gutowski (1985). Stone (1978) noted that in mid- and high latitudes in the northern hemisphere, on seasonal and longer time scales, the zonally averaged atmosphere is very close to marginal stability as measured by the critical velocity shear for baroclinic instability in the inviscid 2-layer model, defined by

$$\Delta u_c = A\beta R(\theta_U - \theta_L)/f^2.$$  

Here $f$ is the Coriolis parameter, $\beta = df/dy$ where $y$ is northward distance, $R$ is the gas constant, $\theta_U$ and $\theta_L$ are the potential temperatures of the upper and lower layers respectively, and $A = \Delta \Phi/(R \theta_I)$ (Phillips 1954) is an $O(1)$ quantity. $\Delta \Phi$ is the geopotential difference between the layers and $\theta_I$ is the potential temperature at the interface between the layers. In the present study $A$ is about 0.7. Stone took $A$ to be identically one, and we have done the same for the calculations presented subsequently. The choice of upper and lower layers is somewhat arbitrary. We have followed Stone and split the troposphere (with the tropopause assumed to be at 200 mb for simplicity) into two layers of equal mass. Using the thermal wind relations and (1a) (the quasi-geostrophic approximation being valid in this context) we can equivalently write the critical horizontal temperature gradient as

$$\left(\frac{\partial \theta}{\partial y}\right)_c = -A(\beta R \theta_I/fg)(\theta_U - \theta_L)/(z_U - z_L)$$

where $g$ is the gravitational acceleration and $z$ is the height.

Stone found that the actual shear was very close to the critical shear for all seasons, and that its seasonal variation was much less than the seasonal variation in radiative forcing. Thus Stone's proposed scheme was to adjust the shear (or, equivalently, the meridional temperature gradient using (1b)) so that it was nowhere supercritical.

In models which are continuous or multilevel in the vertical, instability is possible for all shears. A shear comparable to the two-layer critical shear separates a regime of baroclinically unstable modes which fill the depth of the atmosphere from shallow modes which are confined near the lower boundary. Held (1978) argued that when the scale height of the modes, $h = f^2 \bar{u}_z/(\beta N^2)$, is much less than $H$, the atmospheric scale height, the vertically integrated heat flux is proportional to $\bar{u}_z^2$, so that it becomes very small in 'sub-critical' conditions. When $h$ exceeds $H$, the heat flux is proportional to $\bar{u}_z^3$ and so is large. His argument suggests a rapid transition in the strength of the heat flux near
\( u_U - u_L = \Delta u_c \) (see Fig. 1 of Held 1978), so that this mechanism could result in near-critical shear for a wide range of thermal forcing.

This envisaged process involves increased baroclinicity stimulating growth of eddy heat flux, but a study by Stone et al. (1982), where time-lagged correlations between the eddy heat flux and the supercriticality \( S = u_U - u_L - \Delta u_c \) were calculated, found no significant evidence of this, only of the effect of eddy heat flux reducing the supercriticality. However, Ghan (1984) did find evidence of such forcing when a crude high-pass filter was applied.

We present the results of calculations similar to some of those presented in Stone (1978) and Stone et al. (1982), having made use of both numerical model runs and FGGE northern hemisphere data. In particular we highlight the role of zonal variations, ignored by those previous studies. In section 2 we present baroclinicity and eddy heat flux results for the atmospheric data, while section 3 contains the equivalent results for the numerical model runs, together with details of the runs. Section 4 deals with time-lagged correlations between baroclinicity and eddy heat flux, and, finally, section 5 draws together some conclusions and discussion.

2. Northern Hemisphere FGGE Data

(a) The data

The data used in this study were the level IIIB data produced for the First GARP Global Experiment (FGGE) observing year at the European Centre for Medium-range Weather Forecasts (ECMWF). Details of the forecast–analysis–initialization cycle used to produce the analyses can be found in Bengtsson et al. (1982). We used only the 00 and 12 GMT fields, and extracted data on a regular latitude–longitude grid with spacing 5°-62° in each direction.

Winter calculations were based on the January and February 1979 analyses and summer quantities were derived from June, July and August 1979. December 1978 was not included because of the lower reliability of the analyses in the early stages of the FGGE data processing. Even in the months included it is not clear exactly how much faith can be placed in the analyses. For example, Lau (1984) shows that there can be considerable differences between the IIIB analyses produced by ECMWF and by GFDL at Princeton. The largest differences are mostly over data-sparse areas. For example the ECMWF scheme gives larger zonal winds in the entrance and exit regions of the winter Asian jet. Also the ECMWF initialization scheme tends to suppress the tropical divergent component of the wind so that the vertical motion is too weak in the tropics. Of more importance to the present work is the meridional transient eddy heat flux which is stronger, and probably more accurate, in the ECMWF analyses.

(b) Baroclinicity

Following Stone et al. (1982) the supercriticality \( S \) is defined here as \( S = \Delta u - \Delta u_c \) where \( \Delta u = u_U - u_L \), and \( \Delta u_c \) was defined in (1a). For simplicity \( A \) is assumed to be 1. Aside from this simplification \( S \) represents the excess of the vertical shear of velocity over the critical shear for instability in a 2-layer model, so that a negative value of \( S \) indicates stability, and a positive value instability according to that model. In the continuous Charney model a similar measure is \( h/H \) and, as already discussed in section 1, one can expect large eddy heat flux when \( h/H \) is large, and vice versa.

The winter zonal average \( S \) (where the overbar denotes the time mean over the season) is shown in Fig. 1(a). Details of the calculation of \( S \) are given in appendix A. The standard errors were estimated in such a way as to avoid underestimation (resulting
from approximating the effective decorrelation time), as described in appendix B. As with the results of Stone et al. (1982) $\bar{S}$ did not become much greater than zero, especially between 35°N and 70°N where it was everywhere between zero and 0.6 m s$^{-1}$. If a more realistic value of $A$ had been chosen all values of $\bar{S}$ would be larger, and these near-zero values would instead be about 50% of $\Delta \mu$. However, as discussed in section 5, other factors such as horizontal shear probably have an opposite effect, and it may be that there is substantial cancellation.

The rest of Fig. 1 shows $\bar{S}$ for the individual sectors of longitude, each sector comprising 60° of longitude, with the first occupying 0–60°E and each sector to the east
of the previous one. The sectors will henceforth be referred to by the following names: Europe, Asia, West Pacific, East Pacific, North America and Atlantic.

Table 1 summarizes the maximum values of \( \bar{S} \), and the latitudes at which they were attained. The maximum values varied longitudinally by a factor of about 4 between North America and Atlantic, with \( \bar{S} \) strongly positive at 70°–80°N in the latter region. By contrast, in West Pacific there was a pronounced peak in mid-latitudes roughly coincident with the storm track, and a band of negative \( \bar{S} \) to the north. Thus the near-zero zonal mean \( \bar{S} \) in mid-latitudes was the result of averaging some quite different distributions of \( \bar{S} \) at different longitudes.

The summer maximum values of \( \bar{S} \) are given in Table 2, while the zonal mean profile of \( \bar{S} \) is presented in Fig. 2. \( \bar{S} \) was very close to zero poleward of 50°N, quite similar to the winter, but with the near-neutral region about 15° further north. There was relatively little zonal variation. Thus, unlike in the winter, \( \bar{S} \) was near zero in mid- and northern latitudes at all longitudes.

The baroclinic-adjustment-type parametrization would seem to be compatible with these results in the zonal mean, and also locally during the summer, but possibly not locally in winter.

**Table 1. Maximum \( \bar{S} \) (m s\(^{-1}\)) and \( \bar{F} \) (10\(^{15}\)W) values. Northern hemisphere winter**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Europe</th>
<th>Asia</th>
<th>West Pacific</th>
<th>East Pacific</th>
<th>North America</th>
<th>Atlantic</th>
<th>Zonal mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{S} )</td>
<td>5.9 (79°)</td>
<td>4.0 (51°)</td>
<td>5.2 (34°)</td>
<td>5.6 (84°)</td>
<td>2.3 (39°)</td>
<td>8.6 (73°)</td>
<td>2.6 (73°)</td>
</tr>
<tr>
<td>( \bar{F} )</td>
<td>4.0 (51°)</td>
<td>6.7 (51°)</td>
<td>7.5 (45°)</td>
<td>5.9 (45°)</td>
<td>5.9 (39°)</td>
<td>6.4 (45°)</td>
<td>5.4 (51°)</td>
</tr>
<tr>
<td>( \bar{F}_{W} )</td>
<td>0.9 (56°)</td>
<td>1.1 (62°)</td>
<td>3.1 (39°)</td>
<td>2.0 (45°)</td>
<td>3.1 (39°)</td>
<td>2.2 (45°)</td>
<td>1.8 (39°)</td>
</tr>
<tr>
<td>( \bar{F}_{E} )</td>
<td>2.7 (51°)</td>
<td>6.0 (51°)</td>
<td>5.3 (51°)</td>
<td>4.1 (51°)</td>
<td>3.7 (51°)</td>
<td>4.2 (56°)</td>
<td>4.1 (51°)</td>
</tr>
</tbody>
</table>

**Table 2. Maximum \( \bar{S} \) (m s\(^{-1}\)) and \( \bar{F} \) (10\(^{15}\)W) values. Northern hemisphere summer**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Europe</th>
<th>Asia</th>
<th>West Pacific</th>
<th>East Pacific</th>
<th>North America</th>
<th>Atlantic</th>
<th>Zonal mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{S} )</td>
<td>2.2 (79°)</td>
<td>3.4 (84°)</td>
<td>1.8 (84°)</td>
<td>3.2 (79°)</td>
<td>2.7 (51°)</td>
<td>3.2 (56°)</td>
<td>1.4 (79°)</td>
</tr>
<tr>
<td>( \bar{F} )</td>
<td>2.6 (39°)</td>
<td>1.5 (62°)</td>
<td>1.8 (45°)</td>
<td>1.1 (39°)</td>
<td>2.7 (51°)</td>
<td>1.0 (56°)</td>
<td>1.2 (56°)</td>
</tr>
<tr>
<td>( \bar{F}_{W} )</td>
<td>0.4 (56°)</td>
<td>0.5 (51°)</td>
<td>1.3 (51°)</td>
<td>0.3 (45°)</td>
<td>1.5 (51°)</td>
<td>1.0 (56°)</td>
<td>0.8 (51°)</td>
</tr>
<tr>
<td>( \bar{F}_{E} )</td>
<td>2.5 (39°)</td>
<td>-1.4 (39°)</td>
<td>0.6 (34°)</td>
<td>1.5 (39°)</td>
<td>1.4 (52°)</td>
<td>0.9 (34°)</td>
<td>0.5 (62°)</td>
</tr>
</tbody>
</table>

Figure 2. Zonal mean \( \bar{S} \) (with standard error bars) in the northern hemisphere, June–August 1979. Units m s\(^{-1}\).

**Eddy heat flux**

The eddy heat flux was calculated at each latitude by integrating around a latitude circle and throughout the troposphere:

\[
F = (2\pi a e c_p / g) \cos \varphi \int_{P_S}^{P_T} dp [\nu^* \theta^*],
\]

(c) Eddy heat flux
where $a$ is the earth's radius, $q$ is the latitude, $c_p$ is the specific heat capacity of air at constant pressure, $p_s$ and $p_T$ are the pressures at the surface and tropopause respectively, $\left[ \ldots \right]$, the mean around a sector of a latitude circle, and $^*$ indicates the deviation from the entire zonal mean. Details of the calculation are given in appendix A. A Lanczos time filter (Duchon 1979) was applied to $\nu$ and $\theta$ to give the high-pass flux $F_H$ and the low-pass flux $F_L$. The filter cut-off was around 10 days, designed to separate the synoptic time scale eddies, expected to be strongly associated with baroclinic instability in mid-latitudes, from longer time scale eddies perhaps linked to other processes. The low-pass eddy flux includes the steady eddy flux.
(i) **Winter.** The time-averaged results are shown in Fig. 3, while the maximum values of the eddy heat flux components \( \overline{F} \), \( \overline{F}_L \) and \( \overline{F}_H \) are displayed in Table 1.

In the zonal mean the total flux \( \overline{F} \) reached a maximum of about \( 5.4 \times 10^{15} \text{W} \) at about 50°N and was poleward everywhere except south of 20°N, where there was a small equatorward flux. \( \overline{F}_L \) was everywhere larger than \( \overline{F}_H \), because of the large steady eddy flux. This tended to be the case at all longitudes.

\( \overline{F}_H \) was largest in West Pacific and North America regions while \( \overline{F}_L \) showed only one pronounced longitudinal peak, over Asia and West Pacific. These results are consistent with those of data studies from other years, such as Blackmon *et al.* (1977). The combined result is that \( \overline{F} \) was largest in the Pacific storm track region.

There is no clear correspondence between these heat fluxes and the supercriticality, although the region of largest eddy heat flux did coincide quite closely with the region of near-zero supercriticality in the zonal mean. On the other hand, \( (-1/N) \partial \theta/\partial y \) did seem to vary in the same sense as \( \overline{F}_H \), with maximum values at very similar latitudes. This would suggest that a parametrization of the transient eddy heat flux based on such a measure might not perform too badly in a climate model which includes zonal variations (so that the steady eddy heat flux would be explicitly included). The part of the transient eddy flux included in \( F_L \) might, however, be poorly represented by such a parametrization and cause significant errors in such a model, so ideally the low-pass transient eddy flux ought to be treated separately. It would also suggest that \( \overline{F}_H \) is linked to baroclinic instability, but that \( \overline{F}_L \) is probably dominated by other processes. Further evidence for such a view will be presented in the next section.

(ii) **Summer.** The zonal mean eddy heat flux components are shown in Fig. 4, with the maximum absolute values in Table 2. The general level of the fluxes was considerably less than during the winter, with the difference especially marked for \( \overline{F}_L \) with the familiar weakening of the steady eddy flux from winter to summer. In the zonal mean, \( \overline{F}_L \) and \( \overline{F}_H \) were of comparable magnitude, with \( \overline{F}_L \) peaking to the north and south of the single maximum in \( \overline{F}_H \).

![Figure 4. Zonal mean \( \overline{F} \) (solid), \( \overline{F}_H \) (dotted) and \( \overline{F}_L \) (dashed), northern hemisphere, June–August 1979.](image)

Locally \( \overline{F}_L \) dominated over Europe and Asia, and over East Pacific. As during winter, \( \overline{F}_H \) was largest in the two storm track regions, especially the Atlantic one, with values generally about 40–50% of the winter ones. (In fact comparison of the FGGE year with the ECMWF 'climatology' for the 6 years 1978–84 shows that the FGGE year transient eddy heat flux was probably larger than usual in winter, and smaller than usual in summer.) \( \overline{F}_L \) was much smaller than in winter everywhere except Europe.
3. NUMERICAL MODEL RUNS

Numerical model runs were performed to try to generate a storm-track-like region, and to investigate baroclinic adjustment in a zonally varying flow under more controlled conditions than possible for atmospheric data. The runs gave the added advantage of a longer data record, providing more reliable statistics.

(a) The numerical model

The numerical model used was essentially that described by James and Gray (1986). It is a dry, primitive equation, hemispherical model with the fields represented at each of five equispaced $\sigma$ levels by a series of spherical harmonics with triangular truncation at wavenumber 21. The only parametrized processes present were surface drag acting on the lowest model layer, a scale-selective horizontal diffusion within each level, and diabatic heating.

The surface drag formulation was that of a momentum flux at the surface such that

$$ \frac{\partial v_s}{\partial t} = - C_D (|v_s| + U_0) v_s + \text{usual terms} $$

where $v_s$ is the surface velocity, the 'gustiness factor' $U_0$ was 3 m s$^{-1}$ and the non-dimensional drag coefficient $C_D$ was $10^{-3}$, which is a reasonable value over oceans, that over land probably being about $3 \times 10^{-3}$. In the absence of other forces, and for surface speeds 5–15 m s$^{-1}$, this would give e-folding times of about 2–3 days at the lowest level. The diabatic heating relaxed the temperature towards a 'target state' temperature, with an e-folding time $\tau_E$.

The numerical runs were performed with the diabatic heating applied only to the $m = 0$ (zonal mean) and, in the zonally varying cases, the $m = 1$ components of the temperature. The initial state in all cases was the target state itself, with a white noise perturbation applied to the surface pressure.

(b) Details of the numerical model runs

Two pairs of runs were made, with differing target states and time scales $\tau_E$. Each pair comprised a run in which the target state was zonally symmetric, and another in which the target state included a zonal wavenumber-one component. In the first pair of runs $\tau_E$ was 10 days at all levels. In the second pair $\tau_E$ was 1 day at the lowest level, 5 days at the next level, and 10 days at the top three levels. This pattern is consistent with the increase in radiative time scale away from the surface suggested by Prinn (1977). The original motivation for this change in $\tau_E$ was to try to produce a region recognizably like a storm track, something which did not occur in the first pair of runs. It was hoped that a shorter time scale would lead to more of the zonal variation in the target state being sustained throughout the run, as in fact turned out to be the case. The reason for the different choice of target state was also to try to realize a higher degree of zonal variation.

The first pair of runs will be termed SYMLOW and VARSLOW, to indicate the long low-level time scale, and whether the diabatic heating was zonally symmetric or varying. Similarly, the second pair will be termed SYMFAST and VARSFAST. All of these were run for 500 days, except for SYMLOW which was run for 610 days.

(i) Zonally symmetric runs. The target state of SYMFAST is shown in Fig. 5. It was set by specifying the temperature as

$$ T(\lambda, \varphi, \sigma) = T_s(\sigma) - \frac{1}{2} \Delta T(\sigma) \tanh\{(\varphi - \varphi_o)/\Delta\varphi_o\} $$

where $\Delta T(\sigma)$ was approximately the equator–pole temperature difference and $\varphi_o = 40^\circ$, $\Delta\varphi_o = 20^\circ$. The temperature gradient was set to zero near the equator, but small gradients
resulted from the spectral truncation. In this state the maximum growth rate was 0-70 d⁻¹ for \( m = 8 \), the growth rate being calculated by solving the eigenvalue problem as in Hoskins and Karoly (1981). The target state for SYMSLOW was very similar to that for SYMFAST.

Figure 6 shows the zonal mean climatology of SYMFAST. It was broadly realistic, with Hadley and Ferrel cells having been established, and the eddy fluxes showed structure similar to that observed. The eddy heat fluxes (Fig. 6(b)) were of the form of those associated with baroclinic instability, with net transport from low levels in the subtropics to higher levels further north.

The eddy behaviour can be summarized by the EP flux cross-section (see Edmon et al. 1980) and the E-vector plot (see Hoskins et al. 1983). Given the limited resolution of the numerical model, the pattern of EP fluxes (Fig. 6(c)) was quite realistic, with divergence from the surface in mid-latitudes and equatorward-turning fluxes at upper levels. The \( E \) vectors were very similar to those of VARFAST, shown in Fig. 11(b), but without the zonal variation. Those shown were calculated from the high-pass filtered transient eddies and were integrated through the troposphere, so that their divergence gives the forcing of the barotropic wind by those eddies. The \( E \)-vector pattern, including the low-pass \( E \) vectors which tended to point westward, was similar to that in the northern hemisphere (Hoskins et al. 1983).

With drag, the maximum growth rate on the time mean was about 30% of that on the target state, and without drag about 45%. The most unstable wavenumber was \( m = 6 \), compared to \( m = 8 \) for the target state.

SYMSLOW was broadly similar to this run, but the eddy activity was stronger in SYMFAST, the eddy heat fluxes (Fig. 6(b)) being roughly double those of SYMSLOW.

(ii) Zonally varying runs. The target state for VARFAST was the same as for SYMFAST except that in (2) \( \Delta \Phi_o \) was replaced by \( \Delta \Phi_o (1 + 0.5 \sin \lambda) \), where \( \lambda \) is the longitude. Figure 7 depicts the zonal variation of velocity in this target state. The longitudinal variation was between a broad, weak jet near \( \lambda = 90^\circ \), and a narrow, strong jet near \( \lambda = 270^\circ \) (centred near 40°N). Associated with this pattern, north of about 27°N the maximum meridional temperature gradient was found at \( \lambda = 270^\circ \), but to the south it occurred at \( \lambda = 90^\circ \). At 40°N the gradient varied longitudinally by a factor of 3, and the static stability by about 10%. The zonal variation of growth rate, shown in Fig. 8, was by a factor of
Figure 6. SYMFAST time mean. Zonal mean cross-sections: (a) Zonal velocity (contour interval 5 m s\(^{-1}\), velocities greater than 20 m s\(^{-1}\) stippled, zero contour thick) and potential temperature (contour interval 10 K). (b) \(u^*\), maximum 45-4 K m s\(^{-1}\), contour interval is 20% of the maximum. Negative flux stippled. (c) EP flux (arrows) and its divergence (contour interval 4\(\times\)10\(^{15}\) m\(^3\), positive values stippled).

about 2, with a clear wavenumber-1 pattern. The growth rate at a particular longitude was calculated by using an average over 3 longitude points (about 17°) to form an equivalent of a zonal mean. The zonal wavenumber at a particular longitude can be interpreted in terms of a wavelength for a localized instability, provided that this wavelength is small compared with the scale on which the local growth rate varies. Thus the most unstable wavenumber was probably near the longwave limit of the local growth rate.

Figure 9 shows how eddies developed first in the region of largest growth rate and propagated into regions of greater stability, tending to die out before they re-entered
Figure 7. Target state zonal velocity at $\sigma = 0.3$ in VARFAST, contour interval 5 m s$^{-1}$. Negative velocities stippled. Latitude circles are drawn every 20° from the equator, and meridians every 20°.

Figure 8. Growth rate (with drag) on target state (solid line) and time mean (dashed line) of VARFAST, zonal wavenumber 8.

Figure 9. Hovmöller diagram for VARFAST. $\nu^+$ at $\sigma = 0.3$, 41.5°N. Contour interval 8 m s$^{-1}$, positive areas stippled.
Figure 10. Time mean zonal velocity at $\sigma = 0.3$ in VARFAST, contour interval 5 m s$^{-1}$. Latitude circles are drawn every 20° from the equator, and meridians every 20°. Negative velocities stippled.

Figure 11. VARFAST time mean, high-pass filtered transient eddy statistics. (a) Temperature flux (arrows) and temperature (contour interval 5 K) at $\sigma = 0.9$. (b) $E$ vectors (arrows) and zonal velocity (contour interval 5 m s$^{-1}$, negative velocities stippled) both averaged over $\sigma = 0.2$ to $\sigma = 1.0$. (c) Eddy kinetic energy (contour interval 5 m$^2$ s$^{-2}$) averaged over $\sigma = 0.2$ to $\sigma = 1.0$. Values greater than 50 m$^2$ s$^{-2}$ stippled.
the more unstable region. This pattern of eddies appearing in the unstable region, propagating out and dying out was in evidence throughout the run.

In the time mean the zonal average state resembled that of SYMFAST, with the jet and eddy fluxes about 5% stronger. Figure 10 shows the zonal variation of the zonal velocity in the time mean. The maximum steady eddy kinetic energy was about 50% of that of the target state, and about 5% of the maximum zonal kinetic energy, although it was strongest at the equator, unlike in the target state.

Figure 11 shows the zonal variation of some eddy statistics. The high-pass $E$ vector (Fig. 11(b)) had a zonal wavenumber-1 variation with largest values about 50° downstream of the target state jet maximum, and roughly coincident with the time mean jet maximum. The same was true of the high-pass eddy heat flux (Fig. 11(a)), while the high-pass eddy kinetic energy (Fig. 11(c)) had its maximum an additional 60° downstream. A fairly realistic looking storm track (see Blackmon et al. 1977) was thus generated, with largest values of high-pass transient eddy kinetic energy just downstream and poleward of the jet maximum. The time mean state had slightly smaller growth rates than in SYMFAST, the zonal variation in growth rate being shown in Fig. 8.

The run VARSLOW by comparison preserved much less zonal variation. The steady eddy kinetic energy was only about 10% of that of the target state, and there was little zonal variation in transient eddy strength. The growth rate on the time mean also varied less and seemed to exhibit a zonal wavenumber-2, rather than -1, pattern.

(c) **Baroclinicity**

Figure 12 shows $S$ for the runs SYMSLOW, SYMFAST and VARFAST, both in the time mean, $\bar{S}$, and in the target state. Table 3 gives maximum values. All the runs were sufficiently long for the standard errors to be almost negligible. The general pattern in mid- and higher latitudes was of reduction of $S$ where it was positive in the target state, and of increase where it was negative.

The greatest change in $S$ usually occurred where it was largest in the target state. $S$

![Figure 12](Image)

Figure 12. Target state (solid lines) and time mean (dashed lines) $S$, units m s$^{-1}$. (a) SYMSLOW; (b) SYMFAST; (c) VARFAST, WEAK sector; (d) VARFAST, STRONG sector.
TABLE 3. MAXIMUM $\bar{S}$ (m s$^{-1}$) AND $\bar{F}$ (10$^{15}$W) VALUES, NUMERICAL MODEL RUNS

<table>
<thead>
<tr>
<th></th>
<th>SYM-SLOW</th>
<th>SYM-FAST</th>
<th>VAR-SLOW</th>
<th>WEAK</th>
<th>Intermediate</th>
<th>STRONG</th>
<th>Intermediate</th>
<th>Zonal mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{S}_{\text{tar}}$</td>
<td>7-6 (53°)</td>
<td>5-9 (42°)</td>
<td>7-6 (53°)</td>
<td>1-6 (53°)</td>
<td>5-6 (42°)</td>
<td>14-0 (42°)</td>
<td>6-4 (42°)</td>
<td>6-0 (42°)</td>
</tr>
<tr>
<td>$\bar{S}$</td>
<td>3-7 (53°)</td>
<td>1-8 (47°)</td>
<td>4-4 (53°)</td>
<td>1-3 (53°)</td>
<td>0-7 (53°)</td>
<td>4-9 (42°)</td>
<td>3-7 (47°)</td>
<td>2-1 (47°)</td>
</tr>
<tr>
<td>$\bar{F}$</td>
<td>2-7 (47°)</td>
<td>5-5 (42°)</td>
<td>2-2 (53°)</td>
<td>3-7 (53°)</td>
<td>4-2 (36°)</td>
<td>8-8 (42°)</td>
<td>7-9 (47°)</td>
<td>5-9 (42°)</td>
</tr>
<tr>
<td>$\bar{F}_{\text{h}}$</td>
<td>1-8 (53°)</td>
<td>4-1 (47°)</td>
<td>1-7 (53°)</td>
<td>3-2 (53°)</td>
<td>2-9 (42°)</td>
<td>6-3 (42°)</td>
<td>6-0 (47°)</td>
<td>4-4 (47°)</td>
</tr>
<tr>
<td>$\bar{F}_{\text{l}}$</td>
<td>1-1 (36°)</td>
<td>2-0 (30°)</td>
<td>0-8 (30°)</td>
<td>1-1 (30°)</td>
<td>1-7 (30°)</td>
<td>3-1 (30°)</td>
<td>2-1 (36°)</td>
<td>2-0 (30°)</td>
</tr>
</tbody>
</table>

was brought closer to zero in the FAST than in the SLOW runs (with the possible exception of the STRONG sector of VARFAST).

The zonally varying runs had the same zonal mean forcing as the symmetric runs, and indeed the time mean zonal mean supercriticality was very similar in the two sets of runs, although the maximum zonal mean $\bar{S}$ was slightly larger in the varying runs. The relative baroclinicity of the time mean states in the FAST and SLOW runs is not clear, being dependent on what measure of baroclinicity is chosen. Both the supercriticality and $(-1/N)\partial \bar{\theta}/\partial y$ indicated smaller baroclinicity in the FAST run, while the temperature gradient itself and the full linear growth rate calculation suggested the opposite. Perhaps the full growth rate calculation is the most reliable indicator, but, given the subtleties of nonlinear development, it is not obvious which, if any, of these might be a good predictor of the amplitude attained by any baroclinic growth.

A baroclinic adjustment parametrization would probably work fairly well in the FAST pair of runs, but would be less successful in the SLOW pair. It seems likely that the mean state depends on the relative time scales of the destabilizing thermal forcing and of the stabilizing baroclinic instabilities. If the stabilizing effect were much faster one might expect the mean state to be near neutral. In the atmosphere, and in these numerical runs, the time scales are comparable and so a significantly supercritical state might be possible.

(d) Eddy heat flux

The time mean eddy heat flux components for SYMFAST are plotted in Fig. 13, with the maximum values given in Table 3. The pattern for SYMSLOW was similar, and the relevant maxima are also given in Table 3. The regions of large $\bar{F}_{\text{h}}$ coincided with those of greatest $\bar{S}$, while the peaks of $\bar{F}_{\text{l}}$ were well equatorward. The relative positions

![Figure 13. $\bar{F}$ (solid), $\bar{F}_{\text{h}}$ (dotted) and $\bar{F}_{\text{l}}$ (dashed) for SYMFAST. Units 10$^{15}$W.](image-url)
of $F_L$ and $F_H$ and the fact that $F_H$ was larger, were common features of all the runs. The heat fluxes in SYMFAST were roughly double those of SYMSLOW, as noted in section 3(b).

In the run VARSLOW the zonal mean heat fluxes (for the maximum values see Table 3) were very similar to those of SYMSLOW, but smaller, while the baroclinicity, as measured by the supercriticality, $(-1/N)\partial \tilde{\theta}/\partial y$ or linear growth rate, was larger. The zonal mean heat fluxes in VARFAST were also very similar to those in the symmetric case. If anything the introduction of a zonal variation in the forcing reduced the size of $F_L$, while increasing $F_H$. The reduction of $F_L$ with the introduction of zonal variation was also seen to occur in the SLOW runs, so zonally varying forcing of the temperature field does not seem to produce any increase of heat transport by steady or low frequency eddies. Rather, $F_L$ seems to be associated with zonal average processes in the same way as $F_H$, suggesting that in these runs both $F_L$ and $F_H$ might be a result of baroclinic instability. The forcing simply gives a zonal variation in the strength of the fluxes, rather than giving them a radically different character. This is in contrast to the deduction made for the northern hemisphere, especially the winter, where $F_L$ is dominated by the steady eddy heat flux. Furthermore the smallness of the steady eddies in the model runs was evident not just in the heat flux, but also in the kinetic energy. In the model runs $F_L$ may have been a result of longer time scale baroclinic eddies or of low frequency eddies produced by a nonlinear cascade from the primary baroclinic scales, which had probably propagated equatorwards from the strongly baroclinic region. However, time-lagged correlations between $F_L$ and $F_H$ did not reveal any significant relationship. Hendon and Hartmann (1985), using a two-level model, also found significant low frequency variability equatorward of the main baroclinic zone when zonally symmetric forcing was applied.

In the WEAK sector of VARFAST the eddy heat flux was fairly small, associated with relatively small temperature gradients, while in the STRONG sector $F_L$ and $F_H$ were 3 and 2 times larger, respectively. In the intermediate sectors the eddy heat flux was intermediate in strength, but close to that in the sectors immediately upstream. This was because of the downstream propagation of eddies. Both Hovmöller diagrams (e.g. Fig. 9), and a life cycle run performed for the STRONG sector, suggest that the most unstable eddies reached their largest amplitude about 45–90° downstream of their ‘source’.

Again, as in the northern hemisphere winter, a parametrization of $F_H$ proportional to a positive power of baroclinicity would capture the zonal variation quite well. Also, although it is not clear how the time mean baroclinicity varied between the SLOW and FAST runs (since it depends on the choice of measure), it does seem that a forcing time scale might play some role in the parametrization of eddy heat flux. This is plausible since the equilibration of growing baroclinic eddies could be expected to depend on such a scale.

It is worth noting that these results show that although applying a zonally varying thermal forcing can produce a storm-track-like region where transient eddy fluxes are large, a geographically fixed forcing towards large temperature gradients is not sufficient to give large steady eddies.

4. Time-lagged correlations

In order to describe the relationship between baroclinicity and eddy heat flux, this section presents the results of time-lagged correlations between baroclinicity and eddy heat flux, calculated as in Stone et al. (1982) and Ghan (1984).

The correlations shown have been normalized by the correlation required for significance at a 5% level in a two-tailed $t$ test, calculated using the effective number of
degrees of freedom as estimated by the method of appendix B. The measure of baroclinicity used in these calculations was actually \((\frac{-1}{N})\frac{\partial \theta}{\partial y}\), but the results are very close to those obtained with the supercriticality.

(a) Numerical model

Figure 14 shows the correlations for the zonal mean of run VARFAST. The curves shown are at the latitude of largest time mean eddy heat flux for each component. All three components gave a significant negative peak at a positive lag of 1 day (i.e. with baroclinicity lagging behind the heat flux). This is consistent with the anticipated reduction in baroclinicity effected by the heat fluxes. In addition \(F_H\) gave a significant positive maximum at a negative lag of about 6 days, suggestive of \(F_H\) being a result of baroclinic instability.

The correlations for the other runs were broadly similar, the significant peaks being listed in Table 4. The most noteworthy features were the aforementioned ones: the negative correlation when the baroclinicity lagged the flux, generally by 1 or 2 days, true for all three heat flux components; and the positive correlation when the flux lagged the baroclinicity, mostly by 1-6 days, present for \(F_H\) and, to a lesser extent, for \(F\) but completely absent from the \(F_L\) correlations. These features were present in nearly all cases, even locally in run VARFAST. In the case of \(F_H\) the negative correlation was absent in the 2 sectors where the heat flux was weakest. The absence of the positive \(F_H\) correlation in the sector 315-045° might be explained by the heat flux in this sector being largely a result of downstream propagation of eddies, and so controlled by the larger baroclinicity just upstream, rather than by the baroclinicity in the sector itself. The absence of the negative \(F_L\) correlation from the STRONG sector 225–315° does not have an obvious explanation.

In all these cases the number of degrees of freedom was greater than 55, and in most it was greater than 100 (the decorrelation times being of the order of 5 days), which should be sufficient for a reliable \(t\) test.

The clear presence of forcing of the high frequency eddy heat flux by the baroclinicity suggests that one is correct in identifying synoptic-scale eddy heat fluxes as largely the result of baroclinic instability. The lack of evidence for such a direct link between baroclinicity and low frequency eddy heat flux, combined with the suggestion of some association made in the previous section, would support the view of the low frequency flux resulting from a nonlinear cascade from higher frequencies, as outlined in the
previous section. It would also be of great interest to correlate the baroclinicity with the heat flux downstream and, indeed, to investigate other downstream relationships.

(b) Northern hemisphere

In the case of the FGGE data, since the data series for the single seasons were relatively short, the number of degrees of freedom was rather low (in mid-latitudes the typical number of degrees of freedom ranged from about 10 to 60, corresponding to decorrelation times ranging from 1 to 6 days). However, the results obtained using the longer time series output from the numerical model provide a guide to which results can be used with confidence, and results which occurred in several cases are also more likely to be real features of interest. Unfortunately this means that it is not possible to come to any firm conclusions about behaviour which might characterize a single local region, as a result of circumstances special to that region. To overcome this problem data from several years are probably required.

The significant correlations in the northern hemisphere are given in Tables 5 and 6. The main features found from the model results, namely the positive correlation when
### TABLE 5. SIGNIFICANT CORRELATIONS, NORTHERN HEMISPHERE WINTER

<table>
<thead>
<tr>
<th>Lag in days</th>
<th>Positive correlations</th>
<th>Negative correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>$F_i$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zonal mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>1, 8</td>
<td>1</td>
</tr>
<tr>
<td>Asia</td>
<td>15</td>
<td>0.5</td>
</tr>
<tr>
<td>West Pacific</td>
<td>1, 10</td>
<td></td>
</tr>
<tr>
<td>East Pacific</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>North America</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>Atlantic</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zonal mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>West Pacific</td>
<td>2, 4, 8</td>
<td></td>
</tr>
<tr>
<td>East Pacific</td>
<td>1, 5, 4, 9</td>
<td>8</td>
</tr>
<tr>
<td>North America</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Atlantic</td>
<td>1</td>
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</table>

### TABLE 6. SIGNIFICANT CORRELATIONS, NORTHERN HEMISPHERE SUMMER

<table>
<thead>
<tr>
<th>Lag in days</th>
<th>Positive correlations</th>
<th>Negative correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Positive</td>
<td>Negative</td>
</tr>
<tr>
<td>$F$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zonal mean</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Asia</td>
<td>2, 7</td>
<td></td>
</tr>
<tr>
<td>West Pacific</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>East Pacific</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>North America</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Atlantic</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$F_{II}$</td>
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<td></td>
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<tr>
<td>Zonal mean</td>
<td>7, 14</td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>3, 15</td>
<td></td>
</tr>
<tr>
<td>Asia</td>
<td>6</td>
<td></td>
</tr>
<tr>
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<tr>
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<td>North America</td>
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<tr>
<td>Atlantic</td>
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<td></td>
</tr>
<tr>
<td>$F_L$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zonal mean</td>
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<tr>
<td>Europe</td>
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<tr>
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</tr>
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<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Atlantic</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
$F_H$ lagged $S$, and the negative correlation when $S$ lagged $F$, $F_H$ or $F_L$, were also the most prominent features in the atmospheric data, in both seasons. They did not stand out so clearly, however, both because of the shorter data record and because of the greater complexity of the real atmosphere. The forcing of $F_H$ was clearest over Asia and the Pacific in both seasons.

5. DISCUSSION

The hypothesis that the atmosphere adjusts so that in the time mean the supercriticality $S$ is close to zero in mid- and polar latitudes seems to be supported by the zonal mean results for the northern hemisphere presented here. However, locally in the winter there was considerable variation in $S$. These two aspects can perhaps be reconciled within a baroclinic adjustment framework through the concepts involved in a simple model of the interaction of (zonaly varying) baroclinicity and eddy heat flux in the presence of thermal forcing, and with allowance made for downstream propagation of eddies. As eddies grow they propagate downstream so that their heat flux is largest at some distance downstream of the point where they originated. In such a model, if this distance is zero one might expect greatest reduction of baroclinicity by the eddy heat fluxes to occur where the target state baroclinicity is greatest, and if this distance is equal to half the zonal wavelength of the target state baroclinicity the greatest reduction of baroclinicity would occur where it was smallest. In the former case the degree of zonal variation would be decreased, whereas in the latter case it would be increased. It does seem likely, though, that the equilibrium level of $S$ would depend on parameters such as the strength and time scale of radiative forcing and rotation rate, rather than being zero in all cases. This is supported by the difference in time mean levels of $S$ between the two pairs of numerical runs, where the main difference was in the forcing time scales. Shorter time scales at low levels led to smaller time mean $S$ and larger eddy heat fluxes. The results also suggest that forcing time scales might play some role in eddy heat flux parametrization. Further investigation of the role of this time scale, including possibly the effects of its vertical distribution, would be of considerable interest.

It should be emphasized that there are, of course, factors not included in $S$ which affect both the susceptibility of an atmospheric state to baroclinic instability, and also the subsequent evolution of any such instability. Prominent among these factors is the horizontal shear of velocity. James (1987) showed how horizontal shear can reduce the degree of instability and James and Gray (1986) showed that, even at large drag, such shear can reduce growth rates to 55% of the values for the unsheared state. It is thus likely that $S$ overestimates the baroclinicity, although taking $A = 1$ in (1) probably means that the error is not very large (see section 2(b)).

Time-lagged correlations between baroclinicity and eddy heat flux showed clearly different behaviour of the low- and high-pass eddy heat fluxes in the model runs, and similar indications in the FGGE data. On time scales of a few days there were two processes clearly evident from the correlations: increases in both components of the eddy heat flux led to decreases in baroclinicity; and increases in baroclinicity led to increases in high-pass eddy heat flux. Locally the picture was more complex, presumably because of the effects of downstream propagation and, in the atmosphere, because of the important differences in surface condition. To understand what is happening locally it will be essential, especially in the atmosphere, to use considerably more data so that more reliable correlations can be obtained. It seems likely that the high frequency eddy heat flux is mostly a direct result of baroclinic instability, while the low frequency eddy heat flux (excluding the steady flux which has a different character) is not directly forced.
by the baroclinicity, but might be the result of a nonlinear cascade from the baroclinic scales. The fact that there was very little difference between the low frequency eddy heat flux in the symmetric and asymmetric numerical runs supports this kind of conclusion. It is clear that in dealing with processes involving interactions between eddy heat flux and baroclinicity on time scales of days account needs to be taken of the different heat flux components. Further worthwhile work regarding the downstream propagation of eddies and the understanding of the local time-lagged correlations would be the examination of the relationship between the baroclinicity at a given longitude and eddy heat flux further downstream.

The numerical model results further showed that although a storm-track-like region can be produced by using a zonally varying thermal forcing, this is not sufficient to give large steady eddies. The theoretical work of Pierrehumbert (1984) on the instability of zonally varying flows is relevant to the location and size of the storm track. Applying his theory to the target state of VARFAST suggests that a localized instability is possible, with a peak roughly midway between the centres of the STRONG and WEAK sectors. The downstream decay scale of this mode would be about 50° of longitude. Applying the theory to the time mean state of VARFAST gives similar results. These predicted locations and dimensions are broadly consistent with those of the storm track in VARFAST. However, the degree of localization of storm tracks and the strength of zonal variations of baroclinicity and eddy heat flux require further numerical investigation. A suitable method would perhaps be to add varying barotropic superrotations to the basic state to alter the phase speeds of the normal modes without changing their other characteristics too much.

ACKNOWLEDGEMENT

During this research NM was supported by a grant from NERC.

APPENDIX A

Calculation of eddy heat flux and supercriticality

Eddy heat flux was calculated from FGGE data as

\[ F = (2\pi a c_p / g) \cos \varphi \sum_{K=1}^{8} W_K [u_K^s \theta_K^s]_s, \]

where \( K = 1 \) corresponds to level 200 mb, \( K = 2 \) to 250 mb and so on for the levels 300, 400, 500, 700, 850 and 1000 mb. Although data at the 1000 mb level are not fully reliable, it makes very little difference to the structure of the depth-integrated fields. The weights \( W_K \) were given by

\[ (W_K) = (25, 50, 75, 100, 150, 175, 150, 75) \text{ mb} \]

while \([\_\_\_\_\_\_\_\_]_s\) denotes the average along a latitude circle for the sector in question, and the rest of the notation is standard (\( ^t \) being the deviation from the entire zonal mean, not the sector mean). Thus this was the flux integrated through the depth of the troposphere and along a latitude circle within a sector, and scaled by \( 2\pi / \Delta \lambda \) where \( \Delta \lambda \) is the size of the sector.

Supercriticality was calculated as

\[ S = [u_U - u_L]_s - (\beta R / f^2) [\theta_U - \theta_L]_s, \]
where

\[ X_U = \sum_{k=1}^{5} W_k X_k = \sum_{k=1}^{5} W_k \]

\[ X_L = \sum_{k=6}^{8} W_k X_k = \sum_{k=6}^{8} W_k \]

for any quantity \( X \). Thus the troposphere was split into an upper (U) layer 200–600 mb and a lower (L) layer 600–1000 mb.

For the numerical model output the procedure was the same except that the relevant levels were at 300, 500, 700 and 900 mb, with the weights all 200 mb.

**APPENDIX B**

*Estimation of standard errors and numbers of degrees of freedom*

The standard errors were estimated from the formula \( \sigma_E = \sigma/\sqrt{n} \) where \( \sigma_E \) is the standard error, \( \sigma \) the sample standard deviation, and \( n \) the number of effectively independent samples. In the continuous case \( n \) can be calculated (Leith 1973) using

\[ n = T/\tau_d, \quad \tau_d = 2 \int_0^T (1 - \tau/T) \rho(\tau) d\tau \]

where \( T \) is the total time interval, \( \tau_d \) is an effective decorrelation time, and \( \rho(\tau) \) is the autocorrelation of the field in question at lag \( \tau \). In the discrete case the integral is replaced by a summation, and since an integral time scale given by \( \int_0^T \rho(\tau) d\tau \) is much less than \( T \) for the data in this study we made an approximation in which the \( (-\tau/T) \rho(\tau) \) term in the summation was neglected. In most cases this neglect contributes to an overestimation of the standard error. Thus \( \tau_d \) becomes

\[ \tau_d/\Delta = \lim_{N \to N_0} \tau_N \]

where \( \tau_N = 1 + 2 \sum_{n=1}^{N-1} \rho(n\Delta) \)

\( N_0 \) is the total number of data points and \( \Delta \) is the time interval between data points. We only calculated the autocorrelations up to a lag of 15 days so in most cases the convergence was not complete. To avoid underestimation of the standard error we estimated \( \tau_d \) by

\[ \tau_d/\Delta = 1 + \max_{2 \leq N \leq (15/\Delta) + 1} \tau_N \]

which should have been sufficient to ensure that the standard error was overestimated almost everywhere.

The number of degrees of freedom (for use in the calculation of the significance of the cross-correlations) was estimated using a similar procedure, as in Stone et al. (1982). An effective joint decorrelation time for the eddy heat flux and baroclinicity was calculated by

\[ \tau_{jd}/\Delta = \lim_{N \to N_0} \tau_{jn}, \quad \tau_{jn} = 1 + 2 \sum_{n=1}^{N-1} \rho_B(n\Delta)\rho_F(n\Delta) \]

where the subscripts B and F denote the baroclinicity and heat flux respectively. The convergence was much faster than that of the ordinary decorrelation time, so the above formulation was used as it stands with the restriction \( \tau_{jd} \geq \Delta \).
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