Observation and interpretation of wave clouds over Macquarie Island

By R. M. MITCHELL, R. P. CECHET, P. J. TURNER and C. C. ELSUM
CSIRO Division of Atmospheric Research, Station St., Aspendale, Victoria 3195, Australia

(Received 26 May 1989; revised 19 October 1989)

SUMMARY

Images obtained from two NOAA/AVHRR overpasses of Macquarie Island in October 1985 show pronounced cloud bands associated with trapped lee waves. One image reveals striking evidence of an abrupt change in the propagating wave mode. The structure and evolution of the waves is interpreted using a linear wave-model coupled with radiosonde data obtained from the island. The model successfully forecasts the mode change, and accurately predicts the associated wavelengths. An interpretation of the difference between the amplitudes of the two modes is also provided. Infrared channel measurement of the temperature of the cloud in the wave-crests yields a useful constraint on the cloud height.

1. INTRODUCTION

Macquarie Island is 32 km long and 4 km wide with a central ridge 300 m high extending over the length of the island (Fig. 1). It is located at latitude 54°37’S and longitude 158°55’E about 700 km south-east of mainland Australia. The island is oriented
at right angles to the prevailing westerly flow and under certain meteorological conditions gives rise to lee-wave clouds which sometimes extend hundreds of kilometres to the east.

The most spectacular satellite images of these clouds occur when an airstream of distant north-westerly origin brings a warm moist low-level flow across the island. These conditions usually occur in the southern spring and summer when the sub-tropical high-pressure ridge is moving towards its southernmost extent and strong meridional flow is observed in the lower troposphere.

Macquarie Island is an isolated ridge and thus should generate transverse waves in the central region in the lee of the island and diverging waves at each end. The overall wave pattern generated by the island thus results from the interference between diverging waves from the island's extremities and transverse waves generated by the central ridge; however, because of the elongation of the island, the transverse component is dominant. For examples of lee waves where the diverging component is dominant, see Gjevik and Marthinsen (1978).

During October 1985, a sequence of two NOAA/AVHRR satellite images showed spectacular lee waves downstream of Macquarie Island. In this study, the wave structure observed in these images is compared with model results based on atmospheric soundings from the island obtained at times which bracketed the satellite images.

2. Observations

The NOAA/AVHRR satellite images were obtained from the CSIRO reception site in Aspendale, Victoria, Australia. The images were recorded at 2024 GMT on 16 October 1985 (local time 0624) and at 0512 GMT on the following day (local time 1512). The images are hereafter referred to as 'Image 1' and 'Image 2' respectively. Grey scale representations of the images as recorded by AVHRR channel 2 (0.6-0.9 μm) are shown in Figs. 2 and 3. The raw satellite-data were processed using the CSIDA installation (CSIRO System for Interactive Data Analysis). This processing included warping the images to conform to a rectilinear latitude-longitude grid.

The wavelength of the transverse waves was obtained by direct inspection of the images shown in Figs. 2 and 3. As further discussed below, the remarkable feature of Image 1 is the abrupt change in the wavelength and apparent thickness of the cloud bands beginning after the seventh crest downwind from the island. For each part of the wave-train, several profiles of the measured radiance were constructed in a direction at right angles to the cloud bands. The navigational information accompanying the image was then sufficient to allow evaluation of the wavelength. For Image 1, the wavelength of the system near the island was 10.0 ± 0.7 km, whilst for the system downwind of the seventh band, the wavelength was 15.4 ± 0.7 km. For Image 2, where only one mode is apparent, the wavelength was found to be 9.8 ± 0.6 km.

An estimate of the height of the cloud in the prominent bands near the island in Image 1 was made using the brightness temperature derived from a thermal infrared channel of the AVHRR (channel 4, 10.6-11.40 μm). Averaging the minimum temperatures from a profile extending across the first seven downwind bands yielded a temperature of -2.1 ± 0.7°C. From sonde measurements of the vertical temperature profile (see below) this leads to a height estimate of 3.2 ± 0.2 km. This procedure was not attempted for the less pronounced bands further downwind, which appear to be optically thin. In this case, the measured radiance would emanate from a range of altitudes, including from the sea surface, and hence give an unreliable height estimate.

Regular atmospheric soundings are taken at the Macquarie Island meteorological station operated by the Australian Bureau of Meteorology. At the time of the present
Figures 2. Image 1 showing the cloud bands associated with lee waves downstream of Macquarie Island. The central ridge of the island coincides with the western edge of the first cloud band. The data were recorded from AVHRR channel 2 at 2024 GMT on 16 October 1985.

Figure 3. Image 2, as for Fig. 2 but obtained at 0512 GMT on 17 October 1985.
observations, pilot-balloon flights yielding wind velocity were launched every 6 hours, whilst radiosondes recording the temperature at significant pressure levels were flown every 12 hours. The time-sequence of soundings and satellite images is summarized in Fig. 4. The raw data were processed to give vertical profiles of the wind speed at right angles to the island ridge, and the temperature.

![Figure 4. Timing diagram showing the interspersal of atmospheric soundings and satellite image acquisitions. The point marked zero on the time axis corresponds to 1100 GMT on 16 October 1985. The wind profiles were obtained from the radar tracking of pilot balloons launched every 6 hours, whilst the temperature profiles were measured by radiosondes flown every 12 hours.](image)

3. MODELLING OF THE WAVE PROPAGATION

The formation, wavelength and amplitude of lee waves is governed by the vertical wind and temperature profiles and the geometry of the generating obstacle. Scorer (1978) summarizes the relationships between vertical temperature and wind profiles which are conducive to lee-wave formation. For lee waves to propagate to large distances downstream, the atmosphere must contain a trapping mechanism, which usually takes the form of a low-level temperature inversion above a highly stable layer. In addition, mechanisms involving wind shear are also possible (Crock 1988).

More specifically, Scorer (1949) showed that the formation of lee waves is dependent on the vertical profile of the Scorer parameter $l^2$, where

$$l^2(z) = \frac{g \beta}{U^3} - \frac{1}{U} \frac{d^2 U}{dz^2}.$$  \hspace{1cm} (1)

In Eq. (1), $U$ is the horizontal wind velocity normal to the ridge, and $\beta$ is the static stability given by $\theta^{-1} d\theta/dz$ where $\theta$ is the potential temperature. In general, atmospheres in which $l^2$ is a decreasing function of height are conducive to lee-wave formation, although this decline need not be monotonic.

Given a pre-determined $l^2$ profile, the wavelengths and relative amplitudes of viable lee-wave modes may be determined from the solution of the wave equation under appropriate boundary conditions. Subsequently, the resultant streamline displacements may be found, given the forcing due to the ridge profile. Since the analysis of this problem is well established, the details of the adopted solution are reserved for the appendix.
4. Results

(a) General wave morphology

Macquarie Island is ideal for the study of lee waves. Its shape is virtually symmetrical with a ridge lying at right angles to the direction of the prevailing winds. The lee-wave images pictured in Figs. 2 and 3 show a combination of transverse and diverging waves. As expected, the transverse wave crests run parallel to the island ridge, even though the airstream is not at right angles to the long axis of the island. In addition to the transverse waves are three diverging wave trains, most clearly evident in Fig. 2. These may be understood by reference to the topography shown in Fig. 1, in which it is seen that the ridge is not uniform in height but is characterized by a local high point to the north and a more extended ridge to the south, with a valley dividing the two regions. The appearance of the diverging wave trains suggests the genesis of one from the northern peak, and one from each end of the southern ridge. Presumably the expected second train running southeast from the northern peak is obscured by the more dominant transverse component downwind of the island.

The meteorological situation at the time of these events is depicted in the mean sea level synoptic chart shown in Fig. 5. A weak cold front had passed through the region in the previous 24 hours. The air behind the cold front is being directed over the region by a large high-pressure system situated to the north-east of the island, which is generating a north-west surface airflow across the island. By the time of the second image, the centre of high pressure has moved eastwards and an approaching cold front has turned the surface winds in a more northerly direction.

(b) Transverse wave structure

The spatial development of the cloud bands seen in Images 1 and 2 can be interpreted as a time-series containing information about the atmospheric structure downwind of

Figure 5. Synoptic mean sea level chart for the Southern Ocean region, valid for midnight GMT 16 October 1985.
Macquarie Island. In this respect, inspection of Image 1 reveals a clear distinction between the first seven lee-wave crests downwind of the island, and those still further downwind. Not only are the crests near the island more sharply defined, but measurement shows that their wavelength is approximately 50% smaller than those farther off. This suggests that the atmospheric conditions have changed over the period during which the lee waves in Image 1 were generated, in such a way as to cause an abrupt switching from one mode to another.

This phenomenon is investigated by applying the wave model described in section 3 and the appendix to the measured sonde-data. The construction of the Scorer parameter profile requires the first derivative of the potential temperature, and the second derivative of the wind speed. Unfortunately, the height resolution of the wind-speed profile was insufficient to permit placing any confidence in its numerically calculated second derivative. Hence the wind-shear term in Eq. (1) was set to zero. This approach receives plausibility from Burroughs and Larson (1979), who noted that the first term in Eq. (1) is usually the dominant contributor to the Scorer parameter.

The resulting profiles of the Scorer parameter are shown together with the temperature and wind-speed profiles in Fig. 6. Each of these profiles corresponds to a sounding for which both temperature and wind speed were recorded (see Fig. 4). In general terms, these profiles indicate very favourable conditions for lee-wave propagation, exhibiting a region of high $P$ in the range 1–2km, and a decrease with height aloft (see, for example, Palm and Foldvik 1959; Smith 1976). The region of high $P$ in the 1–2km range is a straightforward consequence of the high static stability below the inversion. Figure 6 shows that the inversion subsides and strengthens over the duration of the present observations.

The wave model was applied to the three $P$ profiles shown in Fig. 6. In each case a search for eigenmodes was carried out between wavelengths of 1 and 30 km. A summary of the modes found is presented in Table 1, which indeed predicts a switching of modes during the time frame of the soundings. Diagrams jointly depicting the vertical run of the relative amplitude function $\psi$ and the streamline displacement $\xi$ are presented in
TABLE 1. EIGENMODES DERIVED FROM THE WAVE MODEL

<table>
<thead>
<tr>
<th>Profile source</th>
<th>Mode wavelength (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sounding A</td>
<td>16.1</td>
</tr>
<tr>
<td>Sounding C</td>
<td>16.0, 9.6</td>
</tr>
<tr>
<td>Sounding E</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Fig. 7. For the computation of the streamline displacements, it is necessary to include the forcing by specifying the vertical cross-section of the ridge. The adopted parametrization of the cross-section is described in the appendix. The height of the ridge was assumed to be 300 m, and the half-width at half height to be 1000 m.

(i) Sounding A. The \( l^2 \) profile obtained from sounding A leads to the prediction of a single eigenmode at a wavelength of 16.1 km. The relative amplitude of this mode peaks in the range 2-4 km, but declines only slowly with increasing altitude, as seen in Fig. 7(a). This is reflected in the accompanying streamline plot, which shows significant displacement up to 9 km.

It is of interest to check the predictions of the models against the temporal development of the lee waves implicit in the images. For this it is necessary to estimate the group velocity of the wave trains, which may be substantially less than the mean wind speed, \( U \). Crude estimates based on the assumption of a constant and height-independent Väisälä–Brunt frequency (e.g. Lighthill 1978) and the approximate equality of the horizontal and vertical wave-numbers evident in Fig. 7 suggest that the group velocity will be of order \( U/2 \). Given that the wind speed at cloud level (3-2 km) was \( \sim 20 \text{ m s}^{-1} \), the wave-train speed will be about 10 m s\(^{-1}\). Thus, waves generated by the island at the time of sounding A will have travelled downwind by 300-400 km in the 9-7 hour interval between sounding A and Image 1, and would appear near the eastern edge of Image 1.

Hence it is consistent to identify the wave structure downwind of the mode change with the 16.1 km mode found for sounding A. Not only is the measured wavelength of 15.4 ± 0.7 km in good agreement, but the amplitude of the streamline displacement is small in comparison with that of modes appearing subsequently. This results from the leakage of wave energy to high altitudes, which in turn is due to inefficient trapping arising from the relative weakness of the inversion at this time (see Fig. 6(a)).

(ii) Sounding C. For this profile, two modes were found, with wavelengths of 16.0 and 9.6 km. Figure 7(b) shows that the 9.6 km mode is well trapped, with a peak in amplitude at \( \sim 1.8 \text{ km} \). The 16.0 km mode has two amplitude maxima of opposite phase at 1.8 and 7.0 km, with an intervening node at 3.2 km. The streamline displacements therefore exhibit an interesting variation with height due to interference. Since the derived cloud-top height of \( \sim 3.2 \text{ km} \) is near the node of the 16.0 km mode, the wave-cloud structure should be controlled by the 9.6 km mode.

Given that the mode change is located \( \sim 85 \text{ km} \) downwind from Macquarie Island on Image 1, an estimated group velocity of 10 m s\(^{-1}\) implies that the mode change occurred roughly 140 minutes before the image was recorded, or almost 6 hours before sounding C was obtained. The model prediction of the occurrence of the mode change no later than the time of sounding C is therefore consistent with the observed sequence of events. This conclusion would hold even if the group velocity was substantially different from that estimated above.

The measured wavelength of the new mode is 9.8 ± 0.6 km, in good agreement with the predicted value of 9.6 km. The increased prominence of the cloud bands for this
Figure 7(a). Left panel: Height dependence of the relative amplitude function of the lee-wave mode predicted by the model for the \( I^2 \) profile derived from sounding A. Right panel: Streamline displacements as a function of height. Note that, for display purposes, the scale for both the ridge height and streamline displacement has been exaggerated. A scale factor of 3.3 was chosen so that the plotted ridge height is 1.0 km. The erratic behaviour near \( x = 0 \) is an artefact resulting from the adoption of Scorer's approximation (see appendix).

Figure 7(b). As for Fig. 7(a), but for the \( I^2 \) profile derived from sounding C.

Figure 7(c). As for Fig. 7(a), but for the \( I^2 \) profile derived from sounding E.
mode does not immediately follow from a comparison of the amplitude of the respective streamline displacements at 3 km, which are seen from Figs. 7(a) and (b) to be comparable. However, the cloud height measurement of 3-2 km refers specifically to the cloud top, whereas the appearance of the cloud bands will be influenced by the displacement of the cloud layer at lower levels. Figure 7(b) reveals vigorous wave activity in the 1-2 km range. Since the presence of cloud at this altitude is confirmed by an observer report from the Macquarie Island meteorological station on 16 October 1985, it is concluded that the increased prominence of the cloud bands is caused by the enhanced vertical excursions experienced by atmospheric layers at altitudes below 3 km.

(iii) Sounding E. By the time of sounding E, the inversion had strengthened, resulting in more efficient trapping of the wave energy. This is shown in the model prediction of a single mode of wavelength 8-6 km, with a relative amplitude reaching a maximum at ~2 km, and declining rapidly with increasing height (Fig. 7(c)). The measured wavelength of 9.8 ± 0.6 km shows that the atmospheric structure is changing in such a way as to shorten the wavelength of this mode as time goes on, since sounding E was obtained almost 6 hours after Image 2 was recorded. As a further check, an artificial temperature profile for sounding D was synthesized from a simple average of the profiles from soundings C and E. The predicted wavelength for the resulting $T^2$ profile was 9.8 km, in excellent agreement with the observed value.

5. Conclusion

It has been shown that the lee waves generated by Macquarie Island are amenable to quantitative interpretation by the comparison between satellite imagery and linear wave-propagation models. Using profiles of the atmospheric structure as input data, the model successfully predicts the switching in modes apparent in one of the satellite images. In addition, the model not only accurately predicts the measured wavelengths, but also enables the observed change in wave-cloud thickness accompanying the mode change to be understood.

Macquarie Island is a rich source of lee waves, and many additional images displaying a wide variety of wave-cloud morphologies have been obtained since those presented here. Further effort will be directed towards quantitatively estimating the wave-cloud structure. In addition, detailed study of these latter images suggests scope for the investigation of wave-damping rates.

Acknowledgements

The authors wish to thank Mrs J. Bathols for assistance with image processing tasks. The radiosonde data from Macquarie Island were kindly supplied by the Australian Bureau of Meteorology.

Appendix

Description of the procedure used to solve the wave equation

An early exposition on this topic may be found in Scorer (1949). The notation used here closely follows that of Sawyer (1960), who lists the assumptions needed to arrive at the usual form of the wave equation. These include the assumption of two-dimensional
motion, presupposing an infinitely long ridge, and the neglect of nonlinearities. Given the vertical run of the Scorer parameter \( l^2 \) as defined in Eq. (1), the wave equation may be written

\[
\frac{d^2 \hat{\psi}}{dz^2} (k, z) + [l^2(z) - k^2] \hat{\psi}(k, z) = 0
\]  
(A1)

where \( k \) is the wavenumber, and \( \hat{\psi} \) is related to the stream function \( \phi \) by the equations

\[
\hat{\psi}(k, z) = \left[ \frac{\rho(z)}{\rho(0)} \right]^{1/2} \hat{\phi}(k, z)
\]  
(A2)

where \( \rho \) is the density, and

\[
\phi(x, z) = \int_0^z dk \hat{\phi}(k, z) e^{ikx}.
\]  
(A3)

The streamline displacement is given by

\[
\zeta(x, z) = \phi(x, z)/U(z).
\]  
(A4)

Equation (A1) is solved by dividing the atmosphere into \( N + 1 \) layers, labelled 0, 1, \ldots, \( N \), in each of which \( l^2 \) is assumed to be constant. In this case the general solution of (A1) in layer \( n \) is

\[
\hat{\psi}_n(k, z) = A_n \exp(-\mu_n z) + B_n \exp(\mu_n z)
\]  
(A5)

where \( \mu_n = (k^2 - l^2)^{1/2} \) and \( A_n \) and \( B_n \) are constants.

The lee-waves appear as eigenvalues of (A1); that is, values of \( k \) for which \( \hat{\psi} \) satisfies the requisite upper and lower boundary conditions. For an eigenmode characterized by wavenumber \( k_m \), we require the wave amplitude at the ground to be zero, i.e.

\[
\hat{\psi}_0(k_m, 0) = 0
\]  
(A6)

whilst in the uppermost layer, the amplitude must decay with increasing height:

\[
\hat{\psi}_N(k_m, z) = A_N \exp(-\mu_N z) \quad B_N = 0.
\]  
(A7)

The solution is extended from the top layer recursively downwards into lower layers by the conditions that \( \hat{\psi} \) and \( \hat{\psi}/\hat{z} \) must be continuous across the interface between layers. This is equivalent to demanding the continuity of streamline displacement and pressure (Scorer 1949). These conditions allow the recursive definition of the coefficients \( A \) and \( B \) as follows:

\[
A_n = \frac{1}{2 \mu_n} [\alpha_n A_{n+1} \exp(\beta_n z_n) + \beta_n B_{n+1} \exp(\alpha_n z_n)]
\]  
(A8)

\[
B_n = \frac{1}{2 \mu_n} [\beta_n A_{n+1} \exp(-\alpha_n z_n) + \alpha_n B_{n+1} \exp(-\beta_n z_n)]
\]  
(A9)

where \( \alpha_n = \mu_n + \mu_{n+1} \), and \( \beta_n = \mu_n - \mu_{n+1} \).

In the top layer, it is convenient to take \( A_N = 1 \). In the bottom layer, the boundary condition (A6) implies that \( A_0 + B_0 = 0 \). Hence a wavenumber which leads to the satisfaction of this condition corresponds to an eigenvalue of the problem. In practice eigenmodes were located by a direct search procedure using a simple bisection algorithm. Computations were carried out in the lowest 10 km of the atmosphere, which was divided into equal layers each 250 m thick, corresponding to \( N = 40 \).
Once the run of $I^2$ has been specified and any leeward waves found, it is of interest to calculate the resultant streamline displacements. For this it is necessary to specify the forcing due to the ridge, which is assumed to have a profile given by

$$H(x) = \frac{hb^2}{b^2 + x^2} \quad (A10)$$

where $h$ is the height of the ridge, and $b$ is its half-width at half height. Assuming this profile to be a streamline provides a lower boundary condition on the stream function,

$$\phi(x, 0) = -H(x) \left. \frac{\partial \phi}{\partial z} \right|_{z=0} = U(0)H(x). \quad (A11)$$

This corresponds to a boundary condition on $\psi$ by virtue of (A2) and (A3), and the fact that

$$H(x) = \text{Re} \int_0^\infty dk \exp(-kb + ikx). \quad (A12)$$

The resulting condition is

$$\hat{\psi}(k, 0) = U(0)hb \ e^{-kb}. \quad (A13)$$

To extend the solution to all heights, we first find a function $f$ which satisfies (A1) by applying the downward recursive solution (A5) beginning with $A_N = 1$ and $B_M = 0$. The function $\hat{\psi}$ is then constructed by selecting the linear multiple of $f$ which satisfies the lower boundary condition (A13). Hence,

$$\hat{\psi}(k, z) = C f(k, z) \quad (A14)$$

where $C$ is a constant, and

$$\hat{\psi}(k, 0) = U(0)hb \ e^{-kb} = C f(k, 0). \quad (A15)$$

Hence

$$\hat{\psi}(k, z) = U(0)hb \ e^{-kb} \frac{f(k, z)}{f(k, 0)} \quad (A16)$$

and, using (A2), (A3) and (A4),

$$\zeta(x, z) = \text{Re} \left[ \frac{\rho(0)}{\rho(z)} \right]^{1/2} \frac{U(0)}{U(z)} \frac{hb}{f(0, z)} \int_0^\infty \frac{f(k, z)}{f(k, 0)} \exp(-kb + ikx) \quad (A17)$$

where the adoption of the real part of the right-hand side is valid since the real and imaginary parts of $\hat{\psi}$ will separately satisfy (A1). The integrand in (A17) is characterized by the presence of singularities arising from lee waves for which $f(k, 0) = 0$, and thus the integral is evaluated using contour integration and the theorem of residues. Although Sawyer (1960) developed a full numerical solution, for our purpose the approximate solution of Scorer (1949) is sufficiently accurate. Sawyer has shown that Scorer’s approximation is poor only in the immediate vicinity of the ridge. Hence we take

$$\zeta(x, z) = \text{Re} \left[ \frac{\rho(0)}{\rho(z)} \right]^{1/2} \frac{U(0)}{U(z)} \frac{hb}{f(0, z)} \frac{(b + ix)}{(b^2 + x^2)} \left[ \sum_m \frac{f(k_m, z)}{f(k_m, 0)} \exp(-k_mb + ik_mx) \right] \quad x \geq 0 \quad (A18)$$
\[ = \text{Re} \left[ \frac{\rho(0)}{\rho(z)} \right]^{1/2} \frac{U(0)}{U(z)} \frac{\hbar}{f(0, 0)} \frac{f'(0, z)}{(b^2 + x^2)} \quad x < 0 \]  

(A19)

where \( f'(k_m, 0) \) denotes the derivative of \( f \) with respect to wavenumber \( k \).

The asymmetry in the upwind \((x < 0)\) and downwind \((x > 0)\) terms arises since the lee waves contribute only to the downwind solution. In (A18) the summation over \( m \) includes all lee-wave modes found in the search procedure described above.

REFERENCES


Lighthill, M. J. 1978 *Waves in fluids*. Cambridge, Cambridge University Press, Chapter 4


Scorer, R. S. 1949 Theory of waves in the lee of mountains. *ibid.*, **75**, 41–55

1978 *Environmental aerodynamics*. Ellis Horwood, England