Simulating giant hailstone structure with a ballistic aggregation model

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SUMMARY

A ballistic aggregation model designed for thin film simulation in microelectronics has been adapted to simulate the rime growth behaviour of hailstones. The model is able to account for both lobe development and the occurrence of large air bubbles and voids. The microstructures predicted by the model and those observed in a thin section of a giant hailstone are seen to be both qualitatively and quantitatively similar, lending some support to the possibility that certain large hailstones may grow by the accretion of low density rime with subsequent soaking.

1. INTRODUCTION

The use of Monte Carlo (MC) or ballistic aggregation models to simulate rime growth has achieved a certain popularity in recent years. Perhaps the first example of this was a ping pong ball model by Buser and Aufermayer (1973). More recently, computer models have been developed to do the same thing, following a suggestion by Lozowski et al. (1983). These hard disc (2D) or hard sphere (3D) models are constructed by firing discs in two dimensions, or spheres in three, at a target. The point of origin of the discs or spheres is usually random and the trajectories are typically straight lines. The discs, or spheres, are assumed to stick at their point of impact on the target or on the overlying structure of discs/spheres. To the extent that rime ice consists of an aggregation of frozen spherical cloud droplets, these models are capable of representing certain of its features, particularly growth angle and density (Personne and Durore 1987; Gates et al. 1988; Personne et al. 1988; Rimbaldi et al. 1988). These results have been interpreted in terms of the growth of graupel, and of rime feathers on the edges of ice accretions on cylinders. Because these models require considerable computing time, they have typically been restricted to accretions of around 10000 or fewer particles. Since a cubic millimeter of rime could conceivably contain as many as $10^6$ cloud droplets, the ability to model realistic structures in three dimensions is limited. Consequently, much of the MC stimulation of rime growth has been performed in two dimensions.

Until recently, MC modelling of rime deposits in cloud physics has proceeded with little knowledge of related work in other fields. For example, in the simulation of thin film growth in optics and microelectronics, a large body of MC modelling literature has built up following early work by Henderson et al. (1974), and was reviewed by Bartholomeusz et al. (1986). Two recent papers by Brett (1988, 1989) describe a ballistic aggregation model for thin films. One feature of this model which distinguishes it from the current ballistic aggregation models for rime is the inclusion of surface mobility simulation. By allowing the accretion discs to move up to four diameters along the surface in order to find 'cradle' sites with a higher coordination number (contacting the highest number of particles), more compact structures are obtained with higher densities and more clearly defined air voids. This leads to more realistic micro-structures for certain conditions of thin film growth.

In the present paper, we adapt the Brett model to explore the possibility of using it to explain certain features of the internal structure of giant hailstones. Although we have no evidence to believe that cloud droplets impacting on a rime surface exhibit surface mobility, we feel that the inclusion of mobility will, in a crude way, simulate the effects of limited droplet spreading which occur at high impact speeds and high surface temperatures (Macklin and Payne 1968).

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2. COMPUTER SIMULATION

The details of the model are well described by Brett (1988, 1989) and are not reproduced here. Rather, since the hailstone geometry differs from that for surface thin films, we will describe only the unique aspects of the hailstone model.

The target embryo for the model is a ring of discs of unit diameter, spaced at intervals of 1.3 diameters along the perimeter of a circle of diameter 30 (all measurements will be expressed in terms of the diameter of the impacting discs). The discs along the ring do not touch each other, in order to prevent subsequent discs from ending up in predetermined sites during initial growth; the precise spacing does not affect the general structure of the accretion as long as the centre-to-centre separation exceeds unit diameter. Hard discs of the same diameter as those on the ring are fired at the target disc along straight line trajectories from random positions along a line of length 200. In order to simulate rotation of the hailstone, this starting line is rotated around the growing accretion with a constant angular velocity. A total of 10000 discs were fired in this way, leading to a hailstone of mean diameter about 175 and with spherical (or more precisely cylindrical) symmetry (Fig. 1).

If the discs are regarded as individual large cloud droplets of diameter 50 μm, say, then the embryo would be about 1.5 mm in diameter. Since we wish to compare this result with a real hailstone of diameter close to 9 cm, we must either think of the impacting discs as 500 μm droplets or as representing the effect of as many as 1000 of the smaller cloud droplets which arrive more or less simultaneously along similar trajectories. A similar interpretational dilemma arises in the thin film simulations where the individual discs cannot be thought of as single atoms, but rather as representing the effect of large numbers of atoms arriving along similar trajectories.

One difficulty in this type of simulation arises in the representation of time. In a single figure, it is difficult to show the surface except at the end of the simulation. In order to overcome this problem, we visualized the surface at roughly 1/3 and 2/3 of the final time by temporarily increasing the surface mobility from two to four diameters at these times. These layers appear solid black in Fig. 1. They could also be thought of as thin layers produced by temporarily raising the surface temperature of the deposit.

The simulation exhibits a columnar growth almost from the beginning. Some of these columns exhibit continuity throughout the entire accretion, while others are cut off by adjacent columns or

Figure 1. Simulated hailstone with 10000 accreted discs. The dark layers are formed by annealing of the surface at intervals of approximately 1/3 and 2/3 of the total growth time.
split into multiple columns. As a result of this columnar structure, the surface is lobed. These are the cusped lobes described by Knight and Knight (1970). They suggest that this type of lobe results from a collection efficiency/shielding effect and that the conditions favourable to their formation involve tumbling, accompanied by dry growth with little or no migration of material on the hailstone surface. Bailey and Macklin (1968) also suggest that this type of lobe may be produced in dry growth and that deep fissures are frequently formed between them. These ideas are certainly consistent with the behaviour of our model. The mean column widths were estimated by drawing circles of diameter 50, 100, and 150 units and counting the number of gaps along the perimeter. Values of 12.1, 13.8, and 14.5 units, respectively, were obtained suggesting that the characteristic column size does not vary significantly, although there appears to be a slight tendency to increase with time. The columnar structure arises from competitive growth along with shadowing. Like the columns, some of the air gaps are continuous throughout the accretion, while others are cut off and form large ‘bubbles’. The bottom of Fig. 1 exhibits a particularly large cavity which we shall refer to again in the next section.

3. A GIANT HAILSTONE

A severe hailstorm on 27 August 1973 near Cedoux, Saskatchewan deposited hailstones of up to 10 cm in diameter over a large area (Wojtjw and Lozowski 1975). Figure 2 shows a cross-section in reflected light of one of these giant hailstones. The central core is circular and about 1.5 cm in diameter. Within this, a clear lenticular embryo is suggestive of a large, deformed raindrop, but is perhaps a fragment. The surrounding dark layer consisting of clear, bubble-free ice probably grew at a surface temperature near 0°C. There follows a layer of thickness 1.5 to 2.0 cm containing large numbers of small bubbles giving it a very white appearance. This is the layer on which we wish to focus our attention. It consists of a columnar type of structure with radial air gaps or bubble lines and some large air inclusions. The surface structure exhibits distinct lobes. This structure is reminiscent of the simulated structure in Fig. 1. In fact, fortuitously, the very large cavity near the bottom of the figure exhibits a shape quite similar to that of the large cavity at the bottom of Fig. 1. Some of the columns grow and divide, while others are pinched off by their faster-growing neighbours. Drawing a circle of diameter similar to that of the intermediate circle in the last section, it is possible to count about 18 major columns around the perimeter. By contrast, 23 columns were counted subjectively at the same diameter in the simulation model. A more objective autocorrelation calculation gives 18 lobes for the model. The lack of precise agreement between

Figure 2. Thin section of a giant hailstone which fell near Cedoux, Saskatchewan on 27 August 1973, photographed in reflected light. The scale is divided into centimetres and millimetres. (This hailstone was provided to the senior author through the courtesy of Mrs Skorupa, Box 34, Cedoux, Saskatchewan.)
these numbers need not be viewed as a failure of the model since the counting procedure is somewhat subjective and since the model is a stochastic one, so that one would not expect two repetitions to have exactly the same number of columns. The fact that the numbers are at all close suggests to us, in fact, that the model may be capturing the essence of the growth mechanism during this phase of the hailstone's history. Whether or not the number of lobes is an invariant quantity, we are not as yet in a position to say.

It is not new to suggest that giant hailstones may grow, in part, through fast, low-density rime growth followed by soaking. List (1958), Kidder and Carte (1964), Browning (1966), and Prodi (1970) have all alluded to the possibility of liquid water penetrating into previously formed low-density ice. More recently, Pflaum (1984) and Prodi et al. (1986) have conducted wind-tunnel experiments on this type of two-stage growth. Prodi et al. (1986) show that the lobe structure formed in the dry growth stage retains its general characteristics after soaking. They point out that radial lines of bubbles may be formed in the channels between lobes. They also state that large radially oriented bubbles offer evidence of a low-density first stage with large pronounced channels between the lobes.

The similar structures of our modelled and natural hailstones suggest that the low-density rimeing mechanism at work in the model may also operate in the natural world. However, because similar structures may be obtained by different mechanisms, we cannot claim that we have 'proved' that the natural structure grew in this way.

4. Conclusions

A ballistic aggregation model of hailstone growth has been developed by enlisting some of the modelling techniques used in the Monte Carlo simulation of thin films. Comparing the model predictions with a thin section of a giant hailstone, one finds both qualitatively and quantitatively similar structural features.

The evidence provides tempered support for the possibility that some giant hailstones may grow, in part, by fast rime growth with subsequent soaking. With further research, it may be possible to use the model to make inferences about hailstone aerodynamics, as suggested by Browning (1966).

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