The initial value problem for tropical perturbations to a baroclinic atmosphere

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SUMMARY

Large-scale wave propagation ideas have proved extremely valuable in interpreting real and model data. These ideas have been developed in the context of barotropic models, and here the extension to baroclinic atmospheres is investigated. The dispersion of waves from an initial, large-scale, tropical perturbation to a baroclinic basic flow is studied using a direct approach. A 15-level, spectral, primitive equation model is integrated forward in time. Even in baroclinically unstable flows it is found that for a period, normally of order 12 days, the model yields what can be considered to be the direct response to the tropical perturbations. After this time baroclinic instability dominates. The basic flows are a resting atmosphere, a climatological westerly, an equatorial longitudinal modification to this, and a 3-D climatological December–February state. For the perturbations used here, equatorial Kelvin waves play only a very small role. The large-scale equatorial Rossby wave shows only small dispersion and tends to move with a Doppler-shifted phase speed which allows it to ‘accumulate’ to the east of an equatorial maximum of $10 \text{ m s}^{-1}$ or more. Rossby wavetrains propagate into the middle and high latitude westerly regions. With the climatological flow, perturbations in the Indian Ocean–west Pacific sector trigger a modal structure in the North Pacific and wavetrain across North America very much as barotropic theory would suggest and similar to observed Pacific–North America (PNA) patterns. In such a flow the middle latitude waves tend to extend to the equator in the westerly regions of the east Pacific and Atlantic, in agreement with observed behaviour.

1. INTRODUCTION

The theory of large-scale energy propagation in the atmosphere was first developed by Rossby (1945) and Yeh (1949). With a simple $\beta$-plane channel barotropic model, Yeh (loc. cit.) found that Rossby wave energy propagates with the group velocity so that a succession of troughs and ridges can be induced by a stationary vorticity source. The extension of this theory to a spherical domain was made by Longuet-Higgins (1964), who found that Rossby wave energy tended to propagate along great circles. Hoskins et al. (1977) studied various initial and forcing problems in a global shallow water equation (SWE) model with both simple superrotation and more realistic zonal flows. The almost great circle propagation of wave energy was found in all cases, and also in the barotropic calculations of the linear steady response to orographic forcing in Grose and Hoskins (1979). The baroclinic solutions for both thermal and orographic forcings in Hoskins and Karoly (1981) showed equivalent barotropic wave trains whose ray paths were interpreted in terms of simple wave theory.

The observational relevance of these theoretical studies was suggested by the teleconnection patterns detailed by Wallace and Gutzler (1981) and Horel and Wallace (1981). From the one-point-correlation maps, they identified various dominant patterns which were equivalent barotropic and similar to the theoretical great circle wave trains.

Simmons (1982) and Simmons et al. (1983) considered perturbations to the longitudinally varying 300 mb winter climatological rotational flow in a barotropic vorticity model in an attempt to explain the geographic location of the teleconnection patterns. They found that the most unstable eigenmode is similar to the observed PNA pattern. Sensitivity of the pattern to the location of tropical forcing was also found. Branstator (1985) showed through Green’s function calculations that the Indonesian region is the

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most effective area for producing strong response over the mid-Pacific. Similar results have been found by Zhang (1988) through adjoint eigen-vector calculations.

Murakami and Unninayar (1977) showed that the upper tropospheric equatorial atmosphere has maximum perturbation kinetic energy in specific longitudinal regions. Webster and Holton (1982) used an SWE model to test the hypothesis that these maxima are associated with the propagation of barotropic planetary waves into the tropics in regions of westerlies. The further observational study by Arkin and Webster (1985) also seemed to support this.

Equatorial β-plane SWE models have been used to great effect by Matsuno (1966) and Gill (1980), in particular, to give a simple description of transient and forced motions in the tropical atmosphere in terms of the wave motions that are possible. For the primitive equations linearized about a zero basic flow there is a simple separation between the vertical and horizontal structures. In a layered model or with a lid, each discrete vertical mode and, in particular, the first internal mode is described by the SWE. Consequently, waves such as the eastward-moving Kelvin wave and westward-moving equatorial Rossby wave can be isolated. For a non-zero basic flow the simple separation is not possible and one mathematical interpretation is that the vertical modes are coupled (Kasahara and Silva Dias 1986). Salby and Garcia (1987) described the local response to equatorial heating which is stochastic in space and time in terms of equatorial Kelvin and Rossby waves which propagate vertically out of the troposphere and a continuum of horizontally propagating, equivalent barotropic Rossby waves. In Garcia and Salby (1987) they went on to exhibit the evolving global solutions corresponding to such heating in a realistic zonal flow.

The interpretation of an SWE model with longitudinally varying basic flow in terms of the 3-D atmosphere is not straightforward. However, Webster and Chang (1988) showed that, in an SWE model, equatorial Rossby waves tend to 'accumulate' near the zero-wind region to the east of an equatorial westerly wind maximum, and put this forward as an additional mechanism to explain the longitudinal distribution of equatorial upper tropospheric perturbation kinetic energy.

 stead-state solutions for tropical heating in a baroclinic atmosphere linearized about a time-mean general circulation model (GCM) basic state have recently been exhibited by Navarra and Miyakoda (1988), and shown to be significantly different from those obtained when the basic state was zonal. Further, Branstator (1990) has investigated the response of such a model linearized about a 3-D GCM flow to random forcing. An empirical orthogonal function analysis revealed structures generally rather similar to those in the GCM itself.

In this and a subsequent paper (not yet published) many of the ideas developed in the earlier studies are extended to zonal and non-zonal flows in a 3-D spherical atmosphere using the simple approach of time-integrating a spectral numerical model. This technique is used here for initial tropical perturbations and in a subsequent paper its application to the tropical heating problem will be described. Of course, realistic atmospheric flows are baroclinically unstable and the integrations will eventually become dominated by such synoptic systems. However, it is found that for perturbation from the tropical region there is an initial period of about 12 days in which the development of the direct response is seen. In section 2 some details of the model, basic flows and initial perturbations are given. A solution for a perturbation to the simplest basic state, a resting atmosphere, is described in section 3. This is followed by a solution for a climatological zonal flow in which significant wave propagation away from the equator is possible. In section 4 longitudinal variation in the equatorial zonal wind is introduced into the zonal flow to look at the possibilities in a 3-D atmosphere of the accumulation mechanism of Webster
and Chang (1988). A full 3-D climatological basic state is considered in section 5. Some concluding comments are made in section 6.

2. **Model outline and design of experiments**

(a) **Model**

The numerical model used in this study is the primitive equation $\sigma$-coordinate model described by Hoskins and Simmons (1975). The domain is global and the truncation of the spectral horizontal representation is triangular at total wave number 31. Fifteen layers are used in the vertical, the $\sigma$ values of full levels being approximately 0.03, 0.08, 0.13, 0.16, 0.20, 0.24, 0.29, 0.34, 0.40, 0.48, 0.57, 0.67, 0.78, 0.89, 0.97.

The model includes a biharmonic internal diffusion with coefficient $2.338 \times 10^6 m^4 s^{-1}$, the damping time-scale being about 20 h on the smallest retained scale. There is Newtonian cooling with damping time-scales at the 15 model levels (in order) of 10 (levels 1--12), 9, 8 and 7 days for zonal components and 4, 6, 10, 18, 25 (levels 5--12), 18, 12 and 8 days for wave components. The boundary layer is modelled by a drag at the lowest two levels with time-scales of 4 and 1$\frac{1}{2}$ days, respectively. The time-step in all the integrations is 40 minutes.

For the linear calculations described here a finite-amplitude initial perturbation was multiplied by $10^{-6}$ and the resulting perturbation solution multiplied by $10^6$ before output.

(b) **Basic flows**

Most of the experiments use a basic flow derived from a six-year, 1979–84, average of the European Centre for Medium Range Weather Forecasts (ECMWF) operational analyses for the December–February period (DJF). For basic zonal flows only the zonally averaged vorticity data are used. The temperature and surface pressure fields are determined by the balance scheme of Hoskins and Simmons (1975) but are very similar to the 'observed' fields. For full DJF basic flows, both the vorticity and divergence data are specified.

In each calculation the conventional assumption is made that the basic flow is balanced by momentum and heat sources and that these sources are fixed in time and space. Problems solved using this assumption have yielded extremely interesting results, e.g. Simmons et al. (1983), but the lack of real basis for the assumptions must always be recalled.

(c) **Initial perturbations**

The horizontal structure of most of the initial perturbations used in this study was that of a vorticity dipole about the equator, both parts of which are elliptical with semi-axes $30^\circ$ in longitude and $10^\circ$ in latitude. Results from two vertical profiles will be described here. Profile 1 is shown in Fig. 1 as a function of model level. Profile 2 is the upper portion of profile 1, being zero up to level 11 ($\sigma = 0.57$). For the dipole, profile 1 perturbation, the streamfunction at level 6 ($\sigma = 0.24$) is shown in Fig. 2. The field is global, but the implied velocities away from the locality of the vorticity perturbation are comparatively small and appear to be irrelevant. With the scaling of the perturbation used, the equatorial wind extremum is an easterly of $-10 m s^{-1}$, though of course the calculations are linear. The vorticity perturbation was balanced with temperature and surface pressure perturbations in the manner of Hoskins and Simmons (1975).
3. Perturbations to a Zonal Flow

We consider first the simplest situation of an initial tropical dipole perturbation to an atmosphere at rest with respect to the rotating planet. Six days later the upper and lower tropospheric streamfunction as typified by the level 6 ($\sigma = 0.24$) and level 13 ($\sigma = 0.78$) fields are as shown in Fig. 3. The upper level equatorial anticyclonic dipole has weakened and moved about 40° westwards and additional Rossby wave centres of the opposite sign have developed near latitude 20°. There is a weak easterly wind all along the equator. At low levels the structure is similar but with the opposite sign and weaker amplitudes. The equatorial wave propagation nature of the solution as shown by the upper tropospheric equatorial westerly wind ($u$) plotted against time is displayed in Fig. 4(a). The major feature is the westward movement of the maximum at about 62° day$^{-1}$. There is also an eastward-moving maximum travelling at about 32° day$^{-1}$ which can be traced moving through about 480° of longitude in the 15 days shown.

In this ‘Gill-type’ problem we can associate the eastward propagation along the equator with a simple Kelvin wave with a tropospheric ‘first internal mode’ structure. Its
speed of about 41 m s\(^{-1}\) is consistent with this. The initial perturbation can be considered as triggering westward-travelling equatorially trapped Rossby waves of various meridional wavenumbers. Again the vertical structure is the first tropospheric internal mode. The westward speed of about 8 m s\(^{-1}\), approximately \(\frac{1}{3}\) that of the eastward speed of the Kelvin wave, is between that of the first and second symmetric long Rossby waves (Gill 1980). With the balancing of the initial perturbation, the Kelvin wave is relatively weak.

The same time versus longitude picture for \(u\) at \(\sigma = 0.24\) but for a perturbation to a basic DJF zonal flow is given in Fig. 4(b). Very similar equatorial wave propagation features are seen. The main differences are a more rapid decrease in amplitude in the planetary wave, a slightly weaker Kelvin wave and signs of other weak anomalies developing and then propagating westwards. The time development of the global upper tropospheric flow is illustrated in Fig. 5. With the mid-latitude westerly zonal winds, there is now propagation of Rossby wave activity away from the equator which is responsible for both the weakening of the equatorial Rossby wave and the subsequent equatorial developments. The propagation is strongest in the winter (northern) hemisphere.

The nature of the northern hemisphere solution at various times is suggested by the
three plots in Fig. 6. Fig. 6(a) is a longitude–time plot of $u$ at $\sigma = 0.24$ and 30°N. For the first 12 days it is dominated by a wave pattern with westward or zero phase speed and eastward group velocity, as evidenced by the downstream development. After 14 days there is still an eastwards group velocity but now a marked change to eastward phase speed, suggestive of baroclinic instability. Figures 6(b) and (c) are longitude–height (level) plots of $v$ at 30°N at day 6 and 18, respectively. At day 6 the structure has a slight westward tilt with height in the ‘boundary layer’, but is essentially equivalent barotropic. However, at day 18 there is a general westward tilt in the troposphere and signs of maxima at the surface as well as the tropopause, again indicative of baroclinic instability. Similar plots at 70°N (not shown) suggest a smoother transition from equivalent barotropic Rossby wave to baroclinic wave about 180° downstream from the initial perturbation.
Figure 5. Streamfunction perturbation at level 6, $\sigma = 0.24$ for a DJF zonal flow and initial perturbation with vertical profile 1 at (a) day 3, (b) day 6, (c) day 18. Conventions as in Fig. 2.
Figure 6. Other fields for the DJF zonal flow case. (a) Longitude–time diagrams for $\nu$ at level 6, $\sigma = 0.24$, and 30°N. The conventions are as in Fig. 4 except that the contour interval is 0.25 m s$^{-1}$. (b) Longitude–height plot of $\nu$ at 30°N at day 6. The contour interval is 0.25 m s$^{-1}$ with negative contours dashed and the zero contour not drawn. The tick marks are at $\sigma$ levels in the vertical and 30° in longitude. (c) as (b) but for day 18.
during the period 10–14 days as the low wavenumber polar wave train reaches its turning latitude and starts moving equatorwards.

The implication from this analysis is that for the first 12 days or so we are looking at the direct response to the equatorial perturbation and that it is only after this time that baroclinic instability dominates, e.g. in Fig. 5(c). Thus Figs. 4(b), 5(a) and (b), 6(a) and (b) give a detailed view of the time-scale, amplitude and nature of the direct response in the equatorial waves and in the equivalent barotropic Rossby waves. For example, by day 6 the 10 m s\(^{-1}\) perturbation on the equator has been reduced to 4 m s\(^{-1}\) but much of the mid-latitude equivalent barotropic wave train has been established with winds of order 2 m s\(^{-1}\).

Similar solutions have been obtained for initial perturbations with various vertical profiles. The upper level, profile 2, perturbation gives very similar results. A structure with only the lower part of profile 1 yields weaker poleward wave propagation, only into the sub-tropics. Thus the importance of upper tropospheric perturbations in the equatorial regions is stressed. A further solution has been obtained with the levels 14 and 15 drag reduced, their time-scales changing from 4 and 1\(\frac{1}{2}\) days to 5 and 2\(\frac{1}{2}\) days, respectively. It is generally very similar to the higher drag case, but the equivalent barotropic wave train shows less low-level tilt and the subsequent baroclinic instability occurs rather more rapidly.

4. Perturbations to a Flow with Longitudinal Variation in the Tropics

The techniques exploited in the previous section are easily extended to longitudinally varying flows. In this section we isolate the impact of longitudinal variations in the equatorial flow in order to assess in a baroclinic atmosphere the wave accumulation mechanism discussed by Webster and Chang (1988).

The DJF zonal flow is modified by the addition of a flow $\left(\bar{u}, \bar{v}\right)$ with an equatorially trapped wavenumber 0 and 1 $\bar{u}$ but zero $\bar{v}$. The horizontal structure is given by velocity potential ($\chi$) and streamfunction ($\psi$) of the form:

$$
\chi = -A \exp \left(-\frac{\mu^2}{\mu_0^2}\right) \sin(\lambda - \lambda_0),
$$

$$
\psi = A \left(1 - \mu^2\right) \frac{2\mu}{\mu_0^2} \exp \left(-\frac{\mu^2}{\mu_0^2}\right) \left[\cos(\lambda - \lambda_0) - B\right],
$$

where $\mu = \sin(\text{latitude})$, $\mu_0 = \sin(10^\circ)$, $\lambda = \text{longitude}$, $A = 5 \times 10^{-4} \Omega$ and $B = \frac{1}{3}$. The vertical structure is that of profile 1 in Fig. 1, so that all upper tropospheric deviations from the basic zonal flow are reversed at low levels. The equatorial zonal wind at level 6 ($\sigma = 0.24$) is shown in Fig. 7. It varies from $-14 \text{ m s}^{-1}$ to $16 \text{ m s}^{-1}$. The upper troposphere 'wave accumulation' region of Webster and Chang (1988), where $\bar{u} = 0$ and $\partial \bar{u} / \partial \lambda$ is negative, is marked in the figure. However, it should be noted that in the lower troposphere $\partial \bar{u} / \partial \lambda$ is here positive.

Given that the zero basic flow case gives the Kelvin wave speed to be $41 \text{ m s}^{-1}$, it is unlikely to be significantly affected by the variation in $\bar{u}$. However, the Rossby wave speed of $-8 \text{ m s}^{-1}$ would imply regions of Doppler-shifted easterly and westerly phase speeds. These regions at $\sigma = 0.24$ are indicated in Fig. 7.

Given the reversal of basic flow anomalies in the vertical, the initial perturbations for the experiments to be discussed here are chosen to have upper tropospheric amplitude only (profile 2), thus maximizing the possibility of wave accumulation. The initial horizontal dipole perturbations for three experiments are located as shown in Fig. 7. The
corresponding longitude–time plots of the equatorial $u$ at $\sigma = 0.24$ for the three solutions are given in Fig. 8. The Kelvin wave is identifiable in A and B but has little significance. The dominant part of the equatorial Rossby wave moves towards the east in A, slowly at first, then faster and slowing again, it is almost stationary in B and moves towards the west in C. In each case it tends to remain in the region of the initial location of B.

The behaviour shown in Fig. 8 is entirely consistent with a long, almost non-dispersive, equatorial Rossby wave moving westward relative to the flow at 8 m s$^{-1}$. When such a wave is implanted on a flow like that used here, its centre moves with speed $\bar{u} - c$, where $\bar{u}$ is some horizontal and vertical average of $\bar{u}$ in the region of the waves. It remains near the point where its Doppler-shifted group and phase speed are zero. For the cases discussed here, it appears that the $\sigma = 0.24$ equatorial $\bar{u}$ is close to the relevant $\bar{u}$ and that Fig. 7 summarizes the relevant Doppler-shifted Rosby phase speeds. The zero phase speed region west of the westerly wind maximum is one from which the wave activity tends to diverge.

This accumulation mechanism is much simpler than that described by Webster and Chang (1988). With the large equivalent depth they used, the implied Doppler-shifted long wavelength phase speed was always westwards. Their mechanism depended on the convergence in the flow shrinking the zonal wavelength until the phase speed and group velocity were near zero and the wave accumulated near $\bar{u} = 0$. At $\sigma = 0.24$ this point, as indicated in Fig. 7, is some 30° to the east of the zero Doppler-shifted phase speed point. There is no indication in Fig. 8 of zonal length scales decreasing markedly and of small-scale wave activity accumulating at this point.

In all three cases, despite the Doppler-shift accumulation mechanism, the amplitudes in the equatorial region are diminishing with time, though this decay is least marked for the initial perturbation in the accumulation region. In agreement with Webster and Chang (1988) there is no sign of instability associated with the longitudinally varying flow.

Other features of the solutions are suggested by Fig. 9, which is a longitude–time plot of $\nu$ at $\sigma = 0.24$ and 30°N for case C. Compared with Fig. 6(a) it is clear that the Rossby wave train tends to move with the equatorial perturbation that spawned it a
Figure 8. Longitude-time plots of $\sigma = 0.24$, equatorial perturbation for cases A, B and C. The conventions are as in Fig. 4 with the contour interval $0.5 \, \text{m} \, \text{s}^{-1}$.
couple of days earlier. In case B the wave train tends to be stationary and in case A eastward-moving. Apart from the very weak wave train source after two weeks tending to be near point B there is no sign of the emanation from the accumulation region discussed by Webster and Chang (1988). In case C (Fig. 9) the signature of westward-moving baroclinic waves is delayed a few days from the zonal flow case. However, in case A it is advanced by a few days. This suggests the importance of Rossby wave phase speed in determining the efficiencies of the triggering of baroclinic instability, a point stressed in Hoskins (1990).

5. PERTURBATIONS TO A 3-D CLIMATOLOGICAL FLOW

The basic DJF climatological flow used is shown by its $\sigma = 0.24$ streamfunction in Fig. 10. Numerous structures and locations for initial perturbations to this flow have been used but only one (case A) will be described in detail and some results shown for two more (cases B and C). Their locations are indicated in Fig. 10.

Case A has an initial vorticity dipole centred on the equator at 120°E with the upper tropospheric, profile 2, vertical structure. The time development of the $\sigma = 0.24$ perturbation streamfunction is shown in Fig. 11. Comparing day 3 with the zonal flow case (Fig. 5(a)), a similar development is seen, though the 'Kelvin wave' equatorial easterly flow to the east is absent. However, the day 6 results (Figs. 5(b) and 11(b)) are completely different away from the initial perturbation. A meridional dipole has rapidly developed in the central North Pacific, and is accompanied by another centre in the eastern sub-tropical North Pacific. At day 9 the North Pacific pattern is enhanced. By day 12 the sub-tropical Pacific pattern has weakened and the Rossby wave propagation across North America is marked. The southern hemisphere wave pattern is weaker and shows no dramatic change in character during the period.

More details of the solution are given in Fig. 12. The longitude–height section of the 30°N $v$ at day 9 (Fig. 12(a)) emphasizes the upper tropospheric and equivalent barotropic nature of the sub-tropical waves. The longitude–time plot of $v$ at $\sigma = 0.24$ at 30°N (Fig. 12(b)) shows a rapid transition near day 5 from the initial response to the
longer-lasting North Pacific pattern. The relaxation of this and the downstream wave propagation is evident after day 9. This wavetrain has eastwards phase speed but not as large as that of the baroclinic waves which dominate Fig. 12(b) after day 14. The transition between the two appears to be rather smooth.

The whole development is remarkably similar to that anticipated on the basis of studies with the barotropic vorticity equation. The structure in the North Pacific and relaxation across North America is reminiscent of the eigenmodes found by Simmons et al. (1983) as well as the observed PNA pattern. Zhang (1988) has made a thorough investigation of the barotropic initial value problem and the projection of initial states onto the eigenmodes. The optimum initial perturbations for projecting onto the North Pacific modes of Simmons et al. (1983) have their maximum amplitude in the north equatorial region 30°E–150°E. For such initial conditions wavetrains from the source region seemingly trigger the Pacific mode followed by the North American wavetrain, the sign of the pattern being a sensitive function of the longitude of the initial perturbation. The results presented here show that much of this barotropic theory is relevant in the context of a baroclinic model. Indeed, the favourable longitudes for the triggering of the equivalent barotropic North Pacific dipole and North American wavetrain are found to apply also in the baroclinic model. The relatively large amplitudes found after day 5 in the mid-Pacific to North American region in Figs. 11(c) and (d) and Fig. 12 (compare Fig. 6(a)) are indicative of energy extracted from the basic flow in the mid-Pacific. This can be viewed as a manifestation of the barotropic instability found by Simmons et al. (1983).

The development of the u-perturbation for \( \sigma = 0.24 \) on the equator is shown by the longitude–time plot in Fig. 13(a). The equatorial easterly maximum initially migrates westwards but then appears to split with one portion remaining near 60°E and the other moving eastwards before disappearing. It is clear from Fig. 11 that both extrema become associated with sub-tropical rather than equatorially trapped waves. One centre is almost stationary near 60°E, 20°N at a longitude west of the significant upper tropospheric equatorial easterlies. The other centre appears to be part of the Pacific pattern and is centred near 30°N. The most significant equatorial perturbation winds after day 12 are the quasi-stationary easterlies near 120°W and in the Atlantic sector. These are both regions of basic equatorial westerlies in which it is seen from Fig. 11 that the sub-tropical wavetrains extend to the equator.
Figure 11. Streamfunction perturbations at \( a = 0.24 \) for an initial equatorial perturbation centred at 120\(^\circ\)E (case A) on the DJF climatological flow for (a) day 3, (b) day 6, (c) day 9, (d) day 12. The conventions are as in Fig. 2.
Further information on equatorial behaviour is given in Figs. 13(b) and (c) which are similar plots for cases B and C, which have initial perturbations centred at 160°W and 45°W. Both show a weak Kelvin wave propagating rapidly to the east. In B, the equatorial Rossby wave propagates rapidly towards the west in the region of strong equatorial easterlies, much as suggested by the study described in section 4. The subsequent behaviour of splitting followed by equatorial perturbations near 60°E, 120°W and 15°W is similar to that in case A. Case C leads within a few days to a split equatorial Rossby wave pattern with one portion almost stationary near 80°W and the other near 20°W, rather to the east of the two equatorial westerly wind maxima. By day 20 the perturbation maxima are, however, at longitudes similar to those in the other cases and not associated with equatorially trapped waves.

The notion of equatorial Rossby wave Doppler-shift accumulation appears to apply in the case of large-scale perturbations to a climatological flow to the extent that these
Figure 13. Longitude–time plots of $\sigma = 0.24$, equatorial $u$ perturbation for the DJF climatological flow and initial perturbations centred on the equator at (a) 120°E (case A), (b) 160°W (case B), (c) 45°W (case C). The conventions are as in Fig. 4 with the contour interval 0.5 m s$^{-1}$. 

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waves appear to move with a Doppler-shifted phase and group speed: they move rapidly westwards in regions of equatorial easterlies and are more nearly stationary in regions of equatorial westerlies, becoming stationary to the east of significant westerly maxima. In the Indian Ocean a quasi-stationary response occurs in a region west of significant upper tropospheric equatorial easterlies. The upper tropospheric equatorial winds in this region are weak but, averaged over the sub-tropical region of the response, there are significant westerlies. In the east Pacific and Atlantic where there are significant westerlies on the equator, the waves in the mid-latitude Rossby wavetrains extend meridionally to produce anomalies that may be viewed as equatorial Rossby waves which again ‘accumulate’ slightly to the east. This is in some agreement with the theoretical investigation by Webster and Holton (1982). According to the observational studies of Murakami and Unninayar (1977) and Arkin and Webster (1985), maxima of equatorial 200 mb perturbation kinetic energy are indeed observed in the east Pacific and Atlantic sectors. The Indian Ocean maximum is not, however, supported in general, although Murakami and Unninayar (1977) showed such a region for January 1971 slightly north of the equator and Blackmon et al. (1984) showed this to be the tropical end of one of the waveguides for stationary Rossby wavetrains for motion on time-scales of 10–30 days.

6. CONCLUSIONS

The initial value technique has enabled the barotropic vorticity equation wavetrain and eigenmode studies and the equatorial vertical mode plus shallow water equation studies to be extended to a 3-D baroclinic atmosphere. Results for only one horizontal perturbation structure have been shown here, but limited experimentation suggests that the behaviour shown is not dependent on this. However, it is interesting to note that a perturbation limited to one hemisphere leads to a 5-day wave (e.g. Madden 1979; Salby 1984) which is clearly visible in the other hemisphere.

The equivalent barotropic Rossby wavetrains have a behaviour that is qualitatively very similar to that suggested by barotropic investigations. Over the first few days a wavetrain propagates through the sub-tropics of the winter hemisphere in particular. In a zonal flow a sub-tropical wavetrain of scale about zonal wavenumber six becomes established followed by a higher-latitude, lower-wavenumber propagation. In a longitudinally varying climatological flow the location of the tropical perturbation is very important. For perturbations in the Indian Ocean–west Pacific sector, the wavetrain propagating through the sub-tropics triggers a modal structure in the North Pacific corresponding to a meridional dipole which relaxes to a downstream wavetrain over North America.

Equatorial Kelvin waves are identifiable moving eastwards at about 41 m s⁻¹ in a resting atmosphere. However, for the balanced initial perturbations used here their amplitudes are relatively small. The large equatorial Rossby wave has a simple ‘internal mode’ structure in the troposphere. On the large length-scales it is approximately non-divergent and the single equatorial wind anomaly propagates westwards at about 8 m s⁻¹ in the resting atmosphere. In more complicated basic states its speed is approximately Doppler shifted by the upper tropospheric tropical zonal wind. This raises the possibility of trapping to the east of a westerly maximum of more than 8 m s⁻¹. This Rossby wave accumulation study was inspired by the work of Webster and Chang (1988) but the Doppler-shift mechanism is simpler than theirs, which requires the wavelength, group velocity and phase speed to tend to zero near the zero \( \bar{u} \) point to the east of a westerly wind maximum. In some agreement with Webster and Holton (1982), in a longitudinally varying flow, geographically fixed corridors for wave propagation into the equatorial
region are found in the east Pacific and Atlantic sectors because there the upper
tropospheric westerlies extend to the equator. Extra-tropical equivalent barotropic
Rossby waves are able to extend to the equator and trigger equatorial internal Rossby
waves which tend to be trapped slightly to the east.

The results described here appear to be generally consistent with the observations
of the equivalent barotropic patterns associated with anomalous equatorial heating in
the Pacific (Horel and Wallace 1981), of teleconnection patterns in general (Wallace and
Gutzler 1981), and of equatorial perturbation kinetic energy (Murakami and Unninayar

A subsequent paper will describe experiments in which a geographically fixed
equatorial heating is switched on at an initial instant, enabling the direct response to this
heating to be discussed.

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