Lagrangian and $K$-theory approaches in modelling evaporation from sparse canopies

By A. J. Dolman and J. S. Wallace

Institute of Hydrology, Wallingford, Oxon OX10 8BB

(Received 29 November 1990, revised 15 May 1991)

SUMMARY

An evaporation model based on Lagrangian turbulent diffusion principles is developed and compared with simpler single- and dual-source $K$-theory models of evaporation. The performance of the Lagrangian model is assessed against observations of total evaporation by an eddy-correlation instrument and found to be satisfactory for a millet crop in west Africa. Three versions of existing simpler models containing $K$-theory descriptions of within-canopy turbulence are described and their results also compared. The two $K$-theory models that explicitly take into account the soil source perform better than the single-source Penman–Monteith model. A Lagrangian analysis does not seem to be necessary for this kind of crop because the low source-density profile of the crop, associated with a low leaf-area index, caused the near-field effect to be very small. The overall difference between the evaporation estimates of the dual-source and Lagrangian models is therefore small. It is concluded that, for practical purposes, $K$-theory remains an adequate approximation of turbulent transport in sparse-crop evaporation models.

1. INTRODUCTION

Previous developments in modelling the land surface–atmosphere interaction have shown that for vegetation with near-complete canopy cover, the canopy may be represented as a single ‘big-leaf’, with the sources of sensible and latent heat located at the same level $d + z_0$, where $d$ is the zero-plane-displacement height and $z_0$ the roughness length (for examples, see Monteith 1981 and Shuttleworth 1989). When the canopy cover is incomplete, the big-leaf approach is less appropriate as the exposed soil then becomes a significant additional source of heat and water vapour.

The purpose of this paper is twofold. It shows how Lagrangian principles can be applied in practice to calculate evaporation from sparse crops. It also compares the performance of dual-source $K$-theory models with both a modified Penman–Monteith model and the Lagrangian model. All these models are applied to data from a millet crop in Niger, west Africa, for which the necessary measurements of stomatal conductance, leaf area, and weather variables were available. The model predictions are compared with measurements of total evaporation obtained by an eddy-correlation instrument (Shuttleworth et al. 1988).

The dual-source models (Shuttleworth and Wallace 1985; Choudhury and Monteith 1988) attempt to model the partitioning of energy for soil–vegetation systems with more than one effective source. Such models describe the transport of heat and water vapour from the soil, through the canopy, to the reference level above the canopy. Traditionally the transfer of a scalar quantity, $c$, within a canopy has been parametrized with local gradient diffusion ($K$-theory):

$$ F(z) = -K(z) \frac{\delta c}{\delta z} \tag{1} $$

where $K(z)$ is the diffusion coefficient and $F(z)$ the scalar flux.

A number of studies have emerged during the last decade questioning the validity of $K$-theory for within-canopy transfer (e.g. Finnigan and Raupach 1987). The observation of countergradient fluxes (Denmead and Bradley 1985), implying negative $K$-values, is the most dramatic example of the failure of Eq. (1). Generally, $K$-theory...
requires the characteristic length scale of the dominant eddies to be small compared with
the distance over which the gradient changes appreciably. This is violated within most
plant canopies where the length scale of the turbulence is of the same order as the canopy
height. Although these objections have been known for some time, until recently the
lack of a practical alternative has led most modellers to persist in using K-theory.
Alternatives include either random-walk or second- (or higher) order-closure models
which are computationally too demanding to be used in a micrometeorological model
(e.g. Wilson 1989). A promising alternative has been developed by Raupach (1989a)
which opens up the possibility of replacing Eq. (1) with a more realistic theory of withincanopy turbulence. In essence, his theory allows the near-field effect to be modelled in
conjunction with the larger-scale variation (far field), which is described by conventional
gradient-diffusion theory.

Raupach’s ‘localized near-field’ theory of turbulent transfer is made operational in
the current paper to develop a multi-layer evaporation model. To do this, the coupled
set of equations for simultaneous heat and water-vapour transfer are uncoupled, by
introducing a new concentration and flux variable, and solved using matrix algebra. The
predictions of this evaporation model are compared with the simpler single-source and
dual-source K-theory models and three days of eddy-correlation observations.

2. Theory

(a) Lagrangian theory

The Lagrangian approach aims to predict the scalar concentration at a particular
 canopy level from a given source density by tracking an ensemble of ‘marked fluid
particles’ carrying the scalar. This allows the effect of fluid-particle history to be taken
into account, which is of considerable importance in plant canopies, where fluid particles
exist persistence over time intervals (t) smaller than the Lagrangian time scale, \( t_1 \) (a
full list of nomenclature for ‘symbols used’ is given in the appendix). This may give rise
to ‘bumps’ in the concentration profile and decouple the local gradient from the vertical
flux. Consequently, the dispersion of a scalar cloud in a plant canopy is non-diffusive in the
near field (\( t \approx t_1 \)) and diffusive in the far field (\( t \gg t_1 \)). The scalar concentration at any
level in a canopy is then a superposition of both near-field dispersion (scalar emanating
from nearby leaves) and far-field dispersion (scalar from distant source points). Raupach
(1989a) showed how the concentration of a scalar quantity such as temperature or
humidity at a vegetation layer \( i \) can be represented by the sum of the near-field and far-
field contributions:

\[
C_i = C_n + C_{ff}.
\]

In Eq. (2) the near-field effect provides the small-scale variation and the far field provides
the large-scale background variation. The far-field concentration, \( C_{ff} \), at a particular
 canopy level, \( i \), is calculated with classical gradient diffusion theory:

\[
C_{ff} = C_r - C_{nr} + \sum_{j=1}^{n} \left( \frac{F_j}{K_j} \right) z_j
\]

where \( F_j \) is the total flux at source layer \( j \), \( K_j \) is a far-field diffusion coefficient given as
\( K_j = \sigma_{wj}^2 t_{ij} \), with \( \sigma_{wj}^2 \) the variance of the vertical wind velocity, and \( z_j \) and \( \Delta z_j \) are the
height and thickness of the \( j \)th layer. \( C_{nr} \) is the near-field concentration at the reference
height.
The total scalar flux at level $j$ is given as

$$ F_j = \sum_{i=1}^{j} S_i \Delta z_i + F_s $$

(4)

where $S_i$ is the source density at layer $i$ and $F_s$ the flux emanating from the soil. The appearance of the near-field concentration in Eq. (3) is a result of the boundary condition at the reference height

$$ C_r = C_{ar} + C_{fr}. $$

(5)

In the calculation of the near-field contribution it is assumed that the turbulence is locally homogeneous. It can then be shown (Raupach 1989a) that the near-field concentration can be described as the convolution of the source-density distribution with a 'near-field kernel' function:

$$ C_{ni} = \sum_{j=1}^{n} \left( \frac{S_j}{\sigma_{w_j}} \right) |K_n\left\{(z_i - z_j)/\sigma_{w_j t_j}\right\} + K_n\left\{(z_i + z_j)/\sigma_{w_j t_j}\right\}| \Delta z_j $$

(6)

where $z_j$ is the source height and $K_n(\varepsilon)$ the near-field kernel function derived by Raupach (1989a) for $\varepsilon > 0$, as

$$ K_n(\varepsilon) = K_n(-\varepsilon) = -0.3984 \ln(1 - e^{-\varepsilon}) - 0.15623 e^{-\varepsilon}. $$

(7)

It can be seen from Eq. (6) that the near-field concentration at any level involves contributions from all sources in the canopy. However, due to the highly peaked nature of the near-field kernel function only those contributions for which $|z_i - z_j| > \sigma_{w t_j}$ contribute substantially to the near-field effect. To summarize, the two main assumptions used in deriving this theory are that scalar dispersion in the far field can be described by the conventional gradient diffusion equation (Eq. (1)), and the near-field effects can be described by assuming the turbulence to be locally homogeneous. The second assumption led Raupach (1989a) to refer to his theory as a 'localized near-field theory'.

(b) A practical Lagrangian evaporation model

For practical application of this set of equations to the coupled transfer of heat and water vapour in real plant canopies a further set of equations has to be developed. Raupach (1989b) suggests a description of the scalar concentrations in each layer as

$$ C_i - C_r = \sum_{j=1}^{n} m_{ij} S_j \Delta z_j $$

(8)

where $m_{ij}$ is the concentration in layer $i$, relative to the concentration at the reference height, produced by a source of unit strength in layer $j$ ($S_j \Delta z_j = 1$), so that

$$ m_{ij} = (C_i - C_r)/S_j \Delta z_j. $$

(9)

The system described by Eq. (8) relates the sources of $n$ layers to the concentration of these layers. In the case of heat and water vapour transfer, a complication arises because of the coupling of heat and water vapour transfer in a plant canopy through the energy balance. This would lead to a set of $2n$ equations with both temperature and humidity as unknowns; these can only be solved simultaneously. However, following McNaughton (1976) and Chen (1984), and uncoupling the system by introducing new variables (linear combinations of concentrations and fluxes), a linear system of $n$ equations in only one unknown can be obtained and solved algebraically. This is the approach followed here.
The fluxes of heat and water vapour emanating from any one layer of foliage may be expressed as

\[ H_i = \rho c_p (T_i - T_i)/r_{bi} \]  
\[ \lambda E_i = \{(\rho c_p)/\gamma\}(e_s(T_i) - e_i)/(r_{bi} + r_{si}) \]

where \( \rho \) is the density of air, \( c_p \) the specific heat of air at constant pressure, \( \gamma \) the psychrometric constant, \( T_i \) and \( e_i \) the temperature and vapour pressure of the air in layer \( i \), \( e_s(T_i) \) is the saturated vapour pressure at the temperature of the foliage \( T_i \). \( r_{bi} \) and \( r_{si} \) are respectively bulk boundary-layer and bulk stomatal resistance of foliage layer \( i \). At each level the energy balance equation applies

\[ R_{ni} = H_i + \lambda E_i \]

where \( R_{ni} \) is the available energy in the form of net radiation (at the lowest level the available energy is given by \( R_{ns} = G \), where \( G \) is the soil heat flux). Introducing \( s \), the rate of change of the saturated vapour pressure with temperature, yields two equations, similar to the Penman–Monteith equation (Monteith 1973), for latent and sensible heat, with the new variable \( D \), the vapour-pressure deficit:

\[ H_i = (\gamma r_{si} R_{ni} - \rho c_p D_i)/(sr_{si} + \gamma r_{si}) \]
\[ \lambda E_i = (sr_{bi} R_{ni} + \rho c_p D_i)/(sr_{bi} + \gamma r_{si}) \]

where \( r_{si} = r_{bi} + r_{si} \). From the definition of \( s \) it follows that

\[ D_i - D_r = s(T_i - T_r) - (e_i - e_r). \]

Referring to Eq. 8 it can now be shown that

\[ D_i - D_r = \{(s/(\rho c_p))\} \sum_{j=1}^{n} m_{ij} J_j \]

where the new flux variable, \( J_j \), can be obtained from substitution of Eq. (14) into Eq. (13) and follows as

\[ J_j = H_i - (s/\gamma) \lambda E_i. \]

This is the same flux variable as described by Chen (1984) and McNaughton (1976). Substitution of Eq. (15) into Eqs. (12) and (14) yields the desired steady-state concentration of the vapour-pressure deficit in each layer. In the case of evaporation from the soil surface a complication arises, as near the surface the Lagrangian time-scale becomes zero and the flux is only subject to far-field diffusion. In this case a factor \( (s/\rho c_p) J \Sigma[1/\sigma_{wj}^2 t_i] \Delta z \) has to be added to the right-hand side of Eq. (14), where \( J_s \) is the flux from the soil. The uncoupled system can now be written as

\[ b_i = \sum_{j=1}^{n} a_{ij} D_i \]

where the column vector \( \mathbf{B} \) is given as

\[ b_i = D_r + (s/\rho c_p) \sum_{j=1}^{n} (R_{ni} m_{ij})/[1 + (r_{bi}/\beta r_{si})] + s/(\rho c_p) J_s \sum_{j=1}^{n} \{1/\sigma_{wj}^2 t_i\} \Delta z \]

where \( \beta = \gamma/(s + \gamma) \). The elements of the coefficient matrix \( \mathbf{A} \) are defined as

\[ a_{ij} = m_{ij}/(r_{bi} + \beta r_{si}) \quad i \neq j \]
EVAPORATION MODELLING

\( a_{ij} = 1 + \{m_{ij}/(r_{ij} + \beta r_{ij})\} \quad i = j \) \hspace{1cm} (18b)

This system can be solved with standard matrix procedures for systems of linear equations. Substitution of \( D_i \) into Eq. (12) yields the flux-density distribution. Temperature and vapour-pressure distributions can be found from the equivalent versions of Eq. (14) for temperature and sensible-heat-flux distributions and vapour-pressure and latent-heat-flux distributions respectively. The general theory of the multi-layer evaporation model with a Lagrangian within-canopy transfer theory is thus complete.

(c) Auxiliary equations

The linear system as specified in Eqs. (17) and (18(a), (b)) needs information regarding the distribution of available energy in the canopy, boundary layer and stomatal resistances, and the turbulence profiles for the Lagrangian submodel \( \sigma^2_w \) and \( t_i \). Analysis of turbulence profiles for several plant canopies led Raupach (1989b) to suggest that, in the absence of measurements, the profiles of the vertical-velocity variance and the Lagrangian time-scale may be approximated from

\[
\sigma_w(z) = u_\ast 1.25 \quad \text{for} \quad z/h \gg 1 \\
\sigma_w(z) = u_\ast (0.25 + z/h) \quad 0 < z/h < 1 \\
t_i(z) = (u_\ast/h) \left[ \max\{0.3, k(z - d)/1.25h\} \right]
\]

(19a)

(19b)

(20)

where \( u_\ast \) is the friction velocity and \( h \) the canopy height. The boundary-layer resistances are calculated according to Choudhury and Monteith (1988). First the wind speed has to be extrapolated downwards from a reference height, \( z \), to the top of the canopy:

\[
u(h) = u_* \left[ \ln((h - d)/z_0) \right]/\left[ \ln((z_r - d)/z_0) \right].
\]

(21)

Following Choudhury and Monteith (1988), the variation of \( d \) and \( z_0 \) with leaf area, \( L \), is calculated from

\[
d = 1.1h \ln(1 + X^{1/4}) \quad \text{(22a)}
\]

\[
z_0 = z_0 + 0.3hX^{1/2} \quad 0 \leq X \leq 0.2 \quad \text{(22b)}
\]

\[
z_0 = 0.3h(1 - d/h) \quad 0.2 \leq X \leq 1.5 \quad \text{(22c)}
\]

where \( X = C_dL \). The value of the roughness length of the soil substrate, \( z_0 \), is taken as 0.01 m, and following Shuttleworth and Gurney (1990), a value of 0.07 is used for the mean drag coefficient of the leaves, \( C_d \).

The wind-speed profile within the canopy is calculated from

\[
u(z) = u(h) \exp[\alpha'(z/h - 1)]
\]

(23)

where \( \alpha' \) is an attenuation coefficient for wind speed (\( \alpha' = 2.5 \)). The boundary-layer resistance of a single layer is calculated as

\[
r_b = (u(z)/w)^{-1/2}/a
\]

(24)

where \( w \) is a characteristic leaf width, taken to be 0.05 m for millet and the constant \( a = 0.01 \) (Shuttleworth and Gurney 1990). The total boundary-layer resistance per layer is given by dividing Eq. (24) by twice the leaf-area index for that layer. The stomatal resistances in the present paper are measured (see section 3). The attenuation of net radiation within the canopy is assumed to be analogous to the attenuation of solar radiation and is calculated as

\[
R_n(z) = R_n \exp(-\alpha'L')
\]

(25)
where $R_n$ is net radiation measured above the canopy, $\alpha$ the net radiation attenuation coefficient for millet measured as $\alpha = 0.41$ (Wallace et al. 1990), and $L'$ the leaf area between the top of the canopy and the height $z$.

(d) Single- and dual-source models based on K-theory diffusion

The concept of K-theory has been extensively used in physically based evaporation models (Penman 1948; Monteith 1981). The utility of these models is demonstrated by their increasingly wide adoption in the fields of meteorology, hydrology and agriculture. As the most simple model for canopy evaporation from sparse canopies, a modified Penman–Monteith equation suggested by Wallace et al. (1990) is considered:

$$\lambda E_c = \{s f_c R_n + (\rho c_p D_r / r_s)\} / [s + \gamma (1 + (r_c^s / r_s))]$$

(26)

where $D_r$ is the vapour-pressure deficit at reference height and $r_c^s$ a bulk stomatal resistance of the canopy. Soil evaporation is then added to the canopy evaporation to obtain total evaporation. The modification consists of the use of the net radiation absorbed by the canopy, given as a fraction, $f_c$, of the net radiation at the reference height. From Eq. (25), $f_c$ can be expressed as

$$f_c = 1 - \exp(-\alpha L).$$

(27)

The aerodynamic resistance, $r_s$, is calculated from crop height, and wind speed at the reference height, $u_r$, according to

$$r_s = \left[ \frac{\ln^2((z_r - d)/z_0) + \ln(z_0/z_r) \ln((z_r - d)/z_0)}{k^2 u_r} \right]^{-1}.$$

(28)

The second term in Eq. (28) takes into account the differences in transfer of momentum and heat and water vapour. The natural logarithm of the ratio of the roughness lengths for momentum and temperature ($\ln(z_0/z_r)$) is taken as 1.5 following Garratt and Hicks (1973). The simple relationships of Monteith (1973) are used in this case for $d$ and $z_0$ ($d = 0.63h$ and $z_0 = 0.13h$).

The second $K$-theory model involves a modification by Shuttleworth and Wallace (1985) and explicitly recognizes the contribution of the soil substrate. In this model the total evaporation from the crop and bare soil substrate is given by

$$\lambda E = C_c PM_c + C_s PM_s$$

(29)

where the PM terms refer to the Penman–Monteith equations which would apply to a closed canopy (PM$_c$) and a bare soil (PM$_s$). They have the form

$$PM_c = \{s R_n + (\rho c_p D_r - s r_a^c R_n) / (r_a^c + r_s^c)\} / (s + \gamma (1 + (r_c^s / (r_a^c + r_s^c)))).$$

(30a)

$$PM_s = \{s R_n + (\rho c_p D_r - s r_a^s (R_n - R_n)) / (r_a^s + r_s^s)\} / (s + \gamma (1 + (r_c^s / (r_a^s + r_s^s)))).$$

(30b)

$R_n$ and $R_m$ are the available energies above the crop and at the soil substrate respectively, the coefficients $C_c$ and $C_s$ are functions of the various resistances, all of which are defined by Shuttleworth and Wallace (1985). The aerodynamic resistances are necessarily different from those in the Penman–Monteith equation as they apply to different paths of vapour transfer. The bulk boundary-layer resistance of the leaves, $r_s^c$, is related to the total leaf-area index of the crop, $L$, via

$$r_s^c = r_b / 2L.$$

(31)

In this model we follow Shuttleworth and Wallace (1985) and adopt a value of $r_b = 25$ m s$^{-1}$, this being typical for field crops. The values of aerodynamic resistances between the substrate and canopy source height, $r_s^c$, and canopy source height to reference level,
$r_{s}$, are calculated following Shuttleworth and Wallace (1985). They assumed that the values of $z_0$ and $d$ for a fully developed canopy are given by the Monteith (1973) relationships. Then, implicit in the way $r_{s}$ and $r_{d}$ vary between a closed canopy and bare soil substrate, there is a corresponding variation in the values of $d$ and $z_0$.

The third K-theory model used in this paper is the updated version of the Shuttleworth and Wallace model as described by Shuttleworth and Gurney (1990). This follows Choudhury and Monteith's (1988) description of aerodynamic transfer by replacing the simple crop-height relationships of $d$ and $z_0$ with more sophisticated relationships based on second-order closure modelling (e.g. Shaw and Pereira 1982). These relationships have already been given in this paper as Eqs. (22), (22a) and (22b). They show how the effective sink height for momentum varies with leaf area. It is important to note that in both the Shuttleworth–Gurney model, and the Shuttleworth–Wallace model, the source height of water vapour and heat remains fixed at $d + z_0$, calculated according to the relations of Monteith (1973) as previously given. They only vary with height of the crop. The resulting values of the resistances are calculated using Eqs. (45) and (46) of Shuttleworth and Gurney (1990). Also, rather than using a constant value of boundary-layer resistance, as in the Shuttleworth–Wallace model, a boundary-layer resistance which varies with in-canopy wind speed is used (see Eqs. (23) and (24), this paper).

3. SITE DESCRIPTION AND MEASUREMENTS

The site where the measurements were taken was at Sadoré (13°15'N 2°17'E), the experimental farm of the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) Sahelian Center, located about 45 km south of Niamey, Niger. The crop was millet (Pennisetum americanum cv. CIVT) which was planted in rows 0.75 m apart, with a density of about 30,000 plants per hectare. The soil was Dayoubu sand, 2 to 3 m deep, overlying laterite gravel. The climate is typical of the southern Sahelien zone, with summer rainfall and high temperatures throughout the year. Annual potential evaporation exceeds rainfall by almost 1500 mm on average (Sivakumar 1986).

Stomatal conductance of the leaves was measured with an automatic diffusion porometer (AP3, Delta-T Devices, Cambridge, UK). Direct evaporation from the soil was measured using small soil lysimeters. Further details of the specific measurement procedures used for stomatal conductance, soil evaporation, and leaf-area index are given by Wallace et al. (1990). For the current analysis three days were selected to represent a wide range of soil and crop conditions. Details are given in Table 1. Hourly values of net radiation, temperature, humidity and wind speed were recorded, using an automatic weather station (Didcot Instruments, Didcot, UK) with a modified wet- and dry-bulb assembly to allow the thermometers to be continuously aspirated. The measurement height of these instruments was 4.5 m.

<table>
<thead>
<tr>
<th>Date</th>
<th>Leaf-area index</th>
<th>Crop height metres</th>
<th>Soil evaporation millimetres</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 September 1985</td>
<td>0.26</td>
<td>2.3</td>
<td>2.5</td>
</tr>
<tr>
<td>6 August 1986</td>
<td>1.4</td>
<td>2.7</td>
<td>1.75</td>
</tr>
<tr>
<td>13 August 1986</td>
<td>1.10</td>
<td>2.8</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Measurements of total evaporation were obtained with the Institute of Hydrology Mark 2 Hydra eddy-correlation instrument (Shuttleworth et al. 1988). The Hydra uses a
one-dimensional sonic anemometer to measure vertical wind speed and an infra-red hygrometer to measure fluctuations in humidity. The measurement height for this instrument was also 4.5 m.

4. RESULTS

(a) Performance of the Lagrangian evaporation model

In this section the performance of the Lagrangian evaporation model is analysed by comparing the predictions of total evaporation with eddy-correlation measurements of total evaporation for three days. A sensitivity analysis on the model is performed for a number of important parameters. In a later section in this paper the model’s predicted evaporation for three days is compared with a simpler K-theory model.

In Figs. 1(a), (b) and (c) the hourly predicted total evaporation is shown in comparison with the observed evaporation for the three selected days. The diurnal trend follows the measured evaporation quite closely. Although there is scatter in the measurement, the overall agreement is acceptable, bearing in mind the uncertainties in both the evaporation measurements and the input stomatal conductance and soil evaporation data. The poorest agreement was observed on 17 September. On this day the ‘fall’ in measured evaporation around 10.00 h is likely to be a measurement error, since there was no corresponding fall in the level of net radiation. Daily total evaporation is given in Table 2 and the daily total plant evaporation in Table 3. Total predicted evaporation is within 5% of the observed values for the first two days and within 9% for 13 August. Within the conditions of the test data, therefore, these results indicate that the Lagrangian model is capable of predicting acceptable evaporation fluxes.

It is important to assess the sensitivity of the Lagrangian model to changes in its basic input parameters. In Table 4 the results of a sensitivity analysis are shown for a number of key parameters. The sensitivity of the model to each parameter is expressed as the percentage change in total evaporation over a day, in consequence of an imposed change of ±25% in that parameter. Generally the sensitivity is very low, except for the specification of stomatal conductance where a 25% change propagates as a 15% change in the predicted plant evaporation. Sensitivity to the turbulence profiles as described in Eqs. (19) and (20) is remarkably low. This is important as it suggests that detailed within-canopy turbulence measurements are not necessary to use this Lagrangian approach. This sensitivity study suggests that accurate inputs of stomatal conductance and soil evaporation are prime requirements in sparse-crop evaporation models.

(b) Comparison of K-theory evaporation models

Figures 2(a), (b) and (c) show a comparison of values of total evaporation calculated using the Penman–Monteith (P–M), Shuttleworth and Wallace (S–W) and Shuttleworth and Gurney (S–G) models. On 17 September 1985 soil evaporation rates were high and plant evaporation rates very low, and there is little difference in the values of total evaporation calculated by the three K-theory models (Figure 2(a)). The total evaporative flux is in this case dominated by the soil contribution. On this day the crop was at its maximum height, and the leaf-area index was very low. The three models compare reasonably well with observations on that day, bearing in mind the limitations of the observed flux data. On the second day, 6 August 1986 (see Fig. 2(b)), the soil surface was relatively dry. Under these conditions the predictions are in much better agreement with the measurements. The P–M model gave the lowest plant evaporation rates and the S–G model the highest, the differences between them over the entire day being 8%. The under-estimation of plant evaporation by the P–M model in sparse canopies growing in
Figure 1. Comparison of the observed total evaporation (---) with the predicted total evaporation by the Lagrangian model (----) for (a) 17 September 1985, (b) 6 August 1986 and (c) 13 August 1986.
dry soil is a result of neglecting the substantial heat flux from the soil substrate and has already been reported by Wallace et al. (1990). This pattern is repeated on the third day shown in Fig. 2(c), where the difference between the P–M and S–G models is increased to 13%. The results are again summarized in Tables 2 and 3. Changing from the S–W to the S–G model generally increases the calculated plant evaporation by up to 10%, depending primarily on leaf-area index and soil wetness. The S–G model generally compares slightly better with the observations.

(c) **Comparison of the Lagrangian model with the Shuttleworth–Gurney model**

The S–G model as used in this paper, with its leaf-area-and-height dependent \( z_0 \) and \( d \), is arguably the most physically realistic of the three \( K \)-theory models described in this paper, and is used in this section for comparison with the Lagrangian evaporation model. In Fig. 3 the total evaporation predicted by the two models is compared with the observations. Both models agree quite well with the observations, although there is again considerable scatter. The difference between the two models is generally very small and
Figure 2. Comparison of the observed total evaporation (---) with the predicted total evaporation by the Penman-Monteith model (----), the Shuttleworth-Wallace model (-----) and the Shuttleworth-Gurney model (---) for (a) 17 September 1985, (b) 6 August 1986 and (c) 13 August 1986.
well within the scatter of the experimental results, being largest at high evaporation rates, where the S-G model predicts a consistently higher evaporation rate than the Lagrangian model. The reason for these higher evaporation rates is not that the S-G model predicts higher within-canopy vapour-pressure deficits than the Lagrangian model. This is illustrated in Fig. 4, which shows the within-canopy vapour-pressure deficits for 6 August, the day on which the high evaporation rates occurred. The within-canopy vapour-pressure deficit of the Lagrangian model is obtained by averaging the vapour-pressure deficits for each canopy layer. The S-G model produces a slightly lower

Figure 4. Within-canopy vapour-pressure deficit on 6 August 1986 as predicted by the Lagrangian model (---), the Shuttleworth–Gurney model (----) and the Penman–Monteith model (--.--).
within-canopy vapour-pressure deficit than the Lagrangian model. The predicted lower evaporation by the Lagrangian model is, therefore, probably due to the inclusion of the near-field effect, which may become more pronounced with high evaporative fluxes from a high-source-density profile. It should be noted that the main cause for the difference between the single- and dual-source K-theory model and the Lagrangian model in the afternoon is due to a difference in treatment of the within-canopy deficit in these models, as shown in Fig. 4, since the within-canopy deficit calculated by the P–M model (for its calculation see Wallace et al. 1990) starts to diverge substantially from the Lagrangian and S–G model.

5. CONCLUDING REMARKS

The Lagrangian model developed in the current paper has been shown to be capable of predicting evaporation from a sparse crop of millet in Niger, west Africa. Generally good agreement was obtained between Lagrangian model predictions of total evaporation and those measured by an eddy-correlation device. Good agreement was also obtained between observed total evaporation and the S–G model. Of the three K-theory based models, the P–M ‘big-leaf’ model, predicted lower plant evaporation in dry soil conditions than the two models which explicitly take the soil contribution into account (the S–W and S–G models). When a large evaporative flux emanates from the soil, the difference in plant evaporation between the tested models is small. This is because the large soil-evaporation flux dominates the flow of water vapour through the canopy and the resulting within-canopy water-vapour deficits do not differ markedly between the models (and the value at reference height). Conversely, when there is a large heat flux from the soil, the within-canopy deficits are different between the models and hence the models predict different transpiration rates (Fig. 4). It is then necessary to use models which take the soil contribution explicitly into account.

The S–G model, which incorporates a leaf-area-and-height-dependent location of the canopy sink of momentum (the values of d and z₀), generally predicted up to 10% more plant evaporation than the S–W model in which the d and z₀ vary implicitly with the calculated values of aerodynamic resistance. Under the conditions tested in this paper, the enhanced within-canopy turbulence in the S–G model gives rise to higher evaporation rates. Indeed, using a variable height for both momentum and water vapour, and heat sinks and sources, as in Choudhury and Monteith (1988), changed the calculated plant evaporation by about 10%. The sign of the change depended on the soil wetness. It is of interest to note that under the specific conditions used by Shuttleworth and Gurney (1990), this difference appeared to be numerically less important.

Comparison of the S–G model with the Lagrangian evaporation model showed that the S–G model predicted slightly higher evaporation rates than the Lagrangian model when there was a substantial sensible-heat flux from the soil. This result does not depend upon the inclusion of the near-field effect but is mainly a result of the specification of the far-field diffusion coefficients and the fact that a multi-layer model may give, through its enhanced resolution, an improved estimate of evaporation. The low leaf-area index of the crop specified in this study results in low-source-density profiles which tend to reduce the near-field effect. Indeed, setting the near-field effect to zero changed the total evaporation by less than 2%. In crops with high leaf-area indices and high elevated source-density profiles, such as forests, the near-field effect will probably be of more significance.

On the basis of the present results we conclude that the differences between dual-source, K-theory-based models and the Lagrangian model are small. Both are capable
of predicting evaporation to an acceptable accuracy, which depends more on uncertainties in stomatal conductance and leaf area than on added detail of the within-canopy transfer.

The main advantage of the Lagrangian approach may be in the added detail of the concentration and source profiles in the canopy. Such information may be of prime importance in crop disease and photosynthesis modelling and is easily provided with the practical Lagrangian model developed in this paper. The Lagrangian model, furthermore, provides a clear relation between the within-canopy transfer mechanisms and measurable turbulence characteristics like $\sigma_w$ and $t_i$. This relationship is notoriously difficult to interpret in plant canopies with the aid of classical gradient-diffusion theory (e.g. Denmead and Bradley 1985).

ACKNOWLEDGEMENTS

The data from the Sadoré experimental farm were collected as part of a joint project between the Institute of Hydrology and the ICRISAT Sahelian Center Agroclimatology Section under M. V. K. Sivakumar. The authors are pleased to acknowledge the support of D. D. McNeil, C. R. Lloyd and J. H. C. Gash in collecting the eddy-correlation measurements. The experimental results reported in this study were obtained as part of a project funded by the UK Overseas Development Administration. The first author holds a Central Electricity Generating Board Senior Research Fellowship funded by the Joint Environmental Programme of National Power and PowerGen.

APPENDIX

Nomenclature

- $a$: Constant used in the calculation of boundary-layer conductance ($a = 0.01 \text{ m s}^{-1}$)
- $a_{ij}$: $(i, j)$th element of coefficient matrix
- $A$: Coefficient matrix
- $b_i$: ith element of column vector
- $B$: Column vector
- $C_r$, $C_c$: Resistance coefficients used in dual-source models (dimensionless)
- $C_d$: Mean drag coefficient of the leaves (dimensionless)
- $C_r$, $C_i$: Concentration of scalar quantity at reference height and in the ith layer
- $C_{nr}$, $C_{ri}$, $C_{as}$, $C_{r}$: Near-field and far-field concentrations of scalar quantity in the ith layer and at the reference height
- $c_p$: Specific heat of air at constant pressure (J kg$^{-1}$ K$^{-1}$)
- $d$: Zero-plane displacement height (m)
- $D_r$, $D_i$: Vapour-pressure deficit at reference height and in the ith layer (mbar)
- $e_r$, $e_i$: Vapour pressure at reference height and in the ith layer (mbar)
- $e_s$: Saturated vapour pressure (mbar)
- $f_a$: Fraction of net radiation absorbed by the canopy (dimensionless)
- $F_r$, $F_i$, $F_s$: Scalar flux and scalar flux at the ith layer and soil substrate
- $F_j$: Total flux at source layer j
- $G$: Soil heat flux (W m$^{-2}$)
- $h$: Crop height (m)
- $H_i$: Sensible-heat flux from the ith layer (W m$^{-2}$)
- $J$: Flux variable $H - (s/\gamma) \lambda E$ (W m$^{-2}$)
- $k$: Von Kármán's constant (dimensionless)
$K$ Eddy-diffusion coefficient (m$^2$s$^{-1}$)
$K_f$ Far-field diffusion coefficient
$K_n$ Near-field kernel function
$L$ Projected leaf area per unit ground area (dimensionless)
$L'$ Projected leaf area per unit ground area between the reference level and a level in the crop
$M$ Dispersion matrix
$m_{ij}$ $(i, j)^{th}$ element of dispersion matrix (m$^2$s$^{-1}$)
$n$ Number of model canopy layers
$PM_c$, $PM_a$ Penman-Monteith equations for crop and bare soil (Wm$^{-2}$)
$r_a^s$ Aerodynamic resistance between canopy source height and reference level (s m$^{-1}$)
$r_a^c$ Bulk boundary layer resistance of the vegetative elements in the canopy (s m$^{-1}$)
$r_a^s$ Aerodynamic resistance between the soil substrate and the canopy source height (s m$^{-1}$)
$r_a^r$ Aerodynamic resistance from the single source height of the Penman-Monteith equation to the reference height (s m$^{-1}$)
$r_b$ Mean boundary layer resistance per unit area of vegetation (s m$^{-1}$)
$r_{bi}$ Bulk boundary-layer resistance of foliage layer $i$ (s m$^{-1}$)
$r_{s,i}$, $r_{s,j}$ Bulk stomatal resistance of the $i^{th}$ and $j^{th}$ foliage layer (s m$^{-1}$)
$r_a^r$ Bulk stomatal resistance of the canopy (s m$^{-1}$)
$r_a^s$ Surface resistance of substrate (s m$^{-1}$)
$R_n$, $R_{ni}$, $R_{nj}$, $R_{ns}$ Net radiation flux into the complete crop, the $i^{th}$ and $j^{th}$ foliage layer and the soil surface (Wm$^{-2}$)
$r_{vi}$ Total boundary-layer and stomatal resistance of a layer $r_{ad} + r_{bl}$ (s m$^{-1}$) in the $i^{th}$ layer
$s$ Rate of change of saturated vapour pressure with temperature (mb K$^{-1}$)
$S_i$, $S_j$ Source density of the $i^{th}$ and $j^{th}$ layer (Wm$^{-3}$)
$t$ Time (s)
$t_l$ Lagrangian time-scale (s)
$T_{li}$ Leaf surface temperature of layer $i$ (K)
$T_i$, $T_r$ Air temperature at the $i^{th}$ layer and reference height (K)
$u_r$ Wind speed at reference height (ms$^{-1}$)
$u_*$ Friction velocity (ms$^{-1}$)
$w$ Characteristic leaf width (m)
$z_r$ Reference height at which meteorological measurements were taken (4.5 m)
$z$ Height variable (m)
$z'$ Source height (m)
$z_h$ Roughness length for temperature (m)
$z_0$ Roughness length of the complete crop soil complex (m)
$z_s$ Roughness length of the soil substrate (m)
$\alpha$, $\alpha'$ Extinction coefficient for net radiation and wind speed (dimensionless)
$\beta$ $\gamma/(s + \gamma)$ (dimensionless)
$\gamma$ Psychrometric constant (mb K$^{-1}$)
$\lambda E_c$, $\lambda E_i$ Latent heat flux from the complete crop and the $i^{th}$ layer (Wm$^{-1}$)
$\rho$ Density of air (kg m$^{-3}$)
$\sigma_w^2$ Variance of the vertical wind velocity (m$^2$s$^{-2}$)
REFERENCES

Chen, J. 1984 ‘Mathematical analysis and simulation of crop micro-
meteorology.’ Ph. D. Thesis, Wageningen

Choudhury, B. J. and Monteith, J. L. 1988 A four-layer model for the heat budget of homogeneous land

Denmead, O. T. and Bradley, E.F. 1985 ‘Flux-gradient relationships in a forest canopy’. Pp. 421–442 in
*The forest atmosphere interaction*. Eds. B. A. Hutchison
and B. B. Hicks. Reidel Publishing Co., Dordrecht, Holland

E. Zeiger, G.D. Farquhar and I. R. Cowan. Stanford
University Press, Stanford, USA

Garratt, J. R. and Hicks, B. B. 1973 Momentum, heat and water vapour transfer to and from natural
and artificial surfaces. *Q. J. R. Meteorol. Soc.*, 104,
680–687

McNaughton, K. G. 1976 Evaporation and advection. II: evaporation downwind of a
boundary separating regions having different surface
Soc.*, 102, 193–202

Soc.*, 107, 1–27

Penman, H. L. 1948 Natural evaporation from open water, bare soil and grass.

1989b Applying Lagrangian fluid mechanics to infer scalar source distributions from concentration profiles in plant
canopies. *Agric. and Forest Meteorol.*, 47, 85–108

Shaw, R. H. and Pereira, A. R. 1982 Aerodynamic roughness of a plant canopy: a numerical experi-
ment. *Agric. Meteorol.*, 26, 51–65

Sivakumar, M. V. K. 1986 ‘Climate of Niamey’, ICRISAT Sahelian Center, Progress
Report 1

Shuttleworth, W. J. 1989 Micrometeorology of temperate and tropical forests. *Phil.
Trans. R. Soc. London*, B324, 299–334

Shuttleworth, W. J. and Gurney, R. J. 1990 The theoretical relationship between foliage temperature and

Shuttleworth, W. J. and Wallace, J. S. 1985 Evaporation from sparse crops—an energy combination

Shuttleworth, W. J., Gash, J. H. C., Lloyd, C. R., McNeil, D. D.,
Moore, C. J. and Wallace, J. S. 1988 An integrated micrometeorological system for evaporation
measurement. *Agric. and Forest Meteorol.*, 43, 295–317

using stomatal conductance and vegetation-area indices. *Agric. and Forest Meteorol.*, 51, 35–49

Wilson, J. D. 1989 ‘Turbulent transport within the plant canopy’. Pp. 43–80 in
*Estimation of areal evapotranspiration*. Eds. T. A. Black,
D. L. Spittlehouse, M. D. Novak and D. T. Price. IAHS
Publication No. 177, IAHS Press, Wallingford, UK