A study of mountain lee waves using clear-air radar

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SUMMARY

The tropospheric and lower stratospheric response to flow over complex terrain is investigated using observations of vertical and horizontal motions made by a clear-air Doppler radar stationed at Toulon in southern France. Analysis of the data has disclosed that episodic enhancements in the variability of vertical and horizontal velocities are due to lee waves excited by obstacles located within 5 km of the radar. Three separate cases of lee-wave activity are studied. Two-dimensional linear perturbation theory and existing three-dimensional simulation results, for appropriate parameter values, are used as guides to check against the vertical structure observed by radar. The magnitudes of vertical velocities observed during lee-wave incidence are well within the range reported in previous observational and numerical simulation studies.

From a detailed examination of one of the cases of trapped waves, the time variation, observed by the radar, of timescales of less than an hour, has emerged as the likely result of small directional changes in the upstream winds causing a bodily rotation of the downstream three-dimensional wave pattern centred on the wave-generating obstacle. The observed tendency of the vertical-velocity perturbation to change sign with time above the radar location follows as a consequence of the presence of large horizontal gradients in the motion field, which are inherent in the structure of three-dimensional lee waves.

The temporal variability of the lee-wave pattern has allowed the radar to sweep out a partial horizontal cross-section, revealing vertical circulation features that are broadly consistent with gravity-wave behaviour. However, analysis of the effects of finite antenna beam spacing on the sampled data indicates that a spurious phase-shift might have been introduced between the vertical and horizontal velocity perturbations observed in the cross-section. In contrast, the measured vertical velocities are not seriously affected by finite beam-width effects.

The lee waves examined in this study are quite moderate, nevertheless, this circumstance has provided a favourable opportunity to conduct a critical evaluation of the capabilities and limitations of this radar technique for observing lee-wave phenomena in general. We have been able to show clearly how the incidence and vertical structure of the waves respond to the synoptic-scale situation. In particular we have been able to distinguish between trapped and vertically propagating waves and to relate a transition between these wave types to synoptic-scale changes in the static stability structure.

1. INTRODUCTION

Lee waves induced by mountains have been the subject of numerous investigations, both theoretical and observational. On the observational side, different aspects of lee waves have been investigated employing a variety of techniques such as balloon-sondes, gliders (e.g. Holmboe and Kliefoth 1957), instrumented aircraft (e.g. Lilly et al. 1982; Brown 1983; Hoinka 1984; Cox 1986; Pitts and Lyons 1989), radar tracking of constant-level balloons (Booker and Cooper 1965; Vergeiner and Lilly 1970), Doppler lidars (Blumen and Hart 1988; Neiman et al. 1988; Banta et al. 1990) and clear-air Doppler...
radars (Ecklund et al. 1982; Balsley and Carter 1989; Sato 1990). In addition, satellite cloud imagery has been used in the visualization of the patterns of lee-waves from diverse obstacles in several geographic areas (Conover 1964; Gjevik and Marthinsen 1978; Mitchell et al. 1990). Some pertinent references in which previous observational and theoretical developments have been reviewed are: Smith (1979, 1989); Atkinson (1981); Lilly (1983); and Durran (1986).

The earliest examination of lee waves using a high-sensitivity 10 cm clear-air radar was carried out by Starr and Browning (1972) who measured the slopes of echo-layers recorded on range–height-indicator (RHI) displays and, with the aid of independent horizontal-wind measurements provided by radiosondes, succeeded in obtaining the orientations of the trough and ridge lines, also the magnitudes of the updraughts and downdraughts associated with lee waves generated by the Welsh mountains. They also recognized that with the addition of a Doppler capability, the clear-air radar could become a valuable tool for the study of the dependence of the lee-wave perturbations on height.

This prediction has borne fruit with the development of VHF clear-air Doppler radars which have the facility of sensing both vertical and horizontal components of the wind velocity vector with high resolution of time and height. See Gage (1990), and Röttger and Larsen (1990) for recent reviews of this technique and its applications.

Observations of lee-waves using these wind-profiling radars have been published by Ecklund et al. (1982), Balsley and Carter (1989) and Sato (1990). Ecklund et al. compared the evidence of lee-wave activity at two stations (Sunset and Platteville) in Colorado located at different distances to the east of the Continental Divide. Balsley and Carter studied orographically generated waves in the mountainous tropical island of Pohnpei (7°N, 158°E). Sato (1990) found evidence of lee-wave excited vertical-velocity disturbances in observations made by the MU radar at Shigasaki in Japan. The occurrences and fluctuations of these disturbances were found to correlate with characteristics of the low-level background wind profiles.

The purpose of this paper is to report on the observations made with clear-air Doppler radar of lee waves generated by moderately sized mountains in the vicinity of Toulon in southern France. The location was ideally suited for the observation of lee waves under a variety of synoptic situations during winter and early spring. The radar used a configuration of three antennae which enabled it to sense all three orthogonal components of the wind vector. Among the unique capabilities of this radar, relevant to the study of lee waves, which are fully described in this paper, is its ability to monitor the vertical structure and the time variation of the waves above a fixed location. We provide examples of lee waves trapped in the troposphere and of waves propagating freely into the stratosphere, and are able to relate the transition from one wave type to the other to the changing synoptic situation and attendant changes, mainly in the static stability structure.

For one of the trapped-wave cases, the parameters of the flow are such that a fruitful comparison with one of the numerical simulation examples published by Sharman and Wurtele (1983) has been possible. This comparison may serve to illustrate the usefulness of computer simulation models in the coordination and interpretation of remotely-sensed data relating to lee waves.

The organization of the rest of the paper is as follows. Section 2 describes the characteristics of the radar and of the radar site. Section 3 presents the basic observational results and identifies three specific examples of lee-wave incidence. The background synoptic situation is described and the vertical structures of the disturbances are related to the atmospheric structural parameters in accordance with two- or three-dimensional
lee-wave theory. For one of the examples representing a trapped lee wave (Case 3), a marked correlation is observed between systematic short-period (about one hour) fluctuations in vertical velocity and horizontal wind direction. The meaning of this correlation is the main topic of section 4. Section 5 dwells briefly on the significant conclusions of our study. Appendix 1 examines the limitations on the radar wind retrievals brought about by the finite beamwidth and beam spacing of the antenna system. This question has an important bearing on the radar measurements of lee waves in which large horizontal gradients of the wind perturbations are likely to be present. Appendix 2 outlines the procedure adopted for obtaining a space–height cross-section (Fig. 12) of the lee-wave perturbation of Case 3 from our single-station radar data.

2. THE CHARACTERISTICS OF THE RADAR AND THE RADAR SITE

(a) Radar Provence

In principle, the clear-air VHF Doppler radar transmits at a frequency which is not significantly influenced by atmospheric hydrometeors, and tracks the three-dimensional wind velocity by measuring the Doppler shifts in the signal returns from refractive index structures in the atmosphere of half the size of the radar wavelength. Such refractive-index structures are almost always present in the atmosphere, as stated earlier, and are supposed to participate freely in the motions of the ambient air. These motions are, therefore, impressed on the echoing signal as Doppler shifts.

The radar that was employed in our experiment (known as Radar Provence) had a transmitting frequency of 45 MHz (wavelength of 6.7 m) and a radar pulse length of 2400 m which was oversampled to give an effective gate spacing of 750 m. The power level and signal-processing characteristics were preselected to probe the altitude range from 2.2 km to 20 km for ideal conditions, although, in practice, the ceiling altitude was often reduced to somewhere between 10 and 18 km, depending upon the absence or diminution in echo levels from layers immediately above.

The radar used a beam-swinging technique to transmit and receive signals through three separate antennae, one pointed vertically, and the other two obliquely (125 degrees from vertical) in two orthogonal directions. This arrangement allowed for the complete determination of the three-dimensional wind vector. The vertical velocity was directly obtained from the Doppler-shift measurement of the radial velocity by the vertical antenna. In order to deduce the horizontal velocity with adequate accuracy, measurements from all three antennae were required (Strauch et al. 1987).

A sampling interval of 7.5 minutes was adopted, which meant that eight height profiles of the three velocity components were obtained every hour. It was necessary to suppose that the wind profile remained steady during each sampling interval. Furthermore, it was important to recognize that the finiteness of the antenna beam-width (6 degrees at half-power) and the separation between the two oblique antenna beams used in the horizontal velocity measurements, unavoidably introduced spatial smoothing of the data (Appendix 1).

(b) The radar site

Figure 1 shows the location of Radar Provence during the period of observation of lee waves reported in this paper. The radar station was at an altitude of 50 metres on the narrow coastal plain between the Mediterranean Sea to the south and several distinct mountain peaks in the north-west quadrant, 3 to 14 km away from the station and ranging in height between 540 and 800 m (see Fig. 1(b), which is a plot of the maximum terrain
height within 15 km of the radar as a function of azimuth). Hence, lee-wave activity at this site occurs predominantly with an airflow from the north-west quadrant, as frequently happens in winter and spring. The heights and more exact orientations of five of these peaks are given in Table 1.

One incidental advantage of the radar location for lee-wave studies is made apparent from Fig. 1(b) and Table 1, namely that small changes in wind direction can bring about the abrupt appearance or disappearance of lee-wave activity above the radar owing to the resulting changes in the orientation of a pre-existing lee wave or to the development of lee waves under the new conditions. For example, sudden transitions are likely to occur when the flow is from the west, since south-westerly flow should produce less lee-wave activity at the radar site than would north-westerly flow. This sensitivity of lee-wave incidence above the radar site to wind direction is helpful in identifying specific periods of lee-wave activity amongst a large amount of data displayed in a time–height cross-section. This will be evident from the data presented in the next section.

### TABLE 1. TERRAIN CHARACTERISTICS

<table>
<thead>
<tr>
<th>Name of peak</th>
<th>Height (m)</th>
<th>Distance from ST radar (km)</th>
<th>Azimuth from ST radar (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Le Coudon</td>
<td>702</td>
<td>3.0</td>
<td>340</td>
</tr>
<tr>
<td>La Valette</td>
<td>539</td>
<td>5.0</td>
<td>280</td>
</tr>
<tr>
<td>Mt. Faron</td>
<td>542</td>
<td>6.7</td>
<td>285</td>
</tr>
<tr>
<td>Le Grande Cap</td>
<td>783</td>
<td>9.3</td>
<td>320</td>
</tr>
<tr>
<td>Mt. Caume</td>
<td>801</td>
<td>10.8</td>
<td>300</td>
</tr>
</tbody>
</table>

Locations and heights of the five tallest mountain peaks in the vicinity of the radar. Note that all the peaks are in the north-west quadrant, and that *Le Coudon* is closest to the radar.
3. Observations of lee waves and their vertical structure

(a) The data

The primary data for this lee-wave study consisted of approximately 25 days of radar winds obtained during the winter and spring of 1986. Figure 2(a), forming part of this data-set, is a 5-day time-series of the vertical velocity, comprising more than 15,000 individual data-points. Data like these had to be checked for point-to-point consistency after correction for errors due to aircraft noise, sea echoes, ground clutter, etc. Implementation of these corrections generally involves critical examination of Doppler spectra or signal-to-noise ratios etc., for unusual features; and, in cases when such features are obviously inconsistent with other measurements that are close in space and time, the data may have to be discarded. Such considerations have resulted in the elimination of vertical-velocity data for 4.3 km altitude throughout this study (Figs. 2(a), 3(a), 4(a), 5(a)), because saturation by ground clutter was persistent in the Doppler power spectra recorded for this height, by the vertically-pointing antenna. For all other altitudes combined, however, a total of less than 1% of the vertical-velocity data-points were discarded.

Additional observations used in this study include hourly surface reports from the Toulon weather-service office (7 km west of the radar, Fig. 1(a)) and from all other surface stations within about 300 km of Toulon. Vertical profiles of static stability,

![Figure 2](image-url)

Figure 2. (a) Time-height cross-section of radar-observed vertical velocity from 17 to 22 April 1986. Scale is shown at bottom right. Altitude 4 is omitted owing to ground clutter effects. (b) Radar-observed wind direction at 5.8 km altitude (dashed), and surface wind direction observed at the station marked W in Fig. 1(a) (solid).
moisture and winds are determined from standard rawinsonde ascents at 12-hourly intervals from the two upper-air stations nearest the radar (i.e. Nimes and Ajaccio in Fig. 7(a)).

(b) Nature of disturbances

Figure 2(a) shows both quiescent and disturbed periods of vertical velocity. The periods before 00 UT on 18 April and after 06 UT on 21 April are representative of quiescent periods in our 25-day record, with typical vertical velocities in the range of 30–50 cm s\(^{-1}\) or less. In contrast, the intervening period shown in Fig. 2(a) is clearly disturbed, with maximum vertical velocities of up to 2–3 m s\(^{-1}\). The onset and disappearance of the disturbances are quite abrupt, as would be the case if the disturbances were perturbations generated by isolated mountain peaks, since the presence of such waves above the radar site depends sensitively on the upstream wind directions. Inspection of wind directions from the surface station at Toulon and from radar winds at 5.8 km confirm this hypothesis (Fig. 2(b)). The surface wind direction switches abruptly to the north-west quadrant at the same time as the commencement of the vertical-velocity disturbances, then remains remarkably steady in that quadrant for the entire period of the disturbances, and, equally abruptly, switches back to being south-easterly with the cessation of the disturbances. Although the radar winds at 5.8 km are also generally in this quadrant for most of the disturbance period (indicating weak directional shear of the airstream in the lower troposphere), they do not appear to be as well correlated with the generation of the disturbances as are the winds nearer the surface. These results are readily understood if the disturbances are identified as lee waves generated by the mountain peaks to the north-west of the radar station in an airstream crossing the mountains in a direction which is nearly the same as the surface wind direction recorded at Toulon. It should also be noted that surface weather observations within 300 km of Toulon were used to determine that no significant precipitation events occurred during the disturbed intervals, indicating that it is unlikely that the vertical-velocity disturbances shown in Fig. 2(a) were due to strong convective activity.

(c) Vertically propagating waves and trapped waves

Further examination of Fig. 2(a) reveals two distinct types of vertical structures for the lee waves constituting the disturbed part of the record. In the time interval between 12 UT on 19 April and 00 UT on 20 April wave activity is observed in the entire troposphere and extending well above the tropopause, which is at a height of about 11 km. This is more readily apparent in the expanded version of the velocity records for this time period shown in Fig. 3 where there is indication of the reversal in sign of the vertical velocity with height. This characteristic of vertically propagating waves is seen most clearly in Fig. 3(b) which contains altitude profiles of vertical motion at two specific times during this interval. From Fig. 4, which is used primarily to examine the transition in wave type at about 00 UT, 20 April, it is also apparent that there is a tendency for the waves to penetrate more readily into the stratosphere when the horizontal stratospheric winds are observed to weaken slightly after 18 UT (Fig. 4(b)). These features of wave activity, and also some relevant airflow characteristics for this period, are summarized under case 1 in Table 2. The same Table contains similar information for Case 2 and Case 3 described below.

In the subsequent time interval between 00 UT on 20 April and 06 UT on 21 April, the character of wave activity changed in the respect that it extended less and less to higher altitudes and showed no evidence of phase oscillation with height (Fig. 4(a)). In
other words, the waves betrayed the character of being trapped. The trapping of the waves cannot be attributed directly to the strengthening of the upper tropospheric winds, because, if at all, these winds are weaker than in the previous day (Fig. 4(b)).

In addition to the two cases described above, a third example of lee-wave activity from our radar record is presented in Fig. 5. In this case the activity was again primarily confined to the troposphere as in Case 2, with no evidence of phase variation in the oscillation of the vertical velocity with height. This instance of trapped lee-wave activity commenced at about 18 UT on 29 April and lasted until about 24 UT. As can be seen from the radar-observed wind directions for this day (Fig. 5(b)), the appearance of wave activity above the radar coincided with a transition from north-easterly to northerly flow through most of the troposphere. Inspection of the nearby terrain (Fig. 1) suggests that it is likely that Le Coudon was the mountain peak primarily responsible for this particular
Figure 4. Radar observations for Cases 1 and 2. (a) Vertical velocities at 7.5 minute time resolution (altitude 4 is omitted owing to ground clutter). Scale is at bottom right. Notice the reversal of sign with altitude at approx. 21–22 UT and at approx. 18 UT, for example. (b) Two-hour averaged horizontal winds using standard wind-barb notation (full barb = 5 m s\(^{-1}\)). The loss of data because of weak signal is marked. Notice that there was no corresponding loss of vertical-velocity data. This is because of the aspect sensitivity of the backscatter which favours vertically oriented beams.
Figure 5. Same as Fig. 4, but for Case 3.
TABLE 2. Wave characteristics and case definitions

<table>
<thead>
<tr>
<th>Case</th>
<th>Beginning time</th>
<th>Ending time</th>
<th>Wave type</th>
<th>Maximum vertical velocity</th>
<th>Froude number ( Fr = U/Nh )</th>
<th>Vertical limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12 UT</td>
<td>00 UT</td>
<td>Vertically propagating</td>
<td>2.5 m s(^{-1})</td>
<td>4.5</td>
<td>Lower stratosphere</td>
</tr>
<tr>
<td></td>
<td>19 April</td>
<td>20 April</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>00 UT</td>
<td>06 UT</td>
<td>Trapped</td>
<td>3.0 m s(^{-1})</td>
<td>1.0</td>
<td>Upper troposphere</td>
</tr>
<tr>
<td></td>
<td>20 April</td>
<td>21 April</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>18 UT</td>
<td>00 UT</td>
<td>Trapped</td>
<td>0.6 m s(^{-1})</td>
<td>0.8</td>
<td>Upper troposphere</td>
</tr>
<tr>
<td></td>
<td>29 April</td>
<td>30 April</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Definition and brief description of the three intervals, or cases, of lee-wave activity described in the text. See section 3(d) for details concerning calculation of the Froude number.

appearance of lee-wave activity above the radar site. *Le Coudon*, situated 3 km away from the radar station at 340° azimuth (Fig. 1, Table 1), would have been the first major obstacle encountered close to the radar, as winds changed from north-easterly to northerly.

(d) Froude number

The Froude number \( Fr = U/Nh \) where \( h \) is the height of the obstacle and \( U \) and \( N \) are the speed and buoyancy frequency of the air ascending over the obstacle) is an important parameter in the lee-wave problem. The square of this number is the ratio of the kinetic energy of the airstream to its potential-energy loss due to its ascent over the obstacle. Alternatively, this number is a measure of the ratio of the mean to the perturbation wind speed. These two definitions make it clear that the Froude number has to exceed unity in order to ensure that

1. flow over the obstacle is not inhibited, and
2. the perturbations on the flow introduced by the obstacle are of small amplitude so that a linear theory can account for the observed wave perturbations.

However, the exact value of the critical Froude number is dependent on the circumstances and cannot be prescribed exactly. In a recent numerical simulation Smolarkiewicz and Rotunno (1989), for example, showed that for \( Fr > 0.6 \) flow was predominantly over the obstacle in their study, although some flow around the obstacle had also set in.

The estimates of Froude number for the airstream perturbed by the lee waves has to be obtained ideally from observations of the air flow upstream. However, in the absence of such observations, this information (shown in Table 2) was obtained as follows. The average of the surface winds reported from the Toulon meteorological station and the radar wind from the lowest altitude of measurement is assumed to be the best available guess of the upstream velocity. Similarly, the profile of the buoyancy frequency, \( N \), obtained as an average from the twice-daily radiosonde observations at Nimes and Ajaccio (Fig. 7(a)), is taken to be a reasonable representation of the upstream stability structure. For these reasons, the estimated Froude number \( (Fr) \) values for the flows are somewhat unreliable.

Table 2 shows that Case 1 satisfies the criterion of large Froude number flow but Cases 2 and 3 do so, if at all, only marginally. However, we shall suppose, in what follows, that, even in these two cases, the two conditions specified above are not seriously violated.
(e) Wavelengths and vertical velocities: comparison with theoretical models

Figures 2–5 show that lee-wave-related vertical velocities measured by the radar vary with time. Some aspects of the observed time variation are considered in the next section. The goal here is to seek some measure of validation for our observational results on vertical wave-structure, described in section 3(c), through comparison with simplified, steady-state solutions, either theoretical or numerical, of the standard lee-wave problem.

The Fourier amplitude of the vertical velocity of steady-state, linearized lee waves from an isolated mountain is governed by the vertical-structure equation (Sharman and Wurtele 1983), namely

$$\frac{d^2 \tilde{W}}{dz^2} + \frac{k_y^2}{k_x^2} \left( \frac{N^2}{U^2} - \frac{k_x^2}{k_y^2} \cdot \frac{1}{U} \frac{d^2 U}{dz^2} - k_x^2 \right) \tilde{W} = 0$$ (1)

where $\tilde{W} = \tilde{W}(k_x, k_y, z)$, $k_x$ and $k_y$ are horizontal wavenumbers in the $x$ (downstream) and $y$ (cross-stream) directions, respectively, and $k^2 = k_x^2 + k_y^2$. Both $N$, the buoyancy frequency, and $U$, the background wind velocity, are allowed to be dependent on the height, $z$, but $U$ is assumed to be unidirectional (the influence of the directional shear on our observations is discussed in section 4). The linearized boundary conditions for the problem are the kinematic condition $\tilde{W} = U d h/dx$ at the lower boundary ($z = 0$), where $h$ is the mountain-height profile, and some suitable upper boundary condition such as $\tilde{W} \to 0$ as $z \to \infty$, where $\tilde{r}(z)$ is the mean air density.

Using a background wind-profile of scale-height $L$, so that $U = U_0 e^{z/L}$, and an $N$-profile that is constant with height, Sharman and Wurtele (1983) derived several numerical solutions for the case of three-dimensional lee waves produced by a Gaussian-shaped obstacle. They discuss their solutions for parametric values of $R = Ri^{1/2} = NL/U_0$ where $Ri$ is the upstream Richardson number. Their solution for $R = 5.6$ is especially appropriate for comparison with our observations for case 3 (section 4).

For $R = 5.6$, Sharman and Wurtele (1983) have displayed the vertical-velocity fields for both three-dimensional and two-dimensional solutions of the problem (see their Fig. 23). They find that the calculated wavelengths for these two solutions are comparable, although the calculated amplitudes are greater and the decay of amplitude with distance is less in the two-dimensional case. There is, therefore, some justification for comparing our observations with the more amenable two-dimensional model calculations.

For the two-dimensional case the lee-wave vertical-structure equation reduces to the form:

$$\frac{d^2 \tilde{W}}{dz^2} + (l^2 - k_x^2) \tilde{W} = 0$$ (2)

where

$$l^2 = \frac{N^2}{U^2} - \frac{1}{U} \frac{d^2 U}{dz^2}$$ (3)

and $l$ is known as the Scorer parameter. The term $(d^2 U/dz^2)/U$ involving the curvature of the $U$-profile is often considered to be negligible compared to $N^2/U^2$, so that effectively, $l = N/U$. We have calculated each of these terms and satisfied ourselves that this is the case. For these calculations, the values for $N$ were derived from the rawinsonde data from Nimes and Ajaccio. For the winds, the surface observations from Toulon and the 3 to 6 hour averages of the radar data at 750 m vertical spacing above 2 km were used. With this simplification, $2\pi l^{-1} = \lambda_2$, where $\lambda_2$ is an intrinsic length-scale: physically the
horizontal distance traversed by an air parcel during one period of its buoyancy oscillation. In this sense it can be considered as a horizontal wavelength. In fact it is the critical horizontal wavelength for vertical propagation in a layer in which $l$ is uniform (see below).

It has been shown, by Scorcer (1949, 1956), Scorcer and Wilkinson (1956), Corby and Sawyer (1958), Palm and Foldvik (1960) and several others, that the shape of the $l(z)$ profile is a critical factor in the selection of the prevalent lee-wave modes. These authors have found, in particular, that a rapid decrease of $l$ with height in the lower troposphere sets up a wave-guide in which the wave amplitudes can become large and in which the waves are more or less effectively trapped.

Figure 6. (a–c) Vertical profiles of the Scorcer parameter calculated from appropriate 3–6 hour averaged radar-observed wind profiles and temperature profiles from the nearest rawinsonde sites (Nimes and Ajaccio, see Fig. 7(a)). For Case 3 an idealized profile of the Scorcer parameter is shown, as well as a profile based on the average of four static stability profiles from soundings just before and after the event. See text for details of analysis procedure.

Figure 6(a–c) shows calculated height-profiles of the Scorcer parameter, $l$, for dates and times relevant to the three observed cases of lee-wave activity (Figs. 2–5). Two distinct types of $l(z)$-profile can be noticed in Fig. 6(a–c), including profiles in Cases 2 and 3 that are characterized by a rapid decrease in the Scorcer parameter with height. In contrast to Cases 2 and 3, at the beginning of the period corresponding to Case 1 (12 UT on 19 April), when upward propagating waves were observed in the radar vertical-velocity data (Fig. 3), values of $l$ oscillate about a mean value of 1.2 km$^{-1}$ (corresponding to values of $\lambda$, of about 5.5 km), with no systematic height variation. The Scorcer parameter is related to the vertical wavenumber, $m$, by the dispersion relation

$$m^2 = l^2 - k_x^2$$  \hspace{1cm} (4)

which is obtained by inserting $\vec{W} \propto e^{i m z}$ into the vertical structure equation (Eq. (2)). For vertical propagation $l^2 > k_x^2$. Inspection of Fig. 3 reveals that, early during Case 1, the vertical wavelength in the mid to upper troposphere was near 8 km (event A), while somewhat later during Case 1 the most prominent vertical wavelength has become about 12.5 km (event B) and the height variation of the vertical velocity is somewhat more complex than for event A. The more complicated structure found in event B may result from an increase in the variation of $l$ with height that is the result of a changing wind profile. The observed increase in the vertical wavelength indicates that the waves are
progressively coming closer to being trapped. In making these calculations for events A
and B we have used the three-hourly averaged winds at the surface and aloft around the
times of these events, and the static stability profile from Nîmes at 12 UT 19 April. If,
on the other hand, the static stability profile at 00 UT 20 April from Nîmes is used, then
the wave comes very close to being trapped by the layer above 5 km above ground level.
These results are consistent with the subsequent transition from vertically propagating
waves to trapped waves, which marks the end of Case 1 and the beginning of Case 2.

Inserting the observed values of $l$ and $m$ in the mid to upper troposphere for events
A and B yields horizontal wavelengths of 6.2 and 7.7 km, respectively. Although it
would be desirable to have independent measurements of the horizontal wavelength for
comparison, we can only conclude that these calculated wavelengths most likely cor-
respond to spectral components of the topography, which also appear to have given rise
to the 6 km horizontal wavelength found in the trapped wave of Case 3.

In contrast to the type of Scorer parameter profile found in Case 1, which is
appropriate for the vertically propagating waves seen there, at the end of the period of
Case 1 (00 UT on 20 April), peaks in the $l(z)$ profile have developed in the stratosphere
and in the lower troposphere. Correspondingly, the wave activity has decreased at upper
levels and the waves have become trapped in the troposphere. This is seen in the vertical
velocity record for Case 2 (Fig. 4(a)). By 00 UT on 21 April the $l(z)$ peak in the
lower troposphere had sharpened considerably, suggesting that the wave duct has been
maintained throughout Case 2, as was initially inferred from the radar data alone. The
transition of wave type was correlated with the passage of a baroclinic wave and with
the associated changes in wind and static-stability profiles, as discussed in section 3(f).
Conditions for wave ducting are even more favourable in the $l(z)$-profiles illustrated as
representative of conditions that should have prevailed in the period (12–24 UT) on 29
April, corresponding to Case 3.

Thus the radar data seem to show, in all the three cases studied, the expected
dependence of the vertical lee-wave structure on the stability and shear of the basic flow.
In particular, the ability of the radar to distinguish between ducting and non-ducting
cases appears to have been demonstrated and the observed ducting has been shown not
to be the effect of the loss of radar signal at stratospheric heights.

Further evidence in support of the reasonableness of the vertical lee-wave structures
observed by the radar may be obtained from a quick comparison with a two-dimensional
model developed by Palm and Foldvik (1960) and Foldvik (1962). The advantage of this
model is that it allows the identification of the wavelengths of the possible wave modes
from two parameters $l_0$ and $c$. These two parameters are chosen preferably from wind
and temperature soundings that make it possible to represent the height variation of the
Scorer parameter in the form $l(z) = l_0 e^{-cz}$. The criterion for the existence of waves in
this model is that $l_0/c > 2.5$, which is equivalent to the condition that the upstream
Richardson number, $Ri_0$, based on the surface wind shear is greater than 5.8 or that
$R = Ri_0^{1/2} > 2.4$ (Foldvik 1962; Sharman and Wurtele 1983). Another practical con-
sideration in the application of this model is that the decrease of $l$ with height in the
lower troposphere should be fairly rapid, with $c$ roughly within the range 0.1 to 0.3 km$^{-1}$.
Both these conditions are clearly met for Cases 2 and 3 of our radar data.

Specifically for Case 3, $l_0 = 2.5$ to 3.0 km$^{-1}$ and $c = (0.20 \pm 0.02)$ km$^{-1}$, based on
average values from available wind and temperature soundings (Fig. 6). According to
the model solution given in nomographic form in Fig. 1 of Foldvik (1962), two transverse
lee-wave modes with horizontal wavelengths $L_1 \approx 3$ km and $L_2 \approx 6$ km are expected to
be excited for these values of $l_0$ and $c$. Recalling that the peak of $Le Coudon$, which is
presumably the wave-generating obstacle in this case, is 3 km away from the radar station
(Table 1), that \( \cos k_1 x = +1 \) for the mode with wavelength \( L_1 \) and that \( \cos k_2 x = -1 \) for the mode with wavelength \( L_2 \) \( (k_1 = 2\pi / L_1; \ k_2 = 2\pi / L_2 \) and \( x \) is the downstream distance of the radar station from the mountain peak). Turning next to Fig. 2 of Foldvik and bearing in mind that Foldvik's diagram has been drawn for \( \cos k_1 x = \cos k_2 x = -1 \), it can be concluded that, in our case, both the \( L_1 \) and \( L_2 \) modes are associated with downward velocities at altitudes higher than about 2.5 km and that, below this altitude, the downward velocities of \( L_1 \) and the upward velocities of \( L_2 \) tend to be in opposition. This conclusion is consistent with the radar observations (Fig. 5(a)) which show an excess of downward over upward velocities between 2 and 10 km during most of the period of lee-wave activity in Case 3.

Solutions of the two-dimensional and three-dimensional vertical-structure equations for similar boundary conditions and similar assumptions about the environmental velocity shear and static stability, give comparable results for the number and type of allowed transverse lee-wave modes (Sharman and Wurtele 1983). However, two-dimensional models almost invariably overpredict the magnitudes of the vertical velocities of the lee waves. This was found to be the case on comparing the measured radar velocities (approx. 0.6 m s\(^{-1}\)) for Case 3 (Table 2) with the velocities (approx. 3 to 4 m s\(^{-1}\)) calculated according to Foldvik's model. Thus the observed values tended to be a factor of two or more smaller than the calculated values for Case 3. This discrepancy may highlight the moderate development and the intrinsically three-dimensional character of the lee waves observed in Case 3.

(f) Synoptic situation

The favourable wind directions for the generation of lee waves in the three cases cited above were produced under differing synoptic situations. At the beginning of the five-day interval shown in Fig. 2 there was initially south-westerly flow associated with an approaching long-wave baroclinic trough. By 00 UT 18 April a short-wave trough had passed over the radar (see the 500 mb winds in Fig. 2(b)) resulting in westerly and even north-westerly flow beneath a 50 m s\(^{-1}\) jet streak. At about 00 UT 19 April the long-wave trough axis at 300 mb was positioned roughly above the radar with general north-westerly flow in the lower and mid troposphere. These north-westerly winds provided the background for vertically propagating waves seen above the radar site between 12 UT on 19 April and 00 UT on 20 April (Fig. 3). Following this, the situation depicted in Fig. 7(a) prevailed. This upper-air map coincides with the period when the propagating lee waves over the station gave way to trapped waves which then persisted from between 00 UT on 20 April to 06 UT on 21 April, as the wind direction shifted first to northerly and eventually to westerly. This evolution, corresponding to the interval shown in Fig. 4, is associated with the passage of a weak upper-level jet on the west side of the long-wave trough, late on 19 April, followed by the approach of a ridge of high pressure early on 21 April. These features can be traced in the radar-observed wind profiles of Fig. 4(b). The appearance of flow with a southerly rather than a northerly component, subsequent to passage of the ridge, coincided with the cessation of wave activity above the radar.

The observations described above suggest that the transition from vertically-propagating to trapped waves was related to the transit of a baroclinic system across the radar site. Results from section 3(e) (Fig. 6) indicate that this transition was also clearly marked in the profiles of the Scorer parameter, which reflect the changing static stability and wind profiles. The radar data have in fact enabled us to examine closely the influence of the rapidly changing wind profiles on the wave structure. From the wind profiles at about the time of transition (18 UT 19 April to 6 UT 20 April in Fig. 4(b)) it is apparent that
Figure 7. Synoptic 300 mb map of wind and temperature fields: (a) at the time of the transition from vertically propagating waves of Case 1 to trapped waves of Case 2. Rawinsonde profiles used in this study were from Ajaccio (marked by a triangle) and Nimes (a square), while the radar site at Toulon is also shown (solid circle). (b) Corresponding conditions for Case 3. Temperatures are in degrees Celsius, geopotential heights are in dam, and winds are as in Fig. 4. The maps are based on the European Meteorological Bulletin.
the transition was marked by cold advection between 10 and 14 km altitude and by warm advection below 8 km, as revealed, respectively, by the counterclockwise (backing) and clockwise (veering) rotation of the wind direction with height. This type of advection pattern is conducive to the reduction of the static stability aloft, thereby increasing the likelihood that waves will be trapped. At the same time the transition was also marked by decreasing wind speeds aloft, which would have acted to reduce wave trapping. On balance it would appear that the static-stability changes turned out to be the overriding factor in accounting for the transition.

Possible systematic relationships between baroclinic activity and the trapping (untrapping) of lee waves and other types of internal gravity waves need to be investigated further with more extensive observations.

The synoptic situation for the trapped-wave case of 29 April was substantially different from that of Cases 1 and 2. The map in Fig. 7(b) corresponds to the end of this period. It shows a cut-off low over northern Italy producing northerly to north-easterly flow over Toulon. A meso-α scale disturbance with characteristics of a shortwave trough was observed to propagate around the cut-off low, giving rise to the switch-over from generally north-easterly flow, from 12–18 UT, to northerly flow after 18 UT, marking the onset of lee waves in Case 3.

4. HORIZONTAL STRUCTURE OF LEE WAVES OBSERVED BY THE RADAR

(a) Observed correlation between vertical and horizontal lee-wave perturbations

The vertical velocities observed by the radar during the three cases of lee-wave activity (Figs. 2–5) show several fluctuations of periods shorter than the typical timescale for changes in the dynamical response of the atmosphere to lee-wave perturbations. Such variability is a characteristic feature of clear-air radar and other types of observational studies of lee waves referred to in the Introduction. The most natural and likely source of these shorter-period fluctuations is the variability in speed and direction of the upstream low-level winds. However, because no direct observations of upstream winds are available, we are forced to seek the nature of the short-period fluctuations indirectly by looking for correlations, if any, between the horizontal-velocity and vertical-velocity variations measured by the radar itself. This approach is followed using data from Case 3 for which the lee-wave forcing is from an isolated mountain peak, whereas, for the other two cases, the more complex topography to the west of the radar is involved. In addition, the small amplitude of the vertical-velocity disturbances in Case 3 allows for greater accuracy in the horizontal wind retrievals, as described in Appendix 1.

Figure 8 is a scatter plot of vertical velocity versus horizontal wind direction at four altitudes for the 6-hour period of lee-wave activity during Case 3 (Fig. 5). The point values of vertical velocity are 2.5-minute averages obtained 7.5 minutes apart, while the wind directions are determined every 7.5 minutes (using radial velocities from all three beams) and are expressed as deviations from the 6-hour mean value at each altitude.

The sensitive dependence of vertical-velocity changes on wind directional changes as well as the almost altitude-independent nature of this dependence are clearly brought out in Fig. 8. The tendency for upward (downward) velocities to be associated with positive (negative) directional deflections, albeit with a slight offset, is also readily noticeable. The correlation coefficients for these variables are between 0.7 and 0.8.

Figure 9 is a similar scatter plot, but for vertical velocity versus horizontal wind speed, for the same four altitudes shown in Fig. 8. In contrast to the well-marked correlation between vertical velocity and wind direction, there is hardly any detectable relationship between vertical velocity and wind speed at any of the four altitudes, as is indicated by the low values of the correlation coefficient (viz. ±0.1).
Figure 8. Scatter plots of radar-observed vertical velocity versus wind direction deviations for altitudes of (a) 2.9 and 3.6 km, and (b) 5.9 and 6.6 km. Deviations are from the average wind direction at each altitude. Best-fit lines, drawn using the least squares method, are shown. The correlation coefficients at 2.9, 3.6, 5.9 and 6.6 km are 0.80, 0.82, 0.73 and 0.68, respectively.

Figure 9. Scatter plots of radar-observed vertical velocity versus wind speed for altitudes of (a) 2.9 and 3.6 km, and (b) 5.9 and 6.6 km. The correlation coefficients at 2.9, 3.6, 5.9 and 6.6 km are 0.02, −0.07, −0.13 and −0.05, respectively.

(b) A suggested explanation

One possible way in which the results of the scatter plots of Figs. 8 and 9 can be explained is to suppose that the most conspicuous effect of upstream wind directional changes is to cause a bodily rotation (without significant change in shape) of the three-dimensional lee-wave pattern downstream about a fixed point located at the mountain peak that is forcing the wave. The radar could then sense the velocity pattern along an arc whose radius is equal to the distance from the radar site to the same fixed point, with the angular extent of the arc determined by the extreme deviations in wind directions sensed by the radar during the observation period.

The foregoing scenario is depicted in Fig. 10. The average wind direction at the lowest altitudes of radar wind measurements was 11 to 13 degrees east of north for the 6-hour period of lee-wave activity denoted as Case 3. Assuming (in the absence of more direct observations) that this average direction was also representative of upstream conditions, the radar was located at a 'wedge angle' of 34 degrees as measured from the downstream axis of the lee-wave pattern (Fig. 10(a)). The lee-wave pattern is drawn in an idealized fashion as consisting of alternating parabolic-shaped regions of upward and downward motion. The standard deviation of the radar wind direction in the 6-hour
period (Fig. 5) was ±25 degrees at 2.9 km and ±10 degrees at 7–8 km. Based on the standard deviation at the lower level, it may be assumed that, for the extreme westerly and easterly deviations from the mean wind, the wedge-angle locations of the radar were 14 and 54 degrees, respectively, as shown in Fig. 10(b, c). Consequently, the radar scanned a vertical cross-section of the three-dimensional lee-wave pattern along a baseline in the form of an arc segment between the wedge angles of approximately 14 and 54 degrees, as shown in Fig. 10(d).

We seek support for the afore-mentioned hypothesis and consequent scenario in the computational results of Sawyer (1962) and Sharman and Wurtele (1983) and in the lidar observations reported and discussed by Blumen and Hart (1988). In a three-dimensional simulation of lee waves, in which the background wind direction was allowed to vary with height, Sawyer found that the dominant effect of a shift in the background wind direction between adjacent atmospheric layers was to rotate the axis of the lee-wave
pattern into a direction which is intermediate between the wind directions in the two layers.

Accepting the validity of Sawyer's result, we have made use of the detailed calculations of Sharman and Wurtele (1983) to illustrate the nature of variation of the lee-wave vertical velocity with the relative wedge angle of the point of observation. As mentioned earlier in section 3, their calculations for $R = 5.6$ are parametrically the closest for comparison with the radar results for Case 3 (Table 2).

Figure 11(a) shows the lee-wave vertical-velocity pattern as reconstructed from Sharman and Wurtele's (1983) Fig. 24 for $R = 5.6$. The variation of the vertical velocity with wedge angle $\theta$, that would be observed according to this pattern at two hypothetical locations of the radar, 12 and 16 km away from the wave source, are shown for two selected altitudes, 2 and 4 km respectively, in Fig. 11(b). The nature of this variation brought about by contributions from the superposition of several diverging and transverse modes, confined to differing wedge angles, is found to depend far less on altitude than on distance from the source. More significantly, for each distance from the peak, there exists a substantial range of wedge angles for which an approximate linear relationship between the two variables is obtained.

Similar results can be inferred from Blumen and Hart (1988), who obtained the horizontal wind divergence from airborne Doppler lidar measurements of lee-wave patterns. As is shown in Fig. 11(c), the variation of horizontal wind divergence with wedge angle, reported by Blumen and Hart (1988), is similar to the variation of the vertical velocity with wedge angle calculated by Sharman and Wurtele (1983). It is significant to note that these variations bear a striking resemblance to our radar vertical-velocity measurements interpreted according to the hypothesis that they refer to differing wedge angles of a lee-wave pattern whose orientation is responsive to fluctuating directions of the upstream basic flow.

(c) Radar-sensed space–height cross-section of lee-wave structure

The foregoing conclusion, that the observed time variations of the radar winds might have resulted from the bodily rotation, without substantial change of form, of the three-dimensional lee wave, suggests the possibility that the wind data could be used to construct a space–height cross-section through part of the wave. The procedure adopted for constructing such a cross-section is described in Appendix 2, and has been implemented for Case 3; the results are shown in Fig. 12.

For the cross-section, use was made of a total of 48 wind-vector measurements for each of the five altitudes 2.9, 3.6, 5.9, 6.6, and 7.3 km, which have been grouped in 5-degree intervals according to wind direction, resolved into vertical, zonal and meridional components, averaged, and plotted along a baseline which is effectively a 40-degree arc at a radius of 2.5 km from the mountain peak. The zonal and vertical components (represented by arrows on different scales) are plotted against wind direction (wedge angle) and altitude in Fig. 12(a). It should be emphasized that the orientation of this 'raw' cross-section is not exactly east–west. Moreover, the altitude dependence of the directional variability of the wind (with standard deviations of 25 degrees at 2.9 km, 20 degrees at 3.6 km, and about 10 degrees at the higher altitudes plotted in the cross-section) have been ignored. This defect has been rectified in the 'modified' cross-section of Fig. 12(b) (in a manner described in Appendix 2). A schematic of this cross-section is shown in Fig. 13, where plane-parallel and plane-perpendicular disturbance components are more prominently marked.

Inspection of Fig. 13 shows the vertical alignment of perturbation components expected in a trapped wave whose horizontal half-wavelength is approximately the arc-
Figure 11. (a) Schematic of the horizontal distribution of vertical motion in a three-dimensional trapped lee wave in the troposphere well above an isolated mountain peak (derived from the numerical simulations of Sharman and Wurtele 1983). Areas of upward motion greater than 10 cm s⁻¹ are shaded. Bold arrow represents the mean upstream wind direction while bold arcs represent hypothetical locations of a radar relative to the wave pattern under the scenario described in Fig. 10. The obstacle is shown as a circle. Inset shows the relative wedge angle ($\theta$) of the ST radar. (b) Variation of vertical motion as a function of relative wedge angle from the $R = 5.6$ simulation of Sharman and Wurtele (1983). Results are shown for the 12 and 16 km distances of the two arcs in (a), and for two altitudes. (c) Same as (b), but instead of vertical velocity from a numerical simulation, Doppler lidar measurements of horizontal wind divergence near Mount Shasta are plotted from Blumen and Hart (1988). The results are shown for three distances from the peak.

length of the baseline (1.75 km). Thus the distance of the radar station from the forcing obstacle (Le Coudon) is slightly in excess of one disturbance wavelength, taking into consideration the fact that the cross-section in Fig. 13 is not exactly perpendicular to the wave. This wavelength is in agreement with the intrinsic length scale (2.6 km) for the lee wave deduced from the mean static stability and winds of the lowest 2 km above ground level. Also, the deduced wavelength corresponds to one of the subsidiary maxima in the topographic spectrum of Le Coudon.
Figure 12. Space–height radar–wind cross-sections for Case 3 derived according to the analysis procedures described in section 4 and in Appendix 2. (a) In this cross-section vertical arrows represent vertical velocities ($w$) and horizontal arrows represent the zonal wind component ($u_w$). Values are averages of 5 degree data bins. (b) Modified space–height cross-section. Vertical arrows are the same as in (a). Horizontal arrows are perturbation horizontal velocities in the plane of the cross-section, and numbers are perturbation velocities (m s$^{-1}$) perpendicular to the cross-section (positive into the page). See Fig. 13 for a schematic version of (b).
Figure 13. Schematic of the modified space–height cross-section (Fig. 12(b)) of the lee-wave perturbation. Bold arrows represent flow in the plane of the cross-section, while dashed contours represent the flow perpendicular to the plane of the cross-section. (See Appendix 2 for details).

On further examination, Fig. 13 shows an apparent mismatch, in the sense that the regions of maximum horizontal (vertical) divergence (convergence) are not aligned as would be expected within the upper half of a trapped lee wave. There are two probable reasons for this mismatch. First, the cross-section is not exactly perpendicular to the diverging wave disturbance and, consequently, the horizontal divergence pattern has additional contributions from the plane-perpendicular flow. Second, the analysis of finite beam spacing effects, carried out in Appendix 1, indicates that the retrieved horizontal wave perturbations can be phase-shifted with respect to the actual perturbations when the radar antenna beam spacing becomes comparable to the disturbance wavelength. It should be noted that the phase shifting is more significant for short wavelengths and large disturbance amplitudes, each of which result in larger horizontal gradients in the disturbance velocities.

5. Conclusions

In this paper we have used clear-air radar observations to monitor the incidence and structural characteristics of lee waves generated by isolated mountain peaks. The influence of the synoptic weather situation on the observed wave characteristics has been illustrated and is suggested as a topic that could bear further investigation.
Sustained, short-period (about one hour) fluctuations of lee waves have been observed and related, in one particular instance, to the rotation of the three-dimensional lee-wave pattern centred on the wave-generating obstacle. This rotation is presumably caused by directional changes in the upstream flow. The importance of monitoring upstream flow directly, to interpret the time variations of lee waves, is clearly brought out in this example.

Because of the relatively small spatial scales of the disturbances, the accuracy and horizontal resolution of the retrieved winds is a matter of concern. A method is presented by which the basic measurement errors caused by sampling in an environment containing lee waves can be estimated. The limitations imposed by the finite antenna beam spacing and finite beamwidth are examined in general, while results derived using the observed wave parameters and the radar configuration in this study are described in detail.

The availability of computer-generated model results for the range of parameters to hand was found to be invaluable for the coordination and interpretation of observations.

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APPENDIX 1

The effects of a stationary gravity wave on the accuracy of clear-air Doppler radar wind retrievals

In this appendix we use prescribed conditions to represent the perturbation winds in a plane gravity wave that has a horizontal wavelength comparable to the half-power beam width of the clear-air Doppler radar. Further, it is assumed that

1. the prescribed gravity-wave pattern is stationary during the dwell time or sampling interval,
2. the scatterers are uniformly distributed within the scattering volume and have identical scattering cross-sections,
3. contamination of radial velocities by non-radial velocities is negligible,
4. the power transmitted by the radar has a Gaussian distribution.

First, the effect of horizontal variations of vertical velocity within a vertically-pointing beam is determined. Then, the effect on horizontal wind retrievals in a three-beam system, such as the one used in this study, is discussed.

(a) Radial velocity in a single vertical beam

The total back-scattered power is calculated as the sum of many small elements within the scattering volume (Fig. A1). The scattering volume is considered to be the region within the half-power beam width, which in this study is 6 degrees. Each element contributes back-scattered power proportional to the power incident on that element times the cross-sectional area of the element. Moreover, each element will contribute energy at a specific Doppler shift, depending on its position within the assumed wind
field. The contributions from all the elements within the volume are summed to create a pseudo-Doppler power spectrum. The first moment of this pseudo-spectrum is determined using a standard statistical approach, as by Woodman and Guilling (1974). This process is repeated for various ratios of the horizontal wavelength of the disturbance, $\lambda_x$, to the half-power beam width, $D_1$, and for different orientations of the beam within the wave. When the beam width is relatively small (i.e., $D_1 < \lambda_x/4$) the retrieved radial velocity is at least 90% of the actual vertical velocity at the centre of the beam and shows no dependence on orientation. This behaviour is shown in Fig. A2 for $\lambda_x/D_1 = 3.0$, which is valid for our case at about 8 km altitude since $\lambda_x = 2.5$–3 km. As the beam width

Figure A2. Retrieved vertical velocity (solid line) and actual vertical velocity at the centre of the radar beam (dotted line), as functions of position within a hypothetical wave of $\lambda_x = 2.5$ km. Parameter $\lambda_x/D_1$ is 3.0, with $D_1 = 0.83$ km, which in our case is at 8 km altitude.
increases relative to the horizontal wavelength, the retrieved velocity becomes more of an average and thus has a smaller amplitude than the actual radial velocity at the centre of the beam, although there is still no dependence on orientation.

The results from this analysis suggest that even though the scattering volume may contain within itself systematic variations of vertical velocity associated with a stationary plane gravity wave, the retrieved velocity is most representative of the actual vertical velocity at the centre of the beam. This conclusion is in agreement with Clark et al. (1986) who considered linear rather than sinusoidal variations of the wind field. On the basis of the method presented here, we can identify two basic features leading to the apparent enhancement of the horizontal resolution of the vertical beam. Firstly, the maximum power incident on the scattering volume is at its centre. Secondly, for a radar beam exposed to a plane wave the largest area of exposure to the smallest range of vertical velocities is the area crossing the centre of the beam (Fig. A1). In essence there is an 'effective beam-width' which is narrower than the half-power beam width. Since Case 3 has \( \lambda_v/D_z > 3.0 \), it appears that the retrieved vertical velocities would not have been strongly affected by the variation of vertical velocities within the beam resulting from the lee waves.

The primary limitations in this analysis arise from the various assumptions mentioned at the outset. It should be noted that possible effects of tilted refractivity structures (i.e., contamination by horizontal winds) as discussed by Palmer et al. (1991) are not included. Such effects, it was suggested, contributed to possible vertical velocity errors of the order of 10-20 cm s\(^{-1}\), which is smaller than the vertical velocities observed here. In addition, if the scatterers are not uniformly distributed within the scattering volume and/or have different back-scattering cross-sections, the retrieved radial velocity will be biased by the distribution of scatterers and their characteristics; as has been discussed by Fukao et al. (1988) in terms of the finite-range volume effect. Both of these errors tend to become largest in relatively strong winds and are likely to be small for the weak winds of our study.

\[(b)\] **Effects on horizontal-velocity retrievals using three beams**

The basic measurement errors resulting from sampling in a non-uniform, yet systematically varying, windfield has been considered briefly by Carbone et al. (1986). In this study, the approach taken is to postulate a gravity wave which remains stationary with respect to the antenna positions during the sampling interval of the radar (7.5 minutes in this study). The perturbation velocities, horizontal wavelength and orientation with respect to the antenna beams are prescribed. The wind retrieval is performed with the conventional assumption that the measured wind parameters correspond to the centres of the antenna beams. The retrieved winds are then compared with the prescribed winds directly above the radar. The case of waves propagating horizontally across a radar has been examined previously by Reid (1987).

Figure A3 illustrates the relevant geometry. Figure A3(a) defines the nadir angle \( \phi \), while Fig. A3(b) defines the azimuthal orientation \( \gamma \), and shows the positions of the scattering volumes of the antenna beams within a hypothetical wave pattern. Based on this geometry, the measured (retrieved) winds can be shown to be related to the actual (prescribed) winds according to the formulae:

\[
u_m = -\eta_1 \cos \gamma + \eta_2 \sin \gamma \quad \text{(A1)}
\]
\[
u_m = \eta_1 \sin \gamma + \eta_2 \cos \gamma \quad \text{(A2)}
\]
where

\[ \eta_1 = -u_1 \cos \gamma + v_1 \sin \gamma + (w_1 - w_0) \cot \phi \]

\[ \eta_2 = u_2 \sin \gamma + v_2 \cos \gamma + (w_2 - w_0) \cot \phi; \]

\( u_m, v_m \) are the measured horizontal wind components; and \( u_i, v_i, w_i \) are the actual winds at each of the three antenna beam positions (\( i = 0, 1, 2 \)) shown in Fig. A3(b).

For the actual winds we prescribe a plane gravity wave with north–south oriented phase lines, and exclude the background winds since they would not change the retrieved perturbation winds, namely

\[ w_i = W \sin\left(\frac{2\pi x_i}{\lambda} \right) + c \]  
(A3)

\[ u_i = -U \cos\left(\frac{2\pi x_i}{\lambda} \right) + c \]  
(A4)

where

\( W \) is the vertical velocity perturbation amplitude.
\( U \) is the horizontal velocity perturbation amplitude.
\( \lambda \) is the horizontal wavelength.
\( c \) is a phase adjustment factor used to examine the effects of shifting the phase of the wave.
\( x_i \) is the east–west position of the relevant scattering volume (see Fig. A3(b)).
\( x_0 = 0. \)
\[ x_1 = -r_1 \sin \phi \cos \gamma. \]
\[ x_2 = r_2 \sin \phi \sin \gamma. \]
\( r_{1,2} \) is the radial distance from the radar to the centre of the appropriate scattering volume.

The corresponding measured zonal (meridional) wind can now be obtained by inserting A3 and A4 into A1 (A2), yielding A5 (A6).

\[ u_m = -U\{\cos(\Psi_1 + c) \cos^2 \gamma + \cos(\Psi_2 + c) \sin^2 \gamma\} + \]
\[ + w'\{\sin(\Psi_2 + c) \sin \gamma - \sin(\Psi_1 + c) \cos \gamma\} + \]
\[ + w'\{(\cos \gamma - \sin \gamma) \sin(\Psi_0 + c)\} \]  
(A5)
\[ u_m = U \cos \gamma \sin \gamma (\cos (\Psi_1 + c) - \cos (\Psi_2 + c)) + \\
+ w' \{ \sin (\Psi_1 + c) \sin \gamma + \sin (\Psi_2 + c) \cos \gamma \} - \\
- w' \{ (\cos \gamma + \sin \gamma) \sin (\Psi_0 + c) \} \]  

(A6)

where

\[ \Psi_i = \frac{2 \pi x_i}{\lambda} \quad \text{and} \quad w' = W \cot \phi. \]

The right-hand sides of Eqs. (A5) and (A6) each have three primary terms; as in \( u_m = I + II + III \), although the exact forms of I, II, and III differ from \( u_m \) to \( v_m \). In both Eqs. (A5) and (A6), term I is due to horizontal motions, term II to the presence of vertical motions at the oblique beams, and term III to the inclusion of vertical velocity information from the vertically-pointing beam.

Figures A4(a) and A4(b) show, for an altitude of 3 km, the measured \( (u_m, v_m) \) as well as the actual wind components, where the actual winds are prescribed according to the wave parameters: \( W = 0.5 \text{ m s}^{-1}, U = 2 \text{ m s}^{-1}, \lambda = 2 \text{–} 3 \text{ km}, \gamma = 60^\circ \), and the phase factor \( c \) varies from 0 to \( 2\pi \).

It is seen that when the perturbation wavelength \( \lambda \) is as small as 2 km, errors in the measured values are of the order of as much as 4 m s\(^{-1}\) for the zonal component, \( u_m \), and less than 1 m s\(^{-1}\) for the meridional component, \( v_m \). The amplitude of these errors does not depend on the azimuthal orientation, \( \gamma \), but their phases are dependent on \( \gamma \) (not shown).

Such measurement errors arise from the fact that the phase of the prescribed wave varies from one antenna to another, with the variation increasing as the spacing between

---

Figure A4. (a) Retrieved zonal perturbation velocities calculated from Eq. A5 for an altitude of 3 km for a prescribed perturbation with \( U = 2 \text{ m s}^{-1}, W = 0.5 \text{ m s}^{-1}, \gamma = 60^\circ \), and for \( \lambda = 2, 2.5 \) and 3 km. Results are shown as a function of the phase factor \( c \). The actual zonal perturbation velocities directly above the radar are marked as a dotted line. (b) Same as (a), but for meridional perturbation velocities from Eq. A6. Note that the prescribed meridional wind is zero. (c) Same as (a) but including horizontal wavelengths from 2 to 25 km at 0.5 km increments. (d) Same as (c) but excluding the vertical velocity correction term.
the antenna beams becomes a significant fraction of the perturbation wavelength. For the antenna configuration of the radar used in this study, this spacing is roughly \( \lambda/4 \) to \( \lambda/3 \) for a perturbation wavelength of 2–3 km at an altitude of 3 km. This spacing, of course, increases with altitude.

When the ratio of the oblique beam spacing to the wavelength decreases, the measured value \( u_m \) converges to the actual (prescribed) value (Fig. A4(c)). However, this convergence occurs only when term III in Eqs. (A5) and (A6), which contains information from the vertical beam, is included. Figure A4(d) is shown as a comparison when this term is not included.

It should be emphasized that the size of the measurement errors is greatly affected by the vertical-velocity terms (terms II and III) due to the factor, \( \cot \phi \), multiplying \( W \) in Eqs. (A5) and (A6), as has been discussed by Clark et al. (1985). In our experiment \( \cot \phi = 4.51 \). Thus the vertical velocity terms can be of the same magnitude as term I and can dominate the measured value when the beams are simultaneously sensing different phases of the wave.

In conclusion, it would appear from the analysis in this Appendix that the amplitudes of the \( u \) and \( v \) components of the retrieved winds in our radar experiment are not greatly in error. However, it is possible to have a significant phase difference between the actual and retrieved horizontal wind components.

**APPENDIX 2**

*Data analysis procedure used to create a space–height cross-section*

This Appendix describes the method employed to construct the space–height cross-sections in Fig. 12. The method assumes that the three-dimensional lee-wave pattern does not significantly change shape during the six-hour study interval, although it does change orientation with respect to the fixed radar position. The changing orientation, it is suggested, is the result primarily of changes in direction of the upstream wind forcing the lee-wave pattern. This appears to be a reasonable hypothesis if we consider the observed systematic relationship between the radar-observed wind directions and vertical velocities (Fig. 8) and make the further assumption that the changes in these wind parameters are most likely to be correlated with upstream flow conditions for which we have no observational confirmation. The hypothesis allows us to use the wind direction measured by the radar as an indicator of the radar location with respect to the lee-wave pattern.

The first step in the analysis consists in grouping the radar observations for each altitude according to wind direction. The 5-degree bins that were employed contained from zero to twenty data points each. For each bin that contained more than one sample, the bin-averaged zonal, meridional and vertical velocities are calculated and are denoted by \( u_b(z, \alpha) \), \( v_b(z, \alpha) \) and \( w_b(z, \alpha) \), respectively, where \( z \) is altitude and \( \alpha \) is the wind direction at the centre of the data bin. These are the values used in Fig. 12(a).

The standard deviations of the fluctuations in the downstream wind direction, in response to changes in the upstream conditions, will change with height because of the altitude dependence of amplitudes and horizontal gradients associated with the three-dimensional lee-wave perturbations. Even for an upstream shift in winds that is uniform with height, the resulting downstream deflections need not be uniform. For this reason it is desirable, as a practical matter, to adjust the wind directions in the plot of altitude versus wind direction according to the expression

\[
\alpha' = \alpha^* + \sigma_N^{-1}(\alpha - \alpha^*)
\]  

(A7)
where \( \alpha = \alpha(z) \) and \( \alpha' = \alpha'(z) \) are the original and adjusted directional labels for a particular data bin, \( \alpha^* = \alpha^*(z) \) is the six-hour average wind direction and \( \sigma_\alpha = \sigma_\alpha(z) \) is the normalized standard deviation of wind direction at altitude \( z \). The normalization is in terms of the limiting standard deviation of 10 degrees observed at the higher altitudes (see section 4(c)). For example, at \( z = 2.9 \) km which has \( \alpha^*(z) = 11.5^\circ \) and \( \sigma_\alpha(z) = 2.5^\circ \), an original wind direction label of \( \alpha(z) = 22.5^\circ \) is transformed into \( \alpha'(z) = 15.9^\circ \).

Since the space-height cross-section in Fig. 12 is oriented roughly north-east–south-west, the zonal and meridional wind components are transformed into components parallel \( (u') \) and perpendicular \( (v') \) to the straight line that is tangent to the mid-point of the arc which is used as the baseline for the cross-section. This rotational transformation can be expressed simply as

\[
\begin{align*}
   u' &= u_b \cos \beta + v_b \sin \beta \\
   v' &= -u_b \sin \beta + v_b \cos \beta
\end{align*}
\]  

(A8a)

where \( u' = u'(z, \alpha') \), \( u_b = u_b(z, \alpha') \) etc. and \( \beta \) is the angle of orientation of the tangent as illustrated in Fig. A5.

Similarly transformed six-hour averaged values of the zonal and meridional velocities \( u''(z) \) and \( v''(z) \) are subtracted from \( u'(z, \alpha') \) and \( v'(z, \alpha') \) to yield the perturbation values \( u''(z, \alpha') \) and \( v''(z, \alpha') \). These perturbation values along with the bin-averaged vertical velocities \( w_b(z, \alpha') \) are then used to produce the modified cross-section shown in Figs. 12(b) and 13.

![Figure A5. The angle, \( \beta \), used to define the orientation of the space–height cross-section in Figs. 12 and 13.](image)

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