Solar radiative transfer through clouds possessing isotropic variable extinction coefficient

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SUMMARY

Solar radiative fluxes were computed for heterogeneous clouds using an extension of the Monte Carlo method of photon transport which assumes that clouds possess isotropic variability. Hence, computation of fluxes for three-dimensional (3-D) clouds can be achieved with only an extended, characteristic one-dimensional (1-D) transect of extinction coefficient, $\beta$. These are easily obtained by aircraft measurements. In order to validate the new scheme, fluxes for 3-D, stochastic multiplicative cascade clouds were computed by the conventional approach to Monte Carlo simulation. 1-D transects through these clouds were then strung together and used in the new scheme. Not only do both methods of calculation yield statistically identical flux estimates, but they also produce virtually identical distributions of photon optical pathlengths and number of scattering events. Furthermore, the new technique may require an order of magnitude less computation time, depending on the desired level of accuracy.

Cloud microphysical data obtained by aircraft were used to represent characteristic transects of $\beta$, and corresponding fluxes were computed with the new scheme. Results suggest that internal inhomogeneity reduces cloud albedo below homogeneous values by about 5–10% for overcast and isolated cubic clouds. Also, it is predicted that for overcast clouds of optical depth $\tau < (>) > 40$, inhomogeneous clouds absorb less (more) radiation relative to their homogeneous counterparts. Three individual and trivial modifications of a plane-parallel, homogeneous two-stream solution of the radiative-transfer equation appear to be capable of capturing the essential effects of inhomogeneity. This is promising for incorporation of inhomogeneous cloud effects into existing climate-model solar-radiation routines. Little evidence, however, was found to suggest that the effects of inhomogeneity alone can explain the spectral aspects of the cloud absorption/albedo anomaly problem.

1. INTRODUCTION

Satellite data (Stephens and Greenwald 1991) and global climate model (GCM) simulations (Cess et al. 1989; Slingo and Slingo 1991) support the long held claim (Schneider 1972; Stephens and Webster 1981) that clouds are important regulators of the earth’s radiation budget and climate. Operational radiative-transfer models in GCMs and remote sensing assume that all clouds are plane-parallel and homogeneous, though observations (Lovejoy 1982; Stephens and Platt 1987; Spinhirne et al. 1989; Cahalan and Joseph 1989) show that clouds are very inhomogeneous. Recent studies (Stephens 1988; Cahalan 1989; Davis et al. 1990) suggest that solar radiative fluxes can be sensitive to cloud geometry. Furthermore, it has been speculated that the cloud absorption/albedo anomaly problem may be resolved by taking account of inhomogeneity in radiative-transfer calculations (Davis et al. 1990; Stephens and Tsay 1990).

Aside from stratified plane-parallel models (e.g. Wiscombe 1977), the form of inhomogeneity addressed most frequently in atmospheric radiative-transfer studies is horizontal mesoscale geometry which defines the ‘broken/finiteness’ of cloud fields (e.g. Welch and Wielicki 1984; Kobayashi 1989). Recently, however, attention has been directed towards the effects of internal inhomogeneity of cloud on radiative transfer. Stephens (1988) developed a semi-analytic solution of the radiative-transfer equation and showed that a thin cloud with a Guassian spatial distribution of vertically integrated optical depth in one direction reflects and absorbs less solar radiation than its plane-parallel counterpart. Cahalan (1989) used the Monte Carlo method to compute fluxes for a horizontally extensive, moderately thick cloud possessing extreme scaling variability in one direction. He found reduced albedo relative to the plane-parallel case (similar to Ronholm et al. 1980) except that they did not account for lateral transfer of photons. Davis et al. (1990) used the Monte Carlo method also and computed fluxes for optically
thick, horizontally finite and infinite clouds characterized by extreme three-dimensional (3-D) variability. Again, they found that (for overhead sun) internal inhomogeneity has the potential to reduce drastically the cloud albedo to below homogeneous estimates. Thus, all evidence suggests that internal inhomogeneity suppresses cloud reflectance and absorptance.

The actual importance of internal inhomogeneity, however, remains obscured by the fact that all simulated clouds used thus far for radiative-transfer calculations have been structurally quite different from real clouds. In this study, solar fluxes were computed with the Monte Carlo method of photon transport for a class of clouds with internal structure based on actual microphysical data, and the additional assumption that the clouds possess isotropic variability. Such inhomogeneity is expected to be approximated in some forms of cumuli owing to the turbulent cascade.

The outline and aims of this study are as follows. In the second section a new technique of applying the Monte Carlo method is presented, and validated with results from the conventional Monte Carlo approach. In the third section high-resolution microphysical data obtained by aircraft are used to characterize clouds, and radiative fluxes are computed with the new technique. In particular, questions are examined regarding the importance of internal 3-D inhomogeneity for both radiative-transfer calculations and the absorption/albedo anomaly. An assessment is given on how well three trivial modifications of a two-stream approximation account for the effects of inhomogeneity.

2. METHOD OF CALCULATION: DEVELOPMENT AND VALIDATION

This section has two main parts. First, the conventional approach to Monte Carlo simulation is reviewed and a new approach is presented. Second, a conventional Monte Carlo scheme is used to validate the new one.

(a) Development

The conventional procedure for computing radiative fluxes for internally inhomogeneous clouds is to construct a 3-D lattice of elementary cells, assign fixed optical properties to each cell, and trace photon trajectories through the lattice until they cross the boundary of the experiment (e.g. McKee and Cox 1974; Welch et al. 1980). Thus, this method yields flux estimates for clouds with a specific geometry. The conventional 3-D Monte Carlo code used in this study was developed by Barker (1991). It uses continuous-angle photon scattering, and the atmosphere is defined by a rectangular lattice of elementary rectangular cells.

These kinds of Monte Carlo simulations are notorious for their computer time and memory demands which increase dramatically with the resolution and mass of the lattice. An equally serious problem, however, is prescribing the 3-D distribution of cloud optical properties. As yet, there is no way to map the full 3-D density of cumuloform clouds. This is compounded by a paucity of mathematical procedures for generating realistic 3-D clouds. Only discrete, isotropic multiplicative cascade models have been used for the purpose of computing solar fluxes (e.g. Davis et al. 1990). The remainder of this section describes an alternative representation of inhomogeneous clouds for use in Monte Carlo simulations of photon transport.

As in the conventional procedure, a region of space designated to contain cloud droplets is defined. In this study, clouds are housed in either cubes or plane-parallel slabs. In principle, however, any shape can be used. Then, with random uniformity, irradiate the region with simulated photons. The photons are traced through the cloud
as they undergo scattering events with cloud droplets. Photon position in space is determined as though the cloud was homogeneous. Cloud droplet single-scattering phase functions are represented by the Henyey–Greenstein (1941) function with an asymmetry factor of 0.86. For flux calculations, this reproduces results obtained with detailed Mie phase functions and is computationally efficient.

The novelty of this procedure is in the assignment of the extinction coefficient, $\beta$. It is assumed that the internal 3-D variability of $\beta$ is isotropic or invariant with respect to rotation and reflection. One-dimensional (1-D) transects of $\beta$ through such media are statistically similar regardless of path (e.g., 3-D white noise and homogeneity are limiting examples). Thus, one may assume that the extinction coefficient along any transect in the cloud is represented by $\beta(l)$ where $l$ is the geometric length. Therefore, as usual, if $RN$ is a pseudo-random number distributed evenly between 0 and 1, a photon will travel a distance $f$ between successive scattering events defined by:

$$\exp\left\{-\int_{l_{0}}^{l_{0}+f} \beta(l)dl\right\} = RN$$

(1)

where $\beta(l_0)$ is the extinction coefficient at the first of the two scattering events or at the point of entry into the cloud. In general, $\beta(l)$ will be approximated by a discrete and finite set of values $\{\beta_i\}$ of resolution $\Delta l$. Ideally, $\{\beta_i\}$ should be a lengthy series (i.e., much longer than the cloud's vertical extent) in order to typify statistically a random walk through the cloud, and $\Delta l$ should be as small as possible. Therefore, once injected into a cloud, photons begin traversing $\{\beta_i\}$ at randomly chosen starting points. They are then traced through the cloud and along $\{\beta_i\}$ as they undergo scattering events by cloud droplets. A schematic portrayal of this process is shown in Fig. 1.

Because of the discrete nature of $\{\beta_i\}$, three forms of solving for $f$ in Eq. (1) are possible. In the first case, two successive scattering events take place in the same 1-D cell and

$$e^{-\beta_0 f} = RN$$

(2a)

hence

$$f = \frac{-\ln(RN)}{\beta_0} < \Delta l - l_0$$

where $l_0 < \Delta l$ is the distance inside the cell that the first scattering event takes place. This is the usual homogeneous solution. Second, the second scattering event takes place in the cell adjacent to the one in which the first scattering event occurred. Thus,

$$e^{-(\Delta l - l_0)\beta_0 + l_1\beta_1} = RN$$

(2b)

and

$$f = \Delta l - l_0 + l_1 < 2\Delta l - l_0$$

where $l_1 < \Delta l$ is the distance travelled through the adjacent cell before the second scattering event occurs. In the third, and most general, case, the photon traverses $n$ complete cells such that

$$e^{-((\Delta l - l_0)\beta_0 + \Delta l\sum_{i=1}^{n} \beta_i + l_{n+1}\beta_{n+1})} = RN$$

(2c)

and

$$f = (n + 1)\Delta l - l_0 + l_{n+1}$$
Figure 1. Schematic illustration of the characteristic transect procedure. Photons enter the cubic region containing cloud at points a and c (chosen at random) and begin travelling along the characteristic transect at the respective vertical dashed lines (also chosen at random). Photons make their exit at points b and d after having traversed (for the sake of example) geometric distances of 0.3 and 0.09 km. The corresponding homogeneous cloud would have an extinction coefficient independent of distance and equal to about 25 km$^{-1}$.

where $l_{n+1} < \Delta l$ is the distance travelled into the cell in which the second scattering event takes place. The value of $f$ in Eqs. (2b) and (2c) must be obtained by stepping through the sum in the exponent until RN is accumulated. If the end of $\{\beta_i\}$ is reached, one simply wraps around and continues on at the beginning of $\{\beta_i\}$. The error introduced by this discontinuity is small if $\{\beta_i\}$ is long. As usual, photons are tracked until they leave the prescribed cloud volume.

While droplet single-scattering albedo, $\omega_0$, is set to unity, the number of scattering events experienced by each photon is saved, thus enabling post-simulation fluxes to be computed for any value of $\omega_0$ (provided the corresponding asymmetry factor, $g$, remains $\sim 0.86$). Also, the total pathlength traversed by each photon is saved for computation of gaseous transmittances (Davies et al. 1984). In order to focus on cloud radiative properties, all simulations in this paper were conducted with surface albedo of zero.

(b) Validation

Unlike the static lattice clouds used in conventional Monte Carlo simulations, the full 3-D structure of clouds used in the method just described is never actually specified. Rather, a cloud is portrayed as a constantly fluctuating medium existing in a prescribed
volume of space yet having a stationary (i.e. statistically invariant in space and time) characteristic transect of $\beta$. This fundamental difference makes for imprecise validation of the new method with conventional methods. Regardless, there appears little choice but to work with the conventional methods, and recognize that a perfect one-to-one correspondence of fluxes should not necessarily be expected. In this section, a variant of the multiplicative cascade $\beta$-model (Mandelbrot 1974; the $\beta$ in $\beta$-model should not be confused with extinction coefficient) is used to produce static lattices of inhomogeneous media (clouds). Then, all of the horizontal transects of $\beta$ in these ‘clouds’ are strung together to form a 1-D characteristic transect. Fluxes are then computed and compared for both cloud forms.

The cascade process begins with a homogeneous cube which is repeatedly subdivided into eight subcells such that after $N$ steps, $2^{3N}$ subcells exist. At each step of the cascade, the probability of newly formed cells having $\beta$ set to zero is $1 - (1/2)^c$ where $3 - c$ is the Hausdorff dimension of the fractal set that would form as $N \to \infty$. At this stage, assume that all surviving cells have the value 1 (i.e. mass is not being explicitly redistributed). After $N$ steps, an ensemble average of $2^{3-Nc}$ identical cells remain designated as occupied by liquid water droplets. Because radiative fluxes for different geometries are the only concern, the total mass of liquid water in the cloud is normalized for each realization. Thus, if the average optical depth of the cloud is to be $\bar{\tau}$, then after $N$ cascade steps the optical depth of non-empty cells is

$$\tau_{\text{cell}} = \frac{2^{2N}\bar{\tau}}{M}$$

(3)

where $M$ is the number of surviving cells.

Despite the complex geometric structures this model is able to generate (see Davis et al. (1990) Fig. 5(c)), clouds simulated in this manner are still unrealistic because subcells are either empty or have optical depth $\tau_{\text{cell}}$ only, and an ensemble of transects through these cascade clouds yield a wave-number spectrum of $\beta$ no steeper than $k^{-1}$ where $k$ is the wave number (Cahalan 1989). Real clouds, on the other hand, are typified by a continuous range of $\beta$ (actually liquid water content) and corresponding spectra ranging from $k^{-1.5}$ to $k^{-3}$ (King et al. 1981; Barker 1991). It is anticipated, however, that the extremely erratic, scaling nature of $\beta$-model clouds will provide a stringent test for the new procedure.

A further problem for $\beta$-model clouds of practical and theoretical importance is when to stop the cascade. Theoretically, scaling should proceed down to at least the centimetre level. However, for a 1 km cubic cloud, this implies about 20 cascade bifurcations which is currently impossible to handle. In this study, as in Davis et al. (1990), 3-D variability is pursued down to five cascade steps, thus producing cubic clouds consisting of 32768 cells. Also, results presented here are for $c = 0.25$ which produces extremely irregular clouds having about 58% of their volume empty.

Table 1 lists average total reflectances and standard deviations calculated by the conventional and new Monte Carlo schemes for ten realizations of isolated cubic $\beta$-model clouds with $\bar{\tau} = 50$. For each solar zenith angle, $\theta_0 (\mu_0 = \cos \theta_0)$, differences between the model estimates are statistically indistinguishable from zero (the same level of agreement was achieved using $c = 0.5$ in which on average about 83% of the cloud is set to zero). For each case, the standard deviation is about 10% of the reflectance. This illustrates the potential importance of cloud structure for radiative transfer, since $\bar{\tau}$ was fixed in all realizations. As a reference, Table 1 lists reflectances for the corresponding isolated homogeneous cube. These values are about 10–17% greater than their inhomogeneous
TABLE 1. ENSEMBLE-AVERAGED, SOLAR ZENITH ANGLE, $\theta_0$ ($\mu_0 = \cos \theta_0$), DEPENDENT REFLECTANCES CALCULATED BY THE CONVENTIONAL AND THE NEWLY PRESENTED MONTE CARLO SCHEMES

<table>
<thead>
<tr>
<th>$\mu_0$</th>
<th>Conventional</th>
<th>New method</th>
<th>Homogeneous cube</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.461 ± 0.043</td>
<td>0.456 ± 0.048</td>
<td>0.516</td>
</tr>
<tr>
<td>0.4</td>
<td>0.439 ± 0.047</td>
<td>0.437 ± 0.042</td>
<td>0.517</td>
</tr>
<tr>
<td>0.6</td>
<td>0.426 ± 0.050</td>
<td>0.425 ± 0.038</td>
<td>0.513</td>
</tr>
<tr>
<td>0.8</td>
<td>0.427 ± 0.046</td>
<td>0.426 ± 0.039</td>
<td>0.516</td>
</tr>
<tr>
<td>1.0</td>
<td>0.540 ± 0.066</td>
<td>0.537 ± 0.046</td>
<td>0.621</td>
</tr>
</tbody>
</table>

Values are for ten realizations of isolated, cubic $\beta$-model clouds with $\bar{\tau} = 50$ and $c = 0.25$. Fluctuations represent ± one standard deviation for the ten realizations. The last column is reflectance for the corresponding homogeneous cube. Each simulation used $10^5$ photons (leading to errors of at most ±0.0015).

counterparts. It is worth noting that fluxes out of individual faces of the inhomogeneous finite clouds, as predicted by the two schemes, agree as well as do the results listed in Table 1.

Figure 2 shows the ensemble-average distribution of photon optical pathlengths for photons reflected out of the top face of the inhomogeneous cubes used to produce Table 1 with $\mu_0 = 0.8$ for both Monte Carlo procedures. This remarkable agreement, which extends to distributions of the number of scattering events (not shown here), implies that both methods behave similarly at a much more fundamental level than just conservative flux calculation. In other words, if gaseous or droplet absorption takes place, the quality of agreement displayed in Table 1 will be altered insignificantly.

Table 2 lists reflectances for $\beta$-model clouds with $\bar{\tau} = 20$, but now the horizontal boundary conditions are periodic such that a $32 \times 32 \times 32$ cube is repeated infinitely many times in all four horizontal directions thus rendering an overcast cloud (with the exception of holes which may form occasionally). As in Table 1, reflectances produced by both procedures are statistically identical. This time, however, the reflectances are only 3–8% less than their corresponding homogeneous values. It is noted (though not listed here) that both methods of calculation predict the same fluxes for 3-D arrays of white noise which are similar to the clouds used by Welch et al. (1980).

Figure 2. Ensemble-average distribution of photon optical pathlengths for photons reflected out of the top face of the $\beta$-model clouds used in Table 1 at $\mu_0 = 0.8$. The null hypothesis stating that these distributions are from the same population cannot be rejected at the 95% confidence level.
TABLE 2. As Table 1 except clouds are \( \beta \)-model clouds, with \( T = 20 \), repeated an infinite number of times in all four horizontal directions (cyclic boundary conditions)

<table>
<thead>
<tr>
<th>( \mu_0 )</th>
<th>Conventional</th>
<th>New method</th>
<th>Homogeneous slab</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.794 ± 0.006</td>
<td>0.788 ± 0.008</td>
<td>0.814</td>
</tr>
<tr>
<td>0.4</td>
<td>0.729 ± 0.005</td>
<td>0.721 ± 0.011</td>
<td>0.753</td>
</tr>
<tr>
<td>0.6</td>
<td>0.669 ± 0.014</td>
<td>0.660 ± 0.005</td>
<td>0.698</td>
</tr>
<tr>
<td>0.8</td>
<td>0.603 ± 0.007</td>
<td>0.606 ± 0.011</td>
<td>0.646</td>
</tr>
<tr>
<td>1.0</td>
<td>0.543 ± 0.011</td>
<td>0.552 ± 0.011</td>
<td>0.594</td>
</tr>
</tbody>
</table>

Finally, if the ten transects in each of the above experiments are strung together to create two very long series of \( \beta \), and these are used in single simulations of the new scheme, reflectances shown in the 3rd column of Tables 1 and 2 are essentially reproduced exactly. This suggests that if a long enough time series of \( \beta \) is available, ensemble averaging of fluxes is effectively done in a single experiment. This represents a significant reduction in computation time. In this case, stringing the realizations together leads to an order of magnitude reduction in computation time; information regarding the variance of results is, however, lost.

In conclusion, therefore, despite the fact that inhomogeneous media in the two Monte Carlo schemes are conceptually quite different, they transport photons very similarly. This supports the validity of the new technique.

3. RESULTS

It may now be hypothesized that as long as a medium possesses isotropic variability, and a characteristic transect of \( \beta \) is available, overall radiative properties of the medium can be estimated accurately by the Monte Carlo scheme proposed in this paper. In this section, cumuloform clouds are assumed to be isotropic, and transects of \( \beta \) from high-resolution cloud microphysical data are used to estimate the importance of inhomogeneity for calculating solar fluxes. At present, algorithms for generating corresponding specified lattice clouds do not exist.

(a) Cloud microphysical data

Cloud droplet concentration, \( N_0 \) (cm\(^{-3}\)), and liquid water content, \( W \) (gm\(^{-3}\)), data used in this study were obtained by the forward scattering spectrometer probe (FSSP) onboard the Atmospheric Environment Service (AES) Twin Otter aircraft. While data of resolution \(~1\) m (64 Hz) were available, \( 4 \) m averages were used. This is because of instrument noise at high-frequency sampling. Since the AES cloud physics division routinely provide \( N_0 \) and \( W \), it is assumed that, regardless of position and time, the droplet-size spectrum is approximated by the modified gamma-size distribution (Welch et al. 1980; \( \gamma = 1 \)). Therefore, letting the extinction efficiency be 2.0, local extinction coefficient (km\(^{-1}\)) is approximated as

\[
\beta = 24.18 \left( \frac{\Gamma^3(\alpha + 3)}{\Gamma^2(\alpha + 4) \Gamma(\alpha + 1)} \right)^{1/3} N_0(W/N_0)^{1/3}
\]

where \( \Gamma \) is the gamma function and \( \alpha \) is its parameter. Figure 3(a) shows 8 kilometres of \( \beta \) determined directly from the FSSP droplet-size spectra and by Eq. (4) with \( \alpha = 4 \). For the most part, Eq. (4) does well and, if anything, makes the clouds appear to the Monte Carlo simulation as too inhomogeneous.
Two types of clouds are considered: stratocumulus (Sc) and fair-weather cumulus (Cu). All data were collected in August 1988 about 200 km north of Toronto, Ontario. The characteristic transect of $\beta$ used for the Cu case is a concatenation of several cumuli that were traversed near their mid point during flight 25 (which was in cloud about 40% of the time). This constitutes about 30 km of sampled cloud. At $\alpha = 3$ (determined from a short stretch as in Fig. 3(a)), the average extinction coefficient $\beta$ is 24.91 km$^{-1}$ which is similar to the values of Stephens and Platt (1987). The wave-number spectrum of this series of $\beta$ follows $k^{-1.8}$ down to 8 m (approx.) where $k$ is the wave number (King et al. 1981). For the Sc cases, data from two flights through the middle of overcast cloud were used (see Fig. 3). At $\alpha = 4$, $\beta$ for flights 32 and 36 are 36.41 and 31.91 km$^{-1}$. Wave-number spectra for both series of $\beta$ follow $k^{-5/3}$ very closely down to 8 m. Since the corresponding temperature and liquid water content wave-number spectra follow $k^{-5/3}$ closely, it may be safe to assume that the associated turbulence is approximately isotropic.

![Figure 3](image-url)

Figure 3. (a) Extinction coefficient, $\beta$, (solid line) estimated from FSSP cloud droplet size spectra obtained on AES flight 36. Dashed line is $\beta$ estimated from corresponding droplet number density and liquid water content, and assuming a Gamma droplet size distribution with $\alpha = 4$ (Eq. 4). (b) Extended transect of $\beta$ as determined from the Gamma distribution for the same flight as in (a). Data are 4 m resolution (16 Hz) and were recorded on 17 August 1988 beginning at 2024 CMT. The flight path was through approximately the center of a stratocumulus deck about 200 km north of Toronto. In (b) $\beta = 31.91$ km$^{-1}$.

To simulate a cloud with an average optical depth, $\tau$, in some direction, simply define the geometric size of the cloud in that direction to be $\tau/\beta$. However, because real data are used here, the most appropriate cases will be for clouds with vertical geometric thickness equal to the actual cloud thickness. This was $<1$ km (approx.) in all cases which corresponds to $\tau < 30$ approx. For this study, cloud thickness was allowed to be quite large ($\tau$ in the vertical of 128) which for the horizontally infinite cases corresponds to very thick and solid overcast.
Figure 4 shows cloud reflectance as a function of $\tau$ for homogeneous, plane-parallel, non-absorbing clouds with $\theta_0 = 0^\circ$ and $60^\circ$. It also shows reflectance differences between these clouds and corresponding clouds whose extinction coefficient transects are represented by those measured on flights 32 and 36 (computed by the new technique). Thus, using these examples of authentic internal variability, reflectance is reduced below homogeneous values by a maximum of only about 10% (around 0.04 in absolute terms) which occurs for $\tau$ between 5 and 30. Since stratocumulus clouds in this range of $\tau$ cover a substantial fraction of the earth’s subtropical oceans, this systematic error due to the assumption of homogeneity is likely to be important for climate simulations.

The definition of the co-packing factor, $\delta'$, (Davis et al. 1990) is $\tau' = \tau \delta'$ where $\tau'$ is the optical depth required for a homogeneous cloud to yield the same reflectance as that predicted for an inhomogeneous cloud at $\tau$. For both inhomogeneous cases used in Fig. 4 and $\theta_0$ or $0^\circ$ and $60^\circ$, $\delta' = 0.92$ to 0.96 for all $\tau$. For the example of $\theta_0 = 0^\circ$ and $\tau = 50$, $\delta = 1 - \delta' = 0.06$ which is achieved for the 5th order cascade $\beta$-model clouds with $c \approx 0.1$ (see Davis et al. (1990) Fig. 6) which are characterized by about 30% of their volume being empty. Thus, while Fig. 6 in Davis et al. (1990) shows that inhomogeneity can profoundly alter reflectance, this study implies that only the extreme left side of their plot may be applicable to real Sc.

![Homogeneous, plane-parallel, non-absorbing reflectances as a function of optical depth for $\theta_0 = 0^\circ$ (solid line) and $60^\circ$ (dashed line). Lower set of curves are differences between the homogeneous reflectances (upper set of curves) and the corresponding inhomogeneous reflectances obtained with the new technique presented in this paper with characteristic transects of the extinction coefficient obtained from flights 32 and 36.](image)

In Fig. 5, cloud absorptance is shown for droplets with single-scattering albedo $\omega_0$ of 0.97 (inhomogeneous clouds correspond to those associated with the transect of flight 36). This value of $\omega_0$ is expected for a wavelength of about 2.15 $\mu$m (Wiscombe et al. 1984, g is still $\sim$0.86). For $\theta_0$ of $0^\circ$ and $60^\circ$, absorptance by the inhomogeneous clouds is least for $\tau$ less than 45 and 35. For respectively larger values of $\tau$, however, inhomogeneous clouds absorb more than homogeneous clouds. This is explained as follows. At large $\tau$ and $\omega_0 = 1.0$, both clouds transmit a small number of photons, and those which are transmitted generally experience many (>100) scattering events. Thus, as $\omega_0$ decreases, transmittance rapidly approaches zero. Since in both types of clouds reflected photons experience similar numbers of scattering events, and for $\omega_0 = 1.0$ the inhomoge-
Figure 5. Cloud absorbances when droplets have a single-scattering albedo of 0.97 as a function of optical depth. Solar zenith angles are indicated on the plot. Inhomogeneous cloud corresponds to flight 36.

Inhomogeneous clouds reflect least, reflectance differences for $\omega_0 < 1.0$ are proportionally similar to the $\omega_0 = 1.0$ case. Combining these two points, with conservation of energy, when $\omega_0 < 1.0$, thick inhomogeneous clouds are more absorbive than homogeneous clouds (the smaller $\omega_0$, the smaller the cross-over value of $\overline{\tau}$). For thin clouds, on the other hand, the majority of photons are transmitted through inhomogeneous cloud and experience significantly fewer scattering events than in the homogeneous case. Thus, for inhomogeneous clouds, transmittance decreases slowly and absorbance increases slowly for decreasing $\omega_0$ relative to the homogeneous case. Therefore, thin inhomogeneous clouds absorb less than their homogeneous counterparts. This, however, cannot explain the absorption anomaly; for homogeneous solutions seem to operate well in the near infrared (NIR) regions of very small $\omega_0$. It is in the adjacent windows where $\omega_0 = 0.97-0.99$ that problems arise (Stephens and Tsay 1990).

Figure 6 shows the normalized effective distribution of vertically integrated optical depth, $p(\tau)$, for clouds of different geometric thickness, $h_c$, with $\beta$ transects characterized by the sequence for flight 36. These were derived by choosing random starting points in $\{\beta\}$, averaging $\beta_\tau$ for the subsequent length $h_c$, multiplying by $h_c$ to get optical depth, and binning the result to produce histograms. If the transects of $\beta$ were longer, the curves in Fig. 6 would become smoother. The general structure of the density functions is that of a negative skew (opposite to log-normal) and a precipitous decline in the frequency of occurrence of $\tau$ greater than the mode (especially at small values of $\overline{\tau}$). These density functions could represent a cloud field with vertical extent limited sharply by a temperature inversion at the top of a boundary layer.

Using the density functions $p(\tau)$ in Fig. 6 to compute average direct-beam transmittance as

$$T = \int_0^\infty p(\tau)e^{-\tau}/\mu_0 d\tau \quad \text{(5)}$$

reproduces the values obtained in the Monte Carlo simulations for the fraction of transmitted, un scattered photons. If $T = e^{-\tau_{\text{eff}}}/\mu_0$, then for the cases used here, $\tau_{\text{eff}}$ falls between 0.3$\overline{\tau}$ and 0.5$\overline{\tau}$ for $\overline{\tau} < 128$. In other words, to the direct-beam, inhomogeneous clouds ‘appear’ to be 1/3 to 1/2 as thick as their homogeneous counterparts. This means
that, on average, diffuse radiation is produced deeper inside inhomogeneous clouds than in homogeneous clouds. Thus, one could envisage a modified 1-D, plane-parallel, homogeneous radiative-transfer equation like:

$$
\mu \frac{\partial I(\mu, \tau)}{\partial \tau} = -I(\mu, \tau) + \frac{\omega_0}{2} \int_{-1}^{1} P(\mu, \mu') I(\mu', \tau) d\mu' + \frac{\omega_0}{4\pi} F_0 P(\mu, \mu_0) e^{-a\tau}\mu_0 \tag{6}
$$

where $I$ is radiance, $P$ is scattering phase function, $F_0$ is downward flux at $\tau = 0$, and $a < 1$. The only difference between Eq. (6) and the standard equation ($a = 1$) is that in Eq. (6) more direct-beam radiation reaches any given level; diffuse radiation is treated as usual but it is produced deeper inside clouds. Therefore, in a manner analogous to the packing factor $\delta'$, the most appropriate value of $a$ in Eq. (6) could be estimated numerically. By letting the optical depth for unscattered photons in homogeneous Monte Carlo simulations be $\sim 0.75\tau$ ($a = 0.75$ and then 1 once photons are scattered), the curves in Figs. 4 and 5 corresponding to inhomogeneous clouds are closely approximated.

Another less analytic approach than the use of $\delta'$ and $a$ is the independent pixel approximation (Cahalan 1989) in which reflectance is defined as:

$$
R_{pp}(\mu_0, \bar{\tau}) = \int_0^{\infty} p(\tau) R_{pp}(\mu_0, \tau) d\tau \tag{7}
$$

where $R_{pp}$ is a plane-parallel, homogeneous analytic solution. An analogous expression exists for transmittance. While this approach neglects the problem of horizontal transport, an appropriate distribution of vertically integrated $\tau$ (which may prove ultimately to be an essential ingredient) must be known.

Figure 7 is similar to Figs. 4 and 5 except that it shows reflectance and absorptance for the plane-parallel, homogeneous delta-Eddington method (Joseph et al. 1976) $R_{pp}$. Also shown in Fig. 7(a) are the reflectance differences between $R_{pp}$ and: (i) the packing factor with $\delta' = 0.94$; (ii) $a = 0.75$; and (iii) $R_{pp}$ with $p(\tau)$ determined from flight 36 data. For Fig. 7(a), at $\tau < \sim 16$, the solution with $a = 0.75$ and $R_{pp}$ are almost identical and track the $R_{pp}$ curve similar to the way in which the inhomogeneous reflectances track the homogeneous reflectances in Fig. 4. But for $\tau > \sim 16$, the three solutions begin to merge too quickly relative to those in Fig. 4. The curve for $\delta' = 0.94$, however, deviates from
Figure 7. (a) Homogeneous, plane-parallel reflectances computed with the delta-Eddington method (thin and thick solid lines are for $\theta_0$ of 0° and 60° respectively). Lower set of curves are differences between standard delta-Eddington reflectances and reflectances obtained by modifying the delta-Eddington solution as: packing factor, $\tau \leftarrow \tau^{0.94}$; Eq. (6) with $a = 0.75$; independent pixel approximation (circles) (Eq. 7) with $r(r)$ obtained from flight 36 data (see Fig. 6). (b) Cloud absorptances for the methods used in (a). Solar zenith angles are indicated on the plot and $\omega_0 = 0.97$.

$R_{pp}$ for all $\tau$, very much like the deviations shown in Fig. 4. But note that for $\tau < 1$ the packing-factor approach serves to enhance both optical depth and reflectance for $\delta' < 1$.

In Fig. 7(b), $\omega_0 = 0.97$ and both the $R_{pp}$ and the $\delta' = 0.94$ solutions match one another closely for all $\tau$, and for $\tau > 50$ are virtually identical to the homogeneous solution. For smaller $\tau$, both approximations predict significantly less absorption than the homogeneous case. For $\tau < 20$, the solutions for homogeneity and $a = 0.75$ are essentially equal. For $\tau > 20$, however, absorption for the $a = 0.75$ case is significantly greater than the homogeneous case. This is in accord with the Monte Carlo simulations but the cross-over occurred at $\tau \approx 20$ (Fig. 7(b)) rather than $\tau \approx 40$ (Fig. 5) and the enhancement of absorption as $\tau \to \infty$ is larger than in the Monte Carlo experiments.

In general, therefore, it appears that simple transformations of homogeneous, two-stream solutions can account for some effects of inhomogeneity some of the time, but none of these transformations can account for all the effects all of the time.
Figure 8 shows the normalized distribution of reflected and transmitted photon pathlengths for $\tau = 50$, $\theta_0 = 0^\circ$, and $\omega_0 = 1.0$ and 0.97. The clouds used are 1.57 km deep; one cloud is homogeneous and the other is characterized by the flight 36 transect. Pathlength distributions for $\omega_0 < 1$ were calculated by adding $\omega_0^n$, where $n_i$ is the number of scattering events experienced by the $i^{th}$ photon, to the pathlength bin corresponding to the pathlength travelled by the $i^{th}$ photon. As expected, as $\omega_0$ decreases, the proportion of short pathlengths increases dramatically such that as $\omega_0 \to 0$, reflectance and transmittance distributions approach single- and zero-order scattering distributions. For transmittance, the inhomogeneous cloud is clearly approaching a delta-spike at $l = 1.57$ km (no scattering events) faster than is the homogeneous cloud. Pathlength distributions for reflected photons, however, are almost identical for the two clouds. When the magnitudes of reflectance and transmittance are factored into the distributions, average photon pathlengths differ very little between the two clouds. Thus, it is unlikely that the cloud-absorption anomaly (Stephens and Tsay 1990) is due to enhanced pathlengths through water vapour as a result of inhomogeneity.

Attention is now turned briefly towards isolated cubic clouds with inhomogeneity characterized by the concatenated string of Cu$_1$ transects (Flight 25). Implicit density functions of vertically integrated $\tau$ in this case (cf. Fig. 6) are nearly symmetric and well approximated by either a log-normal or normal distribution (at $\tau = 25$, it is almost identical to Nakajima et al.'s (1990) case for 16 July). Finite clouds are not dwelled upon here because it is difficult to justify what should be considered as cloud when dealing with finite clouds (Rawlins 1989). For example, with $\beta$-model clouds, one starts with a homogeneous region, $\Re$, and after many cascade steps the cloud occupies a region $\Re' \subset \Re$. In a typical Monte Carlo simulation of the radiative fluxes for the cloud defined by $\Re'$, photons are uniformly injected over $\Re$ as though empty sectors are part of the cloud. However, if one were to measure fluxes for an isolated cloud, one would not imagine the cloud to be in a region that could have potentially contained the cloud. Therefore, comparison of fluxes for finite clouds in simulations, and in the atmosphere, suffers much from a lack of well defined experiments which likely lead to underestimations of reflectances in simulations.

![Figure 8](image-url)
Figure 9 shows overall reflectance (out of the top and upwelling out of the sides) for homogeneous cubes as a function of \( \tau \) for \( \theta_0 = 0^\circ \) and \( 60^\circ \). The reflectance differences between the homogeneous and inhomogeneous clouds are an echo of those in Fig. 4; inhomogeneity reduces reflectance below the homogeneous case by at most about 8%.

Figure 10(a) shows the average number of scattering events experienced by reflected photons out of the top face of the clouds used in Fig. 9. As with the horizontally infinite case, inhomogeneous finite clouds experience slightly more scattering events than their homogeneous counterparts. Hence, ratios between reflectances at \( \omega_0 = 1.0 \) and \( \omega_0 < 1.0 \) will be larger for inhomogeneous clouds. Figure 10(b) shows mean photon pathlengths divided by the geometric size of the clouds. For \( \tau < 20 \) with \( \theta_0 = 60^\circ \), and for all \( \tau \) with \( \theta_0 = 0^\circ \), mean pathlengths are not influenced much by inhomogeneity. For \( \tau > 20 \) with \( \theta_0 = 60^\circ \), however, inhomogeneous clouds have slightly longer average pathlengths. As in the horizontally infinite case, however, the differences are rather minor and will have little impact on water vapour absorbances.

Figure 9. As Fig. 4 except for conservative scattering cubic clouds in which the inhomogeneous ones have extinction-coefficient transects characterized by data collected on AES flight 25.

4. SUMMARY AND CONCLUSION

This paper presented a variation on the conventional method of Monte Carlo simulation of photon transport in media possessing isotropic variability. Instead of specifying a cloud as a lattice of cubes with fixed optical properties, a region of space containing cloud is defined, along with a characteristic transect of extinction coefficient, \( \beta \). Photons are injected and transported through the region as usual but they also traverse along the \( \beta \) transect. As long as the assumption of isotropy is adequate, solar fluxes for realistic clouds can be computed, provided a lengthy transect of \( \beta \), measured for example by an aircraft, is available. This approach, therefore, attempts to side-step shortcomings inherent to procedures that generate artificial clouds. Since much controversy has surfaced in the last decade regarding the potential role of inhomogeneity in resolving cloud absorption/albedo anomalies, the method presented here may be helpful since it uses real cloud data. It should be stressed, however, that the strict assumption of full isotropy is likely invalid in most cases. The fact that it has been assumed by most other relevant studies only marginally vindicates its use here. Furthermore, experiments are in progress which attempt to account for systematic trends in \( \beta \), such as smaller values near cloud edges (Coakley 1991) and increasing values with height (Stephens and Platt 1987).
The validity of the new technique was demonstrated using conventional methods and highly inhomogeneous multiplicative cascade $\beta$-model clouds. In all cases tested to date, the ensemble averages of fluxes predicted by the two methods have been statistically indistinguishable. Furthermore, if all the transects through an ensemble of $\beta$-model clouds are concatenated into a single transect, only one simulation with the new method is required to estimate the ensemble-average fluxes accurately. This can represent in excess of an order of magnitude saving of computation time at the expense, however, of losing information regarding variability of fluxes from realization to realization.

Microphysical cloud data were used to produce 'characteristic' transects of $\beta$ for Cu$_1$ and Sc clouds. Thus, these transects exhibit realistic cloud optical fluctuations down to about 4 m resolution. According to the cases used, inhomogeneity may reduce cloud reflectance in the visible by up to only about 10% (at optical depths between 5 and 30) for both overcast and finite clouds. This systematic bias introduced by homogeneous models may be significant, since clouds with $\bar{\tau}$ less than 30 appear to represent a majority of the earth's layered clouds (e.g. Rossow and Lacis 1990). This could be important in GCMs that predict cloud cover and carry cloud liquid water: striking a simultaneous
balance between measured and modelled cloud amount, liquid water path and radiation budgets may prove impossible if inhomogeneity effects are neglected. Furthermore, since the results of Stephens and Platt (1987) suggest that homogeneous transfer solutions perform well in conservative scattering Sc cases, the results presented here should not upset this agreement too much as ±10% is probably near or within experimental error.

For absorption by cloud droplets, it was shown that for the cases considered, inhomogeneous clouds absorb less radiation than do homogeneous clouds for \( \bar{\tau} < 40 \). This is because appreciable numbers of photons are being transmitted through inhomogeneous clouds with relatively few scattering events. For larger values of \( \bar{\tau} \), however, inhomogeneous clouds absorb more than their homogeneous counterparts. This is because the number of scattering events experienced by transmitted photons becomes so large under either assumption that, for \( \omega_0 < 1.0 \), transmittance is essentially eliminated. Because reflectances for the inhomogeneous clouds are less than those for homogeneous clouds for all \( \omega_0 \), inhomogeneous clouds must, therefore, absorb more than homogeneous clouds. This goes partly in the direction of explaining anomalous absorption but, if the observations are correct, the enhanced absorption shown here should occur not only in deep clouds, and not just in the window regions of the NIR, but also in the strong absorption regions too. Stephens and Tsay (1990) speculated that a NIR water vapour continuum may be responsible for the additional absorption. Nevertheless, the results of this study suggest that the effects of inhomogeneity on radiative transfer are insufficient to explain fully the cloud absorption/albedo anomaly; but before solid conclusions are drawn, the procedure presented here should be applied to more clouds and cloud types. On the other hand, as this study and several others have shown, effects of inhomogeneity are systematic and should certainly be accounted for in climate models and remote-sensing techniques.

It was demonstrated that homogeneous, two-stream radiative-transfer models can be very simply modified to capture much of the effects of inhomogeneity. Two modifications considered here were: first, to simply scale down \( \bar{\tau} \) (cf. Davis et al. 1990), and second to allow the direct-beam to penetrate cloud more easily. Both these procedures may prove to be useful starting points for modifying climate-model radiation codes to account for sub-cloud inhomogeneity. This, however, still leaves unaddressed the seemingly less tractable problem of parametrizing the radiative effects of horizontal variations in cloud geometry on scales greater than about 500 m (e.g. Welch and Wielicki 1985; Stephens and Greenwald 1991; Barker and Davies 1992a).

As a final comment, the modification to the Monte Carlo method presented and used here cannot be utilized to study spatial distributions of fluxes and radiances, for the cloud is not rigidly specified. It is expected, however, that this procedure can be used for regular arrays of identical clouds above reflecting surfaces (Welch and Wielicki 1989; Barker and Davies 1992b) since, as demonstrated, it performs well in the finite and plane-parallel limits.

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