Structure and evolution of an isolated semi-geostrophic cyclone

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(Received 8 November 1991; revised 4 June 1992)

SUMMARY

The evolution and structure of an idealized low-pressure system is studied within the framework of the semi-geostrophic dynamics and in the limit of uniform potential vorticity. An isolated cyclone is grown from suitably chosen initial conditions, rather than studying the evolution of a longitudinally periodic train of baroclinic systems.

It is shown that the resulting development is able to produce a range of flow features that compare favourably with observationally based conceptual models of cyclogenesis. These features include in particular: (i) the simultaneous occurrence of both a cold and a warm front whose alignment shows some of the characteristics of occluded frontal systems and is akin to the notion of a frontal fracture, (ii) a dry descending air-stream to the rear of the cyclone, (iii) a narrow region of maximum ascent within the warm front and its bent-back portion, (iv) a poleward travelling air-stream ahead of the cold front, and (v) a Ω-shaped pattern of vertical lifting comparable with the cloud patterns as commonly observed in satellite pictures.

The evolving cyclone is analysed both from Lagrangian and Eulerian viewpoints. It is demonstrated that Lagrangian criteria exist that allow for the objective definition of air-streams and flow patterns within developing cyclones. The structures of cold and warm fronts at low levels are significantly affected by the different nature of the Lagrangian trajectories within each of these regions. In particular, the air parcels in the warm-frontal region are transported rapidly towards the centre of the low, resulting in a low-level warm front with an intrinsically three-dimensional structure and an associated vorticity gradient in the along-front direction.

1. INTRODUCTION

Today’s depiction of frontal zones on operational surface analysis charts still follows closely the classical polar-front theory of the Bergen School (Bjerknes 1919; Bjerknes and Solberg 1922). Several major modifications of this theory have, however, been suggested. Most importantly, fronts are now viewed essentially as the result of cyclogenesis (Hoskins and West 1979, hereafter referred to as HW) rather than being a permanent feature of the general circulation and the unique seat of cyclogenesis. The structure of the frontal system has been heavily debated, with the classical occlusion process (Wallace and Hobbs 1977) and even the existence of the warm front (HW) being questioned. From an operational point of view it has also been pointed out that today’s use of frontal signatures on surface analysis charts is often not objective, and on occasions confusing and inconsistent (Mass 1991).

Recent observational studies have confirmed that the structure of marine extratropical cyclones significantly departs from the schematic displays of the Bergen School (Shapiro and Keyser 1990; Neiman et al. 1991; Shapiro et al. 1991). Their revised scheme in effect abandons the classical occlusion process. The initial baroclinic zone experiences frontogenesis and is ‘fractured’ into warm and cold fronts. The occluded front is further viewed as forming a single dynamical entity with the warm front, and referred to as a ‘bent-back warm front’. These fronts can reveal sharp structures immediately to the north of the surface low, both in observed cases (Kuo and Reed 1988; Kuo et al. 1992; Neiman et al. 1991) and in idealized numerical experiments (e.g. Takayabu 1986; Davies et al. 1991, hereafter referred to as DSW).

Observational studies have also sought to identify individual air-streams within a developing cyclone (see the review of Browning (1990)). The dynamically most interesting streams are linked to baroclinic energy conversions and to cloud formation through significant ascent or descent. These include the dry descending air-stream to the rear of

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the cold front (Danielson 1966), which causes the dry intrusion commonly seen on infrared satellite pictures (Young et al. 1987), and the ascending warm and cold conveyor belts (Carlson 1980; Browning and Mason 1981; Stewart and Macpherson 1989), which are located ahead of the cold front and within the warm frontal zone, respectively. According to Browning (1990) the warm conveyor belt is the 'primary cloud-producing flow' within an extratropical cyclone. On the other hand, Kuo et al. (1992) related cloud production in their real-case numerical simulation to an ascending flow that rose from the boundary layer in the vicinity of the warm front and spread out in a fan-shaped pattern.

The precise linkage between these air-streams and flow patterns with the dynamics of the evolving cyclone has not yet been fully explored. Several studies (e.g. Heckley and Hoskins 1982; Golding 1984; Whitaker et al. 1988; Kuo et al. 1992) have analysed parcel trajectories in order to establish a link between the flow in idealized or real-case numerical simulations and the conveyor belt and air-stream concepts. This approach is not fully satisfactory since it relies in principle on the subjective choice of a few individual trajectories. Here we shall present a new approach which is based on the consideration of an ensemble of trajectories which are selected objectively.

A common procedure used in theoretical investigations is to study the nonlinear evolution of the most unstable normal mode on a jet-like basic state. Analytical treatment (Saltzman and Tang 1972; Hoskins 1976) proved to be difficult and was predated by early numerical primitive-equation experiments (Hinkelmann 1959; Edelmann 1963). In subsequent studies various dynamical approximations were employed, as summarized in the compilation of Table 1. All these experiments produced an easily detectable, elongated cold front, while the warm front proved to be less discernible and stubber.

There is little information in the literature on the relationship between cyclogenesis in an isolated low-pressure system and in an east-west periodic baroclinic wave. Periodic boundaries and monochromatic initial conditions impose various dynamical constraints on the nonlinear evolution of the system. In particular, wave–wave interactions are suppressed beyond the maximum wavelength prescribed by the length of the periodic domain. In this study we investigate the evolution of an isolated low-pressure system from suitable finite-amplitude initial conditions. Similar strategies have recently been adopted in other numerical experiments (Takayabu 1991; Montgomery and Farrell 1992). This attempt greatly reduces interaction effects with neighbouring cyclones, and this will

<table>
<thead>
<tr>
<th>Study</th>
<th>Dynamics</th>
<th>Geometry</th>
<th>Tropopause</th>
<th>Decay phase</th>
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<td>Lalaurette (1990)</td>
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The labels 'OG', 'SG' and 'PE' refer to quasi-geostrophic, semi-geostrophic and primitive-equation dynamics, respectively.
facilitate the study of the frontal system and the related air-streams. Furthermore, the importance of non-modal initial conditions (Farrell 1982; Bishop 1989) and upper-level/ lower-level vortex interaction (Hoskins et al. 1985) for Petterssen type-B developments (Petterssen and Smebye 1971) is to some extent incorporated into the development.

An additional motivation for using finite-amplitude isolated initial conditions is to avoid the potentially large sensitivity of normal-mode experiments with respect to minor changes of the basic state (DSW). This sensitivity leads to one of two classes of nonlinear developments, which are favoured by either cyclonic or anticyclonic shear, respectively. This behaviour is particularly transparent within the semi-geostrophic dynamics, but has also been detected in primitive-equation numerical simulations on the sphere by Hoskins (1990).

The dynamical framework for the present study is the adiabatic semi-geostrophic dynamics of uniform potential vorticity, as developed by HW. It has recently been shown by Snyder et al. (1991) that primitive-equation numerical simulations depart rather significantly from the semi-geostrophic evolution of the HW case. These discrepancies relate to the fact that the semi-geostrophic dynamics are not a complete $O(Ro^2)$ approximation (McWilliams and Gent 1980). There is some reason to believe that the particular case chosen for comparison in the Snyder et al. study emphasizes these discrepancies. For the symmetric HW basic state considered, the leading terms of the north–south momentum flux disappear on the centre line of the baroclinic zone, leaving the momentum flux dominated by terms that are not amenable by semi-geostrophy (DSW; Nakamura 1991). Moreover, additional effects neglected in this study (e.g. $\beta$-plane or spherical geometry, moist dynamics, boundary-layer friction) are also likely to be associated with modifications comparable with that resulting from the stipulation of semi-geostrophy. A recent study of Whitaker and Snyder (1992), for instance, indicates that $f$-plane semi-geostrophic simulations compare in fact more favourably with the full dynamics on the sphere than primitive-equations $f$-plane simulations.

The outline for this paper is as follows. After the discussion of the dynamical system in section 2, the evolution of an isolated semi-geostrophic low-pressure system is portrayed in section 3. Some aspects of the dynamics at mid levels are discussed in section 4. In section 5 an objective scheme is developed and applied, which allows the identification of some basic air-streams and flow patterns within the developing cyclone. Section 6 contains an analysis of the frontal system, along with a discussion of the dynamical distinction between a cold and warm front.

2. DYNAMICAL SYSTEM

(a) Eulerian dynamics

The parent equations of the geostrophic momentum system can be cast in the following dimensionless forms

$$\frac{Dv}{Dt} - Ro^{-1} v = 0, \quad \frac{Du}{Dt} + Ro^{-1} u = 0$$  \hspace{1cm} (1a)

$$(v_g, -u_g, \theta) = \nabla \phi$$  \hspace{1cm} (1b)

$$\frac{D\theta}{Dt} = 0$$  \hspace{1cm} (1c)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$  \hspace{1cm} (1d)
Here the vertical coordinate $z$ is the pseudo-height of Hoskins and Bretherton (1972), $D/Dt = \partial/\partial t + u \partial/\partial x + v \partial/\partial y + w \partial/\partial z$ is the material derivative, the subscripts $a$ and $g$ refer, respectively, to ageostrophic and geostrophic components of the flow, and $Ro = U/(af)$, where $U$ is a characteristic speed, $a$ a characteristic length and $f$ the Coriolis parameter, denotes the Rossby number. The underlying dimensional scales are based upon the horizontal and vertical length-scales ($a$, $af/N$), and the corresponding velocity scales ($U$, $Uf/N$), where $N$ is a constant Brunt-Väisälä frequency. The appropriate scales for the geopotential and potential temperature are then given by

$$\phi = (Ua)^{-1}\hat{\phi} \text{ and } \theta = (UN)^{-1}\frac{g\hat{\theta}}{\Theta}$$

where we make use of the circumflex to denote dimensional variables. The semigeostrophic transformation of Hoskins (1975)

$$(X, Y, Z, T) = (x + Ro \nu_g, y - Ro \nu_g, z, t)$$

renders the horizontal advection purely geostrophic, i.e.

$$\frac{D}{DT} = \frac{\partial}{\partial T} + u^*_a \frac{\partial}{\partial X} + v^*_a \frac{\partial}{\partial Y} + w \frac{\partial}{\partial Z} = \frac{D_g}{DT} + w \frac{\partial}{\partial Z}. \quad (3)$$

In addition, we stipulate uniform potential vorticity

$$q_g = N^{-2}Ro^{-2} \frac{g}{f\Theta} \nabla \theta \cdot \hat{\xi}_g = Ro^{-2} \quad (4)$$

and will neglect, following HW, the nonlinear part of the potential vorticity relation. In this limit, the semi-geostrophic equations are fully isomorphic to the quasi-geostrophic equations (Hoskins and Draghici 1977) and take on the form

$$\frac{D_g u_g}{DT} - Ro^{-1} \nu_g^* = 0, \quad \frac{D_g v_g}{DT} + Ro^{-1} u_g^* = 0 \quad (5a)$$

$$(v_g, -u_g, \theta) = \nabla \Phi \quad (5b)$$

$$\frac{D_g \theta}{DT} + Ro^{-1} w^* = 0 \quad (5c)$$

$$\frac{\partial u_g^*}{\partial X} + \frac{\partial v_g^*}{\partial Y} + \frac{\partial w^*}{\partial Z} = 0 \quad (5d)$$

where

$$\Phi = \phi + \frac{1}{2}Ro (u_g^2 + v_g^2) \quad (6a)$$

and

$$u_a^* = u_a + Ro w\Phi_{XZ}, v_a^* = v_a + Ro w\Phi_{YZ}, w^* = w Ro^{-1} \xi_g. \quad (6b)$$

Here $\xi_g$ denotes the vertical component of the absolute geostrophic vorticity and $\nabla$ is the gradient operator in geostrophic space. Upon subtracting the Boussinesq background state defined by

$$\theta_o = Ro^{-1} Z, q_o = Ro^{-2},$$

the potential vorticity relation simplifies to
\[ q^*_{\xi} = \nabla^2 \Phi = 0. \]  
(7)

The numerical integration of this system employs both the thermodynamic equation (5c) and the hydrostatic relation (5b) on the horizontal bounding surfaces \( z = 0, z_T \), and the conservation of potential vorticity (7) in the interior of the flow. Some details of this procedure are described in HW and Schär and Davies (1990).

(b) Lagrangian dynamics

In addition to the Eulerian integration in geostrophic space an evaluation is also made of the Lagrangian time-history of the flow. To this end, the initial geostrophic space position is used as a convenient tracer. Here we use the function \( R_0(R, T) \) to denote the initial (i.e., \( T = 0 \)) geostrophic space position \( R_0 = (X_0, Y_0, Z_0) \) of an air parcel which is situated at \( R = (X, Y, Z) \) at time \( T \). The initial position is a materially conserved quantity, i.e.

\[ \frac{DX_0}{Dt} = 0 \]  
(8)

for any of the three components of \( R_0 \). Upon transforming this ‘conservation law’ to geostrophic space, we benefit from the simple form of the transformed material derivative (3) and obtain

\[ \frac{DX_0}{DT} = \left( \frac{D_k}{DT} + w \frac{\partial}{\partial Z} \right) X_0 = 0. \]  
(9)

In order to simplify the application of boundary conditions, the label-variable \( R_0 \) is split accordingly to

\[ R_0 = R + R'_0 = (X, Y, Z) + (X'_0, Y'_0, Z'_0) \]  
(10)

into the geostrophic space position \( R \) at time \( T \), and the backward-displacement \( R'_0 \) which occurred between times \( T = 0 \) and \( T \), respectively. With (9) one obtains

\[ \left( \frac{D_k}{DT} + w \frac{\partial}{\partial Z} \right) R'_0 = - (u_k, v_k, w). \]  
(11)

This equation is integrated simultaneously with the Eulerian part of the numerical model. The function \( R'_0(X, Y, Z, T) \) contains a great deal of information, most of which is not directly inferable from the invertibility principle. First of all, it represents the geostrophic space displacement of any air parcel between times \( 0 \) and \( T \). The vertical displacement directly transforms as \( z'_0 = Z'_0 \) into real space Secondly, knowledge of the function \( R_0 \) enables one to derive any trajectory of the flow evolution. The trajectory of an air parcel in geostrophic space, which is located at \( \hat{R}_0 \) at \( T = 0 \), is obtained by finding the root \([X(T), Y(T), Z(T)]\) of

\[ R_0(X, Y, Z, T) - \hat{R}_0 = 0 \]  
(12)

as a function of time. Alternatively, and somewhat more easily, one may solve the root of

\[ \{X_0(X, Y, Z, T), Y_0(X, Y, Z, T), \theta(X, Y, Z, T)\} - \{\hat{X}_0, \hat{Y}_0, \hat{\theta}_0\} = 0 \]  
(13)

where \( \hat{\theta}_0 \) denotes the potential temperature of the air parcel. This latter approach, i.e. solving (13), has been adopted for all the trajectories shown in this paper.

The numerical integration of the geostrophic space displacement \( R'_0 \) involves the following steps:
(i) The diagnosis of \( \xi_g = \partial v_g / \partial X - \partial u_g / \partial Y, u_g, v_g, \partial \theta / \partial X \) and \( \partial \theta / \partial Y \) on a regular three-dimensional grid from the invertibility relation.

(ii) The diagnosis of the geostrophic space vertical wind \( w^* \) from the thermodynamic equation (5b). To this end \( \partial \theta / \partial T \) is computed from the homogeneous Dirichlet problem \( \nabla^2 (\partial \theta / \partial T) = 0 \) with the boundary values on \( z = 0, z_T \) provided by the tendencies of the Eulerian part of the numerical integration.

(iii) The diagnosis of the real-space vertical wind \( w \) from

\[
w = Ro w^* \xi_g = w^* \xi_g (1 - Ro \xi_g)^{-1}
\]  

where \( \xi_g \) and \( \xi_g^* \) refer to the vertical component of the absolute real space and relative transformed space geostrophic vorticity, respectively.

(iv) Finally, \( R_0 \) is advanced by one time step according to (11).

All diagnostic steps are carried out spectrally, while the time step is accomplished with centred finite differences in time. This scheme was selected since it closely follows the spectral techniques as used for the integration of the Eulerian part of the numerical model, and hence assures a high degree of consistency between the various fields. The existence of a unique root to Eq. (12) is in fact a stringent test of the numerical implementation of the whole algorithm. The latter equation is solved using a Newton--Raphson iteration technique.

The scheme described above involves an approximation due to the simplified potential vorticity relation (7); it implies that the Eulerian and Lagrangian computations are not fully consistent. Some indication of the internal appropriateness of this assumption can, however, be provided by comparing trajectories based on (12) and (13), which are differently affected by the simplification of (7). Such comparisons have been carried out for a range of trajectories and show negligible disagreement over most of the integration time. Significant differences occur, however, close to the breakdown of the semi-geostrophic dynamics (i.e. \( t = 8 \) for the simulation to be portrayed in the next section) for some trajectories.

3. Numerical Simulation

Here we describe the flow development of one particular numerical simulation. Further diagnosis of this development will be discussed in sections 4, 5 and 6. In this section we portray the initial conditions, provide an overview of the simulation, coarsely describe the sensitivity of the evolving flow with respect to the initial conditions, and discuss some differences between normal mode and isolated cyclogenesis.

(a) Initial conditions and set-up of the numerical simulation

The initial conditions are selected to be a superposition of a two-dimensional baroclinic jet and both an isolated upper- and lower-level anomaly. These anomalies are not associated with internal potential vorticity but rather derive from thermal boundary anomalies. The two anomalies are characterized by positive relative vorticity and the upper-level anomaly is shifted towards the west, such as to allow for baroclinic development. The baroclinic jet in geostrophic space is the same as used by DSW. In dimensionless notation it is specified as

\[
\Phi(Y, Z) = \frac{1}{2} \left( \arctan \frac{Y}{1 + Z} - \arctan \frac{Y}{1 - Z} \right) - 0.12 YZ
\]  

\[
\tilde{\theta}(Y, Z) = -\frac{1}{2} \left( \frac{Y}{(1 + Z)^2 + Y^2} + \frac{Y}{(1 - Z)^2 + Y^2} \right) - 0.12 Y.
\]
The non-dimensional notation is defined by the scales as summarized in Table 2. The horizontal scale \( a = 2000 \text{ km} \) is directly related to the width of the baroclinic zone. The latter is characterized by a total temperature contrast of \(-18 \text{ K}\) and a total vertical shear of \(-30 \text{ m s}^{-1}\). The rigid lid is placed at the level of \( Z = Z_T = 0.45 \), corresponding to a dimensional height of 9 km.

<table>
<thead>
<tr>
<th>TABLE 2. SETTINGS FOR NON-DIMENSIONAL VARIABLES</th>
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<tr>
<td>Horizontal length-scale ( a )</td>
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<tr>
<td>Vertical length-scale ( a/U )</td>
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<tr>
<td>Horizontal-velocity scale ( U )</td>
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<tr>
<td>Vertical-velocity scale ( U/T )</td>
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<tr>
<td>Rossby Number ( U(a/U) )</td>
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<td>Time-scale ( a(U/a) )</td>
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<tr>
<td>Temperature scale ( U(T/\Theta) )</td>
</tr>
<tr>
<td>Scale of geopotential ( U(\Theta) )</td>
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</table>

The upper- and lower-level anomalies are prescribed in terms of the temperature deviation from the basic state on the bounding surfaces. The invertibility relation then defines the whole initial flow state. The potential temperature on the bounding surfaces is assigned the form

\[
\theta_{1,2} = \bar{\theta}(Y; Z = 0, Z_T) + A_{1,2}h(X - X^0_{1,2}, Y) \tag{16}
\]

where the subscripts 1 and 2 refer to the surface and the rigid-lid level, respectively. The anomaly-function \( h \) is given by

\[
h(X, Y) = \left\{ 1 + \left( \frac{X}{d} \right)^2 + \left( \frac{Y}{d} \right)^2 \right\}^{-3/2} - \frac{1}{2} \left[ 1 + \left( \frac{X - X_s}{d} \right)^2 + \left( \frac{Y}{d} \right)^2 \right]^{-3/2} \left[ 1 + \left( \frac{X + X_s}{d} \right)^2 + \left( \frac{Y}{d} \right)^2 \right]^{-3/2} \tag{17}
\]

i.e. it is the superposition of three circular anomalies—a central anomaly and two flanking anomalies of opposite sign and halved amplitude, which are shifted upstream and downstream by the distance \( X_s \), respectively. This particular form is chosen for two reasons. Firstly, anomalies of the form (17) do not affect the zonal basic state in geostrophic space, since \( h \) gives zero contribution to the integral \( \int h \, dX \). Secondly, the flanking anomalies effectively reduce the amplitude of the Fourier spectrum for wavelengths exceeding \(-2 X_s \), whose dynamics are poorly represented within the present f-plane geometry. The various parameters are set as

\[
d = 0.5, \quad X_s = 1, \quad A_1 = 0.15, \quad A_2 = -0.60, \quad X^0_1 = 0, \quad X^0_2 = -1. \tag{18}
\]

Note that the negative upper-level temperature anomaly is associated with a positive vorticity anomaly.

The resulting initial conditions are portrayed in Fig. 1. At upper levels they are characterized by the presence of a finite-amplitude trough-like feature. The weak perturbation of the zonal symmetry at low levels reflects the small amplitude of the low-level anomaly. The weak closed circulation at low levels is induced by the upper-level anomaly, rather than being the result of the low-level anomaly. The maximum relative vorticity occurs at the rigid lid and has a value of \(-1.9f\).

The horizontal scale of the numerical model domain in the zonal direction is chosen to be four times the wavelength of the most unstable normal mode of the baroclinic
jet (see DSW). The computational domain measures $7.32 \times 3.5$ dimensionless units, corresponding to $14640 \times 7000$ km. The numerical resolution in geostrophic space is $128 \times 64$ spectral components and the time step is 0.0125 (i.e. 522 s). The vertical grid spacing for the Lagrangian computations is $\Delta Z = 0.025$. The lateral boundary conditions are ‘periodic’ in the $X$-direction and ‘periodic continuation’ in the $Y$-direction. These boundary conditions translate into fully periodic perturbation fields, where the perturbation is defined as the deviation from the zonal basic state (15a). Simulations with larger domains result in the same evolution of the flow.

(b) Flow evolution

Figure 2 summarizes the resulting cyclogenesis in terms of the surface pressure minimum and the geostrophic space vorticity maximum. The additional line represents the growth to be expected from the linear evolution of the most unstable normal mode. It can be seen that both curves do not match the normal-mode growth—the vorticity grows faster and the pressure drop occurs slower. The dimensionless growth rate of the most unstable normal mode is 0.26 (from DSW), while the mean growth rate between
Figure 2. Trace of the surface pressure minimum and the geostrophic vorticity maximum, i.e. $Ro_{\text{ge}}$, on the surface level. The vertical axis is in logarithmic units. Normal-mode growth is also shown.

$t = 0$ and $t = 5$ as derived from these curves corresponds to 0.35 and 0.15 for vorticity and pressure, respectively. The evolution hence shows significant non-modal contributions.

Figures 3–5 portray the flow fields at day 2 and day 4 on the surface and the rigid-lid level. At upper levels there is distortion of the initial symmetry of the trough into a trough/ridge feature. Maximum velocities are on the eastern side of the trough. At the surface there is both warm and cold frontogenesis. At $t = 8$ (day 4), the regions of strong thermal gradients and vorticity along the cold front and the warm front (including its bent-back portion) are clearly separated from one another. The air masses behind the cold front do not move towards (as depicted by the polar front theory), but rather parallel to, the warm front. The cold front is associated with a length-scale of roughly $1.5X$ (3000 km), and the strength of the thermal gradient varies only slightly over this distance. The strength of the warm front on the other hand shows significant variability with distance along the front. Ahead of the cyclone it is rather weak, both in terms of thermal gradient and vorticity signature, while its amplitude increases towards the low-pressure centre and reaches maximum amplitude in the bent-back portion of the front immediately to the north-west of the low. At upper levels the warm front exhibits an approximately two-dimensional structure and is associated with a long and narrow band of high values of vorticity. In the terminology of HW this type of upper-level warm front is classified as warm front type-B, and it is this phenomenon which was portrayed in their inverted mode case.

At $t = 8$ an anticyclonic development is evident to the north of the trailing cold front. The deformation field established by this anticyclone and the parent low causes some amplification of the thermal gradient immediately poleward of the high pressure centre, which is responsible for the vorticity feature to the rear of the cold front as apparent in Fig. 5(b). This phenomenon, which appears to be an upstream development in the terminology of Simmons and Hoskins (1979), occurs in a region which is well known from observational studies (e.g. Reed 1979) to be a preferred location for the formation of polar lows and comma-cloud features. In the present simulation the growth of this feature is rather modest. It is, however, conceivable that it could undergo additional amplification if coupling to a pre-existing upper-level anomaly would occur. Such a mechanism for the formation of polar lows has been suggested by Montgomery and Farrell (1992).
(c) Sensitivity with respect to the initial conditions

One disadvantage of adopting finite-amplitude initial conditions (as compared with normal-mode initial conditions) is the potentially huge parameter-space which has to be investigated. For the present class of initial conditions we have carried out a set of experiments to study the sensitivity of the evolution with respect to the initial strength of the anomalies and their initial east-west distance. Some of these experiments are documented in Wernli (1989). For the amplitudes given by (18) the sensitivity to the initial westward tilt is rather weak. This is linked in part to the dominant amplitude of the upper-level anomaly (a factor 4 larger than the low-level anomaly). The low-level anomaly has a comparatively minor, accelerating, effect.

If the two anomalies are of comparable strength, our analysis is in agreement with the study of Warrenfeltz and Elsberry (1989). They carried out a systematic investigation of the response to upper- and lower-level finite-amplitude anomalies which were superposed on a baroclinic jet. They found that maximum growth occurs if the upper level anomaly is shifted towards the west by approximately a quarter of the wavelength of the
most unstable normal mode. This tilt is stronger than that of the normal mode, confirming some of the conclusions of Farrell (1982). If the initial east–west distance is increased beyond a critical value of $X_i^2 - X_s^2 > \sim 1.4$ for our setting, then both the anomalies grow their own cyclone. Such an example is shown in Fig. 6.

The horizontal scale of the evolving cyclone is directly related to the parameter $X_s$ in (17). In our case the cyclone in Fig. 4 is characterized by a horizontal scale which exceeds that of the most unstable normal mode by about 50%. This comparatively large scale arises primarily from long-wave components in the initial conditions. It was verified that lower values of $X_s$ result in smaller-scale cyclogenesis, without affecting the structure of the cyclone itself.

While the growth rate and the horizontal scale show some sensitivity with respect to the initial conditions, the structure of the frontal system does not appear to do so. Provided the anomalies are associated with positive vorticity and result in the formation of a single cyclone, the resulting cyclogenesis is always associated with a frontal system that shows a similar configuration as that discussed for Figs. 3–5.
\( \text{VÖRTICITY} \)
\( T = 8.0 \quad Z = 0.450 \)

(a)

\( \text{VÖRTICITY} \)
\( T = 8.0 \quad Z = 0.000 \)

(b)

Figure 5. Distribution of relative vorticity at \( t = 8 \) (i.e. day 4) on (a) the rigid lid and (b) the surface level. Isolines are given in units of \( f \) for \(-0.4, -0.3, -0.2, -0.1 \) (dashed lines) and \( 0.1, 0.2, 0.5, 1.0, 2.0, 5.0 \) (full lines).

(d) **Comparison with normal-mode evolution**

It is useful to compare our isolated cyclogenesis simulation with the development of a normal mode. A comparison is carried out with the 'cyclonic shear case' of DSW, since our initial conditions entail significant cyclonic vorticity superposed upon the symmetric baroclinic jet. For reference, results in terms of relative vorticity from the latter normal-mode experiment are displayed in Fig. 7. Several of the key features—i.e. the characteristic arrangement of the frontal system, the signs of upper-level cyclonic wrap-up, and the elongated upper-level warm front ahead of the low—can be identified both in the isolated (Fig. 5) and normal-mode (Fig. 7) numerical experiment. This correspondence confirms the value of normal-mode experiments in studying cyclogenesis. Nevertheless, there are some noteworthy differences between the simulations which are discussed below.

Within a periodic train of cyclones there is significant interaction between neighbouring frontal systems. The trailing cold front (TCF in Fig. 7(b)) penetrates into the warm sector of the neighbouring low-pressure system to the west, and finally coalesces
with the cold front of the latter system. This process—sometimes referred to as occlusion—cannot take place in an isolated cyclogenesis event. In the case of Fig. 7, the warm front (denoted WF) is associated with a weak signature in the low-level vorticity field but is easily detectable in the thermal field and at upper levels. There is some disagreement in the literature concerning the future of the warm front at this stage. Sometimes it becomes weak or merges with the cold front (e.g. Takayabu 1986; Polavarapu and Peltier 1990) while it can also remain a prominent feature over a considerable time (e.g. Thornicroft and Hoskins 1990, see their Fig. 4(b)). This inconclusive situation has led to a confusing terminology with respect to the warm front. In some studies the trailing cold front is referred to as 'warm front' (e.g. Takayabu 1986; Polavarapu and Peltier 1990), while our warm front WF is referred to as 'Front F' (Takayabu 1986), or 'leading warm front' (Polavarapu and Peltier 1990), or 'warm front' (HW; Thornicroft and Hoskins 1990).
The most pronounced structural differences between the cyclones shown in Fig. 5 and Fig. 7 appear, however, at upper levels. In the periodic simulation, the upper-level warm front is often directly connected to the vorticity features which take part in the cyclonic wrap-up over the neighbouring system to the east (cf. Fig. 7(a), or Fig. 5(b) of Thornicroft and Hoskins 1990). Such an effect is not possible in an isolated cyclogenesis event, and the upper-level warm front extends far away to the east from the cyclone (see Fig. 4(a) and Fig. 5(a)).

The aforementioned interaction effects between neighbouring cyclones are excluded from our numerical simulation of an isolated cyclogenesis event. Some of the uncertainties concerning existence and structure of the warm front are removed. The warm front is easily detectable in the thermal field of Fig. 4(b) and in the frontogenesis function to be displayed later in Fig. 13(c). These properties of isolated cyclogenesis will significantly facilitate our later analysis. For instance, it is not possible in a normal-mode development to relate clearly the air-streams to be discussed in section 5 with individual frontal structures.

4. Mid-level structure

(a) Vertical displacement and relation to cloud formation

Semi-geostrophic dynamics in the limit of uniform potential vorticity yields only weak fronts near the steering level (e.g. Hoskins 1971). This behaviour is related to adiabatic heating (cooling) in the descending (ascending) portions of the flow, which are out of phase with the temperature field. Strong fronts in terms of thermal gradients or high values of vorticity can only develop along the horizontal bounding surfaces where the compensating effects of the vertical motion are inhibited. Figures 8(a) and (b) show both the potential temperature and vorticity fields at the mid level of the model atmosphere. These fields possess a smooth structure with no indication of a frontal surface. On the other hand, Fig. 8(c) depicts the vertical motion at the same level, and this field is characterized by an up–down dipole arranged from north to south and some rather weak extensions which mark the fronts. These patterns agree well qualitatively with the analysis of Keyser et al. (1989). The maximum value of the vertical wind is 0.062, corresponding to 3 cm s\(^{-1}\). This value is typical for dry semi-geostrophic dynamics, and it contrasts with the high values as reported from numerical studies of moist explosive cyclogenesis (e.g. Kuo and Reed 1988).

Figure 9(a) shows the vertical displacement between time \(t = 0\) and \(t = 8\). This diagram can be used to derive the initial vertical position for any air parcel which is located at mid level at \(t = 8\). The contours show the ascent (full contours) or descent (dashed contours) experienced since \(t = 0\). A \(\lambda\)-shaped pattern of ascent is apparent within which the warm front and its bent-back portion constitute the most prominent feature. It is characterized by a band of vertical displacement, which extends from far ahead of the cyclone to the north of the surface low, and bends back cyclonically around the low. Given the comparatively small values of the vertical velocity, the warm-frontal displacement band must be related to highly coherent ascent, with the ascending air parcels residing for a considerable time within the ascending branch of the warm front. The gradient of the vertical displacement along this band is indicative of a three-dimensional structure of the warm front. The cold front on the other hand shows a rather two-dimensional structure. The vertical-displacement values reached in this band are comparatively small.

Further features to note are the centre of negative vertical displacement immediately to the south of the surface low, which can be related to the dry intrusion as often evident
Figure 8. The fields of (a) potential temperature and geopotential (b) relative vorticity and (c) vertical wind at $t = 8$ (i.e., day 4) and on the mid level $z = 0.225$ of the model atmosphere. Conventions for panels (a) and (b) are as in Fig. 3 and Fig. 5, respectively. The contour interval in (c) is 0.005 corresponding to 0.24 cm s$^{-1}$. The zero contour is suppressed.
Figure 9. The fields of (a) vertical displacement (since $t=0$) and initial position in both (b) meridional and (c) longitudinal direction at $t=8$ (i.e. day 4). Contour intervals are 0.01 ($\sim 200$ m), 0.25 ($\sim 500$ km) and 0.5 ($\sim 1000$ km), respectively.
on infrared satellite pictures (e.g. Young et al. 1987), and the additional blob of positive displacement to the rear of the cold front, which is related to the secondary development briefly mentioned in section 3(b). The maximum vertical displacement associated with the latter feature exceeds the values as obtained in the cold front.

Overall, the field of vertical displacement appears to be consistent with some of the typical cloud formations as often observed in extratropical cyclones (see e.g. Wallace and Hobbs 1977; Browning 1990). Beside the striking $\lambda$-shaped pattern, the ascent in the warm-frontal band occurs in regions of small vorticity and moderate stability, while the low-level ascent along the cold front is located in regions of strong vorticity and weak stability. These features can be related to the commonly observed tendency of these two bands to favour stratiform and convective cloud formation, respectively.

(b) Formation of mid-level frontal characteristics in fields of passive tracers

Next we discuss the horizontal distortion experienced by the flow media. Figures 9(b) and 9(c) present the horizontal displacement in terms of the initial position in meridional and longitudinal direction. These displays are somewhat unusual, but contain interesting information. For illustration, consider Fig. 9(b), which shows contours of the initial latitude of the air parcels which are located at mid level. At $t = 0$ these contours were all aligned in an east–west direction, and the distortion evident at $t = 8$ is the result of air parcels being displaced towards north or south. The initial-position field in the longitudinal direction is shown in Fig. 9(c). Upstream of the low the deformation of the longitudinal lines are characterized by the accumulated effects of the westerly jet, which advances lines of constant initial position towards the east along the jet axis. The resulting signature (see the stippled areas in Fig. 9(c)) is strongly distorted in the vicinity of the low. To the north of the jet axis portions of the upstream ‘$>$’-shaped pattern are both slowed down and accelerated in the direction of the jet.

The ‘marble-cake’ patterns in Figs. 9(b) and (c) are indicative of the severe distortion of the flow media. Interestingly, the initial-position fields both clearly mark the frontal system. In essence, if the frontal surface is defined as ‘a surface separating air parcels of different origin’, then the cold front, and to some lesser degree the warm front, are well defined throughout the model atmosphere. Recall in this respect the inability of the thermal and vorticity patterns to locate the frontal surface at mid levels (cf. Figs. 8(a) and (b)). This property of the initial-position field has various implications. Consider for instance the presence of a field of passive tracers, e.g. aerosol or inert chemical constituents, with an initially smooth but inhomogeneous distribution. Cyclogenesis forces strong gradients of those fields to develop along the frontal surfaces. Even for atmospheric constituents which do not qualify as passive tracers, as for example the moisture field, these characteristics have some relevant implications. The deformation and relocation due to the purely dry dynamics themselves are sufficient to explain the formation of ‘moisture fronts’ from smooth initial fields. Furthermore, wet dynamics are likely to transform—via the thermodynamic coupling of the moisture variable—such a moisture contrast into horizontal thermal gradients. This issue merits further consideration in relation to studies on the vertical structure of cold fronts (e.g. Browning and Monk 1982; Hobbs et al. 1990).

It is worth noting that the dynamical origin of the demonstrated formation of gradients in the initial-position fields does not appear to lie within two-dimensional deformation frontogenesis. A study of this latter type, which is very suited for comparison with the present results, is that of Garner (1989). It is apparent from his Lagrangian numerical simulations that the initial-position coordinate perpendicular to the front does experience only a little extra squeezing within the frontal zone (see, for example, his
Fig. 8(b)). The dynamical reason for the demonstrated effects must hence lie beyond Garner’s framework, and is likely to be related to fully three-dimensional effects. The deformation fields as spanned by our developing system have finite length-scales, and the cold frontogenesis does not classify as pure deformation frontogenesis.

It is noted that a profound comparison with observational studies is difficult for various reasons, chiefly because atmospheric fronts are usually characterized by tropopause folds (Shapiro and Keyser 1990). The resulting distribution of potential vorticity and the descent of stratospheric air will often affect the frontal structure throughout the atmosphere. Nevertheless, the demonstration that cyclogenesis induces relocations and strong deformation of mid-level air in the absence of those processes is of some interest. A related mechanism for the formation of sharp edges of cirrus cloud shields has been discussed by Durran and Weber (1988). They demonstrate that the associated moisture discontinuities are the result of confluent dry and moist air-streams.

5. OBJECTIVE DEFINITION OF AIR-STREAMS

(a) Approach

The objective scheme for identifying air-streams and flow patterns is based on the knowledge of the initial position for any air parcel within the flow, and for several output times of the numerical simulation. The scheme essentially uses the initial position as an artificial tracer, a substitute for natural tracer-like constituents (such as moisture) which are often used in observational studies of atmospheric air-streams.

The knowledge of the initial position allows the computation of quantities such as the vertical displacement. These three-dimensional fields (which were discussed in section 4) are next utilized objectively to select an ensemble of trajectories. This technique is described here for the air-stream shown in Fig. 10(a). It involves two steps:

(i) Firstly, a set of air parcels is selected at $t = 8$ (i.e. day 4) of the numerical simulation by requiring that the vertical displacement since $t = 0$ exceeds some critical threshold value for all the parcels within that volume. In practice, the sampling is done on a regular geostrophic grid. The horizontal resolution is $\Delta X = 0.114$ and $\Delta Y = 0.11$, and in the vertical 19 levels with $\Delta Z = 0.025$ are considered.

(ii) Secondly, associated backward (and forward) trajectories are derived following the methods described in section 2(b).

The results are displayed in Fig. 10(a). The panel to the left shows the whole ensemble of selected trajectories, while the panel to the right displays one typical trajectory along with additional information on the vertical location. In order to elucidate the motion relative to the developing low, the diagrams also contain isolines of surface vorticity at three times.

(b) Results

The selection criterion ‘maximum vertical displacement between $t = 0$ and $t = 8$’ results in a well defined air-stream. Relative to the developing cyclone, the stream originates ahead of the initial low-level vorticity anomaly, rises within the warm-frontal region and to the north of the low, and finally turns cyclonically around the upper-level portion of the low (see Fig. 10(a)). The maximum vertical displacement during this time is $\Delta z/z_T \sim 0.4$ corresponding to a dimensional displacement of 3.6 km. Most of this ascent occurs in the bent-back portion of the warm-frontal baroclinic zone.
Figure 10. Air-streams between \( t = 0 \) and \( t = 8 \) (i.e. day 4) based upon the following selection criteria: (a) maximum ascent between \( t = 0 \) and \( t = 8 \), (b) maximum vertical wind at \( t = 8 \), (c) maximum descent between \( t = 0 \) and \( t = 8 \), (d) maximum poleward displacement between \( t = 0 \) and \( t = 8 \), and (e) maximum westward displacement between \( t = 0 \) and \( t = 8 \). The panels to the left each show the whole ensemble of trajectories, while the panels to the right display one typical trajectory of the air-stream including information on the vertical position. The vertical coordinate is here scaled from 0 (surface level) to 1 (rigid-lid level). The small crosses denote the position of the air parcels at \( t = 0, 1, \ldots 8 \). The dashed contours signify the surface vorticity at day 0, 4 and 8, at a level of \( 0.1f \), \( 0.2f \) and \( 0.4f \), respectively.
Next we select all the air parcels with a large vertical velocity at \( t = 8 \). The resulting ensemble of trajectories is shown in Fig. 10(b). Unlike the previous ensemble, the present set of trajectories does not possess the characteristics of an air-stream but rather encompasses trajectories whose origins are scattered over a large area. Most of the trajectories originate from the warm sector immediately to the south of the maximum baroclinicity. However, some air parcels originate from the west of the initial low-level vorticity anomaly at a level of \( z/z_\gamma \sim 0.6 \). They first experience some descent, and then reach the shafts of ascent, where they stay only for 1 or 2 dimensionless time units. The differences between Figs. 10(a) and 10(b) imply that various air parcels with a significant spread in initial position, initial height and potential temperature are subject to strong vertical winds at some point in the development. Only the parcels that stay in the ascending shafts of vertical motion for considerable time make up the coherent air-stream in Fig. 10(a).

The trajectories based upon the selection criterion of 'maximum descent between \( t = 0 \) and \( t = 8 \)' are shown in Fig. 10(c). They originate from the north-west of the initial low-level anomaly at levels \( z/z_\gamma \sim 0.7 \), descend to the rear of the cold front in an initially coherent air-stream, and finally spread out at levels around \( z/z_\gamma \sim 0.35 \). Maximum descent amounts to \( \Delta z/z_\gamma \sim -0.4 \) and the relevant trajectories end in the dry slot immediately to the south of the surface low, which was already mentioned in relation to the vertical displacement field shown in Fig. 9(a).
The ensemble of trajectories shown in Fig. 10(d) is based on the selection criterion 'maximum poleward displacement between \( t = 0 \) and \( t = 8' \). These trajectories originate to the south of the initial low and ascend from levels between \( z/z_T \sim 0.4 \) and 1.0 by an amount which varies between 0.2 (lower-level trajectories) and 0 (rigid-lid trajectories). The parcels travel polewards immediately ahead of the cold-frontal surface (i.e. roughly above the surface cold front) and reach the northern tip of the low at \( t = 8 \). After reaching this point most of the parcels then execute an anticyclonic turn away from the low and later merge with the upper-level westerly flow (not shown).

Finally we show in Fig. 10(e) the trajectories characterized by maximum westward displacement. The resulting air-stream is entirely located on the surface level and originates to the far east of the initial depression. The air parcels are then advected to the north of the warm front towards the centre of the low, and finally circle the low in a cyclonic turn.

The presented technique appears to be rather insensitive with respect to the details of the selection criteria. For instance, the width of the resulting air-stream is dependent upon the exact value of the threshold displacement which is used to select the ensemble of trajectories. However, it is found, for all the air-streams discussed in this article, that this choice does not affect the structure of the stream itself. For further illustration of this insensitivity, we show an additional stream in Fig. 11(a) which is based on the selection criterion 'maximum vertical ascent between \( t = 0 \) and \( t = 6' \). This criterion differs from the one used in Fig. 10(a) only with respect to the length of the time window. The resulting air-streams for these two selection criteria are constituted by entirely different air parcels, but the qualitative characteristics of the two air-streams are the same (compare Fig. 10(a) with Fig. 11(a)). We take the opportunity of the modified selection criterion to include forward trajectories until \( t = 8 \) into the display (Fig. 11(b)). After the air parcels have completed their phase of maximum ascent at \( t = 6 \), they spread out in a fan-shaped pattern (Fig. 11(b)). Some trajectories execute an anticyclonic turn and later merge with the upper-level westerly flow, while others cyclonically circle the upper-level low-pressure system.

While the selection criterion 'vertical displacement' can be regarded as fully objective, the selection criteria based on longitudinal and meridional displacement are arbitrary to some extent. It would be possible to select alternative ensembles of trajectories by choosing the selection criterion 'maximum horizontal displacement' in any other direc-

![Figure 11. Air-stream based upon the selection criterion 'maximum ascent between \( t = 0 \) and \( t = 6' \) (i.e. day 3). The panels show the trajectories for (a) \( 0 < t < 6 \) and (b) \( 0 < t < 8 \). Dashed contours are as Fig. 10.](image-url)
tion. This feature reduces the objectiveness of the air-streams shown in Figs. 10(d) and (e) for the present simulation.

(c) Discussion

The dynamical framework selected for the present study is at best a highly simplified model of what really happens in the atmosphere, and any comparison with observed cyclogenesis is hence not fully conclusive. These difficulties are particularly apparent at upper levels, where the rigid-lid assumption completely prohibits any phenomena associated with tropopause folding and stratospheric intrusions, whose observational key features are now well established (see the reviews of Shapiro and Keyser (1990) and Reed (1950)), and whose dynamical effectiveness is being recognized (see, for example, Hoskins et al. 1985). Despite these limitations, it is interesting to address the question as to which key features of observed cyclogenesis emerge in the present framework.

Several of the air-streams isolated in this section show satisfactory qualitative resemblance to observational concepts. Firstly, one of the central questions which was focused upon in some recent observational and real-case numerical studies addressed the location and nature of the ascending air. Within the present configuration at least, maximum ascent does not occur ahead of the cold front. Rather it is associated with ascent within the warm-frontal region and its bent-back portion (see Fig. 10(a)). This result is consistent with several recent studies (Kuo and Reed 1988; Kuo et al. 1992) which are based on real-case numerical simulations. This correspondence is remarkable, since these studies addressed an explosive development and showed the warm-frontal ascent to be significantly affected by moist symmetric instability. However, the ascending air parcels form a coherent stream and do not spread out, at least until they have essentially completed their phase of rapid ascent (see Fig. 11). In this respect they show some of the characteristics of the conveyor belts as described by Carlson (1980) and Browning and Mason (1981).

Maximum meridional displacement occurs at mid and upper levels. The trajectories associated with maximum poleward displacement (Fig. 10(d)) are located ahead of the cold front and are associated with some ascent. They show some of the characteristics of the warm conveyor belt. Observed warm conveyor belts (see the review of Browning (1990)) usually, however, originate from lower levels and are characterized by more significant lifting. These discrepancies between observed characteristics and the stream as seen in the present simulation are not surprising. There is both theoretical and observational evidence that moist dynamics are important for the development of the warm conveyor belt. Emanuel et al. (1987) have shown that the two-dimensional Eady model exhibits a considerable intensification of the poleward low-level flow ahead of the cold front if the moist dynamics are assumed to create a neutral environment with respect to symmetric instability. There is also some indication that low-level moist air ahead of the cold front can be subject to very significant ascent in cases of cyclogenesis along the east coast of the United States (Whitaker et al. 1988). It is also possible that the maximum northward trajectories (Fig. 10(d)) and the maximum ascent trajectories (Fig. 10(a)) more nearly coincide when moist dynamics are considered.

The stream defined by maximum descent (Fig. 10(c)) is in good qualitative agreement with the dry descending air which was isolated in numerous observational studies (Young et al. 1987). The stream is located to the rear of the cold front and shows spreading motion close to the ground. Maximum descent occurs to the immediate south of the low and this is consistent with the notion of the ‘dry slot’ as it emerged from studies of water vapour and infrared satellite images. Observational studies (e.g. Smigielski and Ellrod 1985; Young et al. 1987) have often referred to the dry intrusion as a precursor of the
rapid development phase. This phase begins when air with high potential vorticity descends from upper levels and is advected over warm moist surface air. Such a process is a priori suppressed in our simulation by the stipulation of uniform potential vorticity. On the other hand, the current results demonstrate that the descending air-stream can also be viewed as a natural side-effect of cyclogenesis. Both the characteristics of cause and effect of cyclogenesis are hence attributable to the dry intrusion, and these two aspects constitute the elements of a potentially positive feedback mechanism.

6. Analysis of Frontogenesis

(a) Cross-sections

Figure 12 shows a selection of cross-sections whose locations are indicated in Fig. 4(b). The first section in Fig. 12(a) cuts the cold front at a location close to maximum strength. The resulting fields show a frontal structure which is similar to what is obtained from two-dimensional deformation frontogenesis (Hoskins 1971; Williams 1972). At the time displayed ($t = 8$), the frontal collapse is still at an early stage. The maximum vorticity of the surface front amounts to roughly 1 $f$.

A section across the warm front ahead of the cyclone is shown in Fig. 12(b). Maximum strength in terms of the vorticity field occurs at the rigid lid, while the vorticity along the surface front is marginal only. Similarly, the maximum gradient in the vertical-displacement field (lowermost panel) occurs at upper levels. The dynamical generation of this structure is affected by the vertical shear in the along-frontal direction (cf. Fig. 4). At low level there is southerly flow, which induces a relative component towards the cyclone, while at upper levels the flow has a westerly component. Within the warm-frontal upper-level vorticity feature there is advection away from the cyclone.

Finally, Fig. 12(c) shows a section which cuts the centre of the cyclone. The thermal and vorticity fields both portray the bent-back portion of the warm front, which is characterized by a strong thermal gradient and by high values of the relative vorticity on the surface level.

The lower panels in Fig. 12 display the vertical wind and the vertical displacement as obtained from the Lagrangian computations, respectively. Fig. 12(a) shows that the extremes of the vertical displacement correspond to a maximum lifting of 0.04 ($\sim 0.8$ km) and to a maximum descent of 0.14 ($\sim 2.8$ km). The descent on the cold side of the cold front is much more significant than the ascent on the warm side. The opposite is true for the warm front along the section in Fig. 12(b). Here the ascent overpowers the descent by roughly a factor of two. These differences are not explained by the instantaneous structure of the vertical-wind field in the corresponding sections; rather they are associated with the amount of coherence in the shafts of ascent and descent, respectively. The interpretation is that the descending 'dry' air-stream as portrayed in Fig. 10(c) amplifies the net negative displacement in the vicinity of the cold front. On the other hand, the ascending air-streams from Figs. 9(a) and 9(d), enhance the net positive displacement in the vicinity of the warm front.

Both the zero-line of the vertical wind and the vertical-displacement fields for the cold- and warm-frontal sections AA' and BB', respectively, give some impression of a frontal surface. In both cases the two zero-lines roughly agree with one another. The situation along section CC' is rather different. The area with ascent takes on a V-shaped pattern centred over the low-pressure centre. On the other hand, significant vertical displacement is present in a band which is tilted roughly with the bent-back portion of the warm front. In essence there is an ascending volume of air which originates initially from higher levels. The corresponding air parcels were subject to descent within the dry
Figure 12. Cross-section at $t = 8$ (i.e. day 4) across (a) the cold front, (b) the warm front, and (c) the centre of the low. The locations of the three sections are indicated on Fig. 4(b) by the lines AA', BB' and CC', respectively. The uppermost panels each show the distribution of potential temperature (bold lines) and relative vorticity (thin lines). The isolines of the relative vorticity follow the conventions of Fig. 5. The centre and lowermost panels show isolines of vertical wind and vertical displacement, respectively. Positive and negative isolines are identified by full and dashed lines, and the contour increments amount to 0.005 and 0.01, respectively (as in Figs. 8(c) and 9(a)).
descending air-stream, reached the warm side of the bent-back warm front at low levels, and finally got involved in the ascent over the low-pressure centre. Trajectories with these characteristics were already detected in Fig. 10(b), and their origin can be traced back with that diagram to the north-west of the initial low-level disturbance. Overlapping areas of negative vertical displacement and positive vertical motion (and vice versa) are also apparent from the mid-level diagrams in Figs. 8(c) and 9(a). At mid level, the zero-line of vertical displacement to the north of the ‘dry slot’ is roughly located over the surface portion of the bent-back warm front. These features are significant from an observational point of view, since it is the vertical-displacement field which—in the present simplified view—will take responsibility for the formation of cloud boundaries. This interpretation appears to be consistent with the real-case numerical simulations of Kuo et al. (1992). They show in a rather similar manner how the dry descending air-stream produces the dry slot commonly seen in satellite imagery over and ahead (i.e. on the cold side) of the surface occlusion.

The last feature discussed is of some relevance for the parametrization of moist dynamics in idealized theoretical models. It has become common practice in moist semi-geostrophic dynamics (e.g. Emanuel et al. 1987; Montgomery and Farrell 1992) to assume that all ascending air parcels are saturated. It can be seen from Figs. 12(a) and (b) that this assumption is presumably justified for the consideration of warm and cold fronts. However, the peculiar behaviour of the vertical motion versus displacement field in Fig. 12(c) indicates that dry air parcels (i.e. which have experienced significant descent) can take part in the ascent within the central region of the low.

(b) Analysis of surface frontogenesis

In order to keep the subsequent analysis of surface frontogenesis as simple as possible, it is entirely kept in geostrophic space. In essence this allows for an analysis within the quasi-geostrophic set of equations. To set the scene, Fig. 13(a) shows the surface fields of pressure and potential temperature in geostrophic space. Upon comparison with Fig. 4(b), it can be seen that the centre of the low and the frontal regions are expanded. An attractive view of this is provided by Fig. 13(b), which shows the geostrophic space as viewed from physical space. The coordinate lines are densely packed along both the cold front and the bent-back portion of the warm front, showing that the geostrophic space (or computational) resolution in these regions is enhanced. These space deformations describe the accumulated effects of ageostrophic advection, which are implicitly taken care of by the transformation from geostrophic back to physical space.

The prime surface characteristics of fronts are traditionally taken to be the observable signatures of a significant thermal contrast, and the existence of substantial low-level convergence, shear and ascent. These key dynamical characteristics (i.e. \( \nabla \theta, \xi \) and \( w \)) are closely interrelated through the following relationships, valid in geostrophic space:

\[
\frac{D^2 \theta}{DT^2} = Q \quad \text{(for } z = 0) \tag{20}
\]

\[
\frac{D \xi}{DT} = \frac{1}{Ro^{-1}} \frac{\partial w^*}{\partial Z} \tag{21}
\]

\[
\nabla^2 w^* = 2 \nabla \cdot Q. \tag{22}
\]

Here, \( Q \) denotes the geostrophic space \( Q \)-vector (Hoskins et al. 1978; Hoskins and Draghici 1977) i.e.
Figure 13. (a) Surface pressure and temperature fields in geostrophic space with the conventions of Fig. 4(b). (b) Geostrophic space as viewed from real space. The additional panels display: (c) the frontogenetic function as defined by Eq. (24), (d) the $Q$-vector and (e) the deformation, all in transformed space; in (f) a typical real-space surface-level trajectory is shown for both the cold- and the warm-frontal region.
\[ Q = -\left( \frac{\partial u_x}{\partial X} \theta_X + \frac{\partial v_x}{\partial X} \theta_Y, \frac{\partial u_y}{\partial Y} \theta_X + \frac{\partial v_y}{\partial Y} \theta_Y \right) \] (23)

and \( \zeta_{gy} \) is the relative geostrophic vorticity in transformed space (corresponding to the vorticity in a quasi-geostrophic framework).

A convenient overall measure of frontogenesis and frontolysis is Miller’s frontogenesis function (see, for example, Keyser et al. 1988)

\[ \frac{D_g(\nabla \theta)^2}{DT} = 2 Q \cdot \nabla \theta \quad \text{(for } z = 0) \] (24)

and this quantity is displayed in Fig. 13(c). The frontogenesis function appears to allow for an excellent and objective definition of the frontal system. In contrast, the practice of diagnosing the cold front, for instance, as the leading edge of the cold advection is not fully consistent for non-frictional systems (such as ours), since it results in fronts which are not Galilei-invariant. In Fig. 13(c) both the cold- and the warm-frontal band of frontogenesis (including the bent-back portion of the warm front) are apparent. These two bands are not connected to one another, rather the cold front is located to the south-east of the bent-back warm front. This is consistent with the notion of a ‘frontal fracture’.

It can also be seen from the diagram that both the frontal features are linked to a region of frontolysis (dashed isolines) to the west. These regions of frontolysis are purely dynamical, i.e. the local deformation field (to be defined more precisely below) and the thermal gradient aligned such as to be conducive for frontolysis. For later reference, the \( Q \)-vector is displayed in Fig. 13(d). This field compares favourably with the recent quasi-geostrophic analysis of a primitive-equation simulation by Keyser et al. (1992).

The \( Q \)-vector can be interpreted as locally linear mapping from \( \nabla \theta \) to \( D_g(\nabla \theta)/DT \), i.e.

\[ \frac{D_g \nabla \theta}{DT} = Q = Q \nabla \theta \quad \text{(for } z = 0) \] (25)

where the matrix \( Q \) is only dependent upon the local kinematic properties of the flow field. The nature of frontotyical and frontogenetical effects can hence be analysed by studying the properties of the matrix \( Q \). For this purpose, it is convenient to split the latter as

\[ Q = -\frac{\mathcal{B}_g}{2} \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) + \frac{\zeta_{gy}}{2} \left( \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right) - \frac{1}{2} \left( \alpha_2 - \alpha_1 \right) = Q_{\text{div}} + Q_{\text{rot}} + Q_{\text{def}} \] (26)

where \( \alpha_1 = \partial u_g/\partial Y + \partial v_g/\partial X \) and \( \alpha_2 = \partial u_g/\partial X - \partial v_g/\partial Y \) denote the deformation components, \( \zeta_{gy} = \partial v_g/\partial X - \partial u_g/\partial Y \) is the geostrophic space vorticity, and \( \mathcal{B}_g = \partial u_g/\partial X + \partial v_g/\partial Y \) the geostrophic space divergence. For the present framework, we have \( \mathcal{B}_g = 0 \) and \( Q_{\text{div}} = 0 \). However, the divergence term would appear if the analysis were to be carried out in the limit of the primitive equations. This splitting of the frontogenesis terms according to (26) is not identical to that employed by Keyser et al. (1988). In our formulation the Lagrangian rate of change of the thermal gradient vector is related to the invariant kinematic components of the flow, while Keyser et al.’s formulation focuses separately on the strength and direction of the thermal gradient.

The two contributions, \( Q_{\text{rot}} \) and \( Q_{\text{def}} \), refer to the pure rotation and pure deformation of the flow field, respectively. They each affect the thermal gradient in their own particular manner. Rotation does not affect the strength of the thermal gradient, but only its direction. Deformation on the other hand causes an intensification (weakening) of the
component of the thermal gradient which is aligned with the deformation (dilatation) axis of the deformation field. Three-dimensional frontogenesis can always be expressed in terms of these two components (or the three components of (26) if primitive-equation dynamics are considered), even 'pure' deformation and shear frontogenesis. It follows that the term 'shear frontogenesis' is not properly defined beyond the two-dimensional Eady model. The associated difficulties of analysing three-dimensional frontogenesis into one or the other of the idealized 'stretching' or 'shearing' models have already been recognized by Mudrick (1974), who split (24) into various contributions using similar concepts as those employed in our analysis of (25).

The deformation and dilatation axis of the deformation field are given by the eigenvectors $e_+$ and $e_-$ of $Q_{def}$, respectively. The eigenvalues are of opposite sign and relate to the strength of the deformation. They take on the form

$$\lambda_{\pm} = \pm \frac{1}{2} \alpha \quad \text{where} \quad \alpha = [\alpha_1^2 + \alpha_2^2]^{1/2}. \quad (27)$$

The parameter $\alpha$ is usually referred to as the 'deformation parameter'. The orthogonal eigenvectors can be expressed as

$$e_{\pm} = \begin{cases} [\pm \alpha - \alpha_1 - \alpha_2; \pm \alpha - \alpha_1 + \alpha_2] \left(2\alpha \sqrt{1 + \frac{\alpha_1}{\alpha_2}}\right)^{-1} & \text{(for } \alpha_2 \neq 0) \\ [-1; \pm \alpha_1/\alpha_2] 2^{-1/2} & \text{(for } \alpha_2 = 0). \end{cases} \quad (28)$$

The deformation axis scaled with the eigenvalue, i.e. $\lambda_{\pm} e_{\pm}$, is displayed in Fig. 13(c). A similar display has also been utilized in the analysis of HW. The highest values of the frontogenesis function occur in the vicinity of the cold front, where the $Q$-vector is aligned parallel to the thermal gradient. The peculiar pattern of the deformation eigenvectors (Fig. 13(c)) is consistent with (26), i.e. the different orientations of the deformation axis ahead of and behind the cold front are directly due to the presence of anticyclonic and cyclonic vorticity in those regions, respectively. In the same way the contributions of the $Q_{rot}$ matrix will act differently, and add up with the deformation to a roughly unidirectional $Q$-vector (Fig. 13(d)).

This situation along the cold front is comparable with pure two-dimensional deformation frontogenesis, where the $Q$-vector has always the same direction (i.e. is perpendicular) to the front. On the other hand the $Q$-vector is aligned almost parallel to the front in the bent-back portion of the warm front, indicating that the $Q_{rot}$ contribution dominates (26) in those regions. This contribution itself has zero impact on the frontogenesis function, and it merely rotates the thermal gradient such as to cause the bending of the warm front.

(c) Dynamical distinction between warm and cold fronts

There is a stark difference between the structure of cold and warm fronts, both in numerical simulations and in most atmospheric cyclones (e.g. Palmén and Newton 1969). Cold fronts are easily detectable and commonly characterized as sharp features, while the routine identification of warm fronts is often based merely upon the existence of a general band of 'weather' in their vicinity. The interrelation between key dynamical parameters as expressed by (20–22) implies that a suitable ageostrophic circulation could in principle compositantly enhance the ambient thermal gradient, induce low-level ascent, and thereby increase the vorticity in a frontal zone. This has given rise to the common
inference that warm fronts are often benign zones of weak ascent. This view is particularly
germaine to the interpretation advanced by Hoskins and Heckley (1981). Based on
three-dimensional semi-geostrophic simulations (HW) and two-dimensional theory of
deforestation frontogenesis (Hoskins 1971), they concluded that the vertical tilt of the
thermal wave with respect to the low-level thermal gradient is responsible for many of
the distinctions between cold and warm fronts. For the warm-frontal region, the typical
thermal forward tilt of a growing baroclinic wave results in a displacement of the upper-
level thermal pattern towards the cold side of the warm front. This geometry produces
weak $Q$-vector forcing, and the resulting frontogenesis in terms of the low-level vorticity
production is marginal. Keyser and Pecnick’s (1987) interpretation on the other hand
does not attempt to explain the documented weakness (in terms of low-level vorticity)
of warm fronts. Their model warm and cold fronts are characterized by low-level shear
of comparable strength, and the differences between the two are analysed in terms of
the induced ageostrophic circulation.

Here we discuss an alternative interpretation that is based on the premise that both
cold and warm fronts are characterized by a band of significant low-level convergence.
The distinction between the two arises from the different Lagrangian time-histories of
individual air parcels within each of the frontal zones. This interpretation can explain
how warm and cold fronts have a different character in terms of low-level vorticity, while
they both can be associated with significant low-level convergence.

Figure 13(f) shows (in physical space) typical surface-level trajectories within each
of the frontal zones. To relate the motion to the cyclone, vorticity contours are shown
for times $t = 0, 4$ and $8$. The selected cold-frontal air parcel is located immediately ahead
of the surface cold front at $t = 4$. It then tries to penetrate the cold front from the warm
towards the cold side. However, it is very inefficient in doing so, and has just reached
the centre of the low-level vorticity band at $t = 8$. The air parcel hence stays for a long
time within the region of low-level convergence and is thus able to accumulate significant
amounts of vorticity (cf. Fig. 5). The warm-frontal trajectories show rather different
characteristics. Relative to the evolving system, the air parcels travel rapidly from far
ahead towards the centre of the cyclone, following at any instance roughly the warm
front. Again significant accumulated vorticity appears at the end of the trajectories and
in the bent-back portion of the warm front. This Lagrangian view has already been briefly
discussed by HW and carefully studied by Takayabu (1986) in order to discuss his front
‘F’ which corresponds to our warm front.

Here we draw attention to an associated distinction between warm and cold fronts.
The starting point is that the local rate of change of vorticity (i.e. $\partial \zeta_{ll}/\partial T$ in the reference
frame moving with the low) is rather small in the eastern portion of the warm front, while
there is significant $Q$-forcing. Consider now a uniformly translating coordinate system ($X', Y'$), whose $Y'$-axis is roughly aligned with the warm front. We select the
translation rate such that $\partial \zeta_{ll}/\partial T' = 0$ at the origin. Such a coordinate system can easily
be defined for the situation under consideration, and keeps its approximate alignment
with the warm front over several dimensionless units of time. Next we employ (21) with
$\partial \zeta_{ll}/\partial T' \approx 0$. Furthermore, we have $\partial \zeta_{ll}/\partial X' = 0$ on the warm front itself (defined as
the line of maximum vorticity), and the vorticity equation then takes on the form

$$
u_s \frac{\partial \zeta_{ll}}{\partial Y'} = Ro^{-1} \frac{\partial w^a}{\partial Z}. 
$$

(29)

Essentially, this equation states that vorticity production in the warm front is roughly
cancelled out by advection in the along-frontal direction, as already noted by Takayabu
(1986) and Xu (1990). Next we use the fact that the flow is almost parallel to the warm front. It follows that the advective term can only be significant provided $\partial \zeta_{\text{int}} / \partial Y' \neq 0$, i.e. in the presence of an along-frontal gradient of vorticity. Such a vorticity gradient along the warm front can easily be identified in Fig. 5(a). In essence, the argument implies that the warm front has an intrinsic three-dimensional structure, and it is this feature which allows for the presence of significant $Q$-forcing (or ‘weather’) in the absence of significant local vorticity production. The associated shortness of the along-frontal scale of the warm front has been noted in numerous studies, yet the dynamical distinction problem was often discussed in terms of two-dimensional models.

7. SUMMARY AND FURTHER REMARKS

A highly idealized dynamical framework was utilized to study the evolution of an isolated low-pressure system. The simplifications involved the neglect of diabatic and frictional processes and the stipulation of the semi-geostrophic dynamics within a uniform potential-vorticity fluid. These approximations pose significant constraints upon the validity of the results. It is for instance well established, from observational (Gyakum 1983), numerical (Uccelini 1990) and theoretical viewpoints (Emanuel et al. 1987) that moist dynamics can have a pronounced impact on cyclogenesis. Nevertheless, in qualitative terms, the simulated cyclogenesis compares favourably with observed events.

The Lagrangian fields derived for the presented simulation contain significant additional information, which is not discernible in the more commonly used Eulerian fields. The vertical displacement differs significantly from the vertical-wind field, and compares quite favourably with cloud patterns as commonly seen in satellite pictures. In addition, while our model fronts are associated with insignificant thermal gradients and vorticity at mid levels, they can easily be identified in terms of the initial meridional and longitudinal position fields. This feature is of interest since it introduces frontal characteristics into initially smooth but inhomogeneous fields, provided they exhibit tracer-like behaviour (as, for example, the field of water vapour mixing ratio).

It is also possible on the basis of Lagrangian considerations to define objectively two air-streams which are associated with maximum descent and ascent during the formation of the low. The descending air-stream shows several of the characteristics of the dry-descending air-stream as identified in several observational studies (see the review of Browning (1990)). A part of the descending air parcels reach a location above the bent-back portion of the warm front (or occlusion) and then experience significant ascent—a result which is in agreement with a recent study of Kuo et al. (1992). Maximum ascent on the other hand occurs in the form of a coherent air-stream located within the warm-frontal baroclinic zone (or its bent-back portion) throughout the development. It would be of significant interest to apply our criteria to numerical simulations obtained in a less simplified framework.

The presence of maximum ascent within the warm-frontal zone shows that there is significant vorticity production by vortex stretching, and that the relevant air parcels stay in the shafts of ascent for considerable time. The documented weakness of the warm front in terms of low-level vorticity can then be explained from the cancellation of vorticity production by advection. This requires the warm front to have an intrinsically three-dimensional structure. This dynamical distinction between warm and cold fronts is also consistent with the identification of the warm front and the occlusion (or bent-back warm front) as one dynamical entity (Shapiro and Keyser 1990).
ACKNOWLEDGEMENTS

The authors would like to thank Huw Davies for his help in designing the present study, and for valuable suggestions during the preparation of this manuscript. Numerous careful comments of Richard Reed on an earlier version of this manuscript, as well as discussions and correspondence with Keith Browning, Dan Keyser, François Lalaurette, Mike Montgomery, Andy Rossa, Rich Rotunno, Ronald Smith and Izuru Takayabu are also appreciated. We would also like to thank two anonymous reviewers for their constructive comments. CS acknowledges receipt of a Fellowship from the Swiss National Science Foundation (during a stay at Yale University) and was further supported by the U.S. National Science Foundation under the grants ATM-8914138 and ATM-9106494.

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